'Open-loop' tracking interferometer for machine tool volumetric error measurement – Two-dimensional case –

Soichi Ibaraki[a], Goh Sato[a], Kunitaka Takeuchi[a]

[a]*Department of Micro Engineering, Kyoto University, Katsura, Nishigyo-ku, Kyoto 615-8530, Japan. Phone/Fax: +81-75-383-3676*

Abstract

The tracking interferometer, or the laser tracker, is a laser interferometer with a steering mechanism to regulate the laser beam direction to follow a retroreflector ("target"). Applying the multilateration principle, it measures the target's three-dimensional position at an arbitrary location in the workspace. Its application to the volumetric accuracy measurement for coordinate measurement machines or machine tools has been long studied. In this paper, we propose the 'open-loop' tracking interferometer, where the laser beam is regulated toward the command target position. This eliminates the automated tracking mechanism and thus may significantly reduce the manufacturing cost of conventional tracking interferometers. The objective of this paper is to validate this 'open-loop' tracking interferometer concept by investigating its measurement uncertainty both experimentally and analytically. To simplify the problem, this paper focuses on the measurement of the target's two-dimensional position by using a single-axis 'open-loop' tracking interferometer prototype.

Email address: ibaraki@prec.kyoto-u.ac.jp (Soichi Ibaraki)

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1. Introduction

Recent revision of ISO 230-1 [1] defines the term "volumetric accuracy" of machine tools. ISO TC39/SC2, a technical subcommittee in International Organization of Standardization (ISO), has been discussing the publication of a Technical Report (TR) on numerical compensation for machine tool volumetric errors [2]. Such an effort clearly indicates the recognition by machine tool manufacturers/users of the importance of volumetric accuracy. Many major CNC makers have lately commercialized the functionality of numerically compensating for volumetric errors.

Suppose that the command tool center position (TCP) is given by $p^* \in \mathbb{R}^3$ in the machine coordinate system. Denote its actual position by $p \in \mathbb{R}^3$. The three-dimensional positioning error, $\Delta p \in \mathbb{R}^3$, is defined by $\Delta p(p^*) \equiv p - p^*$. The assessment of a machine tool's volumetric accuracy requires the measurement of $\Delta p(p^*)$ at arbitrary p^* in the machine's workspace.

An established direct measurement method of the volumetric accuracy uses calibrated three-dimensional artifacts, e.g. a three-dimensional ball plate. While such an artifact-based calibration is more common for coordinate measuring machines (CMMs) (ISO 10360-2:2009 [3]), its application to machine tools has been also reported [4]. For large-size machine tools, a large artifact of the calibrated geometric accuracy is needed, which is in practice very difficult and/or expensive to make. Indirect assessment of volumetric accuracy based on the assumption of the machine's kinematic model is more common for machine tools. The references [5, 6] present a good review of conventional direct and indirect volumetric accuracy measurement schemes.

The tracking interferometer (the term in [1]), or the laser tracker, is probably only commercially available instrument capable of directly measuring the three-dimensional TCP at an *arbitrary* location within its workspace. It is a laser interferometer with a steering mechanism to change the laser beam direction to automatically follow a retroreflector (referred to as the "target" hereafter). Some commercial tracking interferometers, from, e.g., Leica Geosystems, Faro, and Automated Precision Inc. (API), measure the TCP from the distance (displacement) to the target and the direction of the laser beam [7] (see Fig. 1(a)). Since its angular measurement uncertainty directly contributes to the target position's estimation uncertainty, it is typically difficult to ensure the measurement accuracy high enough to evaluate machine tools.

On the other hand, the multilateration measurement (the term in [1]) estimates the target's three-dimensional position by the distances from typically four or more tracking interferometers to the target (see Fig. 1(b)). As is illustrated in Fig. 2, the instrument's angular positioning error only impose the "cosine error" on the laser displacement. In the multilateration principle, therefore, the laser beam's orientation error does not impose significant contribution on the measurement uncertainty of the target position. Its application to machine tool calibration has been long studied [8, 9, 10]. Its commercial product is available (Etalon AG [11, 12]). Figure 3 illustrates a typical laser beam steering mechanism (an example from [10]). The laser beam direction is controlled so that the laser spot position on the quadrant photo-diode is regulated at the reference point.

In the application to machine tool calibration, the target's command position in the machine coordinate system is given. It is, furthermore, reasonable to assume that the target's positioning error is sufficiently small to make its "cosine error" on the laser displacement negligibly small. In such a condition, the multilateration measurement can be done by regulating the laser beam toward the command target position (see Fig. 4). This eliminates the automated tracking mechanism, i.e. a photodiode and a feedback control system for laser beam direction. This may significantly reduce the manufacturing cost of conventional tracking interferometers; it performs the multilateration measurement by using a laser interferometer and a two-axis numerically-controlled rotary drive only.

The objective of this paper is to validate this idea by investigating its measurement uncertainty. It is particularly important to evaluate the contribution of the laser beam's directional error caused on the target's positioning error from its command position; this is an essential difference from conventional automated tracking interferometers. In this paper, the proposed measuring instrument is called the 'open-loop' tracking interferometer – 'open loop' in the sense that the target's actual position is not fed back to the control of laser direction.

To simplify the problem and to investigate the fundamental validity of the proposed scheme, this paper focuses on the measurement of the target's two-dimensional position by using a single-axis 'open-loop' tracking interferometer prototype. The uncertainty in the estimation of the target's XY trajectory is investigated both experimentally and analytically.

2. Proposed measurement procedure

2.1. Command generation for laser beam direction

Figure 5 illustrates the 'open-loop' tracking interferometer setup in the XY plane. A laser interferometer is mounted on a rotary table such that the laser beam approximately intersects with the rotary table's axis average line. A cat's eye retroreflector is typically used as the target. A cat's eye retroreflector a spherical glass of the pre-calibrated geometric accuracy with its hemispheric surface coated by the total-reflection metal-film deposition [13]. It is attached to the machine spindle. Suppose that the i-th command position of the target retroreflector in the machine coordinate system (X-Y) is given by $p_i^* \in \mathbb{R}^2$ $(i = 1, \cdots, N_i)$. Assume that:

1. The position of the rotary table axis average line, $P_j^* = \left[P_j^*(1), P_j^*(2)\right]^T$, is roughly known.
2. The zero angular position of the rotary table, $\theta_j = 0$, is roughly aligned normal to the reference straight line of the machine tool's Y-axis.

Then, the rotary table's angular position, $\theta_{ij} \in \mathbb{R}$, to direct the laser beam to the target's command position, $p_i^* = [p_i^*(1), p_i^*(2)]^T$, is given by:

$$\theta_{ij} = \arctan\left(\frac{p_i^*(2) - P_j^*(2)}{p_i^*(1) - P_j^*(1)}\right) \tag{1}$$

2.2. Initial estimation of tracking interferometer position and laser direction

In this paper, the rotation center of the 'open-loop' tracking interferometer, P_j^*, is estimated as illustrated in Fig. 6. With the cat's eye retroreflector placed approximately on the rotary axis average line, the laser displacement is continuously measured as the rotary table rotates. The retroreffletor's position is modified such that the variation in the laser displacement is minimized. The retroreffletor's position in the machine coordinate system gives P_j^*. Potential contributors for the estimation uncertainty include the machine's positioning error and the radial error motion of the rotary axis.

The zero angular position of the rotary table is set so that the laser beam is aligned normal to the machine tool's Y-axis reference straight line as illustrated in Fig. 7. As the retroreflector is moved to the Y-direction, the laser beam direction is modified such that the variation in the laser displacement is minimized.

Their estimation uncertainty can potentially contribute to the overall measurement uncertainty. It is to be noted that the estimates are used just as an initial value for the optimization to be presented in the following subsection; interferometer positions, P_j^*, are variables to be identified. The influence of their estimation uncertainty will be numerically evaluated in Section 4.

2.3. An algorithm to estimate target positions

When the target is positioned at the i-th command position, p_i^*, and tracking interferometer's rotation center is located at P_j^*, the laser beam is directed to the angular position, θ_{ij}, given in Eq. (1), and the laser displacement, $\Delta L_{ij} \in \mathbb{R}$, is measured. The tracking interferometer is set at N_j different positions ($j = 1, \cdots, N_j$, where $N_j \geq 4$). This problem inherently differs from the trilateration principle in that 1) the exact position of each tracking interferometer is not known, and 2) the absolute distance from the interferometer to the target cannot be measured (only the relative distance from the initial position can be measured by a laser interferometer). For conventional automated tracking interferometers, a self-calibration approach by using redundant measurements from more than three tracking interferometers has been well developed [11, 14, 10]. The same algorithm can be applied to the 'open-loop' tracking interferometer. This subsection briefly reviews it.

The problem can be written as the following minimization problem:

$$\min_x \sum_{i=1\cdots N_i, j=1\cdots N_j} \left(f_{ij}(x) - \Delta L_{ij}\right)^2 \tag{2}$$

where the function, f_{ij}:$\mathbb{R}^{3N_i+4N_j} \to \mathbb{R}$, is given by:

$$f_{ij}(x) = \|p_i - P_j\| - L_{0j} \tag{3}$$

where $L_{0j} \in \mathbb{R}$ represents the dead path length in the measurement by j-th tracking interferometer [11], i.e. $\Delta L_{ij} = L_{ij} - L_{0j}$, where L_{ji} is the distance from the j-th tracking interferometer to the i-th target position. $x \in \mathbb{R}^{3N_i+4N_j}$ represents a set of unknown parameters to be identified, containing:

$$x = \left[\{p_i\}_{i=1,\cdots,N_i}, \{P_j\}_{j=1,\cdots,N_j}, \{L_{0j}\}_{j=1,\cdots,N_j}\right] \tag{4}$$

The coordinate system can be set up arbitrarily, and total six parameters in x can be constrained according to the coordinate system setup (see [10]). For $3N_i+4N_j-6$ unknown parameters, the number of laser measurements is $N_i \cdot N_j$. Therefore, when $N_i > \frac{4N_j-6}{N_j-3}$, the number of measurements exceeds the number of unknown parameters.

Since the problem (2) is a nonconvex problem, an iterative linearization-based approach is typically used to locally solve it. Define $f := \{f_{ij}\}_{i=1\sim N_i, j=1\sim N_j} : \mathbb{R}^{3N_i+4N_j} \to \mathbb{R}^{N_i \cdot N_j}$ and $\Delta L := \{\Delta L_{ij}\}_{i=1\sim N_i, j=1\sim N_j} \in \mathbb{R}^{N_i \cdot N_j}$. An iterative linearization-based approach can be represented by the following updating law:

$$\hat{x}^{(k+1)} = \hat{x}^{(k)} + \left(A^{(k)T} A^{(k)}\right)^{-1} A^{(k)T} \left(\Delta L - f(\hat{x}^{(k)})\right) \tag{5}$$

where

$$A^{(k)} := \left. \frac{\partial f}{\partial x} \right|_{x=\hat{p}^{(k)}} \tag{6}$$

The initial value for $\hat{x}^{(k)}$ is typically given by the command target positions, p_i^*, and initial estimates of tracking interferometer positions in Section 2.2.

3. Experiment

3.1. Experimental setup

The objective of the experiment is to demonstrate the estimation of the two-dimensional position of the target by using the proposed 'open-loop' tracking interferometer, and to experimentally investigate its estimation accuracy by comparing with direct measurement of target positions.

Figure 8 shows the developed prototype. A laser interferometer, DISTAX L-IH-302A by Tokyo Seimitsu Co., Ltd., is mounted on a rotary table, RT158A by Nippon Thompson Co., Ltd. The laser interferometer's major specifications are shown in Table 1. The laser displacement is counted by a counter board, DISTAX LD-301 by Tokyo Seimitsu Co., Ltd., and logged on a PC. The influence of air temperature, pressure and humidity on the laser wavelength is compensated. Table 2 shows major specifications of the rotary table. Figure 9 illustrates its control system configuration. The current and velocity feedback loops are implemented in the servo amplifier (DrvGIII by Yokogawa Electric Corp.), and the position feedback loop is implemented in the PC-based control system (LT-RTSim by DSP Technology), where the angular position command (1) is generated. A cat's eye retroreflector, by Etalon AG, is attached to the machine spindle. Its major specifications are shown in Table 3. A vertical-type five-axis machining center, NMV1500DCG by Mori Seiki Co., Ltd. is tested.

3.2. Experimental procedure and result

Figure 10 shows command target positions ("Command target positions"), as well as tracking interferometer positions (Pos A to C). Within X100mm×Y100mm, the target is positioned at total 20 nominal positions with 20 mm step. Since only one tracking interferometer is currently available, the same measurement is repeated with the tracking interferometer set at three different positions on the machine table, assuming that the machine's unrepeatable positioning error is sufficiently small. Figure 8 shows the experimental setup at (a) Pos B and (b) Pos C.

As an example, the laser displacement profile measured at Pos A is shown in Fig. 11. The same measurement was repeated three times; ◯ markers in Fig. 11 show their average, and error bars show the variation in three tests, magnified 200,000 times. The largest variation in three tests is about 0.3 μm. The target's XY positions were then estimated by applying the algorithm in

Section 2.3. Figure 10 shows estimated target positions ("Estimated target position ..."), where the error from each command position is magnified 2,000 times.

For the comparison, target positions are measured by using a two-dimensional digital scale (the term in [1]) ("Measured by 2D digital scale" in Fig. 10). KGM182 by Heidenhain was used (see Fig. 12). The estimates by the 'open-loop' tracking interferometer show larger linear positioning error both in X and Y directions by 4 μm at maximum. This difference is largely attributable to the thermal expansion of the machine and the reference grid of KGM 182. The thermal expansion coefficient of KGM182 grid plate is, according to the manufacturer's catalog, about $8 \times 10^{-6}\mathrm{K}^{-1}$, and the grid plate is calibrated at $20°C$. The thermal expansion coefficient of the machine's X- and Y-axes was estimated $10.5 \times 10^{-6}\mathrm{K}^{-1}$. The machine temperature was about $24.5°C$. Since the thermal expansion coefficient of the machine and the grid plate is different only by $2.5 \times 10^{-6}\mathrm{K}^{-1}$, the difference in the KGM and the laser interferometer measurement is 3.6 μm for 100 mm. assuming that the grid plate's temperature was about the same as the machine.

For further comparison, Fig. 13 compares estimated linear positioning error profiles in X- and Y-directions by the 'open-loop' tracking interferometer, taken from Fig. 10, and measured values by the same interferometer fixed to X- or Y-direction (measured position: $(Y, Z) = (52.3, 0)$ mm for X-measurement, $(X, Z) = (71.3, -39.0)$ mm for Y-measurement). The difference is within 2 μm for both directions. Error bars represent the measurement uncertainty ($k = 2$) assessed in Section 4.

4. Uncertainty analysis

4.1. Objective

Some uncertainty contributors, included in the proposed measurement procedure, do not exist in conventional automated tracking interferometers. To show the validity of the proposed 'open-loop' tracking interferometer concept, it is of a particular importance to show that these uncertainty contributors do not impose significant influence on the overall measurement uncertainty. This section presents uncertainty analysis of the experimentation in Section 4.

4.2. Categorization of uncertainty contributors

Uncertainty contributors are categorized based on its influence on the laser measurement as follows:

u_1: Uncertainty contributor on the laser length measurement.

u_2: Uncertainty contributor on interferometer position in the laser direction.

u_3: Uncertainty contributor on interferometer position in the direction normal to the laser direction.

u_4: Uncertainty contributor on the laser beam direction.

Clearly, u_2 directly influences the laser displacement, while u_3 only imposes negligibly small influence. The influence of u_4 is regarded as the "cosine error" (see Fig. 2).

4.3. Error budget for laser displacement uncertainties

Table 4 shows the extended uncertainty, $U(k=2)$, in the measured laser displacement by each potential uncertainty contributor (k: the coverage factor in the uncertainty assessment). The uncertainty is dependent on the distance, and thus is different for each target or interferometer position. As an illustrative example, Table 4 shows the case where the tracking interferometer is at Pos A, and the target is at (X, Y)=(100,100) mm in Fig. 10. 'Type A' uncertainties are assessed by actually measuring the experimental instrument. 'Type B' uncertainties are assessed by using the instrument's catalog or specifications.

The following contributors are inherent in the proposed 'open-loop' tracking interferometer, and thus should be studied carefully:

- *Uncertainty in target position*: When the machine has the positioning error, the laser beam, directed to the target's command position, would not be directed to the exact center of the retroreflector. In conventional automated tracking interferometers, this error can be negligibly small, when the tracking mechanism is ideally effective. For the machine's estimated volumetric accuracy, this error causes the uncertainty in laser beam direction given by u_{44} in Table 4. Its influence on the laser displacement is, however, the "cosine error" and thus negligibly small.

  For example, when the tracking interferometer is at Pos A, and the target is at (X,Y)=(100,100) mm, then the target's position error of 0.8 mm in the direction normal to the laser beam direction causes an error in the laser displacement of about 1.0 μm. This illustrates that a typical machining center's volumetric error would be far from affecting the multilateration measurement.

- *Uncertainty in initial estimation*: When the initial estimation of tracking interferometer position, presented in Section 2.2, has significant error, it also causes the direction error of laser beam. From the experimentation, the uncertainty in the initial estimate is assessed as about 11.5 μm ($k = 2$). This causes the uncertainty in laser beam direction given by u_{43} in Table 4. Its influence on the laser displacement is negligibly small. The influence of the initial estimation of the zero angular position of the rotary table, u_{41}, can be assessed similarly.

The uncertainty contributors, u_1, u_2 and u_{42}, can be in principle present also in automated tracking interferometers. Table 4 indicates that their contribution is significantly larger than u_{41}, u_{43} and u_{44}, which are only present in the proposed 'open-loop' tracking interferometer. The present analysis validates the authors' claim that the 'open-loop' regulation of laser beam direction, without an automated tracking mechanism, does not significantly contribute on the uncertainty of the multilateration measurement.

4.4. Uncertainty in Target Position Estimation

The uncertainty in target position estimation is then assessed. The propagation of the uncertainty in laser displacement to the estimation of each target position is calculated by applying the Monte Carlo simulation to the algorithm presented in Section 2.3. Statistical analysis based on the Monte Carlo simulation is common and well established in the measurement uncertainty assessment [15]. Analogous uncertainty analysis for conventional tracking interferometers can be found in previous publications [11].

Figure 14 shows the extended uncertainty ($U(k = 2)$) in each estimated target position, p_i. The standard deviation ($k = 2$) of the two-norm of an error of the estimated target position to its command position is represented by color. While the laser displacement uncertainty is about 2.5 μm in Table 4 (the uncertainty at other points does not differ much), the maximum uncertainty in estimated target positions is about 8 μm. In the multilateration measurement, it is well known that the target position's estimation uncertainty may significantly vary depending on tracking interferometer positions [11, 10]. The target position estimation uncertainty may be further reduced by modifying tracking interferometer positions.

In Fig. 13, the error bars represent the extended uncertainty ($U(k = 2)$) in estimated linear positioning errors in X- and Y-directions, E_{XX} and E_{YY}. Figure 13(a) shows the uncertainty at $Y = 0$, and Fig. 13(b) shows that at $X = 100$ mm. While Fig. 14 shows the uncertainty in $\sqrt{E_{XX}^2 + E_{YY}^2}$, Fig. 13 shows that in E_{XX} and E_{YY}.

5. Conclusion

Assuming that the machine tool's positioning error is small enough to make its influence on the laser displacement sufficiently small ("cosine error"), the multilateration measurement can be done by regulating the laser beam toward the command target position. This eliminates the automated tracking mechanism and thus may significantly reduce the manufacturing cost of conventional tracking interferometers. This paper shows the validity of this 'open-loop' tracking interferometer concept by investigating its measurement uncertainty. The extension to three-dimensional position measurement, for a larger volume, will be studied as the next step.

The uncertainty analysis in Section 4 showed that the 'open-loop' regulation toward the unknown target position causes the uncertainty in the laser beam direction, but it does not impose significant contribution on the measurement uncertainty. In the experiment, the two-dimensional positioning error of a machining center is estimated within 100×100 mm by using the single-axis 'open-loop' tracking interferometer prototype. The comparison with direct measurement of X- and Y-axes linear positioning errors by a laser interferometer showed a good match within the assessed measurement uncertainty.

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References

[1] ISO 230-1:2012, Test code for machine tools – Part 1: Geometric accuracy of machines operating under no-load or quasi-static conditions.

[2] ISO/CDTR 16907:2013, Numerical compensation of geometric errors of machine tools.

[3] ISO 10360-2:2009 Geometrical product specifications (GPS) – Acceptance and reverification tests for coordinate measuring machines (CMM) – Part 2: CMMs used for measuring linear dimensions.

[4] B. Bringmann, A. Küng, W. Knapp, A Measuring Artefact for true 3D Machine Testing and Calibration, CIRP Annals - Manufacturing Technology, 54(1) (2005) 471-474.

[5] H. Schwenke, W. Knapp, H. Haitjema, A. Weckenmann, R. Schmitt, F. Delbressine, Geometric error measurement and compensation of machines –An update, CIRP Annals – Manufacturing Technology, 57(2) (2008) 560-575.

[6] S. Ibaraki, W.Knapp, Indirect measurement of volumetric accuracy for three-axis and five-axis machine tools: A review, International Journal of Automation Technology, 6 (2) (2012) 110-124.

[7] K. Lau, R. Hocken, W. Haight, Automatic laser tracking interferometer system for robot metrology, Precision Engineering, 8(1) (1986) 3-8.

[8] G.N. Peggs, Virtual technologies for advanced manufacturing and metrology, Int'l J. of Computer Integrated Manufacturing, 16(7/8) (2003).

[9] E.B. Hughes, A. Wilson, G.N. Peggs, Design of a high-accuracy CMM based on multi-lateration techniques, CIRP Annals - Manufacturing Technology 2000; 49(1); 391-394.

[10] S. Ibaraki, T. Kudo, T. Yano, T. Takatsuji, S. Osawa, O. Sato, Estimation of three-dimensional volumetric errors of machining centers by a tracking interferometer, Submitted to Precision Engineering, 2013.

[11] H. Schwenke, M. Franke, J. Hannaford, H. Kunzmann, Error mapping of CMMs and machine tools by a single tracking interferometer, CIRP Annals - Manufacturing Technology, 54(1) (2005) 475-478.

[12] H. Schwenke, R. Schmitt, P. Jatzkowski, C. Warmann, On-the-fly calibration of linear and rotary axes of machine tools and CMMs using a tracking interferometer, CIRP Annals – Manufacturing Technology, 58(1) (2009) 477-480.

[13] T. Takatsuji, M. Goto, S. Osawa, R. Yin, T. Kurosawa, Whole-viewing-angle cat's-eye retroreflector as a target of laser trackers, Measurement Science and Technology, 10(7) (1999) 87-90.

[14] Umetsu K, Furutnani R, Osawa S, Takatsuji T, Kurosawa T, Geometric calibration of a coordinate measuring machine using a laser tracking system, Measurement Science and Technology 2005; 16; 2466-2472.

[15] JCGM 100:2008, Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement (GUM).

Table 1: Major specifications of the laser interferometer (Distax L-IH-302A by Tokyo Seimitsu Co., Ltd.).

Laser	He-Ne laser (vacuum wavelength 633.0 nm)
Measurement range	10 m
Measurement resolution	$\lambda/64$ ($\approx 0.01\mu$m)
Maximum response speed	630 sec^{-1}
Measurement uncertainty	$\pm(\lvert L\rvert \times 10^{-7} + 0.005 \times 10^{-6})$ m where L is the measurement length.

Table 2: Major specifications of the rotary table (RT158A by Nippon Thompson Co., Ltd.)

Transmission	direct drive
Stroke	360° (endless)
Max. torque	4 Nm
Max. rotational speed	2.5 sec^{-1}
Encoder resolution	655360 pulse/rev
Positioning accuracy[1]	20 arcsec
Positioning repeatability[1]	±5 arcsec
Table size	ϕ 220 mm

[1]: calibrated by the manufacturer.

Table 3: Major specifications of the cat's eye retroreflector (by Etalon AG).

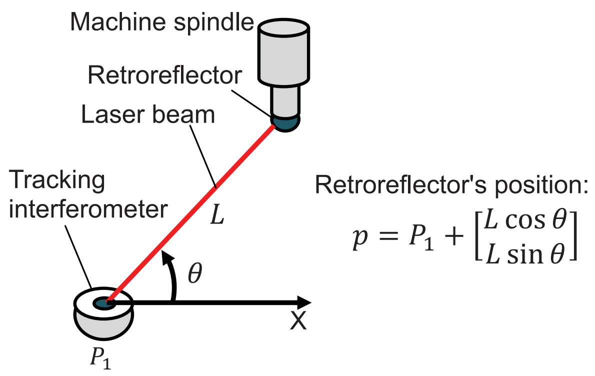
Viewing angle	$\pm 80^\circ$
Optical form deviation[1] (circularity)	$< 0.2\mu$m

[1]: calibrated by the manufacturer.

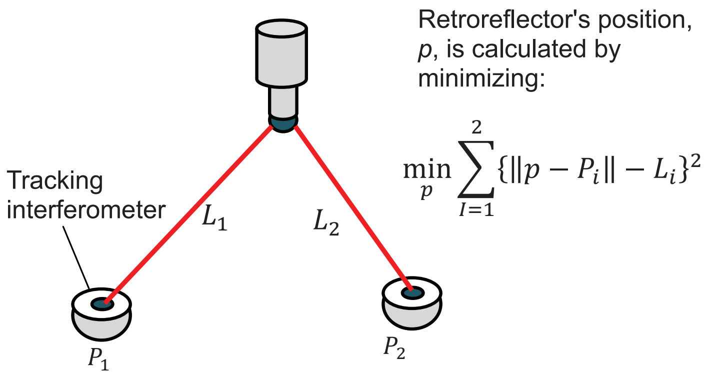
Table 4: Error budget for laser displacement uncertainty ($k = 2$) at the target position $(X, Y) = (100, 100)$ mm and the interferometer position A.

Symbol		Source of uncertainty	Contribution in laser displacement uncertainty	Type
u_1		Uncertainty in laser length	0.43 μm	–
	u_{11}	Wavelength accuracy	0.02 μm	B
	u_{12}	Resolution of interferometer	0.006 μm	B
	u_{13}	Wavelength correction	0.07 μm	B
	u_{14}	Dead path accuracy	0.12 μm	B
	u_{15}	Environmental change	0.04 μm	A
	u_{16}	Machine's Repeatability	0.18 μm	A
u_2		Uncertainty in interferometer position in laser direction	2.2 μm	–
	u_{21}	Radial error motion of rotary axis	2.2 μm	A
u_3		Uncertainty in interferometer position error in direction normal to laser	≈ 0 μm	
u_4		Uncertainty in laser beam orientation	$< 0.01\mu$m	–
	u_{41}	Uncertainty in c-axis zero angular position	$(2.8 \times 10^{-6}$ rad$)^*$	A
	u_{42}	Angular positioning error of c-axis	$(22.0 \times 10^{-6}$ rad$)^*$	A
	u_{43}	Uncertainty in c-axis center position	$(35.2 \times 10^{-6}$ rad$)^*$	A
	u_{44}	Uncertainty due to machine tool positioning error	$(60.6 \times 10^{-6}$ rad$)^*$	A

*: shows the uncertainty in the laser beam direction.



(a)



(b)

Figure 1: Two principles for estimating the retroreflector position, p, by tracking interferometers. P_i: tracking interferometer's position, L_i: distance between the tracking interferometer and the retroreflector's center; (a) Measurement by distance and angle to the retroreflector. (b) Measurement by distances (trilateration).

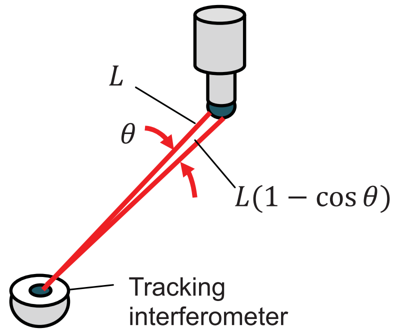


Figure 2: Influence of laser beam's direction error on the laser displacement ("cosine error").

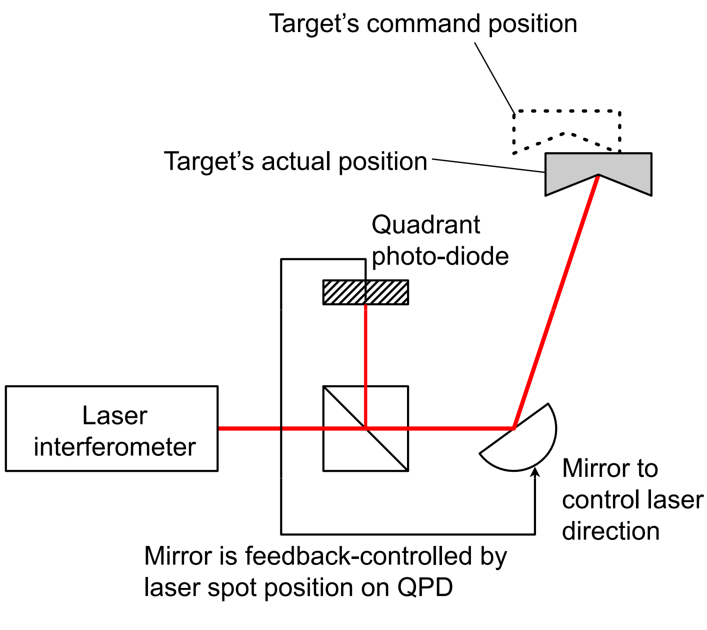


Figure 3: Typical tracking mechanism in automated tracking interferometer [10].

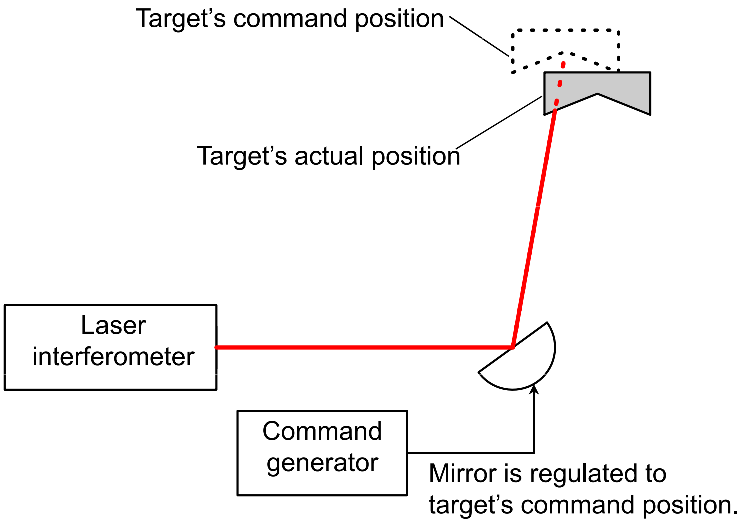


Figure 4: Principle of 'open-loop' tracking interferometer.

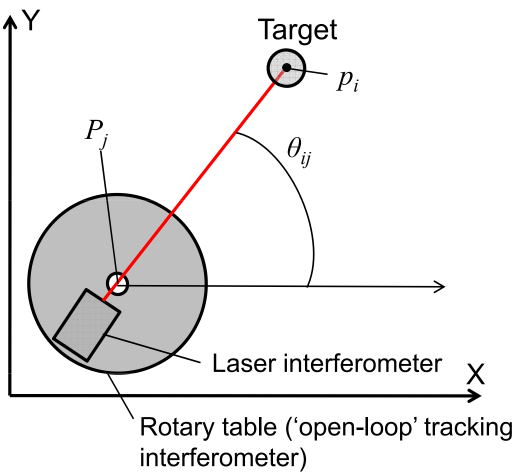


Figure 5: Setup of 'open-loop' tracking interferometer and target in XY-plane.

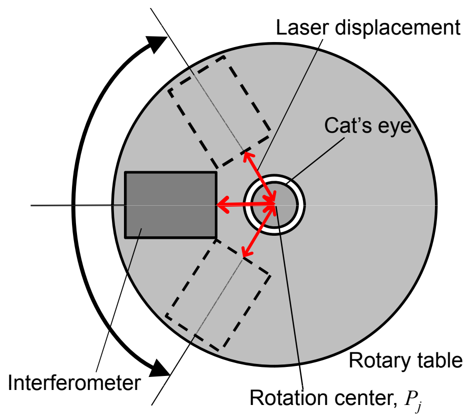


Figure 6: Initial estimation of rotation center position of 'open-loop' tracking interferometer, P_j^*.

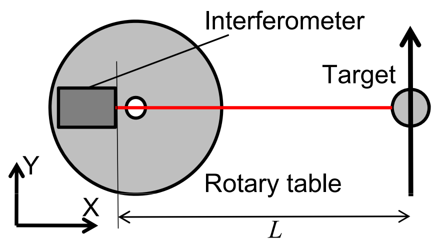
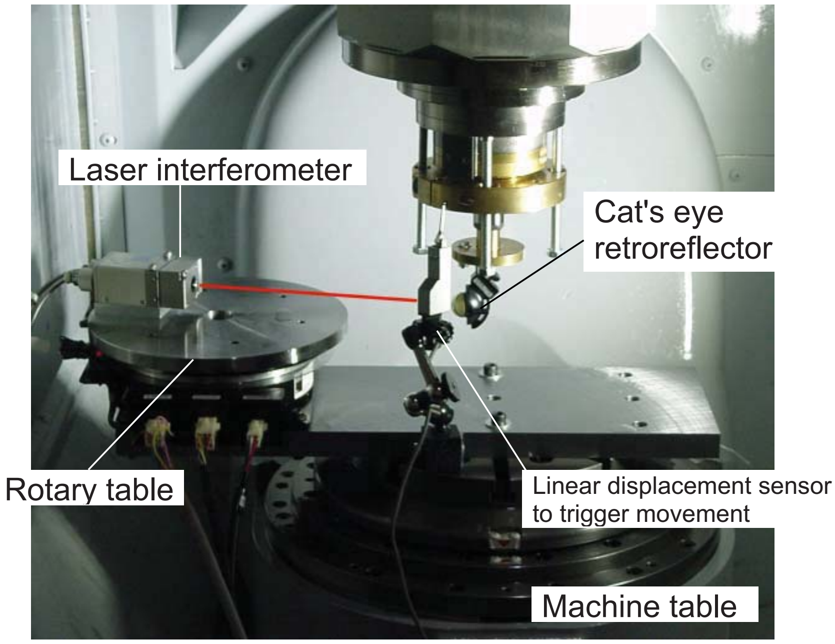


Figure 7: Alignment procedure of 'open-loop' tracking interferometer's zero angular position.



(a)

(b)

Figure 8: Experimental setup, (a) at Pos B, (b) at Pos D.

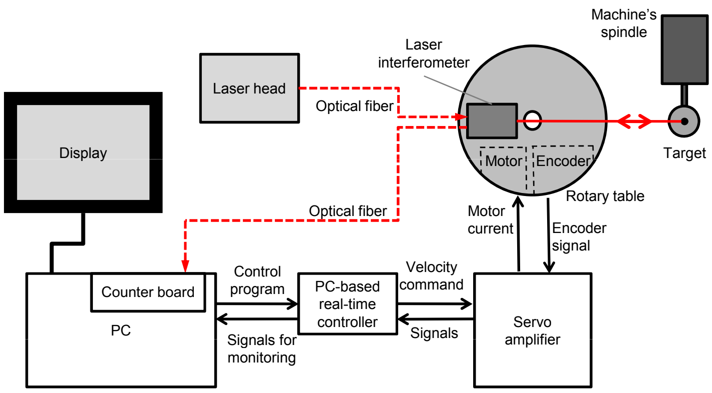


Figure 9: Controller configuration of the prototype.

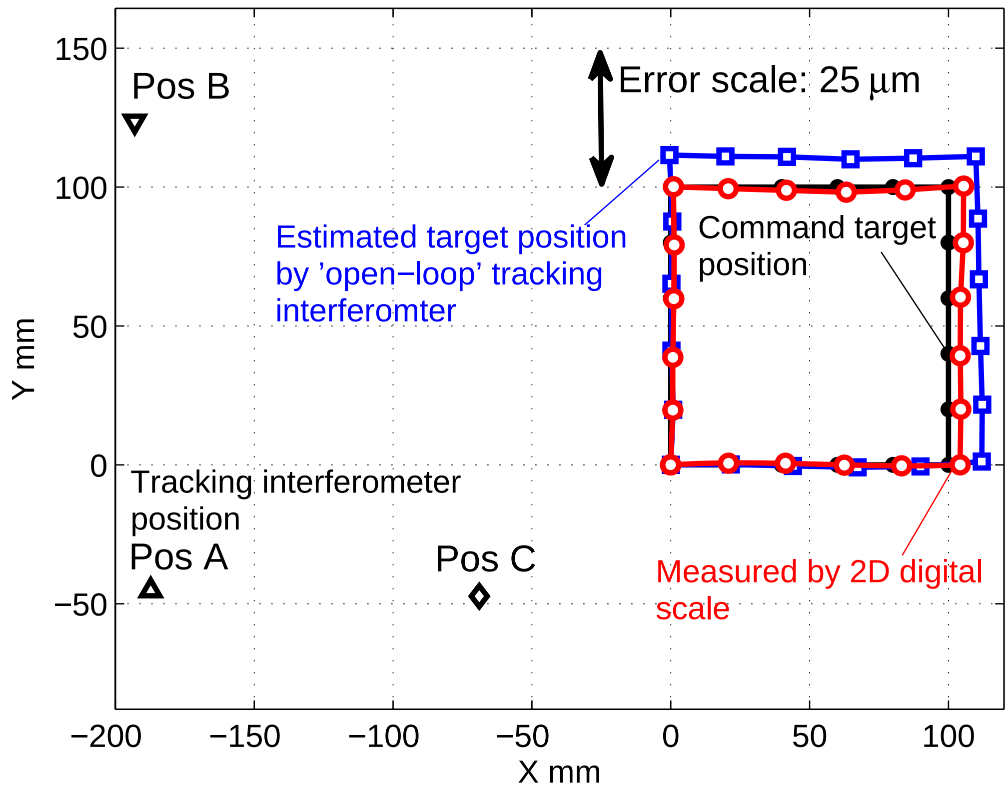


Figure 10: Target positions estimated by the 'open-loop' tracking interferometer, in comparison with measured positions by a two-dimensional digital scale (KGM182). An error from the command position is magnified 2,000 times.

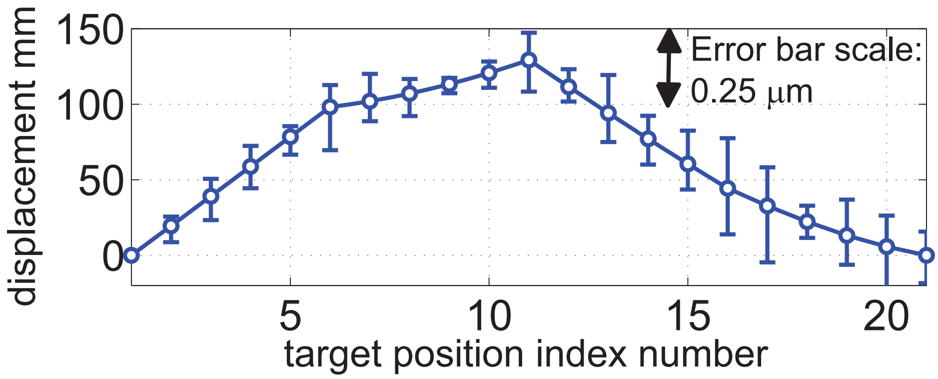


Figure 11: Measured laser displacement profile at the tracking interferometer position Pos A. Error bars show the variation in three tests, magnified 200,000 times.

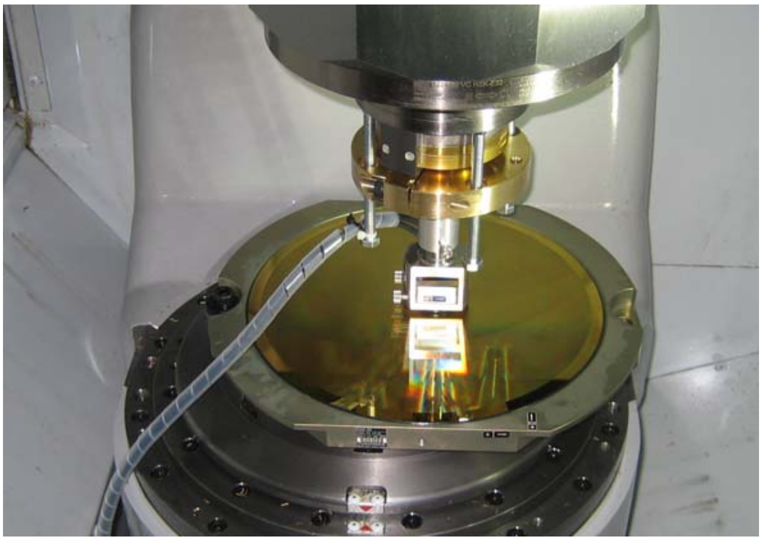
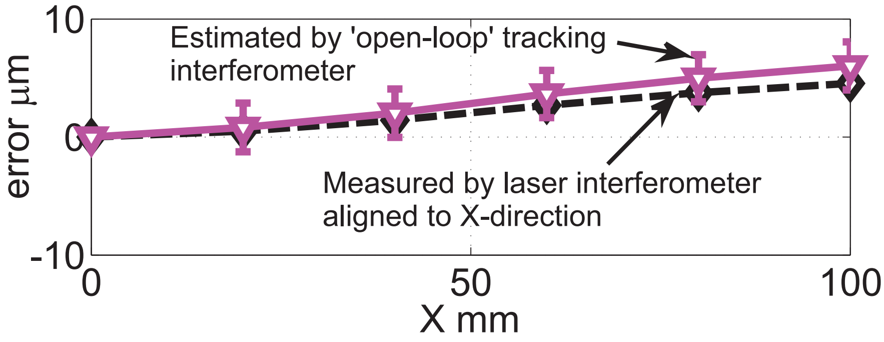
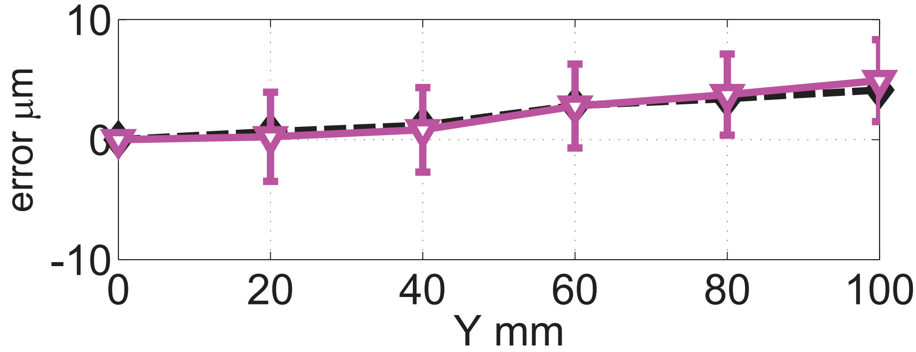


Figure 12: Measurement setup with two-dimensional digital scale (KGM182 by Heidenhain).



(a) Linear positioning error in X, E_{XX}



(b) Linear positioning error in Y, E_{YY}

Figure 13: Comparison of estimated and measured linear positioning errors in X- and Y-directions (estimated values by the 'open-loop' tracking interferometer are taken from Fig. 10, and measured values are by a laser interferometer aligned to X- or Y-direction). Error bars represent the measurement uncertainty ($k = 2$) at each position calculated in Section 4.4.

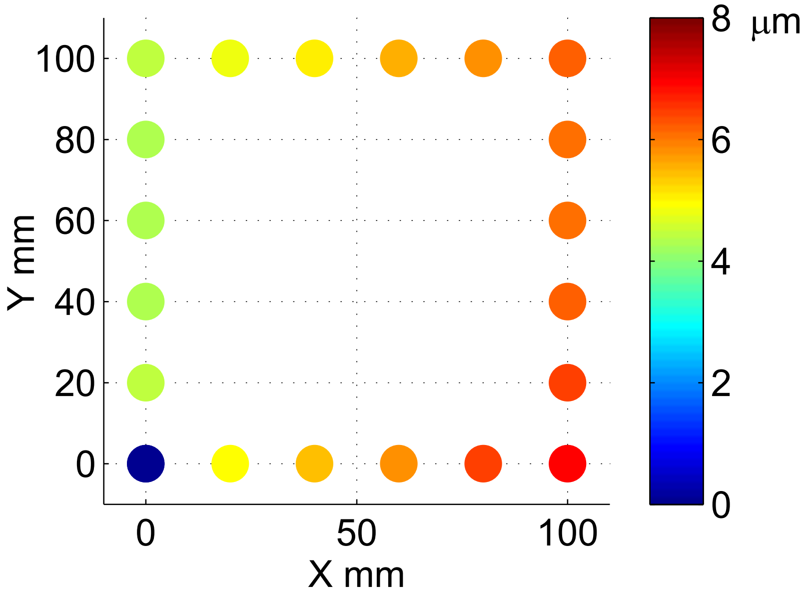


Figure 14: Assessed uncertainty in estimated target positions (the color at each target position represents the estimation uncertainty $U(k = 2)$).