A Self-consistent Model of the Black Hole Evaporation and Entropy in Gravity

Yuki Yokokura

In this thesis, we construct a self-consistent model which, at the semiclassical level, describes a black hole from formation to evaporation including the back reaction from the Hawking radiation, under the conditions: (a) using the eikonal approximation, (b) considering only s-wave of the matter, and (c) assuming massless scalar fields with N degrees of freedom $(N \gg 1)$. Then, we try to understand the entropy and information problems.

In section 2, we will begin with reviewing the Hawking radiation and the information problem based on the original papers by Hawking, whose purpose is to make this thesis self-contained and to define the problems explicitly. It is emphasized that his discussion is based only on the outside of the static vacuum black hole, and that time evolution without considering back reaction from the radiation leads to the information loss. Here information is defined as an initial state of quantum fields for the radiation and collapsing matter, and its loss means that expectation values of time-evolved operators with respect to the state are not determined uniquely on a chosen final time slice.

In section 3, first we will look at a naive picture of time evolution: An observer far away sees a collapsing matter approaching the Schwarzschild radius, which would last forever without quantum effects, and during this process the particle creation occurs to extract the energy given by the matter to the outside as the Hawking radiation. Then, due to the reduction of the mass, the radius shrinks so that the matter cannot catch up with it completely. Repeating this process, the whole energy is transformed into the radiation, while the observer does not see the horizon or singularity. We will show below that this is true.

Next, we will set up a model under the above three assumptions. A continuously-distributed null matter collapses and quantum massless scalar fields on this back ground play roles of the Hawking radiation, which will appear as a result of dynamics later. Then, the information is represented by a pure state which is almost the Minkowski vacuum state, except for the region covering the collapsing matter. The time evolution of this system is

determined by the semi-classical Einstein equation and the wave equation. Here each equation is also equipped.

We investigate motion of the collapsing matter and its effect on the geometry. One shell in the continuous matter approaches, exponentially in the local time, to its effective Schwarzschild radius, which would be determined by the ADM energy under the shell if the shells outside did not exist. If the shell emits the radiation it cannot catch up completely with the radius due to the back reaction. In this thesis, we will call by an asymptotic state the situation in which each shell has approached to the radius sufficiently so that a relation between the position of the shell and the radius holds.

Then, we build a covariant flux formula which evaluates energy flux from particle creation in the dynamical geometry, and set up a system of equations which determines time evolution of the geometry, collapsing matter, and radiation in a self-consistent manner which preserves the energy conservation law by anomalous terms.

Under the situation, we obtain an asymptotic solution for the inside of the hole, and show that the black hole itself can evaporate without horizon or singularity in the usual lifetime. It has an onion-like internal structure, and each layer emits the radiation following the Planck distribution with the Hawking temperature which is given by the effective Schwarzschild radius. Then, a new definition of the Hawking temperature appears, in which the temperature of the hole is defined as a response to the energy change like the Einstein relation of the Brownian motion.

In section 4, we reproduce exactly the entropy area law, by counting microstates of the matter fields in the stationary interior geometry, which tells that matters and their self-gravitational effects contribute to the black hole entropy. Furthermore, the picture of the black hole entropy in this model explains Bekenstein's gedankenexperiment including also the case of evaporation, and then, unify the view of the entropy by Hawking, Gibbons-Hawking, Bekenstein. Furthermore, the obtained flux formula, which can determine corrections to the Hawking flux due to the back reaction of the evaporation, and the dynamical definition of the temperature bring to the entropy area law a logarithmic correction term which depends on details of the matter fields as "hairy effects". In this sense this correction comes from dynamical or non-equilibrium fluctuations.

In section 5, we introduce interactions between the collapsing matter and radiation into the internal structure to show that the initial information of the collapsing matter, such as the baryon number, comes back immediately, which can be checked by evaluating explicitly the recovery time scale. Next, we explain that quantum state of the collapsing matter dissipates by emitting the radiation, identify the origin of the information loss, and propose a conjectured scenario for the complete recovery. The key points are including the back reaction of the matter and radiations time-dependently, and considering interactions between the matter and geometry quantum-mechanically, which automatically lead to interactions between the ingoing collapsing matter and the outgoing radiation to produce correlations between them.