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Maximum Likelihood Approach for RFID Tag Set Cardinality Estimation with Detection Errors

Chuyen T. Nguyen · **Kazunori Hayashi** · **Megumi Kaneko** · **Petar Popovski** · **Hideaki Sakai**

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Abstract Estimation schemes of Radio Frequency IDentification (RFID) tag set cardinality are studied in this paper using Maximum Likelihood (ML) approach. We consider the estimation problem under the model of multiple independent reader sessions with detection errors due to unreliable radio communication links and/or collisions. In every reader session, both the detection error probability and the total number of tags are estimated. In particular, after the R-th reader session, the number of tags detected in j ($j = 1, 2, ..., R$) reader sessions out of R sessions is updated, which we call observed evidence. Then, in order to maximize the likelihood function of the number of tags and the detection error probability given the observed evidences, we propose three different estimation methods depending on how to treat the discrete nature of the tag set cardinality. The performance of the proposed methods is evaluated under different system parameters and compared with that of the conventional method via computer simulations assuming flat Rayleigh fading environments and framed-slotted ALOHA based protocol.

Keywords RFID · tag cardinality estimation · maximum likelihood · detection error

1 Introduction

Radio Frequency IDentification (RFID) technology has become very popular in many applications of identifying objects automatically. Some of them are inventory control

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and tracking, medical patient management and security check [1–5]. One of the fundamental tasks in such RFID systems is to estimate the total number of tags fast and reliably [6]. For example, this task can be met in inventory management applications where the total number of products in a warehouse should be known. Other applications could be Intelligent Transportation Systems (ITS) [7] and indoor stadium systems that track and monitor the population distribution of metropolitan vehicles and visitors, respectively [8]. We can also see this estimation application in factories that store one kind of products, or in conferences, where thousands of participants need to be monitored. Besides, giving an accurate estimate of the tag cardinality could improve the efficiency of other operations such as key assignment, updating [9], and categorization [10].

Collisions happen when multiple tags simultaneously respond to the reader, which results in the detection error of the tags involved in the collisions. Several estimation methods of the tag set cardinality have been proposed in order to cope with collisions in the framed-slotted ALOHA based protocol. In particular, the method proposed in [11] minimizes the distance between the observed and analytical reading results as presented by vectors $[E,S,C]^{\mathrm{T}}$ and $[\bar{E},\bar{S},\bar{C}]^{\mathrm{T}}$, where E ($\bar{E}$), S ($\bar{S}$) and C ($\bar{C}$) denote the observed (analytical) numbers of empty, singleton and collision slots in the frame, respectively. Another simple estimation method is presented in [12] by assuming that the number of tags responding in each time slot is Poisson distributed with mean one which is valid when the number of slots in the frame is the same as that of tags. Besides, [13] has proposed a method using Maximum a Posteriori (MAP) approach.

Apart from the collision problem, transmission errors due to multi-path fading, obstacles in the radio path, blind spot phenomenon [14] or materials to which the tags are affixed [15], can be another limiting factor of not only the tag reading but also the tag cardinality estimation in RFID systems. In [16], the tag cardinality estimation is considered under unreliable radio channels by modeling the tags' spatial distribution and the corresponding channel fading effects. Specifically, a tag responds to the reader only if it receives sufficient power to process the reader's request, which depends on the tag's location and the channel propagation model that is assumed to be Lognormal or Rayleigh fading. However, the spatial distribution of the tags is difficult to know in many practical applications, and also errors due to collisions are not considered. Another solution, in this case, could be deploying multiple readers with overlapping interrogation zones [17], but the disadvantages of this method are high cost, system complexity and reader-to-reader interference [18]. On the other hand, assuming uniform transmission error probability for every tag, a practical method named *Remove Element Greater than Mean* (REGM) is recently proposed in [19–21], where multiple independent reader sessions and capture-recapture approach are employed. In particular, in the R-th reader session, the transmission error probability and the total number of tags are estimated by finding appropriate weights for the observed evidence, which is defined as the number of tags detected in j ($j = 1,2,...,R$) reader sessions out of R sessions. Although only the transmission errors are assumed in [19–21], REGM can cope with collision errors as well. However, since the weights in REGM are found in a heuristic manner, there might be room to improve the estimation performance.

In this paper, we employ the multiple independent reader sessions model and consider the tag set cardinality estimation problem of RFID systems under both the transmission errors and collisions phenomena, and to the best of our knowledge, this is one of the first works dealing with such a scenario. Due to the phenomena, each tag is assumed not to be detected with a probability p, which we call a detection error probability, in each reader session. The detection error probability and the tag cardinality are then, estimated by using a maximum likelihood (ML) approach. In particular, the likelihood function of the detection error probability p, which is assumed to be the same for all tags, and number of tags N given the current observed evidences is firstly derived. In order to maximize the likelihood function, three different methods, namely, *Exhaustive ML (EML)*, *ML with Sample Mean for N (MLSM)*, and *ML with search Stopping Criterion (MLSC)* are considered depending on the ways of treating the discrete nature of N. In EML method, the likelihood function is evaluated for all possible values of N, thus, it can achieve the best performance among all the methods but with high computational complexity due to the exhaustive search. In order to reduce the complexity, MLSM method is proposed as the simplest one, where p is firstly estimated using a rough initial estimate of N, and then the estimate of N is updated by the sample mean of the observed evidences using the estimated p. To obtain MLSC method, the behavior of the likelihood function with respect to N is analyzed based on the continuous relaxation for N. Based on the analysis, MLSC method evaluates the likelihood function for discrete values of N in ascending order starting from the minimum possible value, and it stops searching as the value of the likelihood starts decreasing. The performance of the proposed methods is evaluated and compared with that of the conventional REGM method via computer simulations, both with simple detection error models and practical Rayleigh fading channel model, as well as the collision model with framed-slotted ALOHA based protocol.

The rest of this paper is organized as follows. In Section 2, the system model and the conventional approach are described. Section 3 provides the proposed methods in detail and simulation results are shown in Section 4. Finally, we conclude in Section 5.

2 System Model and Conventional Approach

2.1 System Model

The considered RFID system consists of a reader and N unknown tags in the communication range where multiple independent reader sessions are performed. A reader session is defined as a reading of the tag set in which the reader broadcasts a message to all tags and receives responses from them. Note that if a tag responds to the reader, its IDentity (ID) is obtained correctly, and it can respond in different reader sessions even after being detected, which is valid for passive RFID systems [1,6]. The detection error probability p, which is the probability that a tag is not read in each reader session, is assumed to be unknown a priori and identical for all the tags. After R reading rounds, the number of tags observed in $R+1-j$ ($j = 1,..,R$) reader sessions k_j is updated, which is denoted as an observed evidence. Our problem is

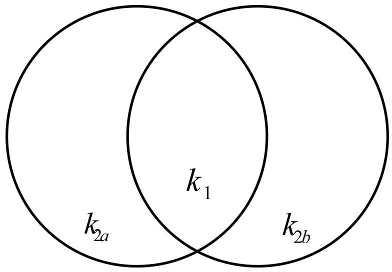

Fig. 1 Venn diagram of observed evidences ($R = 2$)

to estimate the detection error probability p and the tag set cardinality N using the observed evidences at each reader session.

Figure 1 shows a Venn diagram of the observed evidences for the case with two reader sessions, where k_1 is the number of tags that have been read in both reader sessions, and $k_{2\mathrm{a}}(k_{2\mathrm{b}})$ is the tag set cardinality that is read only in the first (second) reader session. In this case, k_1 and $k_2 = k_{2\mathrm{a}} + k_{2\mathrm{b}}$ are the observed evidences, which will be used for the estimation of N and p.

2.2 Conventional Approach-REGM Method

For the case of two reader sessions ($R = 2$), REGM method utilizes the relations of

$$E[k_1] = Np_1 = N(1-p)^2, \tag{1}$$

$$E[k_2] = Np_2 = 2N(1-p)p, \tag{2}$$

where p_1 and p_2 are the probabilities of detecting a tag in both reader sessions and only one reader session respectively. Let $\hat{p}$ and $\hat{N}$ denote the estimates of p and N respectively obtained by replacing $E[k_j]$ with the observed evidence k_j ($j = 1,2$). Then, with the replacement, Equations (1) and (2) become a set of linear equations of $\hat{p}$ and $\hat{N}$, which can be easily solved as

$$\hat{p} = \frac{k_2}{2k_1 + k_2}, \tag{3}$$

and

$$\hat{N} = \frac{k_1 + k_2}{1 - \hat{p}^2}. \tag{4}$$

The problem is also extended to the model with R (> 2) independent reader sessions in which the corresponding observed evidences are $k_1, k_2, ..., k_R$. In this case, N can be estimated for given $\hat{p}$ as

$$\hat{N} = \frac{\sum_{j=1}^{R} k_j}{1 - \hat{p}^R}, \tag{5}$$

while REGM estimates $\hat{p}$ by using a generalized form of Equation (3) as

$$\frac{\sum_{j=1}^{R}\phi_{\mathrm{n}}(j)k_j}{\sum_{j=1}^{R}\phi_{\mathrm{d}}(j)k_j} = \frac{\sum_{j=1}^{R}\phi_{\mathrm{n}}(j)\binom{R}{R-(j-1)}(1-\hat{p})^{R-(j-1)}\hat{p}^{j-1}}{\sum_{j=1}^{R}\phi_{\mathrm{d}}(j)\binom{R}{R-(j-1)}(1-\hat{p})^{R-(j-1)}\hat{p}^{j-1}}. \tag{6}$$

Two window functions $\phi_{\mathrm{n}}(j)$ and $\phi_{\mathrm{d}}(j)$ determine the observed evidences used to compute $\hat{p}$, which are chosen as

$$\phi_{\mathrm{n}}(j) = \begin{cases} 1 & \text{if } w_j \neq 0, \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

and

$$\phi_{\mathrm{d}}(j) = \begin{cases} 1 & \text{if } w_j \neq 0 \,\text{and}\, w_j < m_{\mathbf{w}}, \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where $\mathbf{w} = [w_1, w_2, ..., w_R]^{\mathrm{T}} = \left[\frac{k_1}{\binom{R}{R}}, \frac{k_2}{\binom{R}{R-1}}, ..., \frac{k_R}{\binom{R}{1}}\right]^{\mathrm{T}}$ and $m_{\mathbf{w}}$ is the sample mean of the nonzero elements in $\mathbf{w}$. The basic strategy here is to utilize the observed evidences with smaller normalized values except for zero elements, although no explicit justification is given in [19]. While it has been shown that the REGM can outperform the estimator based on [22] with Capture-Recapture model [23], there might be room to improve the estimation performance using the multiple reader sessions model, because Equation (6) and the window functions are chosen in a rather heuristic manner.

3 Proposed Method

Here, we propose three different estimation methods using ML approach for the model of multiple independent reader sessions.

3.1 Likelihood Function

The probability of detecting a tag in $R+1-j$ reader sessions is given by

$$p_j = \binom{R}{R-(j-1)}(1-p)^{R-(j-1)}p^{(j-1)}. \tag{9}$$

Therefore, if random variables representing the observed evidences obtained after R reader sessions are denoted by $K_1, K_2, ..., K_R$, they will follow the multinomial distribution. Thus, the likelihood function of N and p can be represented as

$$\begin{aligned} P(k_1, ..., k_R|N, p) &= P(K_1 = k_1, ..., K_R = k_R|N, p) \\ &= \frac{N!}{k_0!k_1!k_2!\cdots k_R!}\prod_{j=0}^{R} p_j^{k_j}, \end{aligned} \tag{10}$$

where $p_0 = 1 - p_1 - \cdots - p_R$ and $k_0 = N - k_1 - \cdots - k_R$ are the probability that no tags are detected and the number of undetected tags after R reader sessions, respectively. Then, the log likelihood function is obtained as

$$\ln P(k_1, k_2, ..., k_R|N, p) = \ln \frac{N!}{k_1!k_2!\cdots k_R!k_0!} + \sum_{j=1}^{R} k_j \ln\left(\frac{R}{R-(j-1)}\right) + \sum_{j=1}^{R}(R+1-j)k_j \ln(1-p) + \sum_{j=1}^{R} k_j(j-1)\ln p + k_0 R \ln p. \quad (11)$$

Our problem is thereby equivalent to finding the values of N, p, which maximize the log likelihood function (11) as

$$(\hat{N}, \hat{p}) = \arg \max_{N\in\mathbb{N}, p\in[0,1]} \ln P(k_1, k_2, ..., k_R|N, p). \quad (12)$$

Setting the derivative of (11) with respect to p to be equal to zero, we obtain

$$\hat{p} = \frac{RN - \sum_{j=1}^{R}(R+1-j)k_j}{RN}. \quad (13)$$

On the other hand, due to the discrete nature of N, we cannot obtain an estimate $\hat{N}$ by using the same approach with derivation. Thus, in the sequel, we propose three different approaches depending on how to deal with the discrete nature of N.

3.2 Exhaustive ML (EML)

An exhaustive search algorithm will be employed in this method. In particular, by substituting $\hat{p}$ of Equation (13) into p of the log likelihood function (11), we obtain

$$\ln P(k_1, ..., k_R|N) = \ln N! - \ln\left(N - \sum_{j=1}^{R} k_j\right)! - \ln k_1!k_2!\cdots k_R! + \sum_{j=1}^{R} k_j \ln\left(\frac{R}{R-(j-1)}\right) + \sum_{j=1}^{R}(R+1-j)k_j \ln \frac{\sum_{j=1}^{R}(R+1-j)k_j}{RN} + \left(RN - \sum_{j=1}^{R}(R+1-j)k_j\right)\ln \frac{RN - \sum_{j=1}^{R}(R+1-j)k_j}{RN}. \quad (14)$$

Then, for each discrete value of N, the log likelihood function (14) is evaluated. In the search algorithm, the total number of tags observed in the reader sessions

$$\hat{N}_0 = \sum_{j=1}^{R} k_j, \quad (15)$$

can be used as the minimum possible value of N. The optimal value $\hat{N}$ is the one that maximizes the likelihood function. Hence, EML method can achieve the best performance among the three methods, which will be further discussed via simulation results. However, this method requires a high computational complexity due to the exhaustive search.

3.3 ML with Sample Mean of N (MLSM)

To reduce the computational complexity, MLSM method is proposed in this section. In this method, $\hat{p}$ is firstly estimated by using Equation (13) where $\hat{N}_0$ is used as a rough estimate of N. Then, $\hat{N}$ will be updated by the sample mean of the observed evidences using Equation (5). MLSM method can obtain the estimates of $\hat{p}$ and $\hat{N}$ from the observed evidences with very low computational cost. Although there will be some performance degradation compared as EML, we can expect good performance for a large number of reader sessions R, since $\hat{N}_0$ will be close to the actual total number of tags.

3.4 ML with search Stopping Criterion (MLSC)

EML method requires a high computational complexity because in the exhaustive search algorithm, all possible values of N have to be evaluated. To overcome this inconvenience, in this section, we derive a stopping criterion of the search algorithm by analyzing the log likelihood function. In particular, by using the continuous relaxation, we can consider the derivative of (14) with respect to N as

$$\begin{aligned} f_{\mathrm{P}}(N) &= \frac{\partial \ln P(k_1, k_2, ..., k_R|N)}{\partial N} \\ &= \Psi(N+1) - \Psi\left(N - \sum_{j=1}^{R} k_j + 1\right) + R \ln \frac{RN - \sum_{j=1}^{R}(R+1-j)k_j}{RN} \\ &= f_{1\mathrm{P}}(N) + f_{2\mathrm{P}}(N), \end{aligned} \tag{16}$$

where

$$f_{1\mathrm{P}}(N) = \Psi(N+1) - \Psi(N - \hat{N}_0 + 1), \tag{17}$$

$$f_{2\mathrm{P}}(N) = R \ln\left(1 - \sum_{j=1}^{R} \frac{R+1-j}{RN} k_j\right), \tag{18}$$

and $\Psi(x)$ is the digamma function [24] i.e., $\Psi(x) = \frac{d}{dx} \ln \Gamma(x)$ and $\Gamma(x) = (x-1)!$.

In order to see the behavior of the log likelihood function, the existence and the uniqueness of the solution $f_{\mathrm{P}}(N) = 0$ have to be considered by analyzing $f_{1\mathrm{P}}(N)$ and $f_{2\mathrm{P}}(N)$ in what follows.

Since we have $\frac{d}{dx}\Psi(x) = \sum_{n=0}^{\infty} \frac{1}{(x+n)^2}$ [24], we obtain

$$\frac{df_{1\mathrm{P}}(N)}{dN} = \sum_{m=1}^{N-\hat{N}_0} \frac{1}{m^2} - \sum_{m=1}^{N} \frac{1}{m^2} = -\sum_{m=N-\hat{N}_0+1}^{N} \frac{1}{m^2} < 0, \tag{19}$$

and we also have $f_{1\mathrm{P}}(N) > 0$ because the digamma function is monotonically increasing. Hence $f_{1\mathrm{P}}(N)$ is a positive monotonically decreasing function of N.

On the other hand, the differentiation of $f_{2\text{P}}(N)$ with respect to N gives

$$\frac{df_{2\text{P}}(N)}{dN} = R\frac{\sum_{j=1}^{R}\frac{R+1-j}{R}k_j}{1-\sum_{j=1}^{R}\frac{R+1-j}{RN}k_j}\frac{1}{N^2}. \tag{20}$$

Here, by using the fact that $N \geq \sum_{j=1}^{R} k_j (= \hat{N}_0)$, we have

$$1 \geq 1 - \sum_{j=1}^{R}\frac{R+1-j}{RN}k_j \geq 1 - \frac{\sum_{j=1}^{R}(R+1-j)k_j}{\sum_{j=1}^{R}Rk_j} \geq 0. \tag{21}$$

Therefore, $f_{2\text{P}}(N)$ is a negative monotonically increasing function of N.

Although $\sum_{m=1}^{N}\frac{1}{m^2}$ in (19) is intractable[1], the negative of $\frac{df_{1\text{P}}(N)}{dN}$ can be upper and lower bounded as

$$\int_{N-\hat{N}_0+1}^{N}\frac{1}{x^2}dx < \sum_{m=N-\hat{N}_0+1}^{N}\frac{1}{m^2} < \int_{N-\hat{N}_0}^{N}\frac{1}{x^2}dx,$$

or equivalently

$$\frac{\hat{N}_0-1}{N(N-\hat{N}_0+1)} < \sum_{m=N-\hat{N}_0+1}^{N}\frac{1}{m^2} < \frac{\hat{N}_0}{N(N-\hat{N}_0)}. \tag{22}$$

Thus, by using (20), (22) and L'Hôpital's rule [26], we have

$$\begin{aligned}\lim_{N\to\infty}\frac{-f_{1\text{P}}(N)}{f_{2\text{P}}(N)} &= \lim_{N\to\infty}\frac{\frac{-d}{dN}f_{1\text{P}}(N)}{\frac{d}{dN}f_{2\text{P}}(N)} < \lim_{N\to\infty}\frac{\frac{\hat{N}_0}{N(N-\hat{N}_0)}}{R\frac{\left[\hat{N}_0+\sum_{j=1}^{R}\frac{1-j}{R}k_j\right]}{N\left(N-\left[\hat{N}_0+\sum_{j=1}^{R}\frac{1-j}{R}k_j\right]\right)}} \\ &= \frac{\hat{N}_0}{\hat{N}_0+R\hat{N}_0-\sum_{j=1}^{R}jk_j} \leq 1. \end{aligned} \tag{23}$$

The inequality in the last line holds since $R\hat{N}_0 > \sum_{j=1}^{R} jk_j$. Therefore, we obtain $\lim_{N\to\infty}\frac{-f_{1\text{P}}(N)}{f_{2\text{P}}(N)} < 1$ and hence $f_{\text{P}}(N)$ has a negative value as $N \to \infty$.

Then, we consider the shape of $f_{\text{P}}(N)$ from the derivative. By using (20) and (22) again, we obtain

$$\frac{df_{\text{P}}(N)}{dN} < \frac{df_{2\text{P}}(N)}{dN} - \frac{\hat{N}_0-1}{N(N-\hat{N}_0+1)} = \frac{A}{B},$$

[1] $\sum_{m=1}^{\infty}\frac{1}{m^2}$ is known as the Basel problem and the value is known to be $\pi^2/6 = \zeta(2)$, where $\zeta(s)$ is the Riemann zeta function defined as $\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s}$ [25].

where $A = N\left(R\hat{N}_0 - \sum_{j=1}^{R} jk_j + 1\right) - (\hat{N}_0 - 1)(R-1)(\hat{N}_0 + \sum_{j=1}^{R} \frac{1-j}{R} k_j)$ and $B = N(N - \hat{N}_0 - \sum_{j=1}^{R} \frac{1-j}{R} k_j)(N - \hat{N}_0 + 1)$. It is observed that $B > 0$ for $N \geq \hat{N}_0$ and $R\hat{N}_0 - \sum_{j=1}^{R} jk_j + 1 > 0$. Thus, $\frac{df_{\mathrm{P}}(N)}{dN} < 0$ if $N \leq N^*$, where

$$N^* = \frac{(\hat{N}_0 - 1)(R-1)(\hat{N}_0 + \sum_{j=1}^{R} \frac{1-j}{R} k_j)}{R\hat{N}_0 - \sum_{j=1}^{R} jk_j + 1}. \tag{24}$$

Moreover, we also obtain

$$\frac{df_{\mathrm{P}}(N)}{dN} > \frac{df_{2\mathrm{P}}(N)}{dN} - \frac{\hat{N}_0}{N(N - \hat{N}_0)} = \frac{C}{D},$$

where $C = N\left(R\hat{N}_0 - \sum_{j=1}^{R} jk_j\right) - \hat{N}_0(R-1)(\hat{N}_0 + \sum_{j=1}^{R} \frac{1-j}{R} k_j)$ and $D = N(N - \hat{N}_0 - \sum_{j=1}^{R} \frac{1-j}{R} k_j)(N - \hat{N}_0)$. In this case, $\frac{df_{\mathrm{P}}(N)}{dN} \geq 0$ if $N \geq N^{**}$, where

$$N^{**} = \frac{\hat{N}_0(R-1)(\hat{N}_0 + \sum_{j=1}^{R} \frac{1-j}{R} k_j)}{R\hat{N}_0 - \sum_{j=1}^{R} jk_j} \tag{25}$$

Therefore, $f_{\mathrm{P}}(N)$ is monotonically decreasing for $N < N^*$ and is monotonically increasing for $N > N^{**}$. It is also easily verified that $N^{**} > N^* > 0$ and $N^{**} > \hat{N}_0$ while $N^* \approx N^{**}$ for sufficiently large $\hat{N}_0$.

In summary, assuming sufficiently large $\hat{N}_0$, we can conclude that $f_{\mathrm{P}}(N)$ has a unique minimum at $N^* \approx N^{**}$ in the region $N > 0$ and has a negative value as $N \to \infty$. This means that, if $f_{\mathrm{P}}(\hat{N}_0) > 0$, the equation $f_{\mathrm{P}}(N) = 0$ has a unique solution N^{***} where $N^{***} > \hat{N}_0$. In other words, the log likelihood function (14) is monotonically increasing with respect to $\hat{N}_0 \leq N < N^{***}$ and then is monotonically decreasing with respect to $N > N^{***}$ after reaching to the maximum value at $N = N^{***}$. On the other hand, if $f_{\mathrm{P}}(\hat{N}_0) \leq 0$, $f_{\mathrm{P}}(N)$ is negative in all the range $N \geq \hat{N}_0$ and hence, the log likelihood function is monotonically decreasing with respect to $N \geq \hat{N}_0$. It implies that the log likelihood function obtains the maximum value at $N = \hat{N}_0$. Thus, we evaluate the value of the log likelihood function for each discrete N in an increasing order starting from $N = \hat{N}_0$, and, if we observe the decrease of the likelihood, then N in the previous step is selected as the estimated value $\hat{N}$. We call this method MLSC.

4 Simulation Results

In this section, we will show the performance of the proposed methods under different system parameters via computer simulations. The performance of the methods is also compared with that of REGM method [19,20]. The simulation results are obtained by Monte Carlo method with $S = 10000$ runs.

We first plot the function $f_{\mathrm{P}}(N)$ in MLSC for a certain simulation run in Figures 2 and 3 with $p = 0.2$ and R =2, 4, 8, where the actual tag set cardinalities are set to be 10 and 100, respectively. We can see that $f_{\mathrm{P}}(N) < 0$ as N gets large for all the cases, which supports the validity of our analysis. The value of N at the leftmost point of

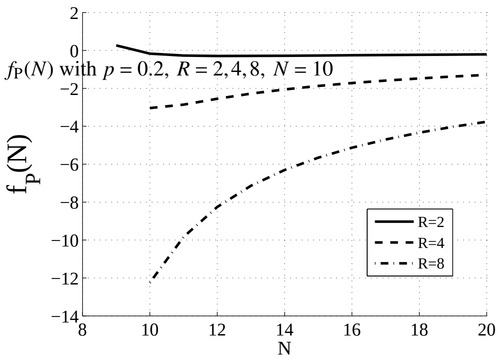

Fig. 2 $f_P(N)$ with $p = 0.2$, $R = 2,4,8$, $N = 10$

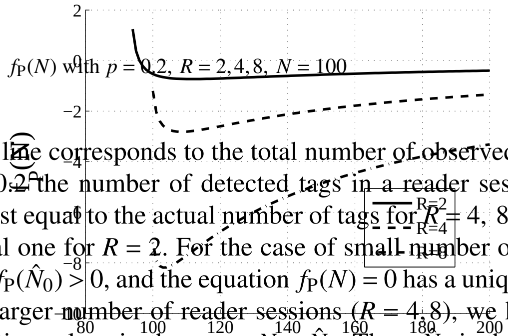

Fig. 3 $f_P(N)$ with $p = 0.2$, $R = 2,4,8$, $N = 100$

each line corresponds to the total number of observed evidences $\hat{N}_0$. Note that, since $p = 0.2$ the number of detected tags in a reader session is large, and hence, $\hat{N}_0$ is almost equal to the actual number of tags for $R = 4,\ 8$, however, it does not obtain the actual one for $R = 2$. For the case of small number of reader sessions R, we can see that $f_P(\hat{N}_0) > 0$, and the equation $f_P(N) = 0$ has a unique solution in the range $N > \hat{N}_0$. For larger number of reader sessions ($R = 4, 8$), we have $f_P(\hat{N}_0) < 0$, and $f_P(N)$ has negative values in the range $N > \hat{N}_0$. Thus, $\hat{N}_0$ is the optimal estimate of N, which accurately reflects the actual number of tags. It should be noted that, although our analysis is valid only when $\hat{N}_0$ is large enough, the results could be applicable for the case of relatively small $\hat{N}_0$, because $N = 10$ means $\hat{N}_0 \leq 10$ in Figure 2.

In Figure 4, we show the root mean-square-error (RMSE) performance of the estimated probability obtained by the proposed methods and the conventional REGM method with $p = 0.2$ and $N = 10$. The RMSE is defined as

$$e_p = \sqrt{\frac{1}{S}\sum_{i=1}^{S}(\hat{p}_i - p)^2}, \tag{26}$$

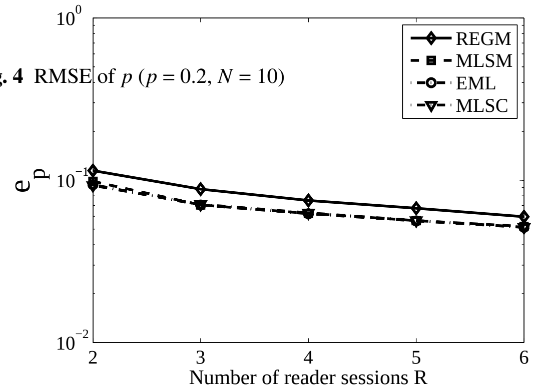

Fig. 4 RMSE of p (p = 0.2, N = 10)

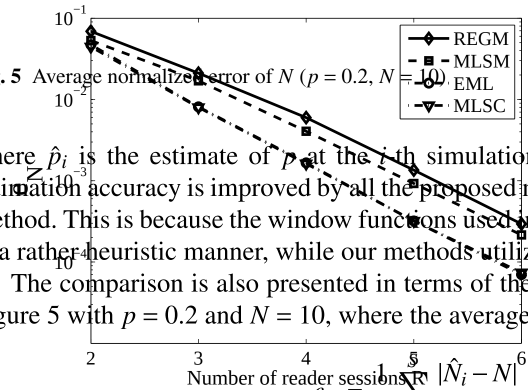

Fig. 5 Average normalized error of N (p = 0.2, N = 10)

where $\hat{p}_i$ is the estimate of p at the i-th simulation run. We can observe that the estimation accuracy is improved by all the proposed methods compared to the REGM method. This is because the window functions used in REGM method are determined in a rather heuristic manner, while our methods utilize ML approach.

The comparison is also presented in terms of the estimation of the cardinality in Figure 5 with $p = 0.2$ and $N = 10$, where the average normalized error is given by

$$e_N = \frac{1}{S}\sum_{i=1}^{S} \frac{|\hat{N}_i - N|}{N}, \tag{27}$$

where $\hat{N}_i$ is the estimate of N at the i-th simulation run. From the figure, we can see that MLSM and conventional REGM have almost the same performance, while MLSM achieves better estimation accuracy of p. The reason is that they share the same estimation method of Equation (5) for $\hat{N}$, and the improvement of $\hat{p}$ does not have a large impact on $\hat{N}$. However, MLSM method requires much smaller number of computations than that of REGM method, where $\hat{p}$ is computed by numerical least

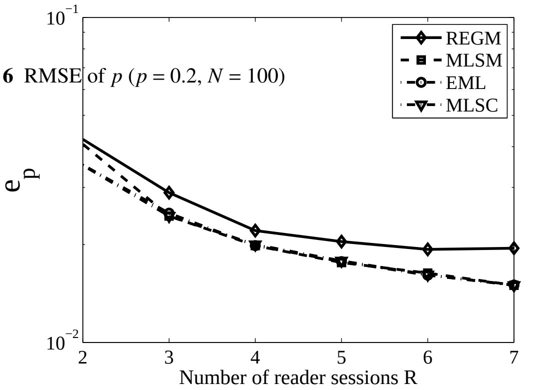

Fig. 6 RMSE of p ($p = 0.2$, $N = 100$)

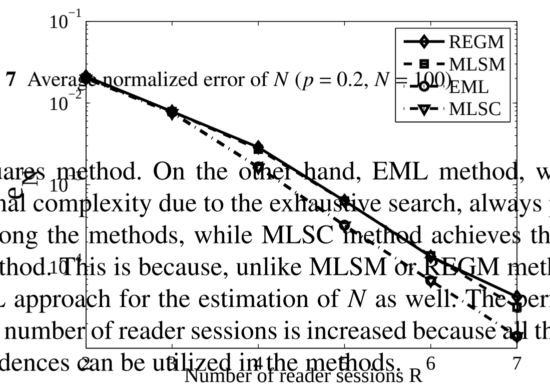

Fig. 7 Average normalized error of N ($p = 0.2$, $N = 100$)

squares method. On the other hand, EML method, which requires high computational complexity due to the exhaustive search, always presents the best performance among the methods, while MLSC method achieves the same performance as EML method. This is because, unlike MLSM or REGM methods, EML and MLSC utilize ML approach for the estimation of N as well. The performance gain gets greater as the number of reader sessions is increased because all the information of the observed evidences can be utilized in the methods.

We also evaluate the performance of the methods for a larger number of tags ($N = 100$) in Figures 6 and 7. We can observe the same tendency as in the case of $N = 10,\ p = 0.2$, however, there is a performance degradation in e_p of MLSM for $R = 2$, which is shown in Figure 6. This is because $\hat{N}_0$ is not an accurate estimate of N due to its large value and small number of reader sessions. Besides, the comparison of computational complexity (CC) between MLSC and EML is also shown in Table 1 where $p = 0.2,\ N = 10,\ 100$. The results tell us that as the searching range is fixed ($N_{max} = 20,\ 170$), the computational complexity of MLSC is much smaller than that

Table 1 Computational complexity of MLSC compared to EML (%) for $p = 0.2$

Case:	$CC_{MLSC}/CC_{EML}(\%)$: $R = 2 \to 7$					
$N = 10$ ($N_{max} = 20$)	9.95	2.88	2.43	2.43	2.43	2.43
$N = 100$ ($N_{max} = 170$)	5.92	1.71	1.41	1.40	1.40	1.40

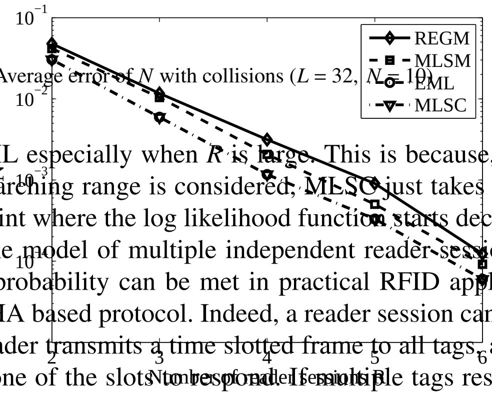

Fig. 8 Average error of N with collisions ($L = 32$, $N = 10$)

of EML especially when R is large. This is because, unlike EML method where all the searching range is considered, MLSC just takes a few of searching steps to find the point where the log likelihood function starts decreasing.

The model of multiple independent reader sessions with the identical detection error probability can be met in practical RFID applications using a framed-slotted ALOHA based protocol. Indeed, a reader session can be considered as a process that the reader transmits a time slotted frame to all tags, and then, each tag randomly selects one of the slots to respond. If multiple tags respond in the same slot, collision happens and hence tags in the slot are unreadable, which results in the identical detection error for every tag in every reader session assuming that every tag responds to the reader's request in every reading round. The average error of N of all the methods using the framed-slotted ALOHA based protocol with collisions is evaluated in Figure 8, in which a frame size L and the actual number of tags are set to 32 and 10, respectively. The expected value of the number of singleton slots $\bar{S}$ (slots with one transmission) is determined by $\bar{S} = N(1-1/L)^{N-1}$ [13]. Therefore, the detection error probability is found as

$$p = \frac{N-\bar{S}}{N} \approx 0.25. \tag{28}$$

We can see that the proposed methods also outperform the conventional REGM in term of estimation accuracy of N.

The assumption of independent reader session will be also valid in time-selective fading environments, which could be seen in practical indoor applications with moving target tags typically on conveyor belt systems. In RFID indoor applications, the

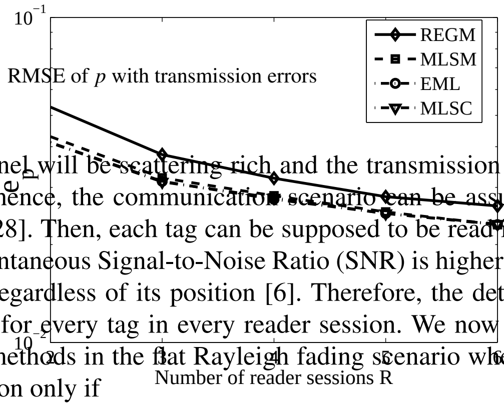

Fig. 9 RMSE of p with transmission errors

channel will be scattering-rich and the transmission is considered to be short range, and hence, the communication scenario can be assumed to be flat Rayleigh fading [27,28]. Then, each tag can be supposed to be read in each reader session only if its instantaneous Signal-to-Noise Ratio (SNR) is higher than the tag's sensitivity threshold regardless of its position [6]. Therefore, the detection error probability is identical for every tag in every reader session. We now evaluate the performance of all the methods in the flat Rayleigh fading scenario where tag j is identified in a reader session only if

$$|h_j|^2 > \eta, \tag{29}$$

where h_j is the tag j's channel gain and $h_j \sim \mathcal{CN}(0,1)$, and η is a given threshold set to 0.2 in what follows. In the channel model, the probability p that a tag is not read in a reader session is identical for every tag, and it can be calculated as

$$p = \Pr\left(|h_j|^2 \leq \eta\right) = \int_0^{\eta} e^{-x} dx = 1 - e^{-\eta}. \tag{30}$$

The performance of all the methods is plotted in Figures 9 and 10. We can see that the proposed methods also give more accurate estimates of p and N than the conventional REGM method in the flat Rayleigh fading scenario as well.

Moreover, in Figure 11, we show the average error of N of all methods in both the framed-slotted ALOHA based protocol with collisions and the flat fading channel with transmission errors, where the frame size, the actual number of tags and the threshold are set to 32, 10 and 0.1 respectively. In particular, each tag responds to the reader when its instantaneous channel power is greater than the threshold, and if the tag responds, it randomly selects one of the slots to transmit its ID. The detection error probability p can be easily obtained by using Equations (28) and (30). Once again, we can see that our methods show a better performance compared to REGM.

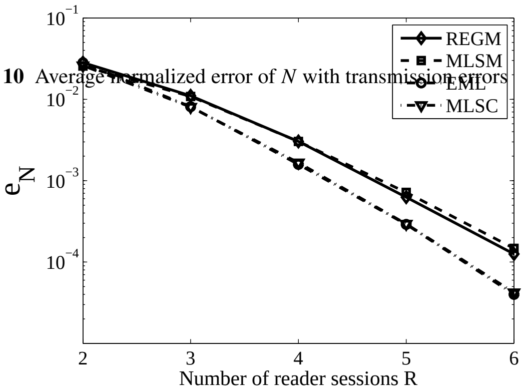

Fig. 10 Average normalized error of N with transmission errors

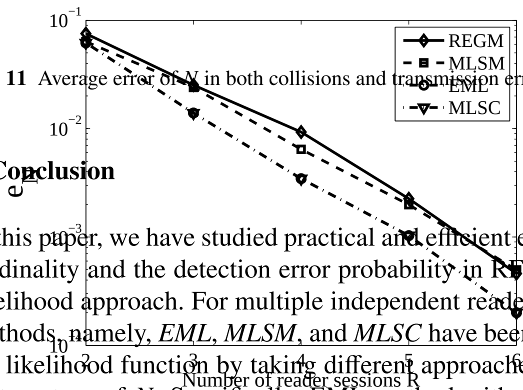

Fig. 11 Average error of N in both collisions and transmission errors ($L = 32,\ N = 10,\ \eta = 0.1$)

5 Conclusion

In this paper, we have studied practical and efficient estimation methods of the tag set cardinality and the detection error probability in RFID systems using the maximum likelihood approach. For multiple independent reader sessions model, three different methods, namely, *EML*, *MLSM*, and *MLSC* have been proposed in order to maximize the likelihood function by taking different approaches in the way of treating the discrete nature of N. Specifically, EML method with the exhaustive search algorithm achieved the best performance among the three, however, it required high computational complexity. To reduce the computational complexity, two methods were proposed where the first one was MLSM in which $\hat{N}$ was obtained by the sample mean of the observed evidences. The other method, MLSC, utilized the continuous relaxation for N to analyze the behavior of the log likelihood function and then, gave a stop-searching criterion. Computer simulation results showed that the estimates of p and N of the proposed methods were more accurate than those of the conventional

REGM method, which also implied that the proposed methods enabled us to determine the number of required reader sessions accurately in order to meet a required missing tag probability. In particular, MLSM method achieved a slightly better estimate of N than REGM method for small number of tags N, while MLSM method outperformed REGM method in terms of the accuracy of $\hat{p}$ regardless of N. MLSC method achieved the same performance as EML method, providing the performance bound for the ML approach. Moreover, the proposed methods achieved similar gains compared to the REGM method under a more practical scenario with flat Rayleigh fading and framed-slotted ALOHA based protocol with collisions. In future work, we intend to consider the proposed approach with framed-slotted ALOHA based protocol under both capture effect and detection errors phenomena, where tags might be detected even in collision time slots, while they might not be identified in singleton slots.

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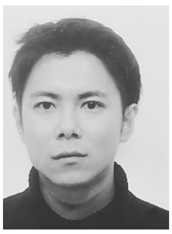

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