Research Paper

# The Simple View of Reading and Multiple Regression:
## Comparing and Compromising Multiplicative and Additive Models

Masayasu AOTANI

**Abstract**

Controversies abound regarding the choice between linear regression and the Simple View of Reading (SVR) to describe reading comprehension ($R$), particularly in relation to decoding ($D$) and linguistic comprehension ($C$) (Høien-Tengesdal, 2010; Savage, 2006; Savage & Wolforth, 2007). This is despite the stark qualitative difference between multiplicative SVR and additive linear regression. In this comparative study, I (1) examine the different fit criteria, validity, and merits as well as demerits of each instrument, (2) explain why they are mathematically equivalent with optimal zero-point adjustments for $D$ and $C$ in SVR and the inclusion of $D \times C$ in linear regression, and (3) generate simulated datasets to examine the contribution of $D \times C$ based on significance levels and effect sizes. The equivalence in (2) means that SVR with optimal zero-point adjustments can always explain more variance in R than multiple regression with only $D$ and $C$ as the predictors. However, the mathematically optimal adjustments may not always make good linguistic sense. These issues are discussed algebraically followed by an illustrative numerical example. This paper concludes with a set of criteria involving the significance, effect size, nullity adjustments, and commonality analysis to assess the applicability of SVR in a given situation.

**【Keywords】** simple view of reading, SVR, multiple regression, additive, multiplicative

## 1. Introduction

The Simple View of Reading (SVR) (Hoover & Gough, 1990) has become a thriving industry due to its explanatory power and simplicity, as the name suggests. In fact, support for SVR comes from a wide variety of cross-disciplinary investigations ranging from learning difficulties to genetics to brain imaging (Goswami, 2008; Keenan, Betjemann, Wadsworth, DeFries, & Olson, 2006; Tan, Wheldall, Madelaine, & Lee, 2007; Vellutino, Tunmer, Jaccard, & Chen, 2007). In this view, reading comprehension $R$ is expressed as the product of decoding ability $D$ and linguistic comprehension $C$ ($R = D \times C$).

It is indicated by numerous studies that both multiple linear regression and the Simple View of Reading can successfully account for variance in reading comprehension (Høien-Tengesdal, 2010; Savage, 2006; Savage & Wolforth, 2007). Even the advocates of the Simple View of Reading were aware early on that multiple regression explained variance in reading comprehension with similar levels of accuracy as the Simple View of Reading (Gough & Tunmer, 1986, p. 7), citing this fact as one difficulty in assessing the soundness and applicability of SVR. It appears customary to classify studies of the dependence of reading comprehension ($R$) on decoding ($D$) and linguistic comprehension ($C$) into three categories (Conners, 2009, p. 603), where the independent variables are $D$ and $C$ (additive), only $D \times C$ (multiplicative), and $D$, $C$, and $D \times C$ (combination). In its simplest formulation, with only $D$ and $C$ as the independent variables and without the interaction term $D \times C$, multiple linear regression takes the form

$$R = aD + bC + f;$$

where $a$, $b$, and $f$ are constants determined by the method of least squares.

> Parenthetically, I would like to draw the readers' attention here to the worrisome prevalence of misuse of the term "linear combination" for $aD + bC + f$ when, in fact, it is a linear expression and not a linear combination. An expression $aD + bC + f$ becomes a linear combination if and only if the constant term $f$ is zero (Weisstein, n.d.).

On the other hand, the Simple View of Reading contains two constants denoted by $s$ and $t$ to adjust the origins for $D$ and $C$ (Hoover & Gough, 1990, p. 142), with the origins corresponding to no decoding skill and no linguistic comprehension, respectively. With these adjustments, we have

$$R = (D + s)\ (C + t)$$

instead of the simplest $R = D \times C$ ; where $D$ and $C$ signify the raw measurements/scores of decoding skill and linguistic comprehension. Details of this adjustment are discussed later in the Simple View of Reading section. Thus, there are irreconcilable differences between SVR and multiple linear regression in the way the relation between dependent variable R and independent variables D and C is formulated; i.e. multiplicative vs. additive as well as the number of adjustable coefficients and constants. There are also problems of ambiguity regarding the adjustments $s$ and $t$. Some of these issues were pointed out already in the original paper by Hoover and Gough (1990, pp. 142-143) but were not explained or investigated in detail. The main differences between $R = (D + s) \times (C + t)$ (the Simple View of Reading) and $R = aD + bC + f$ (multiple linear regression) are summarized in Table 1. Note that substituting $D = D + s$ and $C = C + t$ into $R = aD + bC + f$ generates $R = aD + bC + as + bt + f$ where $as + bt + f$ is some constant. Hence, by abuse of notation, I write $R = aD + bC + f$ either with or without the adjustments to $D$ and $C$.

While multiple linear regression with $D$, $C$, and $D \times C$ as independent variables is not in Table 1, this relation is discussed in detail in the section that analyzes the equivalence between the Simple View of Reading and the linear regression with $D$, $C$, and $D \times C$ as independent variables.

**Table 1.** ***Comparison of the Simple View of Reading and Multiple Linear Regression***

| Formula for the Model $R =$ | Differences | | | |
|---|---|---|---|---|
| | Arithmetic Operation | Relation between $R$ and $D$, $C$ | Parameters to be adjusted/ determined | Ranges of $D$ and $C$ |
| $(D+s)\times(C+t)$ Simple View | multiplication | $R = 0$ if $D+s$ or $C+t = 0$ | $s$ and $t$ to adjust for the origins | [0, 1] for $D+s$ and $C+t$ |
| $aD + bC + f$ Regression | addition | $R$ may not be 0 even if $D+s$ and/or $C+t = 0$ | $a$, $b$, and $f$ by the least squares method | Ranges of the raw scores |

*Note*. $R$ = reading comprehension; $D$ = decoding ability; $C$ = linguistic comprehension; $s$, $t$ = adjustments for the origins of $D$ and $C$; $a$, $b$, $f$ = coefficients and a constant obtained by multiple linear regression. Details of the entries in this table are discussed in the main text.

In this paper, I first discuss various issues and problems surrounding the Simple View of Reading, with a view to assessing its explanatory power and making comparisons with multiple linear regression. This is as a preliminary to my discussions of 1 to 4 below.

1. The Simple View of Reading is mathematically equivalent to multiple linear regression with $D$, $C$, and $D\times C$ as independent variables provided we allow any values for the adjustments $s$ and $t$. This purely mathematical fact applies to any randomly generated datasets $R$, $D$, and C.
2. As a corollary to 1 above, the Simple View of Reading can almost always explain more variance than multiple regression with $C$ and $D$ as the only independent variables, and not including $D\times C$ as an independent variable. The only exception to this is a rare case in which the correlation between $D\times C$ and $R$ is zero.
3. Despite these mathematical facts, only certain values of $s$ and $t$ may be acceptable on linguistic grounds, and this can be used as a measure of the predictive power and theoretical soundness of the Simple View of Reading.
4. In addition, when we compare randomly generated datasets $R$, $D$, and $C$, with those that came from actual measurements of reading comprehension, decoding skill, and linguistic comprehension (Hoover & Gough, 1990), the latter showed clearly significant contributions of the term $D\times C$ in accounting for the variance of $R$ by multiple linear regression, while the former tended to exhibit statistically insignificant contributions at $p < .05$ and with unrecognizable effect sizes.

## 2. Discussion

### 2.1 The Simple View of Reading

As stated already, the Simple View of Reading (SVR) claims that reading comprehension ($R$) can be expressed as a product of decoding ability ($D$) and linguistic comprehension ($C$); $R = D\times C$ symbolically. This is based on the separability of decoding skills and linguistic comprehension as verified, for example, by principal component analysis (Kendeou, Savage, & van den Broek, 2009, p. 354). The two-stage or two-component view of reading comprehension itself, where orthographic word recognition is followed by local and global processing for meaning, appears basically sound and widely accepted even by the critics of the Simple View of Reading (Høien-Tengesdal, 2010, p. 466). However, there are two major areas of controversy when one tries to explain the variance in reading comprehension from this point of view: ambiguities in and problems with the measurement scales themselves as well as the adjustments made to the raw scores of $R$, $D$, and $C$.

## 2.2 Ambiguity in Measurement

The first issue is the ambiguity in the measurement scale of the skills involved. Following the formulation by Hoover and Gough (1990, p. 132), $R$, $D$, and $C$ are supposed to range from 0 (nullity) to 1 (perfection). In actuality, however, it is not clear what nullity and perfection really mean. It is certainly not the raw score. A score of zero only means the student missed every item on a particular test or a set of tests, and it does not necessarily mean possession of no target ability. Likewise, a perfect raw score cannot be interpreted as representing perfection for the corresponding skill. Therefore, conversion and rescaling of the raw scores to obtain appropriate $R$, $D$, and $C$ scores involve much ambiguity and uncertainty. In particular, the product formula necessarily implies that $R$ is zero if $D$ or $C$ is zero. It may not be easy to locate a clear cutoff for $D$ or $C$ such that nullity for either decoding ($D$) or linguistic comprehension ($C$) implies nullity for $R$ no matter how good the other skill is. In addition, the minimum skill levels of $D$ and $C$ required for nonzero reading comprehension ($R$) most likely depend on such parameters as the characteristics of the text and background knowledge of individual readers. Hoover and Gough (1990, p. 131) advocated controlling for textual variation by using parallel materials for reading and listening comprehension, where listening comprehension was their measure of linguistic comprehension as the target of their study was kindergarten and grade school children through the fourth grade. These children were either beginning to read or learning to read fluently. Despite these considerations, it is still unclear how well-defined the locations of the origins are for $D$ and $C$, respectively. Just as the absolute zero degrees kelvin, the state of zero decoding and zero linguistic comprehension may be very difficult to reach or define for ordinary learners of reading, both in the case of native and nonnative speakers. Needless to say, we have the same problem with the score of 1 (perfection). Finally, the same adjustments for nullity and perfection as well as rescaling to [0,1] are needed for $R$. However, this linear transformation applied to $R$ is of no consequence as far as the correlation coefficient with $D \times C$ is concerned. Here is the first problem.

*Problem 1: It is necessary to convert the raw scores to appropriate R, D, and C that one can use in the formula of the Simple View of Reading. One needs to define the lower and upper bounds, corresponding to nullity (0) and perfection (1), for the new converted scale.*

However, we will see later that the location of nullity affects the validity of SVR in a much more significant manner than the location of perfection.

## Goodness of Fit

The goodness of fit criterion for multiple linear regression is, of course, the multiple correlation coefficient (multiple R), defined as the correlation coefficient between the observed and predicted values. On the other hand, for the Simple View of Reading, the criterion is the correlation coefficient between R and D × C. Multiple linear regression takes the data and tries to find the coefficients for the variables as well as the constant term that minimize the sum of the squared differences between the observed values and the values predicted by the resulting formula. The coefficients and constant thus obtained maximize the correlation coefficient between the observed and the predicted values; that is, they give the highest possible simple correlation between the dependent variable and any linear expression of the independent variables (ASSIGNMENT Help, n.d.; Nagpaul, n.d.).

In contrast, the Simple View of Reading does not have a built-in adjustment mechanism like the method of least squares once the values of $R$, $D$, and $C$ are measured and converted/rescaled as necessary. The goodness of fit of the model $R = D \times C$ is judged solely based on the correlation coefficient between $R$ and $D \times C$ as mentioned above. This means any linear transformation of the original $D \times C$, $a(D \times C) + b$ where $a$ and $b$ are arbitrary constants and $a$ is nonzero, exhibits exactly the same goodness of fit as $D \times C$. In other words, the equality in $R = D \times C$ is not a numerical equality, but a conceptual one achieved when the correlation coefficient is unity. Hence, the difference between multiple linear regression and the Simple View of Reading goes beyond that between additive and multiplicative formulae. Linear regression is as much about numerical equality as the correlation, but the Simple View of Reading only uses the correlation as its fit criterion. Regarding this difference, Hoover and Gough pointed out (1990, p. 142) that multiple linear regression combines each component, such as $D$, $C$, and $D \times C$ with the optimal weight, whereas individual weighting of $D$ and $C$ is not possible in the Simple View of Reading.

*Problem 2: While multiple linear regression tries to achieve numerical approximation/equality by adjusting the weights of individual variables in the process of obtaining high correlation, the Simple View of Reading only attempts to account for the correlation.*

Note here that there are philosophical, theoretical, and methodological differences between the Simple View of Reading and linear regression. With the Simple View of Reading, we have the two-stage view of reading comprehension and independent measurements of $R$, $D$, and $C$. Then, we test the validity of the two-stage view by examining the correlation between $R$ and $D \times C$. While adjustments for the raw measurements of $R$, $D$, and $C$ are to be made before calculating the correlation coefficient, these are not adjustments in order to maximize the correlation coefficient, but are adjustments for nullity and perfection as described above. On the other hand, linear regression's built-in mechanism to maximize the correlation coefficient between the observed and the predicted values of $R$, called the method of least squares, is largely a simple numerical procedure. The coefficients and the constant term are determined such that they numerically maximize the multiple correlation coefficient $R$, and the resulting formula is not necessarily a reflection of a well-defined linguistic model/theory as in the case of SVR.

*Problem 3: Unlike the Simple View of Reading, multiple regression is not necessarily guided by a clear theoretical framework. It is more like a numbers game compared with SVR.*

Of course, this alone does not lead to any significant judgment as to comparative utility, validity, and/or superiority of SVR and multiple linear regression approaches.

I now include $D \times C$ in the list of independent variables and consider linear regression studies where the effect of $D$ and $C$ is compared with that of $D \times C$.

### 2.3 Small Effect Sizes

Hierarchical multiple linear regression analyses have been conducted with reading comprehension $R$ as the dependent variable and decoding skill $D$ (often operationalized as word recognition), linguistic comprehension $C$ (often operationalized as listening comprehension), and their product $D \times C$ as the independent variables. In a typical study, $D$ and $C$ are entered together in one step either followed by or preceded by a step where $D \times C$ is entered. It is very often the case that the additional gain in the explained variance is relatively small whether it is $D \times C$ or $D$ and $C$ that are entered in the second stage. For example, in the study by Hoover and Gough (1990, p. 141), the additional gain in the variance explained in the second step ranged from .011 to .073. The ratio of the additional variance explained in the second step to the variance explained in the first step was in the range .015-.088. As inclusion of any additional variable almost always increases the amount of explained variance because of its nonzero correlation with the dependent variable (Hopkins, n.d.), we need to evaluate the contribution of such additional variables carefully by checking both the significance and the size of their effects.

Hoover and Gough reported the significance level of mostly at $p < .0005$ and at least $p < .01$ for each of the gains in the second stage of their hierarchical multiple linear regression, and Cohen's $f^2$ (1988, p. 410), an indicator of the effect size defined as $R^2/(1-R^2)$, ranged from .0447 to .715. More specifically, $f^2$ ranged from .0447 to .480 when $D \times C$ was added in the second stage and from .0826 to .715 when $D$ and $C$ were added in the second stage. By convention, effect sizes of 0.02, 0.15, and 0.35 are termed small, medium, and large (Cohen, 1988, pp. 410-414), according to which five of the eight effects measured by Hoover and Gough (1990, p. 141) were small, two effects were of medium strength, and one was large. Furthermore, Hopkins (Hopkins, n.d.) claims an increase of 0.05 or smaller may be significant if the correlation is greater than .75, which is the case here. Therefore, one can conclude that the effect size is generally significant, but the addition of $D$ and $C$ and the addition of $D \times C$ in the second step tend to exhibit small effect sizes. One caveat here is that Cohen's $f^2$ and its interpretation is not without critics. I am using it here primarily as a means to compare my results with previous works, but it is also true that no clearly better alternative exists at this point.

Whether it is statistically significant or not, the difference between the contributions made in the second step by a weighted sum of $C$ and $D$, $aC + bD$, and the product $c(D \times C)$, where $a$, $b$, and $c$ are regression coefficients, is small. Detection and assessment of such a difference require very precise measurements. In recent works on general English proficiency, with an emphasis on listening comprehension of Kyoto University students (Aotani, 2009, 2011), Rasch analysis was applied to purge non-performing items from the test and adjust the measurement scales so that the abilities of the examinees are more accurately reflected on the scores. Use of raw scores or standardized raw scores may compromise precise assessments of the validity of the Simple View of Reading. On this ground, I believe employing some data cleaning/treatment procedure (such as Rasch analysis) prior to examining the relationships among $R$, $D$, $C$, and $D \times C$ may be essential in future studies.

Note that the small effect size is a result of high intercorrelations and collinearity among the independent variables. Despite the collinearity, multiple linear regression is still useful and effective if used in combination with a commonality analysis (Nimon, 2010, p. 10) as I attempt to do in the Numerical Examples section.

*Problem 4: While they may still be significant, the effect sizes of D and C or D×C in the second stage of hierarchical multiple linear regression, and necessarily the differences among them, are very small. It is important to assess the participants' abilities with high accuracy and precision.*

Before moving on to concrete numerical examples, I first show how the Simple View of Reading can account for the same variance as the linear regression, where $D$, $C$, and the product term $D \times C$ are independent variables, on purely algebraic/mathematical grounds.

### 2.4 Purely Mathematical Considerations

Suppose we have the measurements $R$, $D$, and $C$, preferably after pre-treating the raw data by subjecting them to Rasch analysis for example. First, we can attempt multiple linear regression with $R$ as the dependent variable and $D$, $C$, and $D \times C$ as independent variables and express the result as

$$R = aD + bC + eD \times C + f \text{ (Multiple Linear Regression);}$$

where $a$, $b$, $e$, and $f$ are the constants obtained by the method of least squares. On the other hand, the Simple View of Reading gives

$$R = D \times C \text{ (the Simple View of Reading)}$$

in its simplest form. Now, as the equality, or the extent to which the equality holds to be more precise, in $R = D \times C$ is assessed by the correlation coefficient between $R$ and $D \times C$, any linear transformation of $D \times C$ is equivalent to $D \times C$ as far as the equality in the context of the Simple View of Reading is concerned. This is because linear transformations do not change the correlation coefficient. Therefore, $D \times C$ can be replaced by $e(D \times C) + f - (ab / e)$. Furthermore, as discussed above, the raw values of both $D$ and $C$ are to be adjusted for the points corresponding to nullity by adding a constant. If we added $b/e$ to $D$ and $a/e$ to $C$, we get

$$\begin{aligned} & e\,(D \times C) + f - (ab / e) \Rightarrow e(D + b / e) \times (C + a / e) + f - (ab / e) \\ &= e[D \times C + (a / e)D + (b / e)C + (ab / e^2)] + f - (ab / e) \\ &= aD + bC + eD \times C + (ab / e) + f - (ab / e) \\ &= aD + bC + eD \times C + f \end{aligned}$$

This agrees with the result of multiple linear regression. Therefore, by adding appropriate constants to $D$ and $C$ respectively and applying a necessary linear transformation, it is always possible, at least mathematically, to make the adjusted values of $D \times C$ numerically equal to the values obtained by multiple linear regression. Also note from the series of equalities above that any scalar multiple of $(D + b / e) \times (C + a / e)$ has the same correlation coefficient with $R$ as $aD + bC + eD \times C + f$ does. These observations prove that the Simple View of Reading can account for as much variance in reading comprehension as multiple linear regression if the values of $D$ and $C$ are adjusted appropriately.

However, unlike a straightforward linear transformation of $D \times C$, which may not alter the nature of the relationship between $R$ and $D \times C$ in a significant manner, the direct adjustments made to $D$ and $C$ separately require a more careful discussion. Because it is a mathematical fact that such a set of adjustments can always be found, the problem does not lie in finding such numbers, but in how acceptable and reasonable they are on linguistic grounds. The explanatory power and correctness of the Simple View of Reading as compared with multiple linear regression do not depend on the above mathematical manipulations but on how reasonable the adjustments are from the point of view of applied linguistics.

Hoover and Gough confirm that "[t]he difficulty, of course, is in finding the optimal zero points for the component terms" (1990, p. 143), and this paper provides a purely mathematical means to solving that difficulty.

### 2.5 Numerical Examples

The mathematical equivalence, in the sense of the preceding section, between the productive formula and the additive formula applies to any set of variables $R$, $D$, and $C$. It is not necessary that they come from the measurements of reading comprehension, decoding skill, and linguistic comprehension. In order to further illustrate this fact, I will use three randomly generated sets of 206 numbers each as $R$, $D$, and $C$. Table 2 lists such numbers.

The total number of the participants for this simulated data (206) was the same as that of the second graders in the Hoover and Gough study (1990, p. 140). Following the prescriptions similar to those described by Howell and Ogle (n.d.; n.d.), the correlation between $R$ and $D$ and that between $R$ and $C$ were adjusted to about .80 and .71 to assure that the data sets have the correlation coefficients of a typical study profile: reading comprehension ($R$), decoding skill ($D$), and linguistic comprehension ($C$). These figures were taken from the data for the second graders provided by Hoover and Gough (1990, p. 140). Because Ogle's procedure (n.d.) does not produce datasets exactly with the target correlation coefficient, the actual correlation coefficients are .790 and .715 respectively. Furthermore, the correlation between $R$ and $D \times C$ for this dataset is .837 which is comparable to .85 in Hoover and Gough's datasets for the second graders (1990, p. 140). Note that only the correlations between $R$ and $D$ and between $R$ and $C$ were made similar to those of the Hoover and Gough's data. I did not adjust any other correlation coefficients. As noted above, however, the mathematical arguments presented in the preceding section applies to any sets of data $R$, $D$, and $C$, and it was actually not necessary to adjust the correlations.

In this part of the computation, I ignore the statistical significance issues altogether and focus only on mathematical relations. Statistical considerations such as significance levels are addressed in the next section.

I first conducted multiple linear regression with $R$ as the dependent variable and $D$, $C$, and $D \times$

$C$ as independent variables to obtain $a = 0.705819747$, $b = 0.454159331$, $e = 0.001576832$, *and* $f = -12.77924867$. Hence, the multiple regression formula is:

$R = (0.705819747)D + (0.454159331)C + (0.001576832)D \times C + (-12.77924867)$; where the multiple correlation coefficient R is 0.860803427, which translates to an explained variance of 0.7409825399. I carry 9 to 10 significant digits to assure precision. We have $a / e = 447.6187585$, $b / e = 288.0200457$, $ab / e = 203.2902357$ and $f - (ab / e) = -216.0694844$. Our previous discussion proved mathematically/symbolically that the correlation between $R$ *and* $(D + b / e)(C + a / e)$ is the same as the multiple correlation coefficient R, which is 0.860803427. This was confirmed numerically as well.

**Table 2.** ***Randomly Generated Data Set***

| | | R | | | | | D | | | | | C | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 94 | 25 | 36 | 57 | 32 | 91 | 22 | 25 | 36 | 19 | 73 | 39 | 46 | 39 | 51 |
| 97 | 10 | 14 | 32 | 36 | 93 | 21 | 36 | 43 | 26 | 75 | 12 | 17 | 38 | 50 |
| 25 | 94 | 74 | 57 | 31 | 20 | 98 | 45 | 69 | 56 | 28 | 58 | 46 | 50 | 42 |
| 58 | 90 | 88 | 83 | 3 | 44 | 95 | 55 | 50 | 37 | 47 | 78 | 97 | 81 | 41 |
| 44 | 24 | 25 | 28 | 18 | 46 | 33 | 22 | 45 | 55 | 55 | 10 | 56 | 23 | 19 |
| 60 | 44 | 38 | 86 | 22 | 77 | 62 | 55 | 92 | 13 | 34 | 52 | 16 | 89 | 53 |
| 98 | 21 | 75 | 78 | 68 | 67 | 53 | 54 | 47 | 70 | 49 | 25 | 36 | 74 | 69 |
| 76 | 0 | 32 | 49 | 29 | 75 | 25 | 48 | 60 | 28 | 46 | 34 | 58 | 44 | 15 |
| 65 | 16 | 24 | 18 | 22 | 70 | 39 | 30 | 24 | 51 | 70 | 17 | 40 | 20 | 60 |
| 82 | 43 | 46 | 74 | 31 | 63 | 32 | 65 | 48 | 36 | 68 | 47 | 74 | 76 | 25 |
| 29 | 79 | 28 | 22 | 99 | 46 | 56 | 40 | 57 | 81 | 50 | 90 | 48 | 50 | 80 |
| 84 | 39 | 22 | 74 | 15 | 59 | 44 | 52 | 51 | 25 | 83 | 44 | 11 | 60 | 19 |
| 79 | 90 | 96 | 90 | 84 | 58 | 81 | 100 | 70 | 79 | 38 | 55 | 64 | 63 | 53 |
| 53 | 64 | 78 | 10 | 69 | 33 | 77 | 80 | 41 | 55 | 49 | 74 | 77 | 24 | 73 |
| 75 | 33 | 70 | 0 | 38 | 72 | 50 | 72 | 6 | 41 | 42 | 49 | 83 | 4 | 43 |
| 48 | 33 | 63 | 19 | 17 | 40 | 53 | 49 | 22 | 48 | 64 | 27 | 38 | 44 | 31 |
| 23 | 18 | 40 | 2 | 22 | 49 | 24 | 29 | 44 | 22 | 36 | 49 | 36 | 34 | 39 |
| 82 | 56 | 13 | 73 | 55 | 70 | 66 | 38 | 57 | 38 | 77 | 39 | 18 | 74 | 50 |
| 66 | 50 | 79 | 5 | 100 | 44 | 36 | 64 | 31 | 81 | 78 | 52 | 56 | 38 | 67 |
| 96 | 63 | 16 | 86 | 74 | 91 | 45 | 22 | 94 | 43 | 100 | 63 | 56 | 44 | 56 |
| 68 | 40 | 49 | 73 | 63 | 62 | 55 | 50 | 57 | 81 | 73 | 58 | 72 | 73 | 46 |
| 98 | 33 | 2 | 35 | 20 | 76 | 40 | 31 | 25 | 11 | 91 | 27 | 6 | 56 | 24 |
| 84 | 46 | 28 | 51 | 63 | 76 | 62 | 48 | 64 | 56 | 77 | 20 | 43 | 29 | 64 |
| 59 | 7 | 22 | 5 | 14 | 78 | 29 | 44 | 23 | 48 | 40 | 35 | 43 | 0 | 11 |
| 7 | 2 | 56 | 17 | 23 | 33 | 26 | 55 | 30 | 34 | 41 | 3 | 47 | 50 | 14 |
| 29 | 42 | 6 | 92 | 92 | 34 | 37 | 14 | 59 | 66 | 16 | 33 | 35 | 77 | 96 |
| 22 | 75 | 36 | 6 | 23 | 48 | 48 | 31 | 38 | 35 | 11 | 54 | 39 | 32 | 35 |
| 24 | 70 | 48 | 37 | 2 | 40 | 62 | 70 | 55 | 23 | 14 | 82 | 36 | 70 | 24 |
| 67 | 57 | 17 | 11 | 75 | 61 | 70 | 18 | 39 | 48 | 54 | 46 | 5 | 8 | 46 |
| 34 | 27 | 80 | 4 | 25 | 32 | 56 | 58 | 14 | 23 | 21 | 48 | 81 | 27 | 49 |
| 52 | 11 | 87 | 9 | 36 | 46 | 47 | 77 | 14 | 60 | 39 | 26 | 47 | 45 | 52 |
| 43 | 81 | 68 | 71 | 48 | 57 | 62 | 61 | 79 | 52 | 29 | 90 | 48 | 63 | 67 |
| 10 | 56 | 34 | 99 | 51 | 18 | 55 | 49 | 80 | 60 | 42 | 52 | 63 | 68 | 58 |
| 92 | 85 | 85 | 95 | 7 | 67 | 69 | 76 | 67 | 3 | 66 | 43 | 84 | 47 | 23 |
| 70 | 1 | 97 | 95 | 40 | 57 | 18 | 92 | 64 | 65 | 81 | 44 | 86 | 84 | 61 |
| 42 | 35 | 61 | 51 | 87 | 32 | 30 | 65 | 46 | 53 | 60 | 42 | 82 | 39 | 65 |
| 11 | 58 | 19 | 46 | 54 | 13 | 74 | 12 | 53 | 47 | 7 | 45 | 9 | 53 | 56 |
| 56 | 56 | 73 | 48 | 29 | 43 | 36 | 63 | 58 | 42 | 27 | 64 | 40 | 48 | 22 |
| 9 | 78 | 32 | 54 | | 11 | 62 | 54 | 41 | | 0 | 47 | 33 | 52 | |
| 25 | 41 | 57 | 49 | | 28 | 51 | 66 | 47 | | 42 | 59 | 33 | 61 | |
| 2 | 57 | 12 | 31 | | 0 | 34 | 17 | 20 | | 47 | 72 | 33 | 57 | |
| 4 | 21 | 61 | 88 | | 22 | 16 | 60 | 51 | | 52 | 25 | 57 | 67 | |

*Note*. While no digits under decimal are shown in the table, I generated and carried 10 significant digits in my computations to assure sufficient precision. In this table, each threesome of *m*-th row ($1 \le m \le 42$ ) and the *n*-th column ($1 \le n \le 5$ ) entries for *R*, *D*, and *C* corresponds to the three measurements for one participant.

At this point, $R$ ranges from 0 to 100, $D + b / e$ from 288.0200457 to 388.0200457, and $C + a / e$ from 447.6187585 to 547.6187585. We can divide $R$ by 100, $D + b / e$ by 388.0200457, and $C + a / e$ by 547.6187585 to adjust the ranges of all three variables to [0, 1] without changing the correlation between $R$ and $(D + b / e) \times (C + a / e)$ as these divisions are all linear transformations. One should note here that regarding 388.0200457 and 547.6187585 as representing perfection for decoding and linguistic comprehension, respectively, is nothing but a convenience choice due to the lack of any other numerical indicators. One should also note that use of any other numbers greater than or equal to 288.0200457 for decoding and 447.6187585 for linguistic comprehension, respectively, to represent perfection produces the same correlation coefficient between $R$ and the product of decoding and linguistic comprehension because different choices for the points corresponding to perfection simply mean different linear transformations, and a linear transformation does not alter the correlation coefficient. Therefore, the problem is not with the process of rescaling, which is a linear transformation, but with actually finding the points corresponding to perfection. However, actual values regarded as representing perfection do not affect the correlation coefficient between $R$ and $D \times C$, repeating the point made already. Viewed from a different angle, my analyses and discussions so far lead to a conclusion that it is the location of nullity, and not where perfection lies which is more significant in the Simple View of Reading. A change in the location of perfection does not cause the correlation coefficient to change.

### 2.6 Statistical Considerations

For the data set presented in Table 2, multiple liner regression showed that the significance level of the coefficient $e$, the coefficient for $D \times C$, is .493, which is well above the typically acceptable upper limit of .05 and is in stark contrast to $p < .0005$ for the Hoover and Gough's data (1990, p. 141). In fact, when the term $D \times C$ is removed from the list of independent variables, $D$ and $C$ alone can explain 74.03767360857% of the variance as opposed to 74.09825407095% for $D$, $C$, and $D \times C$. This small difference of 0.0006058046238 in the explained variance is about 100 times smaller than the difference of .067 obtained by Hoover and Gough for second graders. In terms of Cohen's $f^2$, it is .00199 and .0819 for the randomly generated data and the Hoover and Gough data for second graders, respectively. According to Cohen's conventional criterion, .00199 means the effect size is negligible, and .0819 signifies a small but recognizable effect size. This tendency is to be expected from a randomly generated data. It can be seen from the inter-correlations among $D$, $C$, and $D \times C$ that there is significant collinearity, indicating a large shared variance and a small unique variance for $D \times C$, both for my randomly generated data and Hoover and Gough's data (1990, p. 140). For the data I created (Table 2), this is clear from the result of a commonality analysis presented in Table 3. As the necessary raw data are not available, commonality analysis could not be carried out for Hoover and Gough's data.

However, commonality analysis involving $D \times C$ is available in a study by Johnston and Kirby (2006), in which they compared the contributions of $D \times C$ with that of phonological awareness and naming speed, that is, the independent variables were $D \times C$, phonological awareness, and naming speed. Despite the contributions smaller by one order of magnitude, compared with $D$ or $C$, of phonological awareness and naming speed in explaining the variance in $R$, the unique variance to total variance ratio for $D \times C$ was only

.238 (Johnston & Kirby, 2006, p. 353). This result seems to underscore the importance of commonality analysis, which has hitherto not been taken much advantage of in the study of SVR.

**Table 3.** ***Unique and Common Effects for the Randomly Generated Data (Table 2)***

| Variable | Unique | Common | Total | % of $R^2$ |
|---|---|---|---|---|
| *D* | 0.0402 | 0.5842 | 0.6244 | 0.8426 |
| *C* | 0.0168 | 0.4943 | 0.5111 | 0.6898 |
| $D \times C$ | 0.0006 | 0.7001 | 0.7007 | 0.9456 |

*Note*. Unique means the unique effect for each of the predictors. Common is the total of all common effects for which each of the predictors, *D*, *C*, and $D \times C$ was involved. These figures were computed using the SPSS script provided by Nimon (2010).

In order to further confirm that this difference is observed consistently between randomly generated data and Hoover and Gough's data, I next created ten sets of data which more closely simulate Hoover and Gough's data with Ogle's procedure (n.d.) to examine the descriptive statistics, correlation coefficients, and the results of regression analyses. In my data sets, the correlation between *R* and *D*, the correlation between *R* and *C*, the means of *R*, *D*, and *C*, and the standard deviations of *R*, *D*, and *C* were approximately those of the second graders in Hoover and Gough's paper (1990, pp. 140-141). As Ogle's procedure does not adjust these parameters in a pin-point fashion, but the resulting data only exhibit a distribution centered on the target value, some fluctuations and deviations were observed, as shown in Table 4. There were a small number of negative scores generated by this process, and they were all converted to zero.

Two types of two-stage hierarchical linear regression study were conducted for the randomly generated data. In one type, *D* and *C* were included in the first stage and $D \times C$ was added next. In the other, inclusion of $D \times C$ in the first stage was followed by the inclusion of *D* and *C* in the second stage. The results are presented in Table 5.

**Table 4.** ***Descriptive Statistics and Intercorrelations of Variables for Hoover and Gough's Data for Second Graders and Ten Randomly Generated Datasets***

| Variables | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Grade 2 (Hoover and Gough)** | | | | |
| 1. Reading | | 0.80 | 0.71 | 0.85 |
| 2. Decoding | | | 0.59 | 0.95 |
| 3. Comprehension | | | | 0.70 |
| 4. Product | | | | |
| Mean | 3.22 | 3.68 | 5.00 | 21.11 |
| Standard deviation | 2.11 | 2.40 | 1.92 | 15.47 |
| **Set 1** | | | | |
| 1. Reading | | 0.79 | 0.69 | 0.81 |
| 2. Decoding | | | 0.55 | 0.91 |
| 3. Comprehension | | | | 0.77 |
| Mean | 3.27 | 3.73 | 5.00 | 21.06 |
| Standard deviation | 2.02 | 2.31 | 1.91 | 17.94 |
| **Set 2** | | | | |
| 1. Reading | | 0.80 | 0.69 | 0.81 |
| 2. Decoding | | | 0.61 | 0.91 |

| | | | | |
|---|---|---|---|---|
| 3. Comprehension | | | | 0.80 |
| Mean | 3.28 | 3.76 | 5.00 | 21.37 |
| Standard deviation | 2.00 | 2.21 | 1.92 | 17.54 |
| **Set 3** | | | | |
| 1. Reading | | 0.79 | 0.73 | 0.85 |
| 2. Decoding | | | 0.51 | 0.89 |
| 3. Comprehension | | | | 0.76 |
| Mean | 3.29 | 3.77 | 5.00 | 21.01 |
| Standard deviation | 1.96 | 2.21 | 1.91 | 16.22 |
| **Set 4** | | | | |
| 1. Reading | | 0.79 | 0.67 | 0.82 |
| 2. Decoding | | | 0.50 | 0.90 |
| 3. Comprehension | | | | 0.74 |
| Mean | 3.26 | 3.73 | 5.00 | 20.86 |
| Standard deviation | 2.03 | 2.31 | 1.91 | 17.51 |
| **Set 5** | | | | |
| 1. Reading | | 0.81 | 0.65 | 0.82 |
| 2. Decoding | | | 0.52 | 0.89 |
| 3. Comprehension | | | | 0.78 |
| Mean | 3.26 | 3.74 | 5.00 | 20.97 |
| Standard deviation | 2.03 | 2.28 | 1.92 | 17.39 |
| **Set 6** | | | | |
| 1. Reading | | 0.76 | 0.68 | 0.81 |
| 2. Decoding | | | 0.54 | 0.90 |
| 3. Comprehension | | | | 0.78 |
| Mean | 3.27 | 3.75 | 5.00 | 21.06 |
| Standard deviation | 2.01 | 2.26 | 1.91 | 17.14 |
| **Set 7** | | | | |
| 1. Reading | | 0.80 | 0.71 | 0.82 |
| 2. Decoding | | | 0.58 | 0.92 |
| 3. Comprehension | | | | 0.77 |
| Mean | 3.28 | 3.75 | 5.01 | 21.21 |
| Standard deviation | 1.98 | 2.25 | 1.90 | 17.44 |
| **Set 8** | | | | |
| 1. Reading | | 0.80 | 0.68 | 0.84 |
| 2. Decoding | | | 0.49 | 0.89 |
| 3. Comprehension | | | | 0.75 |
| Mean | 3.26 | 3.72 | 5.00 | 20.78 |
| Standard deviation | 2.03 | 2.32 | 1.92 | 18.16 |
| **Set 9** | | | | |
| 1. Reading | | 0.77 | 0.72 | 0.82 |
| 2. Decoding | | | 0.54 | 0.90 |
| 3. Comprehension | | | | 0.78 |
| Mean | 3.30 | 3.74 | 5.01 | 21.08 |
| Standard deviation | 1.95 | 2.28 | 1.90 | 18.02 |
| **Set 10** | | | | |
| 1. Reading | | 0.76 | 0.65 | 0.81 |
| 2. Decoding | | | 0.45 | 0.90 |
| 3. Comprehension | | | | 0.73 |
| Mean | 3.26 | 3.74 | 5.00 | 20.67 |
| Standard deviation | 2.04 | 2.28 | 1.92 | 16.79 |

**Table 5.** ***Summary of Regression Analyses***

| Variable | Multiple $R$ | $R$ square change | $F$ change | $p$ | $f^2$ |
|---|---|---|---|---|---|
| **Grade 2 (Hoover and Gough)** | | | | | |
| Linear | .853 | .728 | 271.87 | .000 | |
| Product | .865 | .020 | 16.16 | .000 | .0819 |
| **Set 1** | | | | | |
| Linear | .852 | .726 | 268.75 | .000 | |
| Product | .852 | .000 | .10 | .748 | .000513 |
| **Set 2** | | | | | |
| Linear | .840 | .705 | 242.59 | .000 | |
| Product | .840 | .000 | .17 | .679 | .000853 |
| **Set 3** | | | | | |
| Linear | .879 | .772 | 344.37 | .000 | |
| Product | .880 | .002 | 2.16 | .143 | .010699 |
| **Set 4** | | | | | |
| Linear | .847 | .717 | 257.49 | .000 | |
| Product | .848 | .001 | .80 | .374 | .003937 |
| **Set 5** | | | | | |
| Linear | .855 | .731 | 275.88 | .000 | |
| Product | .856 | .001 | .91 | .340 | .004521 |
| **Set 6** | | | | | |
| Linear | .831 | .691 | 226.56 | .000 | |
| Product | .831 | .001 | .44 | .508 | .002181 |
| **Set 7** | | | | | |
| Linear | .859 | .738 | 285.93 | .000 | |
| Product | .859 | .000 | .22 | .642 | .001073 |
| **Set 8** | | | | | |
| Linear | .867 | .752 | 308.01 | .000 | |
| Product | .868 | .002 | 1.74 | .188 | .008628 |
| **Set 9** | | | | | |
| Linear | .849 | .722 | 263.12 | .000 | |
| Product | .850 | .000 | .25 | .617 | .001245 |
| **Set 10** | | | | | |
| Linear | .832 | .693 | 228.73 | .000 | |
| Product | .832 | .000 | .18 | .670 | .000902 |
| **Grade 2 (Hoover and Gough)** | | | | | |
| Product | .848 | .720 | 523.27 | .000 | |
| Linear | .865 | .029 | 11.55 | .000 | .116 |
| **Set 1** | | | | | |
| Product | .815 | .664 | 403.15 | .000 | |
| Linear | .852 | .062 | 22.85 | .000 | .226258 |
| **Set 2** | | | | | |
| Product | .805 | .649 | 376.47 | .000 | |
| Linear | .840 | .057 | 19.43 | .000 | .192423 |
| **Set 3** | | | | | |
| Product | .855 | .731 | 553.16 | .000 | |
| Linear | .880 | .044 | 19.82 | .000 | .196203 |
| **Set 4** | | | | | |
| Product | .824 | .679 | 431.78 | .000 | |
| Linear | .848 | .039 | 14.07 | .000 | .139313 |

| | | | | | |
|---|---|---|---|---|---|
| | | | **Set 5** | | |
| Product | .815 | .665 | 404.08 | .000 | |
| Linear | .856 | .068 | 25.55 | .000 | .252957 |
| | | | **Set 6** | | |
| Product | .806 | .650 | 378.36 | .000 | |
| Linear | .831 | .042 | 13.60 | .000 | .134659 |
| | | | **Set 7** | | |
| Product | .823 | .678 | 429.76 | .000 | |
| Linear | .859 | .060 | 23.23 | .000 | .229994 |
| | | | **Set 8** | | |
| Product | .844 | .712 | 505.33 | .000 | |
| Linear | .868 | .042 | 17.20 | .000 | .170342 |
| | | | **Set 9** | | |
| Product | .821 | .674 | 421.33 | .000 | |
| Linear | .850 | .048 | 17.51 | .000 | .173367 |
| | | | **Set 10** | | |
| Product | .808 | .653 | 384.14 | .000 | |
| Linear | .832 | .040 | 13.08 | .000 | .129527 |

*Note*. Linear means $D$ and $C$ are entered at this stage of hierarchical regression. Product means $D \times C$ is entered at this stage. $p$ is the significance level of **F.** $f^2$ is the square of Cohen's effect size index. $f^2$ values are given only for the second stage of hierarchical regression because all $f^2$ values were very large for the first stage and did not merit any further discussion. **Grade 2 (Hoover and Gough)** means it is Hoover and Gough's data for 2nd graders. **Set *n*** stands for the ***n***th randomly generated data set.

The $R$ square change upon the addition of $D \times C$ in the second stage ranged from 0.000 to 0.011 compared with 0.02 for the Hoover and Gough data. More importantly, the significance level for $D \times C$, with $p$ ranging from .004 to .988, was consistently smaller than that for Hoover and Gough's level of $p < .0005$. The effect size $f^2$ ranged from .000513 to .0107 for the simulated random dataset compared with .0819 for the Hoover and Gough data of the 2nd graders in this case. On the other hand, when $D$ and $C$ were added in the second stage, the $R$ square change ranged from .039 to .068, and this time $p < .0005$ for all ten sets of randomly generated data. Cohen's squared effect size indicator $f^2$ ranged from .130 to .253 in contrast to .116 for the Hoover and Gough data. In sum, when added in the second stage of multiple linear regression, $D$ and $C$ cause significant changes in $R$ at $p < .0005$ both for Hoover and Gough's data and the ten randomly generated data sets, but $D \times C$ leads to a significant change at $p < .0005$ only for the Hoover and Gough data. For $D \times C$ added in the second stage, the significance level $p$ for the change in $R$ ranged from .143 to .748 for the ten randomly generated data sets.

As for the effect size, when $D \times C$ is entered second, Hoover and Gough's data showed a "small" effect according to Cohen's criterion, but the ten randomly generated datasets did not show a recognizable effect. When $D$ and $C$ were added in the second stage, however, there was a small effect for Hoover and Gough' s data, while there were seven cases with medium-sized effect and three cases with small effect for the randomly generated data. This indicates a larger predictive power, relative to a combination of $D$ and $C$, of $D \times C$ in the Hoover and Gough data than for randomly generated data. On the other hand, a combination of $D$ and $C$ seem to have similar levels of predictive power both for the randomly generated data and Hoover and Gough's data, which is reasonable because the correlation coefficients between $R$ and $D$ and between $R$ and $C$ were similar by design for Hoover and Gough's data and the randomly generated datasets.

I further generated an additional 90 datasets for a total of 100 sets following the same procedure and conducted the same multiple linear regression study. When the product term $D \times C$ is added in the second stage, the significance level of the $R$ square change had the distribution shown in Table 6. This result shows that the additional contribution made by $D \times C$, after $D$ and $C$ have been entered, is generally insignificant for the randomly generated datasets.

Recall that at least some of the fluctuations resulted from the deviations of the intercorrelations between $R$ and $D$ as well as $R$ and $C$ from .80 and .71, respectively. It was not possible to remove this cause of error built in the process of Ogle's data generation process (n.d.). On the other hand, when $D$ and $C$ are added in the second stage, the significance level of the $R$ square change was at $p < .0005$ for 98 cases out of a hundred. For the remaining 2 cases, it was $p < .0023$ and $p < .00057$; both indicating clear statistical significance. This means the contribution of $D$ and $C$ is significant at $p < .01$ even after $D \times C$ has accounted for a part of the variance in the first stage.

**Table 6.** ***Significance Levels of the R change for One Hundred Simulated Data Sets***

| $p$ | .01 | .05 | .1 | .2 | .4 | .6 | .8 | 1.0 |
|---|---|---|---|---|---|---|---|---|
| Cases | 1 | 8 | 7 | 15 | 16 | 14 | 21 | 18 |

*Note*. $p$ stands for the significance level of the $R$ change. The numbers in the first row mean $p$ is smaller than that number. For example, .05 means $p <$ .05. Cases means the number of cases with the corresponding significance level shown in the first row.

I now examine the effect size using Cohen's $f^2$. When $D \times C$ was entered in the second stage of the hierarchical multiple regression, the effect size ranged from .000000978 to .0345, while $D$ and $C$ entered in the second stage exhibited effect sizes ranging from .0623 to .318. Table 7 shows the breakdown of the effect sizes according to Cohen's convention; threshold $f^2$ values of 0.02, 0.15, and 0.35 for small, medium, and large effect sizes.

**Table 7.** ***Effect Sizes for 100 Randomly Generated Cases***

| | | None | Small | Medium |
|---|---|---|---|---|
| **Cases** | **Product** | 92 | 8 | 0 |
| | **Linear** | 0 | 46 | 54 |

*Note*. The first row is the effect size. None means the effect is not recognizable as $f^2$ is smaller than .02. Small means $f^2$. Medium means .02 $\leq f^2 <$ .15. The terms small and medium are according to Cohen's convention. Cases means the number of cases for each effect size. Product means .15 $\leq f^2 <$ .35. is entered in the second stage. Linear means $D$ and $C$ are entered in the second stage.

Comparing these effect sizes with those of the Hoover and Gough data, .0819 and .116 when $D \times C$ and $D$ and $C$ are entered in the second stage respectively, it can be seen that $D \times C$ does not account for as much variance in $R$ for the randomly generated data. The product term $D \times C$ leaves more variance unaccounted for when entered first and accounts for less additional variance when entered second compared with the Hoover and Gough data. All the above-mentioned observations point to an insufficient contribution made by $D \times C$ whether it is entered first or in the second stage after $D$ and $C$.

## 3. Summary of the Facts and Findings

Here is a list of the main facts and findings:

1. After adjusting for nullity, which usually amounts to adding positive numbers to the raw measurements of the decoding skill $D$ and linguistic comprehension $C$, the Simple View of Reading's productive formula can explain as much variance as multiple linear regression, where the independent variables are $D$, $C$, and $D \times C$. Indeed, the squared multiple correlation $R^2$ represents the maximum amount of variance possibly accounted for by the Simple View of Reading.

   As a corollary to 1 above, I have proved that the product formula of the Simple View of Reading always accounts for more variance in $R$ than the liner regression with $D$ and $C$, but not the product term $D \times C$, as independent variables. This is a direct consequence of the fact that the squared multiple correlation coefficient $R^2$ increases when a new variable is added as an independent variable (Hopkins, n.d.; Norušis, 2008, p. 254).

   Incidentally, one clear difference is that multiple linear regression do not only explain maximum explained variance but also generate numbers which are closest to $R$ as judged by the least squares criterion. This is in clear contrast to the Simple View of Reading that only achieves maximum intercorrelation with $R$, but does not approximate $R$ numerically. However, the former has four parameters, labeled $a$, $b$, $e$, and $f$ in this paper, that can be freely adjusted as opposed to two, called $s$ and $t$ in this paper, for the latter. Therefore, this difference does not necessarily imply that multiple linear regression is a superior instrument to describe $R$. Indeed, an appropriate linear transformation $e(D + s) \times (C + t) + f - (ab / e)$ *yields* $aD + bC + eD \times C + f$ as explained earlier. This means an introduction of two more free parameters for a total of four allows one to reproduce the result of multiple regression starting from the product formula of the Simple View of Reading. It can be concluded that multiple linear regression and the Simple View of Reading are mathematically completely equivalent if the optimal nullity adjustments are made for the Simple View of Reading.

2. While reading comprehension $R$, decoding skill $D$, and linguistic comprehension $C$ are supposed to be in the range [0, 1] in the original formulation of the Simple View of Reading (Hoover & Gough, 1990, p. 132), rescaling does not affect the correlation coefficient. And so, it is the locations of the points on the original scale of measurement representing nullity and perfection that are important. In particular, the points representing nullity are crucial because they determine the adjustments to the raw $D$ and $C$; that is, $s$ and $t$ in $(D + s) \times (C + t)$. Note that $D \times C \rightarrow (D + s) \times (C + t)$ is not a linear transformation and causes the correlation coefficient with $R$ to change.

3. For our purposes of checking the predictive power of the Simple View of Reading, the location of the points corresponding to perfection is not crucial as it only affects the rescaling of the terms $D + s$ and

$C + t$. This rescaling is a linear transformation and does not change the correlation coefficient with $R$, which in turn means that the predictive power of the Simple View of Reading remains the same.

4. When the product term $D \times C$ was added in the second stage of two-stage hierarchical multiple linear regression, where $D$ and $C$ were included in the first stage, the significance indicator $p$ of the $F$ change was larger than .01 for 99 out of 100 randomly generated data, with the one remaining dataset exhibiting .00894. The corresponding $p$ for Hoover and Gough's data for second graders was smaller than .0005. Note that the $F$ change for a single variable, as in this case, is equivalent to the $t$ test for the coefficient of the variable being entered (Norušis, 2008, p. 254). On the other hand, when $D$ and $C$ were entered in the second stage following $D \times C$ included in the first stage, the significance level $p$ of the $F$ change was smaller than .0005 for all 100 datasets and the Hoover and Gough data for second graders.

5. When entered in the second stage of the hierarchical multiple regression, the effect sizes for the 100 randomly generated datasets are unrecognizably small as a whole for $D \times C$, ranging from .000000978 to 0.0345, but small to medium for $D$ and $C$, ranging from 0.0623 to 0.318. The corresponding effect sizes are .0819 (medium) and .116 (medium) for $D \times C$ and $D$ and $C$, respectively, for the Hoover and Gough data. These figures imply that $D \times C$ does not explain as much variance in $R$ for the randomly generated data when entered first, and does not account for as much additional variance in $R$ when entered second, compared with the Hoover and Gough data. The observation is consistent with the Simple View of Reading for the Hoover and Gough data.

## 4. Conclusion

Based on my investigation, the validity and the predictive power of the Simple View of Reading for any study and its target audience can be examined either independently or in comparison to multiple linear regression in the following fashion. Clearly, one should first check the correlations between $R$ and $D$, $R$ and $C$, and $R$ and $D \times C$ in order to assess the importance of these abilities for reading comprehension to begin with. This is particularly important for older readers including adults, for whom the importance of decoding and the validity of the simple and separable component view of reading are often questioned (Macaruso & Shankweiler, 2010; Meneghetti, Carretti, & De Beni, 2006).

Step 1. Multiple Regression

Try multiple linear regression with $R$ as the dependent variable and $D$, $C$, and $D \times C$ as independent variables. The resulting multiple R square, the proportion of variability attributable to the regression equation, represents the maximum amount of variance in $R$ that the Simple View of Reading, $(D + s) \times (C + t)$, could account for when the adjustments for nullity, $s$ and $t$, are optimized. Therefore, if the multiple R square is small, one can conclude at this point that the Simple View of Reading is not a good model. On the other hand, if the multiple R square is considered sufficiently large, one should move to Step 2.

Step 2. Examination of Nullity Adjustments: $s$ and $t$

Once multiple linear regression is performed and the values of $a$, $b$, $e$, and $f$ in

$$R = aD + bC + eD \times C + f$$

are obtained, we can compute $s = b/e$ and $t = a/e$ such that $(D + s) \times (C + t)$ has the same correlation coefficient with $R$ as $aD + bC + eD \times C + f$. Assuming the lowest raw scores for $D$ and $C$ are 0, $s$ and $t$ conceptually mean that the scores of $-s$ and $-t$ correspond to no decoding skill and no linguistic comprehension, respectively. One should assess whether these adjustments are linguistically reasonable. If so, the Simple View of Reading works well for the data at hand. Otherwise, the Simple View of Reading may not be a good model.

However, the correlation coefficient between $R$ and $(D + s) \times (C + t)$ only gradually decreases as $s$ and $t$ deviate from their optimum values. For example, for the data presented in Table 2, the product of the raw data $D \times C$, corresponding to $s = t = 0$, correlates with $R$ with a coefficient of 0.837 and an explained variance of 0.701. Whether this is already judged sufficiently large or not, compared with the optimum values of 0.861 and 0.741, depends on the context and the purpose. For the data, the optimal adjustments are $s = 288.0200457$ and $s = 288.0200457$. These adjustments may seem large and unreasonable because the range for the raw score is from 0 to 100, but more modest adjustments of $s = t = 50$ leads to R = 0.852 and $R^2 = 0.727$, which is already closer to the maximum possible explained variance of .741 than to .701 before the adjustments to $D$ and $C$.

Hence, it may be possible to find a linguistically acceptable compromise guided by the optimal figures. While this process is more like an art than science, it is not necessarily unreasonable as the locations of nullity, if not the notion itself, for decoding and linguistic comprehension are ambiguous to begin with.

Another approach to the assessment of the validity and predictive power of SVR is the two-stage hierarchical regression where the linear terms ($D$ and $C$) and the product term ($D \times C$) are entered in separate steps. It appears from my analysis of randomly generated data and Hoover and Gough data that the raw scores for $R$, $D$, and $C$ could be used without any adjustments for the following criteria.

Criterion 1. When the product term $D \times C$ is entered in the second stage of hierarchical multiple regression and the $p$ value for the $F$ change, or equivalently the $t$ test for the coefficient of $D \times C$, is not significant, one can assume the Simple View of Reading is not a good model. It appears .01 is a reasonable cutoff for $p$.

Criterion 2. When the product term $D \times C$ is entered in the second stage of hierarchical multiple regression and the effect size for $D \times C$ expressed by Cohen's $f^2$ is smaller than .02, it appears one can safely assume the Simple View of Reading is not a good model. According to Cohen's convention, values smaller than .02 indicates no effect.

Certainly, Criterion 1 and Criterion 2 above must be closely related, but are not the same. Furthermore, the criterion values suggested above are simply a rough guide, and actual numbers should be determined depending, for example, on the correlation coefficients between $R$ and $D$ and/or $R$ and $C$. Once the descriptive statistics and actual correlation coefficients are computed, simulated datasets can be generated employing the same procedures as were used in this paper.

In addition, I would like to suggest the possibility of using commonality analysis to assess the contribution made by the product term $D \times C$. There have been few studies of the Simple View of Reading that included commonality analysis (see Johnston and Kirby (2006) for example). However, how much unique variance $D \times C$ can explain is a good indicator of the predictive power of the product term compared with a linear function of $D$ and $C$, and future studies should include commonality analysis as an additional tool to assess the validity of the Simple View of Reading. These considerations will make possible an extended triangulation in the validation of the Simple View of Reading.

Compared with multiple linear regression, the Simple View of Reading may be mathematically more reasonable and theoretically more appealing. Though the linguistic basis for the Simple View of Reading is clear, multiple linear regression is more like numerical matching and may lack a similarly solid linguistic basis. Therefore, it is necessary to continue to work towards formulating a theoretically sound and numerically convincing description of reading comprehension. We should also continue our search for the optimal instruments for that purpose. While the Simple View itself may literally be simple, what it attempts to explain, reading comprehension, is never so.

## References

(1) Aotani, M. (2009). *Proficiency profile of Kyoto University students*. Unpublished study. The International Center, Kyoto University. Kyoto, Japan.

(2) Aotani, M. (2011). Factors contributing to holistic listening of Kyoto University students: A preliminary study. *The International Center Research Bulletin, Kyoto University, 1*, 21-43.

(3) ASSIGNMENT Help. (n.d.). Properties of multiple correlation coefficient  Retrieved 04/22, 2011, from http://www.assignmenthelp.net/assignment_help/properties-of-multiple-correlation-coefficient.php

(4) Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Erlbaum.

(5) Conners, F. (2009). Attentional control and the Simple View of reading. *Reading and Writing, 22*(5), 591-613. doi: 10.1007/s11145-008-9126-x

(6) Goswami, U. (2008). Reading, complexity and the brain. *Literacy, 42*(2), 67-74. doi: 10.1111/j.1741-4369.2008.00484.x

(7) Gough, P. B., & Tunmer, W. E. (1986). Decoding, reading, and reading disability. *Remedial and Special Education, 7*(1), 6-10.

(8) Høien-Tengesdal, I. (2010). Is the Simple View of Reading too simple? *Scandinavian Journal of Educational Research, 54*(5), 451-469.

(9) Hoover, W. A., & Gough, P. B. (1990). The simple view of reading. *Reading and Writing: An Interdisciplinary Journal, 2*, 127-160.

(10) Hopkins, W. G. (n.d.). A new view of Statistics: Multiple linear regression  Retrieved 02/09, 2011, from http://www.sportsci.org/resource/stats/multiple.html

(11) Howell, D. C. (n.d.). Generating data with a specified correlation  Retrieved 03/26, 2011, from http://www.

uvm.edu/~dhowell/StatPages/More_Stuff/CorrGen.html

(12) Johnston, T., & Kirby, J. (2006). The contribution of naming speed to the Simple View of Reading. *Reading and Writing, 19*(4), 339-361. doi: 10.1007/s11145-005-4644-2

(13) Keenan, J. M., Betjemann, R. S., Wadsworth, S. J., DeFries, J. C., & Olson, R. K. (2006). Genetic and environmental influences on reading and listening comprehension. *Journal of Research in Reading, 29*(1), 75-91. doi: 10.1111/j.1467-9817.2006.00293.x

(14) Kendeou, P., Savage, R., & van den Broek, P. (2009). Revisiting the simple view of reading. *British Journal of Educational Psychology, 79*(2), 353-370. doi: 10.1348/978185408x369020

(15) Macaruso, P., & Shankweiler, D. (2010). Expanding the Simple View of Reading in accounting for reading skills in community college students. *Reading Psychology, 31*(5), 454-471.

(16) Meneghetti, C., Carretti, B., & De Beni, R. (2006). Components of reading comprehension and scholastic achievement. *Learning and Individual Differences, 16*(4), 291-301.

(17) Nagpaul, P. S. (n.d.). Guide to advanced data analysis using IDAMS software Retrieved 04/12, 2011, from http://www.unesco.org/webworld/idams/advguide/Chapt5_2.htm

(18) Nimon, K. (2010). Regression commonality analysis: Demonstration of an SPSS solution. *Multiple Linear Regression Viewpoints, 36*(1), 10-17.

(19) Norušis, M. J. (2008). *SPSS16.0 statistical procedures companion*. Upper Saddle River, NJ: Prentice Hall.

(20) Ogle, D. H. (n.d.) Retrieved 03/26, 2011, from http://www.uvm.edu/~dhowell/StatPages/More_Stuff/Ogle.html

(21) Savage, R. (2006). Reading comprehension Is not always the product of nonsense word decoding and linguistic comprehension: Evidence from teenagers who are extremely poor readers. *Scientific Studies of Reading, 10*(2), 143-164.

(22) Savage, R., & Wolforth, J. (2007). An additive Simple View of Reading describes the performance of good and poor readers in higher education. *Exceptionality Education Canada, 17*, 243-268.

(23) Tan, K. H., Wheldall, K., Madelaine, A., & Lee, L. W. (2007). A review of the simple view of reading: Decoding and linguistic comprehension skills of low-progress readers. *Australian Journal of Learning Disabilities, 12*(1), 19-30.

(24) Vellutino, F. R., Tunmer, W. E., Jaccard, J. J., & Chen, R. (2007). Components of reading ability: Multivariate evidence for a convergent skills model of reading development. *Scientific Studies of Reading, 11*(1), 3-32.

(25) Weisstein, E. W. (n.d.). Linear combination Retrieved 06/01/2011, from http://mathworld.wolfram.com/LinearCombination.html

(Associate Professor, The International Center, Kyoto University)

# 読解モデルとしての Simple View of Reading と重回帰分析：
## 乗法モデルと加法モデルの比較と互換性

青谷正妥


**要旨**

読解力 ($R$) の描写においては、decoding ($D$) と linguistic comprehension ($C$) の積で説明しようとする Simple View of Reading (SVR) と、$D$ と $C$ の linear combination と定数の和からなる多項式で説明しようとする multiple linear regression があり、その優劣がさかんに論じられている。本稿では両者を比較し、(1) fit 基準、有効性、利点 (2) SVR における $D$ や $C$ の最適原点調整と linear regression における相互作用項 $D \times C$ の導入による両者の数学的同値性 (3) 有意水準 (significance levels) と効果の大きさ (effect size) のシミュレーション、の 3 点について論じる。(2) の数学的同値性は、最適原点調整を行えば SVR は常に相互作用項 $D \times C$ を含まない multiple linear regression より多くの分散 (variance) を説明することを意味するが、数学的に得られた最適原点調整が言語学的に受け入れられない場合もある。この問題につき、代数的にまた数値化された具体例を使って説明する。最後に、有意レベル、効果の大きさ、原点調整、共通性分析 (commonality analysis) に基づいた SVR の妥当性について論じる。


（京都大学国際交流推進機構国際交流センター・准教授）