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Gluon helicity ΔG from a universality class of operators on a lattice

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We show that the total gluon helicity ΔG in a polarized nucleon can be calculated on a Euclidean lattice through a universality class of QCD operators that describe the helicity or polarization of the on-shell gluon radiation. We in particular find some operators whose matrix elements in a nucleon of momentum P^z are directly related to ΔG with only power-law $(1/P^z)^n (n \geq 2)$ corrections.

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I. INTRODUCTION

The total gluon helicity $\Delta G = \int_0^1 \Delta g(x) dx$ in a longitudinally polarized nucleon (proton or neutron) is an important physical quantity that characterizes the fundamental property of the nucleon. In the last two decades, many high-energy experiments have been carried out to measure the polarized gluon parton helicity distribution $\Delta g(x)$, from which one can estimate the total gluon polarization by integration over the measured region, $\int_{x_{\min}}^{x_{\max}} dx \Delta g(x)$ [1–4]. Since ΔG is intrinsically related to the light-cone physics, it has been impossible to calculate this quantity on a Euclidean lattice. Thus, there has been little interplay between experiment and quantum chromodynamics (QCD) in this field so far [5,6].

In a recent publication [7], a theoretical method has been proposed to allow computing ΔG directly on a lattice for the first time. Instead of the light-cone operator, the matrix element of a time-independent spin operator $\Delta \tilde{G}(P^z, \mu)$ is calculated in a nucleon with finite momentum P^z. The physical quantity ΔG is then obtained through a matching condition

$$\Delta \tilde{G}(P^z, \mu) = Z_{gg}(P^z/\mu) \Delta G(\mu) + Z_{gq}(P^z/\mu) \Delta \Sigma(\mu), \quad (1)$$

where $\Delta \Sigma(\mu)$ is the quark spin and μ is the renormalization scale. Z_{gg} and Z_{qg} are the matching coefficients calculable in QCD perturbation theory. The operator considered in Ref. [7] was $\vec{E} \times \vec{A}_\perp$, where $\vec{A}_\perp$ is the transverse part of the gauge field, or $\vec{E} \times \vec{A}$ in the Coulomb gauge.

In this paper, we show that the gluon spin operator that can be matched to ΔG is not unique. Instead, one can find a universality class of operators which can fulfill the same role. The physics of this phenomenon is easy to understand: According to the Weizsäcker-Williams approximation [8], the gluon field in the nucleon is dominated by quasifree radiation which corresponds to a beam of free gluons with momentum $\vec{k} = (0, 0, xP^z)$. For such radiation, the gluon polarization vector is just $\epsilon^\mu = (0, \epsilon^x, \epsilon^y, 0)$. Thus, the gauge-dependent gluon spin operator $(\vec{E} \times \vec{A})^z = E^x A^y - E^y A^x$ under any gauge choice without changing the transverse polarization can describe the gluon helicity. These operators define a universality class. For instance, in the Coulomb gauge, the gauge condition $\vec{E} \cdot \vec{A} = 0$ yields $\epsilon^z = 0$ which has no effect on the spin operator. Another reason for the existence of a universality class is that the t component and z component of a four vector scale in the same way in the infinite momentum frame (IMF) limit.

In Sec. II, we explore different gluon spin operators that correspond to different gauge choices for $\vec{E} \times \vec{A}$ and show that they all lead to the same light-cone gluon helicity ΔG. We consider physical gauges as well as covariant gauges. In Sec. III, we consider the matrix element of the topological current, leading to some more operators of the universality class which do not even have straightforward gluon spin interpretation. We consider their matrix elements to one loop in the continuum in order to provide a useful input for matching to lattice QCD calculations. We conclude the paper in Sec. IV. In the Appendix, we calculate the one-loop matrix element of the gluon spin operator in the external gluon state.

II. A UNIVERSALITY CLASS OF OPERATORS

In this section, we discuss the matrix elements of the gluon spin operator with different choices of gauges, which asymptotically approach the physical gluon helicity ΔG. We start with the consideration in Ref. [7].

Let us begin with the standard definition of ΔG as the matrix element of a nonlocal operator involving light-cone correlation [9]

$$\begin{aligned} \Delta G \frac{S^+}{P^+} &= \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \\ &\quad \times \langle PS | F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}_{\alpha,b}^{+}(0) | PS \rangle_N \\ &= \frac{1}{2P^+} \langle PS | \epsilon^{ij} F^{i+}(0) A^j_{\text{phys}}(0) | PS \rangle_N, \end{aligned} \quad (2)$$

where $|PS\rangle_N$ is a proton plane-wave state with momentum P^μ and polarization S^μ, $\tilde{F}^{\alpha\beta} = (1/2)\epsilon^{\alpha\beta\mu\nu}F_{\mu\nu}$, and $\mathcal{L}(\xi^-,0) = P\exp[-ig\int_0^{\xi^-} A^+(\eta^-,0_\perp)d\eta^-]$ is a gauge link in the adjoint representation. The light-front coordinates are defined as $\xi^\pm = (\xi^t \pm \xi^z)/\sqrt{2}$.

In the second line of Eq. (2), we defined [10,11]

$$A^\mu_{\rm phys} \equiv \frac{1}{D^+}F^{+\mu} \tag{3}$$

and introduced the antisymmetric tensor in the transverse plane ϵ^{ij} ($\epsilon^{xy} = -\epsilon^{yx} = 1$). The boundary condition for the integral operator $1/D^+$ is related to the $i\epsilon$ prescription for the $1/x$ pole. In the light-cone gauge $A^+ = 0$, $A^\mu_{\rm phys}$ reduces to A^μ.

The matrix element in Eq. (2), being nonlocal in the light-cone direction, cannot be readily evaluated in lattice QCD. However, it has been suggested in Ref. [7] that one can relate ΔG to the following matrix element:

$$\Delta\tilde{G}(P^z,\mu) = \frac{1}{2P^0}\langle PS|\epsilon^{ij}F^{i0}(0)A^j(0)|PS\rangle_N, \tag{4}$$

which is local and time independent, hence measurable on the lattice. In Eq. (4), the momentum P^z is assumed to be large but finite. $\epsilon^{ij}F^{i0}A^j = (\vec{E}\times\vec{A})^z$ is the gluon helicity operator identified by Jaffe and Manohar [12]. As is well known, this operator is not gauge invariant, so the matrix element in Eq. (4) depends on the gauge choice. In Ref. [7] the authors used the Coulomb gauge (see Refs. [13,14] for an earlier discussion)

$$\vec{\nabla}\cdot\vec{A} = 0. \tag{5}$$

The condition in Eq. (5) separates the transverse (or "physical") part from the gauge field which should be kept in the computation of physical quantities like ΔG. While the solution $A^\mu = A^\mu_\perp$ to Eq. (5) in generic frames bears no resemblance to $A^\mu_{\rm phys}$, it has been shown in Ref. [7] that $A^\mu_\perp$ approaches $A^\mu_{\rm phys}$ if one takes the IMF limit.[1]

Naively, then, one might think that the matrix elements $\langle\ldots A^\mu_{\rm phys}\ldots\rangle$ and $\langle\ldots A^\mu_\perp\ldots\rangle$ are simply related by a Lorentz boost as well. However, this is not the case due to the ultraviolet (UV) divergences in field theory. For the external on-shell quark state $|PS\rangle_q$, the one-loop calculation using dimensional regularization (in $D = 4-2\epsilon$ dimensions) yields [7,15]

$$\Delta\tilde{G}(P^z,\mu) = \left.\frac{\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle_q}{2P^0}\right|_{\vec{\nabla}\cdot\vec{A}=0} = \frac{C_F\alpha_s}{4\pi}\left(\frac{5}{3\varepsilon_m} - \frac{1}{9} + \frac{4}{3}\ln\frac{4P_z^2}{m^2}\right)\frac{S^z}{P^0} + \mathcal{O}\left(\frac{m^2}{P_z^2}\right), \tag{7}$$

where we defined $1/\varepsilon_m \equiv 1/\epsilon - \gamma_E + \ln 4\pi + \ln(\mu^2/m^2)$, and m is the quark mass to regularize the collinear divergence. $C_F = (N_c^2-1)/2N_c$ as usual. On the other hand, in the same regularization scheme Eq. (2) is evaluated as

$$\Delta G(\mu) = \left.\frac{\langle PS|\epsilon^{ij}F^{i+}A^j|PS\rangle_q}{2P^+}\right|_{A^+=0} = \frac{C_F\alpha_s}{4\pi}\left(\frac{3}{\varepsilon_m} + 7\right)\frac{S^+}{P^+}. \tag{8}$$

We see that the coefficients of $1/\varepsilon_m$ (anomalous dimension) are different. Moreover, Eq. (7) depends nontrivially on the reference frame. The reason for this discrepancy is that the IMF limit $P^z \to \infty$ and the large loop momentum limit $k^\mu \to \infty$ in the one-loop integral do not commute: One can actually recover the light-cone gauge result in Eq. (8) from the Coulomb gauge calculation by sending $P^z \to \infty$ *before* doing the k integral. On a lattice, P^z is restricted to be less than the cutoff, which is tantamount to taking the $k^\mu \to \infty$ limit first. Thus, the matrix element in Eq. (4), evaluated in the Coulomb gauge, fails to capture the UV properties of ΔG. Nevertheless, since the infrared (IR) divergent part of the matrix elements is not affected by the order of limits, one can correct the discrepancy via the *matching condition* [7][2]

$$\frac{1}{\varepsilon_m} + \frac{16}{3} \leftrightarrow \ln\frac{4P_z^2}{m^2}. \tag{9}$$

This observation paves the way for evaluating ΔG on a Euclidean lattice.

To see the relevance for nonperturbative calculations, we use the general matching formula as given by Eq. (1). According to the result from Eq. (7), we find, for the Coulomb gauge spin operator,

$$Z_{gq}(P^z/\mu) = \frac{C_F\alpha_s}{4\pi}\left(\frac{4}{3}\ln\frac{4P_z^2}{\mu^2} - \frac{64}{9}\right), \tag{10}$$

in the $\overline{\rm MS}$ scheme, which is IR free. The matching coefficient Z_{gg} must be calculated in a gluon state.

We now argue that the Coulomb gauge in Eq. (5) is not the unique possibility in order to match with ΔG. For instance, consider the temporal axial gauge $A^0 = 0$. In this gauge one can identically write

[1]See Eq. (7) of Ref. [7] and notice that in the IMF limit,

$$A^\mu_\perp \to A^\mu - \frac{1}{D^+}\partial^\mu A^+ = \frac{1}{D^+}F^{+\mu} = A^\mu_{\rm phys}. \tag{6}$$

[2]On the lattice, $1/\varepsilon_m$ is replaced by $-\ln(a^2m^2)$ so the matching condition becomes $\ln P_z^2a^2 = \text{const}$.

$$\tilde{A}^\mu = \frac{1}{D^0} F^{0\mu}. \tag{11}$$

Taking the IMF limit, one trivially recovers Eq. (3):

$$\frac{1}{D^0} F^{0\mu} \to \frac{1}{D^+} F^{+\mu} = A^\mu_{\rm phys}. \tag{12}$$

Alternatively, one may choose the $A^z = 0$ gauge in which

$$\tilde{A}^\mu = \frac{1}{D^z} F^{z\mu}. \tag{13}$$

This also becomes $\frac{1}{D^+} F^{+\mu}$ in the IMF limit.

However, as in the Coulomb gauge, the matrix elements of the operator $\vec{E} \times \vec{A}$ are in general different. To one-loop order, we find

$$\Delta\tilde{G}(P^z, \mu) = \left.\frac{\langle PS|\epsilon^{ij} F^{i0} A^j|PS\rangle_q}{2P^0}\right|_{A^0=0} = \frac{C_F \alpha_s}{4\pi}\left(\frac{3}{\varepsilon_m} + 7\right)\frac{S^z}{P^0}, \tag{14}$$

$$\Delta\tilde{G}(P^z, \mu) = \left.\frac{\langle PS|\epsilon^{ij} F^{i0} A^j|PS\rangle_q}{2P^0}\right|_{A^z=0} = \frac{C_F \alpha_s}{4\pi}\left[\frac{2}{\varepsilon_m} + 4 + \ln\frac{4P_z^2}{m^2}\right]\frac{S^z}{P^0} + \mathcal{O}\left(\frac{m^2}{P_z^2}\right). \tag{15}$$

Equation (14) agrees with the previous result in Eq. (8) in the light-cone gauge (see, also, Ref. [16]). On the other hand, Eq. (15) features yet another anomalous dimension together with logarithmic frame dependence. Here again, the order of limits matters: If one takes the $P^z \to \infty$ limit before the loop integration, one recovers Eq. (8) from the $A^z = 0$ gauge calculation. At large but finite momentum, part of the divergence $1/\varepsilon_m$ is transferred to the logarithm $\ln P_z^2$, keeping the sum of their coefficients unchanged. The following matching condition then establishes the connection between Eqs. (15) and (8):

$$\frac{1}{\varepsilon_m} + 3 \leftrightarrow \ln\frac{4P_z^2}{m^2}. \tag{16}$$

The constant term is different from the Coulomb gauge case in Eq. (9). This corresponds to a different matching constant $Z_{qg} = (C_F\alpha_s/4\pi)(\ln 4P_z^2/\mu^2 - 3)$.

Thus, for the purpose of obtaining ΔG, one can broadly generalize the approach of Ref. [7]: Evaluate the "naive" gluon helicity operator Eq. (4) either in the Coulomb gauge, or $A^0 = 0$, or $A^z = 0$ gauge and perform an appropriate matching. However, this does not mean that any gauge choice is allowed. For instance, in the $A^x = 0$ gauge where

$$\tilde{A}^\mu = \frac{1}{D^x} F^{x\mu}, \tag{17}$$

or in the Landau (or covariant) gauge $\partial \cdot A = 0$ where

$$\tilde{A}^\mu = A^\mu - \frac{1}{\partial^2}\partial^\mu \partial \cdot A, \tag{18}$$

$\tilde{A}^\mu$ does not approach $A^\mu_{\rm phys}$ in the IMF limit. This is also reflected in their one-loop matrix elements

$$\left.\frac{\langle PS|\epsilon^{ij} F^{i0} A^j|PS\rangle_q}{2P^0}\right|_{A^x=0} = \frac{C_F\alpha_s}{4\pi}\left(\frac{3}{2\varepsilon_m} + \frac{7}{2}\right)\frac{S^z}{P^0}, \tag{19}$$

$$\left.\frac{\langle PS|\epsilon^{ij} F^{i0} A^j|PS\rangle_q}{2P^0}\right|_{\partial\cdot A=0} = \frac{C_F\alpha_s}{4\pi}\left(\frac{2}{\varepsilon_m} + 4\right)\frac{S^z}{P^0}, \tag{20}$$

which do not agree with the light-cone gauge result.[3] Moreover, the logarithm of P^z is absent so there is no possibility of matching.

The above analysis suggests that there is a class of gauges (similar to the universality class of second-order phase transitions) which flows to the "fixed point" $A_{\rm phys}$ in the IMF limit, and thus can be used to compute ΔG. This class of gauges clearly do not include all possible gauges. To see what gauges are permitted, we consider the Weizsäcker-Williams (WW) approximation [8] in the IMF. The gluon field is dominated by quasifree radiation in the sense that $\vec{B}_\perp \sim \vec{E}_\perp \gg \vec{E}_\parallel$. Thus we have in effect a beam of gluons with momentum xP^z. For these on-shell gluons, the gauge transformation only affects the time component and the third spatial component (we consider only the Abelian part),

$$A^\mu \to A^\mu + \lambda k^\mu, \tag{21}$$

where $k^\mu = (k^0, 0, 0, k^z)$. Thus the transverse part of the polarization vectors is physical:

$$\epsilon^\mu(xP^z) = \frac{1}{\sqrt{2}}(0, 1, \mp i, 0). \tag{22}$$

The gluon spin operator $(\vec{E} \times \vec{A})^z$ is independent of those gauge transformations which leave $A^{x,y}$ invariant. Although Eq. (21) seems to guarantee this for WW gluon field, it contains only a subclass of gauges: There are gauge choices which are incompatible with the notion that WW gluon $A^{x,y}$ shall be left intact by gauge transformations. Those latter gauge transformations will not "flow" into the fixed point light-cone operator in the IMF.

The axial gauge $A^z = 0$ and the temporal gauge $A^0 = 0$ have no effect on the gluon polarization vector. Therefore, they can be used to calculate the gluon helicity. In the

[3]Interestingly, Eq. (19) is exactly one half of Eq. (14).

Coulomb gauge, one has $\vec{k}\cdot\vec{A} = k^z A^z = 0$. This is similar to the axial gauge $A^z = 0$.

The obvious counterexample is $A^x = 0$ or $A^y = 0$ gauges. A less trivial one is the covariant gauge, in which the condition $k\cdot A = k^+A^- = 0$ itself is consistent with having nonzero transverse components. However, actually the WW field in the covariant gauge has only the A^+ component. This can be seen from an example of the WW field associated with a fast-moving pointlike charge. In the covariant gauge we have

$$A^\mu(\xi) = -e\ln\xi_\perp^2\delta(\xi^-)\delta_+^\mu. \tag{23}$$

Equation (23) indeed satisfies $\partial\cdot A = \partial_+A^+ = 0$ but has vanishing transverse components $A^{x,y}$. Therefore the covariant gauge does not belong to the universality class.

III. AXIAL GAUGES, TOPOLOGICAL CURRENT, AND MORE OPERATORS

Our discussion in the previous section is based on the one-loop calculation. It remains to be seen whether the formula Eq. (1) is still valid after including higher loop corrections. Instead of doing this for generic gauges, here we point out that in the axial gauge such a nonperturbative generalization is readily possible.

Indeed, the temporal axial gauge $A^0 = 0$ seems to have a special status since the matrix element in Eq. (14) is the same as that in the $A^+ = 0$ gauge (up to the trivial kinematic factor $S^+ \approx \sqrt{2}S^z$). Therefore, no logarithmic matching is necessary in this gauge. This is actually not a coincidence and is connected to the physics of the topological current in QCD. As we shall see, the matrix element of the topological current allows us to find more operators in the universality class, and some of them do not even have the form of spin operator in a particular gauge.

First, note that in the $A^0 = 0$ gauge, the operator $\epsilon^{ij}F^{i0}A^j$ is the same as

$$\epsilon^{ij}\left(F^{i0}A^j - \frac{1}{2}A^0F^{ij}\right). \tag{24}$$

Likewise, in the $A^+ = 0$ gauge the operator $\epsilon^{ij}F^{i+}A^j$ is the same as

$$\epsilon^{ij}\left(F^{i+}A^j - \frac{1}{2}A^+F^{ij}\right). \tag{25}$$

Actually, the quark matrix elements of these operators are gauge invariant to one loop:

$$\frac{\langle PS|\epsilon^{ij}(F^{i0}A^j - \frac{1}{2}A^0F^{ij})|PS\rangle_q}{2P^0} = \frac{C_F\alpha_s}{4\pi}\left(\frac{3}{\varepsilon_m} + 7\right)\frac{S^z}{P^0}, \tag{26}$$

as can be explicitly checked in all the gauges mentioned in the previous section. (See, also, Ref. [17].) This in particular means that the logarithm of P^z which appears in some gauges is canceled by the contribution from the extra term $\epsilon^{ij}A^0F^{ij}$. The reason is that Eqs. (24) and (25) are a part of the topological current in QCD:

$$\begin{aligned} K^\mu &= \epsilon^{\mu\nu\rho\lambda}\left(A_\nu^aF_{\rho\lambda}^a + \frac{g}{3}f_{abc}A_\nu^aA_\rho^bA_\lambda^c\right),\\ K^+ &= 2\epsilon^{ij}\left(F_a^{i+}A_a^j - \frac{1}{2}F_a^{ij}A_a^+ - \frac{g}{2}f_{abc}A_a^+A_i^bA_j^c\right),\\ K^z &= 2\epsilon^{ij}\left(F_a^{i0}A_a^j - \frac{1}{2}F_a^{ij}A_a^0 - \frac{g}{2}f_{abc}A_a^0A_i^bA_j^c\right), \end{aligned} \tag{27}$$

which satisfies $\partial_\mu K^\mu = F_a^{\mu\nu}\tilde{F}_{\mu\nu}^a$. The forward matrix element of Eq. (27) is perturbatively gauge invariant [12,18] and the $\mathcal{O}(gAAA)$ term starts to contribute only at two loops (for the quark matrix elements). In the Appendix, we provide a similar discussion in the case of the gluon matrix elements and present the one-loop result.

Nonperturbatively, however, there is gauge dependence due to anomaly [12,19,20]. In axial gauges $A\cdot n = 0$, this dependence has been precisely calculated in Ref. [20]. The *nonforward* matrix element of K^μ in a polarized nucleon state is given by

$$\begin{aligned} \langle PS|K^\mu|P+q,S\rangle_N|_{A\cdot n=0} &\overset{q^\mu\to 0}{\to} 4\left(S^\mu - \frac{q\cdot S}{q\cdot n}n^\mu\right)\Delta G(n,P)\\ &+ \frac{in^\mu}{q\cdot n}\langle PS|F_a^{\mu\nu}\tilde{F}_{\mu\nu}^a|PS\rangle_N, \end{aligned} \tag{28}$$

where

$$\int_0^\infty d\lambda\langle PS|n^\tau F_{\tau\nu}(\lambda n)\mathcal{L}\tilde{F}^{\nu\mu}(0)|PS\rangle_N \equiv 2S^\mu\Delta G(n,P). \tag{29}$$

The matrix element in Eq. (29) is the same as in Eq. (2) except for the direction of the Wilson line. Expanding around the deviation from the light cone n^2, one finds the relation [20]

$$\Delta G(n,P) = \Delta G + \mathcal{O}\left(\frac{n^2}{(P\cdot n)^2}\right), \tag{30}$$

which is valid at large momentum (assuming $P\cdot n \neq 0$).

From Eq. (28) one can read off various representations of ΔG. For the $\mu = z$ component in the $A^0 = 0$ gauge, the ambiguity (gauge dependence) in the $q^\mu \to 0$ limit drops out. One can safely take the forward limit and find

$$\langle PS|\epsilon^{ij}A^i\partial^0A^j|PS\rangle_N|_{A^0=0} = 2S^z\Delta G + \mathcal{O}(1/P_z^2). \tag{31}$$

Since Eq. (28) is a nonperturbative formula, Eq. (31) actually extends Eq. (14) to all orders in perturbation theory. Similarly, taking $\mu = 0$ in the $A^z = 0$ gauge, one gets

$$\langle PS|\epsilon^{ij}A^i\partial^z A^j|PS\rangle_N|_{A^z=0} = 2S^0\Delta G + \mathcal{O}(1/P_z^2), \quad (32)$$

which is related to Eq. (15) by replacing F^{i0} with F^{iz}. In the IMF limit, the t component and z component of a quantity have similar scaling properties as they both approach the plus (+) direction. Note that the operator on the left-hand side of Eq. (32) does not have a straightforward gluon spin interpretation.

Moreover, Eqs. (29) and (30) directly give

$$\int_0^\infty d\xi^0 \langle PS|F^0_\nu(\xi^0)\mathcal{L}\tilde{F}^{\nu 0}(0)|PS\rangle_N = \langle PS|\vec{A}^a\cdot\vec{B}^a|PS\rangle_N|_{A^0=0} = 2S^0\Delta G + \mathcal{O}(1/P_z^2), \quad (33)$$

$$\int_0^\infty d\xi^z \langle PS|F^z_\nu(\xi^z)\mathcal{L}\tilde{F}^{\nu z}(0)|PS\rangle_N = \langle PS|\epsilon^{ij}\left(F^{i0}A^j - \frac{1}{2}A^0F^{ij}\right)|PS\rangle_N\Big|_{A^z=0} = 2S^z\Delta G + \mathcal{O}(1/P_z^2). \quad (34)$$

The operator in Eq. (33) is similar to an operator written down by Jaffe [5], except that it includes the z component as well. Equation (34) coincides with the operator introduced in Ref. [21]. All the matrix elements in Eqs. (31)–(34) are measurable on the lattice. In particular, the operators in Eqs. (32) and (33) can be readily transcribed into Euclidean space as they do not contain temporal indices ∂^0, A^0. Note that all these operators yield the gluon helicity ΔG without logarithmic corrections in the large P^z limit.

IV. CONCLUSION

In this paper, we first extended the matching method of Ref. [7] to a broader class of gauges. Not only the Coulomb gauge but also other gauge choices that maintain the $A^{x,y}$ components of the on-shell gluon fields do qualify, and in some of them the gluon spin matrix element does not have logarithmic corrections in the large momentum limit. We then focused our attention on nonlightlike axial gauges. All the matrix elements in Eqs. (31)–(34) can be used to compute ΔG in lattice QCD, and we have computed the one-loop matching coefficients on the continuum theory side.

The implementation of the Coulomb gauge and axial gauges on a lattice may pose technical problems. The usual periodic boundary condition on gauge field configurations is incompatible with the condition $A\cdot n = 0$ because of nonvanishing Polyakov loops. In order to circumvent this and fix the residual gauge symmetry, ideally one should impose antisymmetric boundary condition in the direction specified by the vector n^μ. Or else, one has to confront the problem of lattice Gribov copies [23,24].

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APPENDIX: ONE-LOOP GLUON MATRIX ELEMENT OF $(\vec{E}\times\vec{A})^Z$

In this Appendix we calculate the one-loop gluon matrix element of the operators in (31)–(34) in the axial gauges. Together with the quark matrix element Eq. (8), it is a necessary ingredient for the perturbative matching between $\Delta G_{\rm lat}$ measured on the lattice and $\Delta G_{\overline{\rm MS}}$ defined in the continuum theory in the $\overline{\rm MS}$ scheme [25]. In the case of the operators considered here, the perturbative matching is relatively simple because there are no large logarithms $\ln(P^z/\mu)$ involved.

First we observe that all the operators (31)–(34) should give the same one-loop result (up to $1/P_z^2$ corrections) after factoring out a component of S^μ. *A priori*, this is not obvious especially for the latter two operators Eqs. (33) and (34) because the non-Abelian part of the operator $\mathcal{O}(gAAA)$ would contribute already at one loop for the gluon external state. Yet, this is guaranteed by the general discussion in Sec. III. We then notice that, from the discussion of the topological current, the gluon matrix element of $\epsilon^{ij}A^iF^{0j} = \epsilon^{ij}A^i\partial^0A^j$ in the $A^0=0$ gauge is the same as that of $\epsilon^{ij}A^iF^{+j}$ in the $A^+=0$ gauge. It thus suffices to compute only the latter matrix element.

In order to regularize the infrared divergence, we treat the external gluon to be off-shell $P^2<0$ and define $1/\varepsilon_v \equiv 1/\epsilon - \gamma + \ln 4\pi + \ln\frac{\mu^2}{-P^2}$. After some algebra, the irreducible one-loop diagram in the light-cone gauge $A^+=0$ is evaluated as (see, also, Ref. [16])

$$\frac{\langle Ph|\epsilon^{ij}F^{i+}A^j|Ph\rangle_g^{irr}}{2P^+} \sim h\frac{ig^2N_c}{P^+}\int\frac{d^Dk}{(2\pi)^D} \times \frac{\frac{16}{D-2}k_\perp^2P^+ - k^+(P+k)^2 - 2\frac{P^++k^+}{P^+-k^+}k^+(k^2-P^2)}{k^2k^2(P-k)^2}, \quad (A1)$$

where $h=\pm1$ is the helicity. We use the Mandelstam-Leibbrandt prescription for the pole in the last term of the numerator $1/k^+ \to 1/(k^+ + i\epsilon k^-)$. Using the following formulas:

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 (P-k)^2 (P^+ - k^+)} = \frac{i}{(4\pi)^2 P^+} \frac{\pi^2}{6}, \tag{A2}$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 k^2 (P-k)^2 (P^+ - k^+)} = \frac{-i}{(4\pi)^2 P^+ P^2} \frac{1}{\varepsilon_v'}, \tag{A3}$$

where ε_v' is an IR regulator, we find

$$\left.\frac{\langle Ph|\epsilon^{ij} F^{i+} A^j|Ph\rangle_g}{2P^+}\right|_{A^+=0}^{irr} = h \frac{\alpha_s N_c}{2\pi}\left(2 + \frac{\pi^2}{3}\right). \tag{A4}$$

Note that there is no divergence. The self-energy insertion in the external gluon legs is divergent and reads (cf. Ref. [22])

$$\begin{aligned} &\left.\frac{\langle Ph|\epsilon^{ij} F^{i+} A^j|Ph\rangle_g}{2P^+}\right|_{A^+=0}^{\text{self}} \\ &\quad = h \frac{\alpha_s N_c}{2\pi}\left(\frac{11}{6\varepsilon_v} - \frac{\pi^2}{3} + \frac{67}{18}\right) \\ &\qquad + h \frac{\alpha_s N_f}{2\pi}\left(-\frac{1}{3\varepsilon_v} - \frac{5}{9}\right), \end{aligned} \tag{A5}$$

where the two terms correspond to the gluon and quark loop contributions, respectively. Combining these results, we find

$$\begin{aligned} &\langle Ph|\epsilon^{ij} F^{i+} A^j|Ph\rangle_g|_{A^+=0} \\ &\quad = \left[1 + \frac{\alpha_s}{4\pi}\left(\frac{\beta_0}{\varepsilon_v} + \frac{103 N_c - 10 N_f}{9}\right)\right] \\ &\qquad \times \langle Ph|\epsilon^{ij} F^{i+} A^j|Ph\rangle_g^{\text{tree}}, \end{aligned} \tag{A6}$$

where $\beta_0 = \frac{11 N_c}{3} - \frac{2 N_f}{3}$ is the coefficient of the one-loop QCD beta function. Equation (A6) immediately implies that the same coefficient should appear in the matrix element of all the operators in Eqs. (31)–(34), e.g.

$$\begin{aligned} &\langle Ph|\epsilon^{ij} F^{i0} A^j|Ph\rangle_g|_{A^0=0} \\ &\quad = \left[1 + \frac{\alpha_s}{4\pi}\left(\frac{\beta_0}{\varepsilon_v} + \frac{103 N_c - 10 N_f}{9}\right)\right] \\ &\qquad \times \langle Ph|\epsilon^{ij} F^{i0} A^j|Ph\rangle_g^{\text{tree}}, \end{aligned} \tag{A7}$$

and similarly for the other matrix elements in Eqs. (32)–(34).[4]

[4]The agreement of the divergent part in Eqs. (A6) and (A7) was explicitly checked in Ref. [16].

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