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Highly time-resolved evaluation technique of instantaneous amplitude and phase difference using analytic signals for multi-channel diagnostics[a)]

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A fluctuation analysis technique using analytic signals is proposed. Analytic signals are suitable to characterize a single mode with time-dependent amplitude and frequency, such as an MHD mode observed in fusion plasmas since the technique can evaluate amplitude and frequency at a specific moment without limitations of temporal and frequency resolutions, which is problematic in Fourier-based analyses. Moreover, a concept of instantaneous phase difference is newly introduced, and error of the evaluated phase difference and its error reduction techniques using conditional/ensemble averaging are discussed. These techniques are applied to experimental data of the beam emission spectroscopic measurement in the Heliotron J device, which demonstrates that the technique can describe nonlinear evolution of MHD instabilities. This technique is widely applicable to other diagnostics having necessity to evaluate phase difference. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4891102]

I. INTRODUCTION

On conventional spectral analysis techniques on the basis of Fourier transform, there is a crucial trade-off between temporal and frequency resolutions based on uncertainty principle, $\Delta t \cdot \Delta\omega \sim 1/2$, since the concept is based on the idea that an arbitrary wave is described by the sum of waves having different frequency components. The techniques are a powerful tool to analyze multi-frequency fluctuation such as turbulence and have demonstrated successful results in almost all the research filed in science. However, this is not intuitively plausible to express a single mode with time-dependent frequency and amplitude, for example, MHD phenomena observed in fusion devices. For instance, if we apply short-time fast Fourier transform (FFT) to the signal of cyclic MHD activities appearing repeatedly within shot time period, the spectrum broadening in time and frequency domains would be observed in the spectrogram although the amplitude and frequency of the waveform can simply be expressed and defined as $x(t) = A(t)\exp(\mathrm{i}\int\omega(t)dt))$, where the $A(t)$ and $\omega(t)$ are time dependent amplitude and frequency, respectively.

In this article, a new fluctuation analysis technique using analytic signals is proposed to clarify detailed temporal behavior of fluctuations. The details of the analysis technique using analytic signals, introduction of the concept of instantaneous phase difference and discussion for the error, and an application to experimental data will be shown and discussed.

II. FLUCTUATION ANALYSIS USING ANALYTIC SIGNALS

A. Basic analysis technique using analytic signals

An analytic signal, defined as $X(t) = A(t)\exp(\mathrm{i}\theta(t))$, is a complex function where $A(t)$ and $\theta(t)$ are the mean instantaneous amplitude and instantaneous phase and its time derivative $\omega(t) = \mathrm{d/dt}(\theta(t))$ is instantaneous frequency of the signal, respectively.[1] This expression is suitable to express temporal development of a single mode such as a MHD instability. The analytic signal is expressed as $X(t) = x(t) + \mathrm{i}/\pi\int x(t')/(t - t')dt'$, which means that the real part, $\mathrm{Re}(X(t))$, corresponds to the real signal $x(t)$. In other words, the real signal is the projection of the analytic signal on real axis. The second term, the convolution operation, is known as Hilbert transform. Mathematically this calculation is equal to the following expression:

$$X(t) = 2/(2\pi)\int_0^\infty S(\omega)\exp(\mathrm{i}\omega t)d\omega,$$

which corresponds to the inverse Fourier transform of the real part of Fourier spectrum $S(\omega)$ and is easily achieved by conventional FFT routines. Here the amplitude ($A(t) = |X(t)|$) and the argument ($\theta(t) = \arg(X(t))$) of the analytic signal are the required instantaneous amplitude and the instantaneous phase at a specific moment.

This technique was applied to a test signal and the result was compared to those of other analysis techniques, FFT and wavelet analyses, in Figure 1. Figure 1(a) shows the test signal which is a realistic time-series signal prepared to simulate bursting MHD activity with time dependent amplitude $A(t)$ and frequency $\omega(t)$, described as $A(t)\sin(\mathrm{i}\int\omega(t)dt))$. We assumed that the frequency rapidly chirps down from 90 to

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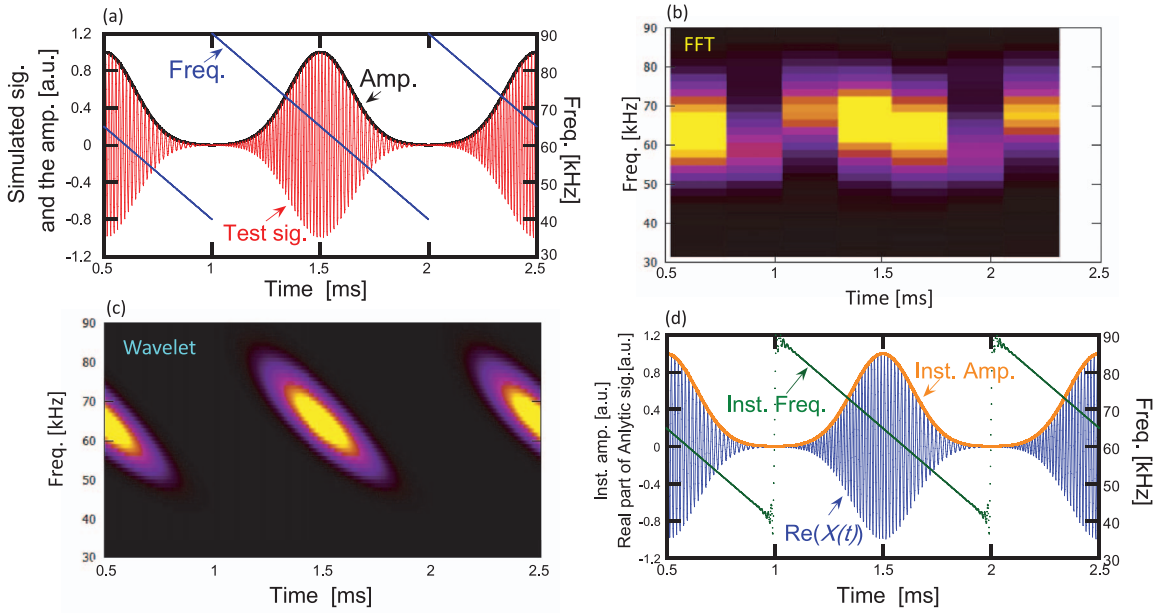

FIG. 1. (a) Test signal to simulated cyclic MHD activities and the spectrogram obtained with (b) short time FFT and (c) wavelet analysis. (d) Instantaneous amplitude and frequency evaluated using analytic signal technique.

40 kHz ($\omega(t) = 2\pi f(t)$, $f(t) = -50 \cdot (t - t_0) + 65$ [kHz]) and the amplitude grows and decays with Gaussian shape ($A(t) = \exp(t - t_0)^2/(0.2)^2$) with a period of 1 ms (here $t_0 = 0.5$, 1.5, 2.5, … ms). The time resolution of the test signal is 1 μs. Figure 1(b) clearly shows that the FFT analysis failed to catch the time evolution of the signal since its time window for FFT is 256 μs, which is a realistic choice on the analysis, and the width is much longer than the time scale of the evolution. The spectrogram of the wavelet analysis is more smoothed compared with the FFT result, however, the broadening of the spectrum in time and frequency domains still remains, as shown in Fig. 1(c), where Morlet wavelet is employed as mother wavelet. It is quite difficult to have a discussion about temporal evolution less than the time and frequency resolutions of ~0.1 ms and ~5 kHz in these analyses. By contrast, analytic signal method can successfully evaluate the instantaneous amplitude and instantaneous frequency, as shown in Fig. 1(d).

B. Evaluation of instantaneous phase difference

Based on the analytic signal method, we newly introduce a concept of instantaneous phase difference, which is useful to clarify the temporal development of phase difference between two signals in detail. The instantaneous phase difference $\varphi(t)$ ($=\theta_2(t) - \theta_1(t)$) between two analytic signals, $X_{1,2}(t) = |X_{1,2}(t)|\exp(i\theta_{1,2}(t))$, can be easily obtained to take inner product of the two signals as $X_1(t)^* \cdot X_2(t) = |X_1(t)| \cdot |X_2(t)|\exp(i\varphi(t))$. This enables us to evaluate phase difference at a specific moment without the limitation of the uncertainty principle.

The instantaneous phase difference was evaluated using two simulated test signals, one is the same test signal used in Sec. II A, $A(t)\sin(i\int\omega(t)dt)$ and the other is the test signal plus Gaussian white noise, $A(t)\sin(i\int\omega(t-\tau)dt) + noise(t)$. Note that the constant of time lag τ was introduced with regard to the coherent component. The standard deviation of the $noise(t)$ is assumed to be 10% of the maximum amplitude of the coherent mode component here. Figures 2(a) and 2(b) show the two simulated signals, and Figs. 2(c) and 2(d) show the instantaneous phase differences between the two signals with different time lag $\tau = 0$ μs and 25 μs. As seen in Figure 2(c), the phase difference was appropriately evaluated to be zero in the case of no time lag when the fluctuation amplitude of the coherent mode is larger than the noise level. Meanwhile, the phase difference varies in time in the case of

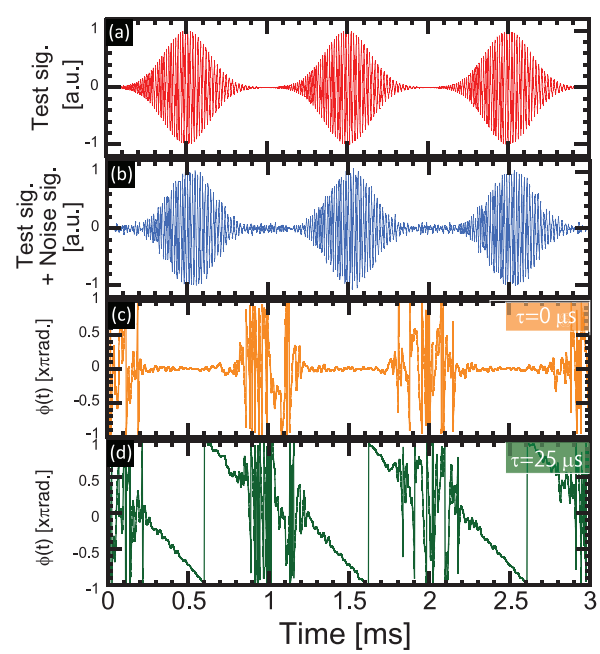

FIG. 2. (a) Test signal defined in Sec. II A, (b) test + Gaussian white noise signal, and (c) instantaneous phase difference between the signal in case of the time lag $\tau = 0$ μs and (d) $\tau = 25$ μs.

$\tau = 25\ \mu$s, and this is because the instantaneous frequency is not constant and changes linearly in time, and the time lag induces linearly time-dependent phase difference between the signals, as $\varphi(t) \sim \tau t$. These results conclude that the technique is effectively utilized to elucidate the temporal behavior of coherent modes.

C. Error estimation of instantaneous phase difference and its reduction with ensemble/conditional average

In this section, error of the phase difference evaluation is discussed. The phase difference evaluated with analytic signals can be expressed as $\tan(\varphi(t)) = I/R$, where I and R indicate the real and imaginary part of $X^*(t) \cdot Y(t)$, respectively. By taking the total differential for the expression, the amount of error can be written as

$$\Delta\varphi = (R_0 \Delta I - I_0 \Delta R)/(R_0 I_0)^2, \tag{1}$$

and R and I are assumed to be $R = R_0 + \Delta R$ and $I = I_0 + \Delta I$. The leading term and perturbation term correspond to the components for the coherent and the noise part, respectively. Similarly, the perturbation terms for the analytic signal of X and Y are also introduced and expressed as $X = X_0 + \Delta X$ $(=|X_0|\exp(i\theta_x) + |\Delta X|\exp(i\varepsilon_x))$ and $Y = Y_0 + \Delta Y$ $(=|Y_0|\exp(i\theta_y) + |\Delta Y|\exp(i\varepsilon_y))$, and as a result we obtain the following expressions:

$$\begin{aligned} R_0 + \Delta R &= \mathrm{R}_e(X^* \cdot Y) \\ &= |X_0||Y_0|\cos(\phi) + |Y_0||\Delta X|\cos(\alpha) \\ &\quad + |X_0||\Delta Y|\cos(\beta), \end{aligned}$$

and

$$\begin{aligned} I_0 + \Delta I &= |X_0||Y_0|\sin(\varphi) + |Y_0||\Delta X|\sin(\alpha) \\ &\quad + |X_0||\Delta Y|\sin(\beta), \end{aligned}$$

where $\varphi = \theta_y - \theta_x$, $\alpha = \theta_y - \varepsilon_x$, and $\beta = \varepsilon_y - \theta_x$. The first term is the required phase difference between the coherent components. By substituting the above expression to Eq. (1), the amount of the error can be expressed as

$$\Delta\varphi = |\Delta X|/|X_0|\sin(\varepsilon_X - \theta_X) + |\Delta Y|/|Y_0|\sin(\varepsilon_Y - \theta_Y).$$

Based on the above expression, ensemble averaging or conditional averaging can be expected to reduce the noise of the evaluated phase difference, since the perturbation term ΔX should converge to zero if we take ensemble or conditional average. The term for the sine function regarding the phase difference between the mode component and random component should also converge to zero.

The results of conditional/ensemble averaging with different averaging number were compared in Figure 3. Note that here the noise component was assumed to be 50% of the maximum amplitude of the coherent mode, which is 5 times larger than that used in Sec. II B. Figure 3(a) is the result of no conditional averaging, in other words, the raw signals of the instantaneous amplitude and phase difference. Figures 3(b)–3(f) are the results with the different numbers of conditional averaging for $N_{CA} = 5$, 10, 20, 100, and 200. It is clearly seen that the results improve as the averaging number increases, and the temporal evolution of the phase difference

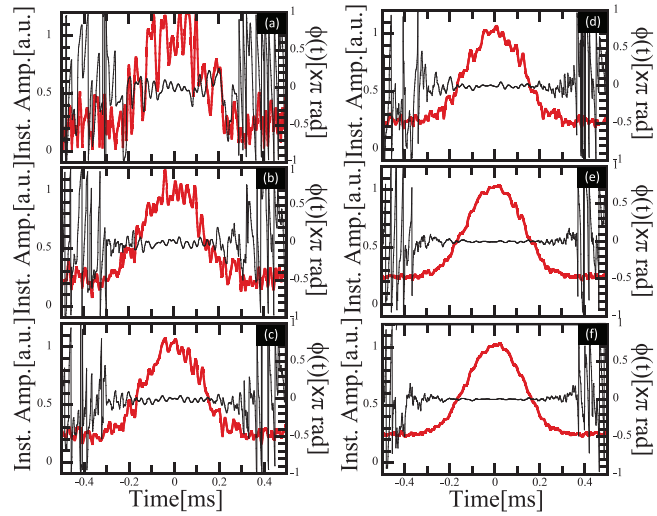

FIG. 3. (a) Instantaneous amplitude (red line) and instantaneous phase difference (black line) with no conditional averaging and (b) those conditional averaging results in the cases of $N_{CA} = 5$, (c) $N_{CA} = 10$, (d) $N_{CA} = 20$, (e) $N_{CA} = 100$, and (f) $N_{CA} = 200$.

can be traced although the noise amplitude is half of the maximum coherent mode amplitude. These results indicate that the conditional/ensemble averaging is an effective technique to improve accuracy on this analysis as well as on other fluctuation analysis techniques.

III. APPLICATION OF THE TECHNIQUE TO EXPERIMENTAL DATA IN HELIOTRON J DEVICE

The technique proposed in this article was applied to beam emission spectroscopy (BES) diagnostic signals to elucidate the dynamical change of a cyclic MHD instability observed in a helical device, Heliotron J.[2–4] The observable range of the BES system covers overall radial region of the Heliotron J plasma from core to edge, and the measurement is useful to investigate radial structure of MHD instabilities. The experiment was conducted in neutral beam injection heating plasma with low electron density of less than $n_e \sim 1 \times 10^{19}$ m^{-3}, and a cyclic energetic-particle driven MHD instability was observed with good reproducibility. The instability is a kind of Alfvén eigenmodes, which is considered as global Alfvén eigenmodes or energetic particle mode.[5] Typical BES signal is shown in Fig. 4(a), and the instability appeared with the period of ~0.6 ms, and the oscillation frequency chirps up from 70 to 90 kHz in each burst. The analytic signal technique was applied to the BES signals. Before generating the analytic signal of the instability, the raw signal is band-pass filtered from 50 to 100 kHz in order to suppress the error caused from unfavorable frequency components. Then, we took conditional averaging of $N_{CA} = 10$ for the instantaneous amplitude at each sightline and the phase difference between the BES signals at a fixed position and at different position.

Figures 4(b) and 4(c) show that the conditionally averaged, instantaneous amplitude and phase shift. It is recognized that the radial structure of the amplitude and the phase shift develops with short time scale less than 0.1 ms. The peak position of the mode amplitude moves from inner to outer

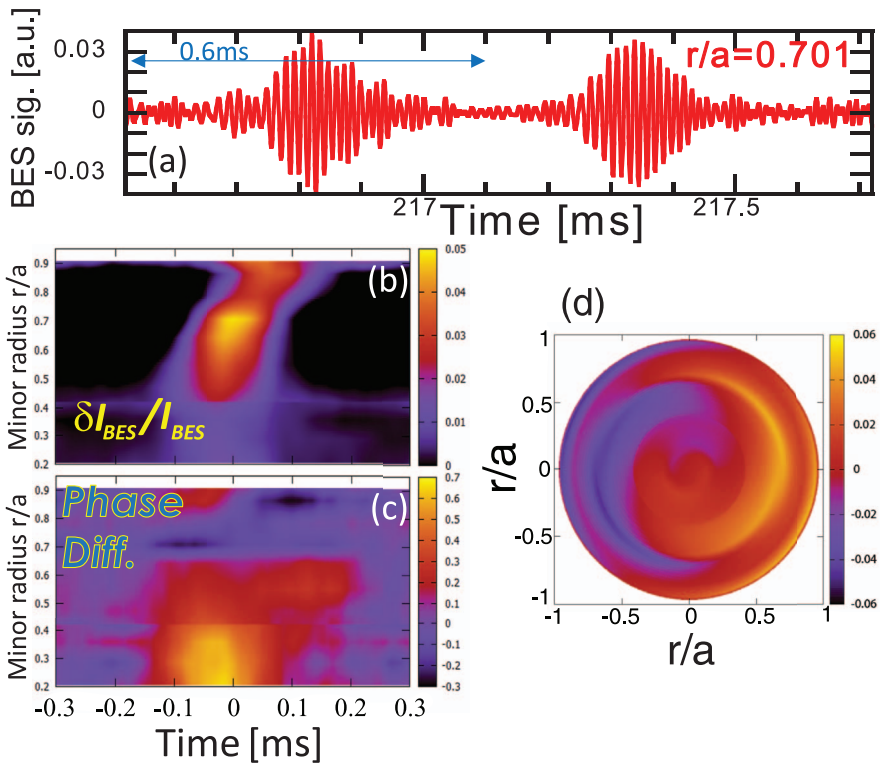

FIG. 4. (a) Typical signal of the burst measured with BES at r/a = 0.701. (b) Conditionally averaged instantaneous amplitude and (c) phase shift of the coherent mode of t = −0.3 ms to +0.3 ms. (d) Reconstructed mode structure of the instability at t = 0.3 ms on a poloidal cross section.

region of the plasma. The structural change of the phase shift means that the mode is distorted within the time scale of each burst activity. These nonlinear behaviors of the mode should have a correlation with the radial transport of fast ions since this coherent mode is driven by fast ions.

Time evolution of the two dimensional mode structure can be reconstructed on the basis of the result shown above. Fig. 4(d) is an example of the reconstructed 2D mode structure on a poloidal cross section at the moment of $t = 0.0$ ms. The mode structure on a poloidal cross section was expressed as a two dimensional function of $n(r, \theta, t) = A(r, t)\exp(i(m\theta - 2\pi f \cdot t + \delta(r,t)))$, where the parameters $A(r, t)$, m, f, and $\delta(r,t)$ are instantaneous fluctuation amplitude as a function of radial position r and time t, poloidal mode number, the mode frequency, and the phase shift of the fluctuation with a fixed magnetic probe signal. The instantaneous fluctuation amplitude and phase shift shown in Figs. 4(b) and 4(c) were used in this calculation. Rotating in ion diamagnetic direction, the mode grows and then the spiral structure is formed at the timing when the mode has maximum amplitude, and finally the structure disappears. These kinds of nonlinear behavior of MHD fluctuations are widely observed in fusion plasma, and in such a situation this technique is useful to characterize the temporal behavior of MHD modes.[6]

IV. SUMMARY

Highly time-resolved analysis technique using analytic signals was proposed. The technique can evaluate fluctuation amplitude, frequency, and phase difference at a specific moment, which can describe temporal development of a single mode such as MHD phenomena. Error of the instantaneous phase difference was discussed, and the result suggests that the error can be reduced by ensemble or conditional averaging. Practically this technique was applied to experimental data in Heliotron J device, and was evidenced that the technique can clarify the time evolution of nonlinear behavior for a MHD instability. This technique can be utilized in all the diagnostics to evaluate phase difference, including a magnetic probe array, an interferometer, and a reflectometer.

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