（ 続紙 1 ）

京都大学	博士（理　学）	氏名	ATARSAIKHAN GANBAT
論文題目	Reducibility of steady-state bifurcations in coupled cell systems		

（論文内容の要旨）

A *coupled cell network* can be represented as a directed graph. We can associate a dynamical system governed by ODEs with any given coupled cell network in such a way that nodes (or cells) represent state variables that evolve over time and edges (or couplings) represent interaction between those state variables. This dynamical system is called a *coupled cell system*. The main motivation of the study of coupled cell system is to understand its dynamics from the network architecture of the associated coupled cell network and not from specific forms of the vector fields.

In this work we focus on *regular* coupled cell networks, in which all cells and all couplings are identical and moreover each cell receives the same number of incoming edges called *input*. For a regular coupled cell network, let $\{1, \ldots, n\}$ be the set of all its cells. Let $x_j \in \mathbb{R}^k$ be the state variable for the cell j, where k is the dimension of the internal dynamics in each cell, which is assumed to be constant. Then a coupled cell system associated with the given regular network is the system of ODEs

$$\dot{x}_j = f(x_j, \overline{x_{\sigma_j(1)}, \ldots, x_{\sigma_j(v)}}) \quad j = 1, \ldots, n$$

where the cell j receives inputs from the cells $\sigma_j(1), \ldots, \sigma_j(v)$. Here $\sigma_j(i)$'s are allowed to be equal to each other and even to j. The number v is called *valency* of the network and it is constant for any choice of the cell j because each cell has the same number of inputs. The overbar indicates that the coupling coordinates are invariant under permutations of the coupling cells. This invariance is assumed, since we assume there is only one kind of coupling. Since there is only one type of node, we assume that the function $f \colon \mathbb{R}^k \times (\mathbb{R}^k)^v \to \mathbb{R}^k$ is independent of j. In Figure 1 we show an example of a regular coupled cell network and its associated coupled cell system.

Every coupled cell system associated with a regular network, when restricted to a flow-invariant subspace defined by an equality of certain cell coordinates, corresponds to a coupled cell system associated with a smaller network called the *quotient network*. A coupled cell system associated with quotient network is called a *quotient system*. Note that fhb dynamics of a quotient system can be “lifted” to the dynamics of the original coupled cell system on the corresponding flow-invariant subspace.

One of important and interesting subjects in coupled cell systems theory is the “synchrony”. We say that two or more cells are *synchronous* if they behave identically the same for all time. The corresponding subspace given by the synchrony defines a flow-invariant subspace, and thus a quotient system on the quotient network.

In this thesis, we consider *synchrony-breaking steady-state bifurcation*, which is a steady-state bifurcation from the fully-synchronous equilibrium in coupled cell systems. The bifurcating solutions are in general not fully-synchronous, and hence not lying in the fully-synchronous subspace. However, it can still be partially synchronous, and thus contained in a flow-invariant subspace given by a certain synchrony relation. In this case, the bifurcation takes place not only in the original coupled cell system but also in a quotient system on the synchronous subspace, which is easier to study. Observe that a given coupled cell system associated with a regular network can have several quotient systems at the same time. Then the following question arises, which is the motivation of this work.

(*) *What is the condition of a regular coupled cell network for which all codimension-one synchrony-breaking steady-state bifurcations in an associated coupled cell system can be lifted from its quotient systems?*

This question leads to a notion of *reducibility* of bifurcations. For certain coupled cell networks, if all bifurcations in associated coupled cell systems can be lifted from its quotient systems then we say that the bifurcation is *reducible.*

The main results of the thesis are summarized as follows. We consider two classes of regular networks, namely, 1-input regular networks, and multiple-input n-cell regular networks with D_n-symmetry. For 1-input regular networks, its architecture is a loop together with trees attached to it. Therefore it suffices to consider a 1-input loop with a 1-input chain attached to it, which we call a (1-input) *loop-chain network.*

Theorem 1 *Under a certain genericity condition, there are only the following three cases of codimension-one synchrony-breaking steady-state bifurcation branches of equilibrium solutions in a loop-chain network:*

(a) a cascading solution of square-root type,

(b) an alternatively synchronous solution,

(c) a fully synchronous solution.

The precise definitions of these three types of solutions are given in the thesis.

From this theorem one can answer to the question (*) for the class of 1-input regular coupled cell networks. In particular, if the loop contains more than one cell, then the codimension-one synchrony-breaking steady-state bifurcation is generically reducible.

For the case of multiple-input regular networks, we obtain:

Theorem 2 *Under a mild condition on the multiplicity of critical eigenvalues, the codimension-one synchrony-breaking steady-state bifurcations in generic coupled cell systems associated with an n-cell regular coupled cell network with D_n-symmetry is reducible for $n > 2$.*

（ 続紙 3 ）

（論文審査の結果の要旨）

申請者の研究は，結合セル力学系の分岐に関するものである．結合セル力学系は，有限有向グラフによって結合のネットワーク構造が指定された常微分方程式の結合系のクラスをいい，M. Golubitsky や I.Stewart らによって 2003 年に導入され，その後，活発な研究が行われている．特に結合セル力学系では，複数のセルの状態が時間的に同一の振舞いをする同期現象に着目し，どのような同期が起るかを常微分方程式の詳細な情報でなく，ネットワークの情報だけから判定するために，バランス・カラーリングという同値関係と，それによる商ネットワークへの還元などのいくつかの興味深いアイディアが提出された.

結合セル力学系のパラメータ族において，すべてのセルの状態が完全に同期した状態が，パラメータの変動に伴って同期状態が変化することを同期破壊分岐と呼ぶ．これは対称性のある系において，完全に対称的な解がパラメータの変動に伴って非対称になる対称性破壊分岐の類似として，結合セル力学系における特徴的な分岐である．申請者は，このような同期破壊分岐をより良く理解するために，可能なすべての商ネットワークによる同期解を考えるという着想により，分岐の還元可能性の概念を導入した．同期破壊分岐が還元性を持つとは，どの分岐解の枝もいずれかの商ネットワークの結合セル力学系の分岐から導かれていることであり，その場合には分岐を調べるには，元の結合セル力学系よりも小さな系を調べれば良いことになり，解析が容易になる.

申請者は，正則な結合セル力学系の１パラメータ族における定常解の同期破壊分岐を精密に考察し，ネットワークのインプット数が１の場合の解の完全な分類と，その帰結としての分岐の還元性を示した．またインプット数が２以上の場合には，ネットワーク構造が二面体群の対称性を持つ場合に，同期破壊分岐に関する還元可能性のための条件を明らかにした．これは部分的とはいえ，同期破壊分岐の特徴的構造の一端を見出した興味深い結果である.

参考論文は，商ネットワークでなく，部分ネットワークに着目して結合セル力学系のダイナミクスを理解しようとする申請者の最近の試みであり，今後の結合セル力学系の研究のための有力な方法に発展する可能性が期待される結果である.

よって，本論文は博士（理学）の学位論文として価値あるものと認める．また，論文内容とそれに関連した事項について平成 26 年 1 月 22 日に試問を行った結果，合格と認めた.

要旨公開可能日：　　　　年　　　月