Numerical Analysis of Nonlinear Acoustic Wave Propagation in a Rarefied Gas

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Abstract. Unsteady motion of a rarefied gas in a half space, caused by an infinitely wide plate when it starts a longitudinal and harmonic oscillation, is investigated numerically on the basis of the Bhatnagar-Gross-Krook (BGK) model of the Boltzmann equation. A deterministic method capable of describing the singularities in the molecular velocity distribution function produced by the oscillating plate, which was developed recently by the authors, is used as a solution method, and the unsteady behavior of the gas is obtained accurately. The streaming motion and the attenuation of the wave, observed in the existing work using the direct simulation Monte Carlo (DSMC) method (T. Ohwada and M. Kunihisa, in *Rarefied Gas Dynamics*, AIP, Melville, 2003, pp. 202–209), are also obtained. In addition, some pieces of numerical evidence that clarify the long-time behavior of the gas are provided. For example, one-period averages of the momentum and energy fluxes across the oscillating plate tend to approach their values for a periodic state (a constant for the momentum flux and zero for the energy flux) slowly, the rate of approach being likely to be inversely proportional to the square root of time.

Keywords: Rarefied gas dynamics, Nonlinear acoustic wave, Moving-boundary problems **PACS:** 47.45.-n, 47.45.Ab, 43.25.-x, 51.40.+p

INTRODUCTION

Acoustic wave propagation in a gas is one of the fundamental problems in gas dynamics. Gas motion in a semi-infinite space induced by an infinitely wide plate oscillating longitudinally is a typical and the simplest setting, for which the linearized Euler equation gives the plane-wave solution (see e.g., [1]).

However, when the frequency of oscillation of the plate is high, or when the gas under consideration is rarefied, the frequency of oscillation can be comparable to the collision frequency of the gas molecules. In such cases, the continuum gas dynamics is not applicable any more and should be replaced by kinetic theory (see e.g., [2]). In fact, this is a classical subject in rarefied gas dynamics and has been investigated by many authors (e.g., [3, 4, 5, 6, 7, 8, 9, 10]). More specifically, the acoustic wave propagation in a half space was studied using linearized kinetic equations, under the assumption that the speed of the oscillating plate is much lower than the sonic speed, and it was reported that the effect of gas rarefaction (or the non-continuum effect) led to attenuation of the propagating wave and spatial inhomogeneity of its wave length. However, when the amplitude of oscillation is increased at a fixed frequency, or when the frequency is increased at a fixed amplitude, the speed of oscillation of the plate may become comparable to the sonic speed, so that the nonlinearity cannot be neglected. In the present study, we consider such cases with full nonlinearity. More precisely, we investigate unsteady motion in a semi-infinite expanse of a rarefied gas caused by an infinitely wide plate when it starts a longitudinal harmonic oscillation. We carry out accurate numerical analysis using the Bhatnagar–Gross–Krook (BGK) model [11, 12] of the Boltzmann equation and give precise description of time-evolution of the macroscopic quantities.

The same nonlinear problem was investigated using classical gas dynamics (the Euler equation) in [13, 14], where the formation of sawtooth-like waves and the streaming motion toward infinity, accompanied by a rarefaction region, were found and attributed to the effect of nonlinearity. In particular, with the help of a long-time computation (up to about 190 periods of oscillation), it was pointed out that a rarefaction region develops near the oscillating plate and extends linearly in time and that a quasi-steady periodic state is formed near the plate. On the other hand, a similar nonlinear problem, i.e., wave propagation caused by a sudden start of a sawtooth oscillation of a plate, was studied for a rarefied gas by the direct simulation Monte Carlo (DSMC) method in [15], where both the effect of gas rarefaction and that of nonlinearity mentioned above were observed. In the present study, we expect to observe more or less the same phenomena, but an emphasis will be put on the quantitative description of the long-time behavior.

Finally, we mention another aspect of the present study. In most of the works based on the linearized kinetic equations mentioned above, the oscillating plate is approximated by a stationary wall, and the oscillation is produced

28th International Symposium on Rarefied Gas Dynamics 2012 AIP Conf. Proc. 1501, 115-122 (2012); doi: 10.1063/1.4769485 © 2012 American Institute of Physics 978-0-7354-1115-9/\$30.00 by imposing an oscillatory macroscopic velocity in the boundary condition. In the present nonlinear problem, however, such an approximation is not valid, so that we have to handle the problem with a moving boundary. It is a challenging subject in kinetic theory because the oscillating plate continuously sends discontinuities in the molecular velocity distribution function in the gas. We tackle this problem by a deterministic solution method, capable of describing the discontinuities caused by a moving plate, that was developed recently by the authors [20].

FORMULATION OF THE PROBLEM

Problem and Assumptions

We consider a semi-infinite expanse of an ideal monatomic gas in a uniform equilibrium state at rest with density ρ_{0*} and temperature T_{0*} , bounded by an infinitely wide plate kept at uniform and constant temperature T_{0*} . The plate is placed at $X_1 = a_* (> 0)$, where X_i (i = 1, 2, 3) denotes the Cartesian coordinate system with the X_1 axis perpendicular to the plate. At time $t_* = 0$, the plate starts an oscillation around $X_1 = 0$ with angular frequency ω_* according to $X_1 = X_w(t_*)$ with $X_w(t_*) = a_* \cos \omega_* t_*$. We investigate the subsequent unsteady motion of the gas numerically on the basis of kinetic theory when the frequency of oscillation of the plate is comparable to the mean collision frequency of gas molecules.

The assumptions for our analysis are as follows:

- 1. The behavior of the gas is described by the BGK model of the Boltzmann equation ([11, 12]; see also [2]).
- 2. The gas molecules undergo diffuse reflection on the plate (see, e.g., [2]). More specifically, the molecules reflected on the plate are distributed according to the half-range Maxwellian distribution characterized by the velocity and temperature of the plate and with the density adjusted in such a way that there is no net mass flow across the boundary.
- 3. The gas motion is one dimensional, that is, the motion is only in the X_1 direction, and the physical quantities do not depend on X_2 and X_3 .

Before formulating the problem, we summarize the notations used throughout the present paper. We first introduce (and repeat) basic dimensional quantities, that is, t_* is the time variable, X_i the Cartesian coordinate system in space, ξ_i the molecular velocity, X_w the position (X_1 coordinate) of the plate, f_* the velocity distribution function of gas molecules, ρ_* the density, ($u_{1*}, 0, 0$) the flow velocity, T_* the temperature, p_* the pressure, p_{11*} the X_1 - X_1 component of the stress tensor (normal stress), and ($q_{1*}, 0, 0$) the heat-flow vector. Then, adopting the time and length scales given respectively by

$$1/\boldsymbol{\omega}_*, \qquad c_{0*}/\boldsymbol{\omega}_*, \tag{1}$$

where $c_{0*} = (2RT_{0*})^{1/2}$ with *R* the gas constant per unit mass, we introduce the dimensionless counterparts, *t*, *x_i*, ζ_i , *x_w*, *f*, ρ , *u*₁, *T*, *p*, *p*₁₁, and *q*₁, as follows:

$$t = t_*/(1/\omega_*), \quad x_i = X_i/(c_{0*}/\omega_*), \quad \zeta_i = \xi_i/c_{0*}, \quad x_w = X_w/(c_{0*}/\omega_*), \quad f = f_*/(\rho_{0*}c_{0*}^3),$$

$$\rho = \rho_*/\rho_{0*}, \quad u_1 = u_{1*}/c_{0*}, \quad T = T_*/T_{0*}, \quad p = p_*/p_{0*}, \quad p_{11} = p_{11*}/p_{0*}, \quad q_1 = q_{1*}/(p_{0*}c_{0*}),$$
(2)

where $p_{0*} = R\rho_{0*}T_{0*}$. In addition, we define the Knudsen number Kn and the Mach number Ma as

Kn =
$$l_{0*}/(c_{0*}/\omega_*)$$
, Ma = $a_*\omega_*/(5RT_{0*}/3)^{1/2}$, (3)

where l_{0*} is the mean free path of the gas molecules at the reference equilibrium state at rest, $a_*\omega_*$ is the maximum speed of the oscillating plate, and $(5RT_{0*}/3)^{1/2}$ is the sonic speed at temperature T_{0*} ; the l_{0*} is defined in terms of the collision frequency v_{0*} of the gas molecules at the reference state as $l_{0*} = (2/\pi^{1/2})(c_{0*}/v_{0*})$; for the BGK model, v_{0*} is given as $v_{0*} = A_c \rho_{0*}$ with a constant A_c .

Basic Equations

The independent variables (i.e., the arguments of f) in the present spatially one-dimensional problem are x_1 , ζ_1 , ζ_2 , ζ_3 , and t. However, in the case of the BGK model, one can eliminate ζ_2 and ζ_3 (the molecular velocity components parallel to the plate) by introducing the following marginal velocity distribution functions g and h [16]:

$$\begin{bmatrix} g(x_1,\zeta_1,t)\\ h(x_1,\zeta_1,t) \end{bmatrix} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \begin{bmatrix} 1\\ \zeta_2^2 + \zeta_3^2 \end{bmatrix} f(x_1,\zeta_1,\zeta_2,\zeta_3,t) \,\mathrm{d}\zeta_2 \,\mathrm{d}\zeta_3.$$
(4)

To be more specific, the BGK model equation for f is reduced to simultaneous integro-differential equations for g and h, which read as

$$\frac{\partial \Phi}{\partial t} + \zeta_1 \frac{\partial \Phi}{\partial x_1} = \frac{2}{\sqrt{\pi}} \frac{1}{\mathrm{Kn}} \rho \left(\lambda M - \Phi \right), \quad \left(\Phi = \begin{bmatrix} g \\ h \end{bmatrix}, \lambda = \begin{bmatrix} 1 \\ T \end{bmatrix} \right), \tag{5a}$$

$$M = \frac{\rho}{(\pi T)^{1/2}} \exp\left(-\frac{(\zeta_1 - u_1)^2}{T}\right),$$
(5b)

$$\rho = \int_{-\infty}^{\infty} g d\zeta_1, \qquad u_1 = \frac{1}{\rho} \int_{-\infty}^{\infty} \zeta_1 g d\zeta_1, \qquad T = \frac{2}{3\rho} \int_{-\infty}^{\infty} [(\zeta_1 - u_1)^2 g + h] d\zeta_1.$$
(5c)

The initial condition for Eq. (5) is given by

$$g = h = M_0,$$
 $M_0 = \pi^{-1/2} \exp(-\zeta_1^2),$ at $t = 0.$ (6)

The boundary condition on the oscillating plate is written as

$$g(x_1, \zeta_1, t) = h(x_1, \zeta_1, t) = M_w(\zeta_1, t), \quad \text{for } \zeta_1 - v_w(t) > 0, \text{ at } x_1 = x_w(t),$$
(7a)

$$M_{w}(\zeta_{1},t) = \pi^{-1/2} \sigma_{w}(t) \exp\left(-[\zeta_{1} - v_{w}(t)]^{2}\right),$$
(7b)

$$\sigma_{w}(t) = -2\sqrt{\pi} \int_{\zeta_{1} - v_{w}(t) < 0} [\zeta_{1} - v_{w}(t)] g(x_{w}(t), \zeta_{1}, t) \mathrm{d}\zeta_{1},$$
(7c)

where

$$x_w(t) = \sqrt{5/6} \operatorname{Ma} \cos t, \qquad v_w(t) = \mathrm{d}x_w(t)/\mathrm{d}t = -\sqrt{5/6} \operatorname{Ma} \sin t,$$
 (8)

that is, $c_{0*}v_w$ is the dimensional velocity of the plate.

As is seen from Eqs. (5)–(7), the present problem is characterized by the two parameters: Kn and Ma. In place of these, we use

$$\mathsf{K} = (\sqrt{\pi}/2)\mathsf{Kn}, \qquad \mathsf{M} = \sqrt{5/6}\mathsf{Ma}, \tag{9}$$

in the following.

The dimensionless pressure p is given by the equation of state, and the dimensionless normal stress p_{11} and heat flux q_1 are expressed in terms of the marginals g and h, i.e.,

$$p = \rho T, \qquad p_{11} = 2 \int_{-\infty}^{\infty} (\zeta_1 - u_1)^2 g \mathrm{d}\zeta_1, \qquad q_1 = \int_{-\infty}^{\infty} (\zeta_1 - u_1) [(\zeta_1 - u_1)^2 g + h] \mathrm{d}\zeta_1. \tag{10}$$

We also denote by \mathscr{P} and \mathscr{E} , respectively, the X_1 component of the momentum and the energy, per unit area and time, transferred to the gas from the oscillating plate. Then, they are expressed as

$$\mathscr{P} = p_{11}|_{x_1 = x_w(t)}, \qquad \mathscr{E} = [q_1 + p_{11}v_w(t)]|_{x_1 = x_w(t)}.$$
(11)

Remarks on Numerical Method

As is well known, the molecular velocity distribution function on the plane boundary is generally discontinuous for the molecular velocity tangential to the boundary (see [2]). Unlike the case of convex boundary (here, "convex" means convex toward the gas), the discontinuity does not propagate into the gas if the plane boundary is at rest or moving at a constant velocity (when no external force acts on the gas molecules), since the characteristic lines of the Boltzmann (or BGK) equation for the molecular velocity equal to the velocity of the boundary stay on the plane. However, when the boundary is accelerated in the direction opposite to the side of the gas, the tangential characteristic lines go into the gas. It decays in time and space because of intermolecular collisions, as in the case of the discontinuity caused by a convex body in steady problems ([2, 17]).

In the present problem, the oscillating plate sends the discontinuities and other types of weaker singularities continuously in the gas (see [18, 19]). In order to obtain an accurate numerical solution, especially near the plate, by a deterministic method, we have to treat the discontinuities carefully. Recently we have developed a method, which is basically a method of characteristics, that is capable of describing the propagation of discontinuities, together with other types of weaker singularities, caused by the oscillating plate [20]. This method is used in the present study (its explanation is omitted here because of limited space).



FIGURE 1. Profiles of ρ , u_1 , T, and p for M = 0.2. (a) K = 0.05, (b) K = 2. In (a), the profiles at $t/2\pi = 19.25$ (*dashed*), 19.5 (*long-dashed*), 19.75 (*dash-dotted*), and 20 (*solid*) are shown, and in (b), those at $t/2\pi = 79.25$ (*dashed*), 79.5 (*long-dashed*), 79.75 (*dash-dotted*), and 80 (*solid*) are shown.

RESULT AND DISCUSSIONS

In the present paper, we show the results only for M = 0.2 and for some different values of K. We restrict ourselves to the macroscopic quantities, leaving the detailed description of the molecular velocity distribution function of complex shape in the forthcoming paper [20].

Unsteady Behavior of Macroscopic Quantities

In Fig. 1, we show the profiles of the density ρ , flow velocity u_1 , temperature *T*, and pressure *p* over a few wave lengths during one period for (K, M) = (0.05, 0.2) [panel (a)] and (2, 0.2) [panel (b)]. In panel (a), the profiles are shown at $t/2\pi = 19.25$, 19.5, 19.75, and 20, whereas in panel (b), they are shown at $t/2\pi = 79.25$, 79.5, 79.75, and



FIGURE 2. Profiles of u_1 and p for M = 0.2 near the oscillating plate (closeup of Fig. 1). (a) u_1 at K = 0.05, (b) u_1 at K = 2, (c) p at K = 0.05, (d) p at K = 2. The two vertical dotted lines indicate the range of the plate motion, i.e., $x_1 \in [-M, M]$. In (a) and (c), the profiles at $t/2\pi = 19.25$ (*dashed*), 19.5 (*long-dashed*), 19.75 (*dash-dotted*), and 20 (*solid*) are shown, and in (b) and (d), those at $t/2\pi = 79.25$ (*dashed*), 79.5 (*long-dashed*), 79.75 (*dash-dotted*), and 80 (*solid*) are shown.

80. In both panels, one can see the time evolution by following the four different types of lines in the order of the *dashed* line, the *long-dashed* line, the *dash-dotted* line, and the *solid* line. Figure 2 is a close-up of Fig. 1 near the plate for u_1 and p; panels (a) (K = 0.05) and (b) (K = 2) are for u_1 , and panels (c) (K = 0.05) and (d) (K = 2) are for p.

Since the Knudsen number is small (K = 0.05) in Fig. 1(a), a sawtooth-like wave is observed as in the case of nonlinear acoustic wave based on the Euler equation [13, 14]. Such a wave is also observed in the DSMC simulation [15]. In Fig. 1(b), where the Knudsen number is intermediate (K = 2), the wave attenuates quickly, and the sawtooth like profile is not observed. The amplitude of the wave near the plate is smaller for larger K, as seen from Fig. 2. The strong attenuation of the wave at intermediate Knudsen numbers is also reported in the previous studies based on linearized kinetic equations.

One-period Averages of Macroscopic Quantities

Let $h(x_1, t)$ stand for the macroscopic quantities, i.e., $h = \rho$, u_1 , T, etc., and $\bar{h}(x_1, t)$ be its time average over one period from $t - 2\pi$ to t, i.e.,

$$\bar{h}(x_1,t) = \frac{1}{2\pi} \int_{t-2\pi}^{t} h(x_1,s) \mathrm{d}s.$$
(12)

We show in Fig. 3 the time evolution of the profiles of the one-period averages, $\bar{\rho}$, \bar{u}_1 , and \bar{T} , for M = 0.2 and K = 2: panels (a) and (d) are for $\bar{\rho}$, panels (b) and (e) for \bar{u}_1 , and panels (c) and (f) for \bar{T} . In the figure, the profiles at $t/2\pi = 20$ (*dashed* line), 30, 40, ..., 70 (*solid* line for all), and 80 (*bold* line) are shown, and panels (a), (b), (c) show the profiles in the near field. In Fig. 3(e), the small circle indicates the profile of the one-period average of the mass flux $\bar{\rho}u_1$. It is seen from Fig. 3 that a weak compression wave, decreasing very slowly, propagates toward infinity, and a high-temperature (low-density) region is developed slowly near the plate. The former wave is called the streaming motion [13, 14, 15], since the one-period average of the mass flux $\bar{\rho}u_1$ is almost the same as \bar{u}_1 in the far field because the deviation of ρ from 1 is small there. Note that $\bar{\rho}u_1$ is almost constant in time in the far field in the case of the Euler equations (see Fig. 9 in [14]). We note here that the behavior shown in Fig. 3 is qualitatively similar to that in the unsteady rarefied gas flow in a half space caused by sudden heating of the stationary plate [21] (or Chap. 4.8 in [2]) though the present computation up to $t/2\pi = 80$ is not long enough to compare with the long-time asymptotes in [21]. In the latter problem, the heating of the gas is caused by the energy supply by the heated plate, which eventually vanishes in the long-time limit. In the present problem, the corresponding energy supply is provided by the oscillation of the plate, and it should also vanish in the long-time limit in which a steady oscillation is established in the near field.

Now we give a close look at the development of the high-temperature region. Let $x_1 = x_{0.02}(t)$ be the position at time *t* of the point at which $\overline{T}(x_1,t) - 1 = 0.02$ holds [see Fig. 3(c); if there are more than one such points, we take the largest one]. Figures 4(a), 4(b), and 4(c) show, respectively, $x_{0.02}$ vs *t*, $dx_{0.02}/dt$ vs *t* (in the double-logarithmic scale), and $\alpha(dx_{0.02}/dt)$ vs *t* at some different K for M = 0.2, where $\alpha(dx_{0.02}/dt) [= d\log_{10}(dx_{0.02}/dt)/d\log_{10}t]$ is



FIGURE 3. Profiles of the one-period averages $\bar{\rho}$, \bar{u}_1 , and \bar{T} for M = 0.2 and K = 2. (a), (d) $\bar{\rho}$; (b), (e) \bar{u}_1 ; (c), (f) \bar{T} . The profiles at $t/2\pi = 20$ (*dashed*), 30, \cdots , 70 (*solid* for all) and 80 (*bold*) are shown. In (a), (b), (c), the region $x_1 \in [0, 100]$ is shown, and in (d), (e), (f), the region $x_1 \in [0, 600]$ is shown with a magnified vertical scale. In (e), $\bar{\rho}u_1$ at $t/2\pi = 80$ is also shown by small circles.

the gradient of a curve in panel (b). The curves in panel (c) exhibit small fluctuations because they are essentially the second derivatives of the curve in panel (a). Nevertheless, they tend to approach -1/2. Therefore, for each K, the velocity of the point, $dx_{0.02}/dt$, tends to approach a straight line with gradient -1/2 in the double-logarithmic plot in panel (b). This means that $dx_{0.02}/dt \approx c_1 t^{-1/2}$ and thus $x_{0.02} \approx 2c_1 t^{1/2} + c_2$ for large t, where $c_1 (> 0)$ and c_2 are some constants depending on M and K. Therefore, we may conclude that the high-temperature region formed near the plate diffuses away from the plate, its width increasing in proportion to $t^{1/2}$.

We also try to measure the rate of decrease of the local peak at the wave front of streaming motion (or the weak compression wave). Let us denote by $\Theta(t)$ the value of $\overline{T}(x_1, t) - 1$ at the local peak of the wave front [i.e., the height of the peak in Fig. 3(f)]. Figure 5(a) is the double-logarithmic plot of $\Theta(t)$ vs t, and the gradients of the curves, $\alpha(\Theta) = d\log_{10}\Theta/d\log_{10}t$, vs t are plotted in Fig. 5(b). The figure shows a tendency that the gradients approach a constant value between -0.16 and -0.2, but we need further computation until much later times to draw some



FIGURE 4. Time evolution of $x_{0.1}$, $dx_{0.1}/dt$, and $\alpha(dx_{0.02}/dt)$ for M = 0.2. (a) $x_{0.1}$ vs t, (b) $dx_{0.1}/dt$ vs t (in the double-logarithmic scale), (c) $\alpha(dx_{0.02}/dt)$ vs t. The results for K = 1 (*dash-dotted*), 2 (*solid*), 5 (*dashed*), and 10 (*long-dashed*) are shown.



FIGURE 5. Time evolution of Θ and $\alpha(\Theta)$ for M = 0.2. (a) Θ vs *t* (in the double-logarithmic scale), (b) $\alpha(\Theta)$ vs *t*. The results for K = 1 (*dash-dotted*), 2 (*solid*), 5 (*dashed*), and 10 (*long-dashed*) are shown.



FIGURE 6. Time evolution of $\hat{\mathcal{P}}$, $\hat{\mathcal{E}}$, $\alpha(\hat{\mathcal{P}})$, and $\alpha(\hat{\mathcal{E}})$ for $\mathsf{M} = 0.2$. (a) $\hat{\mathcal{P}}$ (*solid*) and $\hat{\mathcal{E}}$ (*dashed*) vs *t* (in the double-logarithmic scale), (b) $\alpha(\hat{\mathcal{P}})$ (*solid*) and $\alpha(\hat{\mathcal{E}})$ (*dashed*) vs *t*. The results for $\mathsf{K} = 1, 2, 5$, and 10 are shown.

conclusion. Within the information provided by Fig. 5, we may say that the decay of the peak is very slow and its asymptotic decay rate is something like $\Theta \propto 1/t^{\gamma}$ with γ around 0.2.

One-period Averages of Momentum and Energy Fluxes on the Plate

Let us denote by $\bar{\mathscr{P}}$ and $\bar{\mathscr{E}}$ the time averages (over one period) of \mathscr{P} and \mathscr{E} [Eq. (11)], the X_1 component of the momentum and the energy, per unit area and time, transferred to the gas from the oscillating plate. More precisely, they are defined by

$$[\bar{\mathscr{P}}(t),\bar{\mathscr{E}}(t)] = \frac{1}{2\pi} \int_{t-2\pi}^{t} [\mathscr{P}(s),\mathscr{E}(s)] \mathrm{d}s.$$
(13)

If a periodic state is established near the plate, $\overline{\mathscr{P}}(t) - 1$ and $\overline{\mathscr{E}}(t)$ vanish because of the periodicity. We now investigate the time evolution of $\overline{\mathscr{P}}(t)$ and $\overline{\mathscr{E}}(t)$. Figure 6(a) is the plot of $\overline{\mathscr{P}}(t)$ and $\overline{\mathscr{E}}(t)$ vs t in double logarithmic scale for M = 0.2 and for some different values of K. Figure 6(b) shows the gradients of the curves in Fig. 6(a), i.e., $\alpha(\overline{\mathscr{P}}) = d\log_{10}(\overline{\mathscr{P}} - 1)/d\log_{10} t$ and $\alpha(\overline{\mathscr{E}}) = d\log_{10}\overline{\mathscr{E}}/d\log_{10} t$, as functions of t. We must say that our computation up to $t/2\pi = 80$ is not enough, in particular for large K, to draw some definite conclusions. Nevertheless, it is seen from Figs. 6(a) and 6(b) that $\bar{\mathscr{P}}(t) - 1$ and $\bar{\mathscr{E}}(t)$ tend to vanish as time goes on and that the curves in Fig. 6 tend to approach straight lines with gradient -1/2. In other wards, as time proceeds, $\bar{\mathscr{P}}(t) - 1$ and $\bar{\mathscr{E}}(t)$ tend to vanish in such a manner that $\bar{\mathscr{P}} - 1 \approx c_p t^{-1/2}$ and $\bar{\mathscr{E}} \approx c_e t^{-1/2}$ with positive constants c_p and c_e . This provides a piece of numerical evidence that the gas near the plate approaches steady oscillation.

CONCLUDING REMARKS

In the present study, we have investigated numerically the nonlinear acoustic wave propagation in a semi-infinite expanse of a rarefied gas, caused by an infinitely wide plate that starts longitudinal oscillation, on the basis of the BGK model of the Boltzmann equation and the diffuse reflection boundary condition on the plate. An emphasis is placed on accurate description of the discontinuities in the molecular velocity distribution function in the gas that is produced continuously by the oscillating plate (though the behavior of the velocity distribution function is omitted in the paper because of the limited space). The resulting macroscopic quantities show that a weak compression wave propagates toward infinity (streaming motion) and decays very slowly. It is also shown that a high-temperature region is formed near the oscillating plate and diffuses slowly toward infinity and that the gas near plate approaches a periodic state slowly, the rate of approach being more or less proportional to $t^{-1/2}$.

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