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Kyoto University
On the Probabilities of a waitingtime between the Landworkers and Tractor-transportations in the Landing and Loggingwork.

Koichi KANZAKI

I) Introduction

The logging work with a tractor is divided into three steps:

1) works in the place of felling
2) hauling with a tractor
3) works in the landing.

The kinds of works of each step are different according to the ways of hauling timbers; as the whole tree length or as logs. For example, in case of the former, the works in the felling place are felling and hooking, whereas in case of the latter felling, hookin and conversion (making logs) as well.

Let us consider about the waiting states of the workers in a landing and a tractor.

When a load of timbers is brought to a landing, if any workers are there who are not engaged with other works, it will be disposed at once, but if there are already other timbers and all the workers are engaged, it will be left there and wait its turn of disposal. And when the landing has been filled with timbers which has not yet been disposed... I call this undisposed timber hereafter in this text..., the tractor cannot continue the work any more and be obliged to get into the state of waiting. On the other hand, when there are not any undisposed timbers in the landing, all the workers are in a waiting state, though this is not always happening.

Here comes a problem; what is the probability that each state of waiting occurs? I shall deal with this problem in this text. I take the theory of Stochastic Processes for this problem.

II) The preparation from the theory by D. G. Kendall.

This section is a forest version from Kendall's papers; "Some Problems in the Theory of Queues". (in Journal of the Royal Statistical Society, series B, Vol. XIII, No. 2, 1951)

First of all, I must describe several assumptions for the theory of D. G. Kendall.

It is here assumed that all the workers in the landing must be engaged in disposing of every load of timbers at once without exception, so nobody touches any other loads during the disposal.

Considering the matter from the viewpoint of Macro, this assumption is not conflicted
Suppose that each load brought to the landing is disposed in the order of their arrivals, and there will be a quite number of loads waiting their turns.

Here, let $E_n$ represent for the state of the landing with the $n$ loads of timbers including the one being disposed. And then $E_0$ is for the empty landing when all the workers are waiting, and $E_1$ for the state with only one load being disposed, and so on.

The state $E_n$ is changes all the time. If it can be supposed that the arrivals of tractors belong to the distribution of Poissons type, it is possible to take the fluctuations of $E_n$ as an enumerable Markovian chain if the attention is directed to the epochs at which the disposal of each load is finished (Kendall and Bartlett called these epochs regeneration points).

When the hypothesis that a tractor arrives at random holds, then the number of arrivals in the time $t$ is a Poisson variability of expectation $t/a$, say:

$$P_r = \frac{\left( \frac{t}{a} \right)^r e^{-\frac{t}{a}}}{r!}$$

($P_r$: The Probability of arrivals $\gamma$ in the time $t$)

Note 1):

A sequence of $E_0, E_1, \ldots$, $E_n, \ldots$, is called a Markovian chain, if the probabilities of the states $E_m$ ($m=0, 1, \ldots$) occur at time $t+h$ when the states $E_n$ ($n=0, 1, \ldots$) at arbitrary time $t_0$ depend only on $E$ and have not any relations to the states at any time $t<t_0$.

Now let $a = \frac{1}{\lambda}$ represent for the expectation of the time interval $u$ between two consecutive arrivals of tractors, and $b$ for the expectation of $v$ which denotes the time which elapses while a certain load is being disposed; I call this $v$ the disposing time or the service time. And assume that $v$ has a distributing function $B(v)$.

Then the probability $k_r$ of $r$ arrivals in the time $v$ is

$$K_r = \frac{1}{r!} \int_0^v e^{-\frac{v}{a}} \left( \frac{v}{a} \right)^r d\beta(v)$$

(1)

These $k_r$'s ($r=0, 1, \ldots$) are the probability distribution, being $\sum k_r = 1$.

Let $P_{ij}$ represent for the transition-probability from $E_i$ to $E_j$, and the transition matrix ($P_{ij}$) is

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From the theorems of the theory of Stochastic Processes, the probabilities $u_j$ that the states $E_j$ occur without reference to the initial states will be gained solving the simultaneous the equations;
The equation (3) always holds good if the number of the states $E_n$ is finite, but, if infinite, (3) holds only if $\frac{b}{a} = \rho < 1$, and all $u_j = 0$ if $\rho \geq 1$.

Now, the generating function $k(z)$ of a probability distribution $k_r$ is, from the equation (1),

$$K(z) = \sum_{r=0}^{\infty} Z^r = \sum_{r=0}^{\infty} \left( \frac{z}{r!} \int_0^\infty e^{-\frac{v}{r} z} \frac{v}{r} dB(v) \right) = \int_0^\infty e^{-\frac{z}{r}} dB(v)$$

Then as the Laplace transformation of the disposing time distribution $dB(v)$ is

$$B(s) = \int_0^\infty e^{-sv} dB(v)$$

$$K(z) = \beta \left( \frac{1 - z}{a} \right)$$

Therefore, the function $k(z)$ and the sequence of $k_r$'s and consequently $U_r$'s can be determined, as soon as $dB(v)$ has been specified. And it is better to employ such as $dB(v)$ and can easily be made its Laplace transform.

III) On the service-time distribution function $B(v)$.

Kendall described in his papers especially about two cases of $B(v)$;

1. negative-exponential service-time,

$$dB(v) = e^{-\frac{b}{v}} d(v)$$

2. constant service-time,

$$B(v) = 0 \ (v < b); \quad B(v) = 1 \ (v \geq b)$$

The most remarkable feature of these two distribution functions is that they have only one parameter $b$. But these are not very suitable for the actual cases like the disposing time in a landing. Therefore, though it has two parameters, it should be better to use $\Gamma$-distribution in the $\chi^2$ form

$$dB(v) = ce^{-hv} v^{\nu-1} dv, \quad 0 < v < \infty$$

which was once employed by Erlang, that is seen in Kendall's papers. The equations (7) and (8) are the limiting forms of (9) when $\nu = 1$ and $\nu \to \infty$ respectively.

The expectation and variance of the distribution (9) are respectively

$$E(v) = b = \frac{\nu}{h} \quad \text{and} \quad \nu(v) = \frac{\nu}{h^2}$$

and its Laplace transformation is

$$\int_0^\infty e^{-sv} dB(v) = \frac{h^\nu}{(s + h)\nu}$$

The figures (1, 2, and 3) show the comparisons of the calculated values from the equation (9) with the actual cases which was investigated in the Nagano district.

It is seen in these figures that the more the number of data is, the better the approximation is.
IV) The waiting of the workers in a landing.

It is now supposed that the landing is capable of accommodating \( r + 1 \) loads of timbers, including the one being disposed, and that a tractor can not work any more as long as there are \( r + 1 \) loads in the landing. Consequently, the number of possible states \( E_n \) at a regeneration point (when the disposal of each load has just been finished) is \( r + 1 \); \( n = 0, 1, 2, 3, \ldots, r \), and any other states can not occur.
The formula will be as follows to compute the probabilities $U_n$ of $E_n$ ($n=0, 1, 2, \ldots, r$) at a regeneration point, taking (9) as the disposing-time distribution.

From the equation (6) and (11),

$$K(z) = \beta \left( \frac{1-z}{a} \right) = \frac{h^v}{(1-z+a)^v} \quad \text{[12]}$$

By expanding the generating function $k(z)$, equation (12), in powers of $z$,

$$K(z) = \frac{(ah)^v}{(1+ah)^v} \left\{ 1 + \frac{v(\nu+1)}{1+ah} z + \frac{\nu(\nu+1)}{2!(1+ah)^2} z^2 + \cdots + \frac{\nu(\nu+1)\cdots(\nu+n-1)}{n!(1+ah)^n} z^n + \cdots \right\}$$

$$= k_0 + k_1 z + k_2 z^2 + \cdots, \quad \text{[13]}$$

Therefore,

$$k_0 = \frac{(ah)^v}{(1+ah)^v} \quad \text{[14]}$$

$$k_n = \frac{(ah)^v \nu(\nu+1)\cdots(\nu+n-1)}{n!(1+ah)^{v+n}} \quad \text{[14]}$$

With the equation (14), the transition matrix $(p)$ is

$$\begin{array}{cccc}
0 & 1 & 2 & \cdots \ r \\
0 & k_0 & k_1 & k_2 & \cdots & k_r \\
1 & k_0 & k_1 & k_2 & \cdots & k_r & k'_{r-1} & k'_{r-2} & \cdots & k'_{1} \\
2 & 0 & k_0 & k_1 & k_2 & \cdots & k'_{r-1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
r & 0 & 0 & 0 & \cdots & k'_{1} \\
\end{array} \quad \text{[15]}$$

Here, $U_j$'s ($j=0, 1, \ldots, r$) are now able to be computed, with the equation (2), (3), and (15). $U_n$ is the probability of the waiting of the workers in the landing.

V) The waiting of a tractor.

Here, it will also be supposed the same condition as in IV) about the width of landing.

The $U_j$'s gained in the section IV) are the probabilities of $E_n$'s occurring at epochs when each disposing-time has just been finished (at the generation points). So the probability of the waiting states of a tractor (or the state that the landing is filled with timber) $E_{r+1}$ cannot be gained by the method above mentioned. And $E_n$ only indicates that the waiting state of tractor has just ended.

Then, how can we get the waiting time of a tractor? On my opinion, it must be convenient to compute it from equation (16) supposing that the amounts of imput and disposed timbers during a certain fairly long time are almost equal.

$$\frac{T}{b} (1-U_0) W = \frac{T}{a} (1-U_r) W$$

therefore,

$$U_r = 1 - \frac{a}{b} (1-U_0) \quad \text{[16]}$$

here,

$T$: a time

$W$: mean volume of a load hauled by a tractor
a: expectation of the intervals of the consecutive arrivals
b: mean disposing time (service time) of a load
Ur: probability caused by the tractor waiting

On the other hand, considering it theoretically, the computing equation is as follows.

By the condition that the input is Poissonian \( P_r = \frac{e^{-\lambda t}}{r!} \), when the attention is only directed to the regeneration point at which \( E_{r-n} \) appears, the waiting time of a tractor during the next disposing time \( v \) of a load is

\[
\int_0^v \left( 1 - e^{-\lambda t} \left( 1 + \lambda t + \cdots + \frac{(\lambda t)^n}{n!} \right) \right) (v-t) \, dt
\]

Therefore, the rate of waiting of a tractor (the probability of waiting) during the same time is

\[
R_{r-n} = \int_0^v \left( 1 - e^{-\lambda t} \left( 1 + \lambda t + \cdots + \frac{(\lambda t)^n}{n!} \right) \right) (v-t) \, dt \, dB(v)
\]

Here, we can get \( U_r \) (the probability of tractor waiting without reference to the initial condition) using the equation (18) and \( U_j \) \((j=0,1, \ldots, r)\) gained above.

\[
U_r = \sum_{k=0}^r U_{r-k} R_{r-k}
\]

VI) miscellany

What is above mentioned stands on the assumption that a tractor arrives at a landing at random (Poissonian input). But actually there are often the cases where the assumption is not correct. For instance, when only one tractor works, it is naturally supposed that the intervals of consecutive arrivals have the distribution in the \( X^2 \) form, although, if many tractors work, the assumption must nearly be correct.

However, these problems have not yet been discussed thoroughly so far.

要　約

トラクター集材作業においては、集材工場において土場作業員とトラクター作業員との間に作業手待ちを生ずる。

今土場の広さが \( r+1 \) 荷分（トラクター 1 台が 1 回に集材する構の量を 1 荷と呼ぶ）であるとする。そして、土場が、\( n \) 荷分だけ未処理材でつまった状態を \( E_n \) であらわし、その状態の起る確率を \( P_n \) であらわせば、\( P_r \) は、土場作業員の手持ちの起る確率をあらわす。

トラクター到着間隔がランダムと考えられるときには、\( P_n (n=0,1, \ldots, r) \) は、次の連立方程式によって計算できる。

\[
\begin{align*}
AP &= P \\
\sum_{j=0}^r P_j &= 1
\end{align*}
\]

ここに

\[
P = \begin{pmatrix} P_0 \\ P_1 \\ \vdots \\ P_r \end{pmatrix}, \quad A = \begin{pmatrix} k_0 & k_0 & 0 & 0 & \cdots & 0 \\ k_1 & k_1 & k_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ k_r & k_r & k_{r-1} & \cdots & \cdots & k_0 \end{pmatrix}, \quad k_0 = \frac{(ah)^r}{(1+ah)^r}
\]
ここで、トラクター到着時間間隔の平均値を \( a \) とした。また土場における 1 荷処理時間の平均を \( E(\nu) \)、分散 \( V(\nu) \) とすれば

\[
E(\nu) = \frac{\nu}{h} \quad V(\nu) = \frac{\nu}{h^2}
\]

である。
なお、トラクター手待ち確率 \( P_{\nu-1} \) は、\( P_0 \) と \( E(\nu) \) と \( a \) を用いて本文 (17) 式から間接的に計算できる。