A Many-valued Modal Interpretation for Catuskoti

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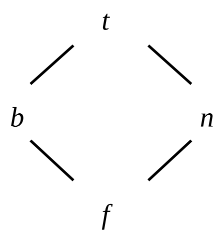
1. Introduction

Catuskoti or tetralemma (四句破，四句否定) is likely to be familiar to any reader of Buddhist philosophical literature. Generally speaking, for any proposition, there is an enumeration of four alternatives or possibilities: that it holds, that it fails to hold, that it both holds and fails to hold, that it neither holds nor fails to hold. The catuskoti is also one of the puzzling features in Buddhist philosophy, since the application of catuskoti is not uniform and incoherence: sometimes one of the four possibilities is chosen as the right one; sometimes, all of the four possibilities are rejected; sometimes all of the four possibilities are affirmed.

Priest (2010) has given a many-valued analysis about catuskoti. This paper tries to develop Priest's analysis into a modal version. In Section 2 and 3, I will give brief summary of Priest's many-valued interpretation for catuskoti. In Section 4, I will point out a problem in Priest's analysis by a citation from the original text. In Section 5 and Section 6, I will show my argument about a many-valued modal interpretation for catuskoti.

2. Many-valued Logic

First Degree Entailment (FDE) is a kind of relevant logic. Its syntax is the same as that of classical logic. And we take negation, disjunction and conjunction as primitives, while implication and equivalence can be defined in the usual way. From the semantic perspective, FDE can be regarded as a four-valued logic; that is, the set of values is {*t*, *b*, *n*, *f*}. Here, *t* means true only; *b* means both true and false; *n* means neither true nor false; *f* means false only. The interpretation for FDE is a structure <*V*, *D*, *v*>, where *V* is the set of values, *D* is the set of designated values, and *v* is a valuation. These values can be ordered in the following way:



A valuation v is a function from formulas to $\{t, b, n, f\}$, and it satisfies the following clauses:

(1) $v(A) \in \{t, b, n, f\}$, for all atomic formula A.

(2) $v(\neg A) = t$, if $v(A) = f$;

$v(\neg A) = b$, if $v(A) = b$;

$v(\neg A) = n$, if $v(A) = n$;

$v(\neg A) = f$, if $v(A) = t$.

(3) $v(A \vee B) = lub\{v(A), v(B)\}$

(4) $v(A \wedge B) = glb\{v(A), v(B)\}$

Here *lub* and *glb* are *least upper bound* and *greatest lower bound* respectively. That is, the truth conditions for disjunction and conjunction give the following truth table:

$\vee$	t	b	n	f
t	t	t	t	t
b	t	b	t	b
n	t	t	n	n
f	t	b	n	f

$\wedge$	t	b	n	f
t	t	b	n	f
b	b	b	f	f
n	n	f	n	f
f	f	f	f	f

Let $\{t, b\}$ be the set of designated values. A formula A is satisfied if and only if there is a valuation v such that $v(A) \in \{t, b\}$; A formula A is valid if and only if, for any valuation v, $v(A) \in \{t, b\}$. For more details about FDE, see Priest(2008, pp. 146-149).

3. Many-valued Interpretation for Catuskoti

Priest(2010) suggests that it be natural to use truth and falsity predicates **T** and **F** to express the four alternatives (true, false, both or neither) in catuskoti. That is,

$T\langle A\rangle$ is $\mathbf{T}\langle A\rangle \wedge \neg\mathbf{F}\langle A\rangle$

$F\langle A\rangle$ is $\neg\mathbf{T}\langle A\rangle \wedge \mathbf{F}\langle A\rangle$

$B\langle A\rangle$ is $\mathbf{T}\langle A\rangle \wedge \mathbf{F}\langle A\rangle$

$N\langle A\rangle$ is $\neg\mathbf{T}\langle A\rangle \wedge \neg\mathbf{F}\langle A\rangle$

where < > is a name-forming operator, and $<A>$ is the name of A. T, F, B and N are called status predicates. $\mathbf{F}<A>$ is defined as $\mathbf{T}<\neg A>$. Thus, $T<A>$, $F<A>$, $B<A>$ and $N<A>$ are actually as follows:

$$T<A> \text{ is } \mathbf{T}<A> \wedge \neg\mathbf{T}<\neg A>$$
$$F<A> \text{ is } \neg\mathbf{T}<A> \wedge \mathbf{T}<\neg A>$$
$$B<A> \text{ is } \mathbf{T}<A> \wedge \mathbf{T}<\neg A>$$
$$N<A> \text{ is } \neg\mathbf{T}<A> \wedge \neg\mathbf{T}<\neg A>$$

The semantic conditions for the truth predicate are as follows:

If the value of A is designated, then $\mathbf{T}<A>$ is t or b
If the value of A is not designated, then $\mathbf{T}<A>$ is f

Then, according to the definitions of status predicates and the semantic conditions for the truth predicate, it is easy to get the following table:

A	$T<A>$	$B<A>$	$N<A>$	$F<A>$
t	t or b	f	f or b	f
b	b or f	t or b	f or b	b or f
n	f	f	t	f
f	f	f	f or b	t or b

The diagonal in the above table shows that the statement, which says that A takes some value, is designated if A takes that value.

Priest argues that catuskoti requires that the four alternatives should be mutually exclusive and exhaustive. That is, catuskoti can be formulated as the following two schemas:

$$C_1: T<A> \vee F<A> \vee B<A> \vee N<A>$$
$$C_2: \neg(S_1<A> \wedge S_2<A>)$$

where S_1 and S_2 are any distinct status predicates. Priest also shows that the many-valued semantics for FDE validates the above formal statements of the catuskoti, that is, whatever value *A* takes, the values of C_1 and C_2 are designated.

4. Back to the Text

Priest has done an excellent job to formulate catuskoti in the framework of FDE; however, there are other points that Priest ignores to formulate. In the original text about catuskoti, the historical Buddha does not give four answers to the question whether the saint exists after death; rather, he believes so:

> How is it, Gotama? **Does Gotama hold** that the saint exists after death, and that this view alone is true, and every other false?
> Nay, Vacca. **I do not hold** that the saint exists after death, and that this view alone is true, and every other false.
> How is it, Gotama? **Does Gotama hold** that the saint does not exist after death, and that this view alone is true, and every other false?
> Nay, Vacca. **I do not hold** that the saint does not exist after death, and that this view alone is true, and every other false.
> How is it, Gotama? **Does Gotama hold** that the saint both exists and does not exist after death, and that this view alone is true, and every other false?
> Nay, Vacca. **I do not hold** that the saint both exists and does not exist after death, and that this view alone is true, and every other false.
> How is it, Gotama? **Does Gotama hold** that the saint neither exists nor does not exist after death, and that this view alone is true, and every other false?
> Nay, Vacca. **I do not hold** that the saint neither exists nor does not exist after death, and that this view alone is true, and every other false. (Radhakrishnan and Moore 1957, p. 289, my emphasis)

That is, Gotama does not believe (hold) that the saint exists after death; he does not believe that the saint does not exist after death; he does not believe that the saint both exists and does not exist after death; he does not believe that the saint neither exists nor does not exist after death. And according to Priest's analysis, these four possibilities should be exclusive and exhaustive. Therefore, we should formulate belief in catuskoti.

5. Many-valued Modal Logic

In modal logic, each proposition is a function from possible worlds to two values (true and false); that is, whether a proposition is true or false is relative to a possible world. A proposition A is necessarily true at world w if and only if, for any world u from which w is accessible, A is true at world u; a proposition A is possibly true at world w if and only if there is a world u from which w is accessible such that A is true at world u.

However, there is no reason that there are only two values for the worlds. Just as the classical logic can be developed into many-valued logics, modal logic can also be developed into many-valued modal logics.

K_{FDE} is the modal version of FDE. The syntax of K_{FDE} is the same as that of FDE, except that the former is augmented by monadic operators $\Box$ and $\Diamond$ in the usual way. An interpretation for K_{FDE} is a structure $<W, R, S, v>$. W is the set of possible worlds; R is an accessible relation on W; S is a structure for FDE; and v is a valuation. Here, a valuation is a binary function from formulas and possible worlds to $\{t, b, n, f\}$; that is, v assigns a formula a value relative to a world. A valuation satisfies the following clauses:

(1′) $v(A, w) \in \{t, b, n, f\}$, for all atomic formula A.

(2′) $v(\neg A, w) = t$, if $v(A, w) = f$;
$v(\neg A, w) = b$, if $v(A, w) = b$;
$v(\neg A, w) = n$, if $v(A, w) = n$;
$v(\neg A, w) = f$, if $v(A, w) = t$.

(3′) $v(A \vee B, w) = lub\{v(A, w), v(B, w)\}$

(4′) $v(A \wedge B, w) = glb\{v(A, w), v(B, w)\}$

(5′) $v(\Box A, w) = glu\{v(A, u): Rwu\}$

(6′) $v(\Diamond A, w) = lub\{v(A, u): Rwu\}$

A formula A is satisfied relative to w if and only if there is a valuation v such that $v(A, w) \in \{t, b\}$; A formula A is valid relative to w if and only if, for any valuation v, $v(A, w) \in \{t, b\}$. It should be noted that there is no logical truth in K_{FDE} and its normal extensions. For more details about K_{FDE}.

(see Priest 2008, pp. 241-242 and pp. 244-247).

6. Belief Interpretation for Catuskoti

Epistemic and doxastic logics are developed from modal logic; that is, knowledge and belief can be interpreted in terms of the necessity operator □. Usually, 'κA' means that it is known that *A*, or that the agent knows that *A*. 'βA' means that it is believed that *A* or the agent believes that *A*. The semantic clause for κ is as follows:

$$(\kappa) \text{ the value of } \kappa A\text{, relative to } w\text{, is true, if and only if,}$$
$$\text{for all } u \text{ with } Rwu\text{, the value of } A\text{, relative to } u\text{, is true}$$

It states that, in an epistemically alternative world *w*, it is known (by the agent) that the formula *A* is true if and only if *A* is true in all worlds *u* that the agent deems epistemic alternatives. The semantic clause for β is similar to that of κ:

$$(\beta) \text{ the value of } \beta A\text{, relative to } w\text{, is true, if and only if,}$$
$$\text{for all } u \text{ with } Rwu\text{, the value of } A\text{, relative to } u\text{, is true}$$

However, since, in many-valued logic FDE, there are four truth-values rather than two values, the semantic clause for β goes as follows:

$$(\beta') \; v(\beta A, w) = glu\{v(A, u): Rwu\}$$

Here, the universal quantifier 'for all' in (β) is regarded as a infinite conjunction over all members of the domain; that is, $\forall u\phi(u)$ is regarded as $\phi(d_1) \wedge \phi(d_2) \wedge \phi(d_3)\ldots$, where $d_1, d_2, d_3\ldots$ are all the members of the domain. Then the infinite conjunction is regarded as the greatest lower bound of each conjuncts; that is, $\phi(d_1) \wedge \phi(d_2) \wedge \phi(d_3)\ldots$ is regarded as $glu\{u: \phi(u)\}$.

Within the framework of many-valued modal logic, the exclusiveness and exhaustiveness requirements in catuskoti should be formulated in the following two ways. One way goes as follows:

$$\mathrm{CBT}_1\text{: } \beta T\langle A\rangle \vee \beta F\langle A\rangle \vee \beta B\langle A\rangle \vee \beta N\langle A\rangle$$

$$\text{CBT}_2\text{: } \neg(\beta S_1\langle A\rangle \wedge \beta S_2\langle A\rangle)$$

Here, $\beta S_1\langle A\rangle$ means that it is believed that A falls into one of the four alternatives; for example, $\beta T\langle A\rangle$ means that *it is believed that it is true that A*. Another way to formulate the two requirements in catuskoti goes as follows:

$$\text{CTB}_1\text{: } T\langle\beta A\rangle \vee F\langle\beta A\rangle \vee B\langle\beta A\rangle \vee N\langle\beta A\rangle$$
$$\text{CTB}_2\text{: } \neg(S_1\langle\beta A\rangle \wedge S_2\langle\beta A\rangle)$$

Here, $S_1\langle\beta A\rangle$ means that the belief of A falls into one of the four alternatives; for example, $T\langle\beta A\rangle$ means that *it is true that it is believed that A*. The difference between the two ways of formulating catuskoti lies in whether truth (or truth value) is prior to belief or not. If truth is prior to belief, then the truth-bearers are beliefs rather than propositions; that is, for any belief or view, there are four possibilities or alternatives. On the other hand, if belief is prior to truth, then the truth bearers are propositions, and, for any proposition, it is believed that there are four alternatives.

According to the truth table for T, B, N, and F,

A	$T\langle A\rangle$	$B\langle A\rangle$	$N\langle A\rangle$	$F\langle A\rangle$
t	t or b	f	f or b	f
b	b or f	t or b	f or b	b or f
n	f	f	t	f
f	f	f	f or b	t or b

It can be easily checked that CTB_1, CTB_2 and CBT_2 are valid, but CBT_1 are not valid. To give a counterexample for CBT_1, consider the case that there are only two possible worlds w and u. To make matters simple, I assume that the accessible relation is total, that is, for any worlds w and v, Rwu. It should be noted that the total relation will lead to the logical omniscience problem; however, I suppose that this issue is not problematic for Buddha. Suppose the value of A in w is t, and the value of A in u is b. Then, the value of $\beta T\langle A\rangle$ is the greatest lower bound of {t or b, b or f}, that is, b or f; the value of $\beta B\langle A\rangle$ is f; the value of $\beta N\langle A\rangle$ is f or b; the value of $\beta F\langle A\rangle$ is f. The value of CBT_1 is the least upper bound of {b or f, f, f or b, f}, that is, b or f, which is not designated.

Semantically, we may interpret the difference between CTB_1 and CBT_1 in terms of possible worlds. The status predicates, *T*, *B*, *N*, and *F*, classify possible individuals in each world into four sets, and call them *T*-set, *B*-set, *N*-set and *F*-set. According to the exhaustive and exclusive requirements, in each world, any two of these sets are disjoint, and the union of the four sets is the domain of all possible individuals in that world. However, any of these sets may have different extensions in different possible worlds. For example, let *A* be a sentence which is both true and false in possible world *w* and false only in possible world *u*. Thus, the denotation of the name of the sentence <*A*> is a cross-world possible individual, that is, in the world *w*, the denotation of <*A*> belongs to *B*-set, but, in the world *u*, it belongs to *F*-set. This shows that CBT_1 is not valid in accordance with the many-valued modal interpretation. As for CTB_1, since the name-forming operator transforms sentence with belief operator into a name, the proof for the validity of CTB_1 is just the same as that for the validity of C_1.

We may conclude from the above that the modal semantics for catuskoti provides a way to combine Buddhist logic with Buddhist cosmology. We may further give a Buddhist cosmological interpretation for the ontological status for possible world; for example, different worlds may be interpreted as formless realm, form realm or desire realm. In this sense, Catuskoti is not only a logical problem but also an ontological or cosmological problem.

Reference

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