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“Time-Varying Wage Risk, Incomplete Markets, and Business Cycles”

Shuhei Takahashi

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Time-Varying Wage Risk, Incomplete Markets, and Business Cycles

Shuhei Takahashi[a,*]

[a] Institute of Economic Research, Kyoto University

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Abstract

Idiosyncratic wage risk exhibits cyclical variation. The present paper analyzes how such risk fluctuations affect business cycles using a heterogeneous-agent model with uninsured idiosyncratic wage risk and indivisible labor. I introduce risk fluctuations as uncertainty shocks and calibrate those shocks to the U.S. micro-level wage data. When moved by both uncertainty and aggregate TFP shocks, the model generates a weakly negative correlation between total hours worked and average labor productivity and large fluctuations in the labor wedge close to those in the U.S. economy. Without uncertainty shocks, hours and productivity comove strongly and the labor wedge varies little.

Keywords: Uninsured idiosyncratic wage risk; Indivisible labor; Uncertainty shocks; Hours-productivity correlation; Labor wedge

JEL classification: E32, E24, D31

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*Takahashi: Yoshidahonmachi, Sakyo-ku, Kyoto 606-8501, Japan. Email: takahashi@kier.kyoto-u.ac.jp, Tel: +81-75-753-7153. I am grateful to Aubhik Khan, Bill Dupor, and Julia Thomas for their comments, encouragement, and guidance. I would also like to thank Paul Evans, Tomoyuki Nakajima, Richard Rogerson, Ken Yamada, Tomoaki Yamada, an anonymous referee, the associate editor, the editor, and seminar/conference participants for their helpful comments. This work is supported by JSPS KAKENHI Grant Number 26780119.

1. Introduction

Idiosyncratic earnings risk exhibits cyclical fluctuations (Storesletten et al. (2004), Heathcote et al. (2010), and Guvenen et al. (2014)). Further, empirical and theoretical analyses find that an increase in wage uncertainty increases labor supply (Parker et al. (2005) and Flodén (2006)). However, the implication for labor market fluctuations has not been studied. How do changes in idiosyncratic wage uncertainty affect business cycles? The present paper examines the question quantitatively using a general equilibrium model.

The model analyzed here is built upon a heterogeneous-agent, incomplete asset markets model with indivisible labor (Chang and Kim (2006, 2007), Alonso-Ortiz and Rogerson (2010), and Krusell et al. (2010, 2011)). Individuals face idiosyncratic wage risk because idiosyncratic labor productivity changes stochastically. Individuals cannot fully insure against this risk because there is only one asset, physical capital. They partially self-insure by holding capital and making employment choice. Importantly, when the process for idiosyncratic productivity is calibrated to micro-level wage data, the model generates realistic heterogeneity in labor earnings and wealth across individuals.

The present paper introduces risk variation into the model using uncertainty shocks in the spirit of Bloom (2009), i.e., time-varying standard deviation of idiosyncratic productivity shocks. I measure the variation in idiosyncratic wage risk in the U.S. using the Panel Study of Income Dynamics (PSID) and calibrate uncertainty shocks to the empirical evidence.

Introducing uncertainty shocks substantially improves the present model's ability in accounting for the U.S. labor market dynamics. One improvement is seen in the correlation between total hours worked and average labor productivity (output per labor hour). With uncertainty and aggregate TFP shocks, the model generates a weakly negative correlation close to that in the U.S. (–0.17 in the model compared with –0.32 in the U.S.).[1] In contrast,

[1] The data on total hours worked is taken from Cociuba et al. (2009). As Shimer (2010) argues, it is the most comprehensive data on hours worked. The data used here is for 1947Q3–2009Q3. The weakly negative correlation after 1984 is widely documented. Gali and Gambetti (2009) find a slightly positive correlation

the model without uncertainty shocks produces a strong, positive correlation (0.83). Hence, introducing varying risk breaks the counterfactually strong hours-productivity comovement.

The other improvement appears in the volatility in the labor wedge. The labor wedge is the gap between average labor productivity and the marginal rate of substitution of leisure for consumption in a representative-agent setting and it is volatile in the U.S. (Chari et al. (2007) and Shimer (2010)). With uncertainty and aggregate TFP shocks, the present model generates the volatility of the labor wedge that is 70% of that in the U.S. The number is only 17% without uncertainty shocks. Thus, introducing risk variation helps the model account for a substantial portion of the labor wedge volatility seen in the U.S. economy.

The main mechanism is the (ex-post) *distribution effect*. As the volatility in idiosyncratic productivity shocks increases, the mobility between different productivity increases. This reduces the positive productivity-wealth correlation, which arises from the persistence in productivity, and it changes the wealth distribution *conditional on* productivity. In particular, the wealth distribution for the low productivity shifts towards larger wealth. Since given productivity individuals below a certain level of wealth choose to work, the low-productivity employment decreases. In contrast, the wealth distribution for the high productivity shifts towards smaller wealth. However, they are so productive that their wealth threshold for employment is near the top tail of the distribution. Thus, the number of individuals below the threshold increases only slightly, so does their employment. Total hours worked decreases. Average productivity increases because the share of high-productivity employment increases. Output and consumption move only slightly because labor input, measured in efficiency units, changes little. Hence, the labor wedge rises.

The (ex-ante) *uncertainty effect* also moves the labor market. An increase in idiosyncratic wage risk increases the incentive to self-insure and labor supply, especially those of

before 1984, while the data used here suggests a weakly negative correlation (–0.30). Gali and Gambetti (2009) use data on the nonfarm business sector, while the data used here includes the farm, government, and military sectors. The present paper uses the most comprehensive data because the labor wedge is computed using the consumption data for the entire economy.

small-wealth individuals. This only increases the low-productivity employment because high-productivity, small-wealth individuals chose employment even before the rise in risk. The high-productivity employment slightly decreases because the low-productivity employment increases and the wage rate falls. Total hours worked and output increase. As the composition of workers shifts towards low productivity, average labor productivity falls. Consumption drops slightly because precautionary saving increases. The labor wedge decreases. I solve the model excluding the distribution effect, i.e., shutting down the ex-post changes in the volatility in idiosyncratic shocks. The hours-productivity correlation is 0.69 and the labor wedge volatility is 22% of that in the U.S. Thus, the distribution effect is dominant.

Uncertainty shocks act as a labor supply shock. Benhabib et al. (1991) and Christiano and Eichenbaum (1992) also consider such shocks in an effort to break the strong hours-productivity comovement in business cycle models (Kydland and Prescott (1982) and Hansen (1985)). However, their models assume a representative agent and without changes in the composition of workers with different productivity, relatively strong hours-productivity comovement remains when calibrated to the U.S. economy.[2] Further, the labor wedge is constant (zero). In contrast, incorporating realistic heterogeneity in wealth and productivity across individuals, the calibrated varying risk model here simultaneously generates a hours-productivity correlation and a labor wedge volatility that are close to those in the U.S.

The present paper contributes to the literature on varying idiosyncratic earnings risk by analyzing its impact on labor market dynamics. While existing studies analyze how time-varying income risk affects aggregate fluctuations (Krusell and Smith (1998) and McKay (2015)), the welfare cost of business cycles (Krusell and Smith (1999), Storesletten et al. (2001), Mukoyama and Şahin (2006), and Krusell et al. (2009)), and asset pricing (Krusell and Smith (1997), Pijoan-Mas (2007), and Storesletten et al. (2007)), they assume exogenous

[2] Benhabib et al. (1991) include home-production technology shocks. Their benchmark model generates a hours-productivity correlation of 0.49. Christiano and Eichenbaum (1992) include government spending shocks. When estimated using establishment hours data, their model generates a correlation of 0.58.

earnings or inelastic labor supply and do not analyze labor market fluctuations.[3]

The present paper is also related to recent work on uncertainty shocks to firm-specific risk. Bloom et al. (2014) and Bachmann and Bayer (2013) investigate how uncertainty shocks interact with input adjustment costs. Arellano et al. (2012) consider financial frictions, while Schaal (2015) analyzes labor search frictions. In these two models, uncertainty shocks trigger heterogeneous changes in labor *demand* across firms, generating negative hours-productivity comovement and a volatile labor wedge. In the present model, the variation in the labor wedge arises solely from the variation in the labor market wedge, i.e., the gap between the real wage and the marginal rate of substitution, while the product market wedge, i.e., the gap between average labor productivity and the real wage, is always zero. In contrast, the variation in the product market wedge also contributes to the movement in the labor wedge in Arellano et al. (2012) and Schaal (2015).[4] Empirical evidence is mixed. While Karabarbounis (2014) finds the dominant role of the labor market wedge in the U.S., Bils et al. (2016) find that the product market wedge is as important as the labor market wedge.

The present paper proceeds as follows. Section 2 analyzes the PSID data and quantifies cyclical variation in idiosyncratic wage risk. Section 3 lays out the model, while Section 4 determines the parameter values. Section 5 examines the impact of uncertainty shocks on business cycles. Section 6 analyzes the implication for labor income risk. Section 7 concludes.

2. Cyclical Fluctuations in Idiosyncratic Wage Risk

This section analyzes the PSID data and provides an estimate for the cyclical variation in idiosyncratic wage risk in the U.S.[5] Idiosyncratic wage risk is computed as the cross-sectional

[3] One exception is Lopez (2010), which assumes *divisible* labor and a different borrowing constraint from that assumed here. His model generates a counterfactually strong hours-productivity correlation of 0.96.

[4] In Arellano et al. (2012), the variation in the labor wedge solely arises from changes in the product market wedge. Both the labor and product market wedges vary in Schaal (2015)'s model.

[5] Appendix A1 explains the data.

dispersion of residuals in a wage regression and its cyclical variation is analyzed.[6]

For each person-year observation of the PSID data, I compute the hourly wage dividing the annual labor income by the annual total labor hours. For each year, I then fit individual wages to the wage process assumed in the present paper, which is derived as follows.[7] First, an individual wage $w^a_{i,t}$ (i: individual and t: year) is $w^a_t x^a_{i,t}$, where w^a_t is the wage rate per efficiency unit of labor and $x^a_{i,t}$ is person-specific labor productivity:[8]

$$\ln w^a_{i,t} = \ln w^a_t + \ln x^a_{i,t}. \tag{1}$$

Second, $x^a_{i,t}$ follows an AR(1) process:

$$\ln x^a_{i,t} = \rho^a_{x,t} \ln x^a_{i,t-1} + \varepsilon^a_{x,i,t},\ \varepsilon^a_{x,i,t} \sim N(0, \sigma^{a2}_{\varepsilon_x,t}). \tag{2}$$

As shown by Chang and Kim (2006), (1) and (2) imply the following wage process:

$$\ln w^a_{i,t} = \rho^a_{x,t} \ln w^a_{i,t-1} + (\ln w^a_t - \rho^a_{x,t} \ln w^a_{t-1}) + \varepsilon^a_{x,i,t}, \varepsilon^a_{x,i,t} \sim N(0, \sigma^{a2}_{\varepsilon_x,t}). \tag{3}$$

I identify idiosyncratic wage risk in three ways. First, I estimate (3) each year for 1969–1991 with ordinary least squares (OLS), replacing $(\ln w^a_t - \rho^a_{x,t} \ln w^a_{t-1})$ with a constant.

In practice, variables such as years of education influence individual wages (Card (1999) and Heathcote et al. (2010)), and individuals could forecast their wage, at least partially. To isolate the pure risk that individuals face, I control for demographic variables:

$$\ln w^a_{i,t} = \rho^a_{x,t} \ln w^a_{i,t-1} + (\ln w^a_t - \rho^a_{x,t} \ln w^a_{t-1}) + Z_{i,t}\beta_t + \varepsilon^a_{x,i,t}, \varepsilon^a_{x,i,t} \sim N(0, \sigma^{a2}_{\varepsilon_x,t}), \tag{4}$$

where $Z_{i,t}$ includes sex, education, experience (defined as age minus education minus six), and experience-squared.[9] I estimate (4) each year as is done for (3), but for 1975–1991 because the data on education is discontinuous in 1974.

[6] This approach is similar to that taken by recent studies that estimate uncertainty shocks to firm-specific risk (Bloom (2009) and Bachmann and Bayer (2013)).

[7] The process is used in Chang and Kim (2006, 2007), Alonso-Ortiz and Rogerson (2010), and others.

[8] A variable with a superscript a indicates an annual value, distinguishing it from its quarterly counterpart.

[9] Controlling for occupation does not change the cyclical risk variation substantially.

Lastly, I take into account the selection effect and follow Chang and Kim (2006) in introducing the selection equation:

$$I_{i,t} = V_{i,t}\gamma_t + v^a_{i,t}, v^a_{i,t} \sim N(0, \sigma^{a2}_{v,t}), \tag{5}$$

where $I_{i,t} = 1$ if the individual worked in both years t and $t-1$. The variables $V_{i,t}$ include marital status, the number of children, education, experience, experience-squared, sex, and a constant. I conduct Heckman-type estimation using (4) and (5) each year for 1975–1991.

The upper panel of Figure 1 plots the idiosyncratic wage risk series estimated in the three ways $\widehat{\sigma^a_{\varepsilon_x,t}} = std(\widehat{\varepsilon^a_{x,i,t}})$. Consistent with Heathcote et al. (2010), they increase over time. To isolate their cyclical variation, I compute the percent deviation from trend using the Hodrick-Prescott filter with a smoothing parameter of 10.[10] The lower panel shows this detrended result. The three series are similar and all the correlations exceed 0.90. Hence, I focus on the series based on OLS, which is the longest.

Four empirical regularities characterize the cyclical component of idiosyncratic wage risk. First, idiosyncratic wage risk varies over time. The standard deviation is 3.2% and the volatility relative to the output volatility is 1.73. Second, idiosyncratic wage risk exhibits some persistence, typically remaining above or below trend for about two years. Its first-order autocorrelation is 0.20, but it is not statistically significantly different from zero. Third, risk variation is approximately symmetric. The size and persistence of idiosyncratic wage risk are similar when it is above and below trend. Fourth, idiosyncratic wage risk is acyclical. The correlation with output is 0.18 and it is not statistically significantly different from zero.[11] Further, idiosyncratic wage risk remained low during the 1981–1982 recession, but it increased during the 1973–1975 and 1990–1991 recessions. Section 4 uses these findings to

[10] The result does not change substantially when using a smoothing parameter of 6.25 or 100.

[11] The finding is consistent with that of Heathcote et al. (2010), who find that the standard deviation of (residual) *wages* shows no clear cyclicality. They also estimate the standard deviation of permanent and transitory shocks to individual wages. I detrend their result based on the 'difference specification' using the Hodrick-Prescott filter with a smoothing parameter of 10. The correlation with (detrended) output is –0.06 for the standard deviation of permanent shocks and 0.00 for transitory shocks. Both correlations are not statistically significantly different from zero. I thank the authors for making their result available.

calibrate uncertainty shocks in the model described below.

3. Model

The model analyzed here is built upon the model of Chang and Kim (2006, 2007). Individuals face idiosyncratic productivity risk and make consumption-saving and employment choice. I introduce risk variation into the environment using uncertainty shocks in the sense of Bloom (2009), i.e., time-varying standard deviation of idiosyncratic productivity shocks.

3.1. Individuals

There is a continuum of individuals of measure one. Individuals differ in labor productivity x. The momentary utility function is $u(c, h)$, where c is consumption and h is hours worked. As in Hansen (1985) and Rogerson (1988), labor is indivisible: $h \in \left\{\overline{h}, 0\right\}$. Individuals earn labor income of wxh, where w is the wage rate per efficiency unit of labor.

Individuals face time-varying wage risk because log of idiosyncratic productivity $\ln x$ follows an AR(1) process, $\ln x' = \rho_x \ln x + \epsilon_x'$, where $\epsilon_x' \sim N(0, \sigma_{\epsilon_x}^2)$, and σ_{ϵ_x} is a Markov chain. As in existing studies (Bloom (2009), Bloom et al. (2014), and Bachmann and Bayer (2013)), individuals learn of the size of σ_{ϵ_x} one period ahead, and σ_{ϵ_x} represents the volatility of shocks not to x, but to x', where a prime denotes the next-period value.[12]

Asset markets are incomplete, and individuals cannot fully insure themselves against idiosyncratic wage risk. As in Aiyagari (1994), individuals partially self-insure by holding physical capital k, which is the only asset. There is a borrowing limit: $k \geq \overline{k}$ ($\overline{k} < 0$).

Define $V(k, x; z, \mu, \sigma_{\epsilon_x})$ as the beginning-of-period value of an individual characterized by (k, x) under the aggregate state $(z, \mu, \sigma_{\epsilon_x})$, where z is aggregate TFP, log of which follows an AR(1) process, and μ denotes the individual distribution over k and x. This beginning-

[12] This timing assumption captures the concept of risk. The business cycle results presented in Section 5 are largely unchanged under the assumption that individuals learn of σ_{ϵ_x} contemporaneously.

of-period value reflects the individual's current employment choice:

$$V(k,x;z,\mu,\sigma_{\epsilon_x}) = \max\left\{V^E(k,x;z,\mu,\sigma_{\epsilon_x}), V^N(k,x;z,\mu,\sigma_{\epsilon_x})\right\}. \tag{6}$$

The individual's within-period value conditional on working $V^E(k,x;z,\mu,\sigma_{\epsilon_x})$ is

$$V^E(k,x;z,\mu,\sigma_{\epsilon_x}) = \max_{c,k'}\left\{u(c,\overline{h}) + \beta E\left[V(k',x';z',\mu',\sigma'_{\epsilon_x})|x,z,\mu,\sigma_{\epsilon_x}\right]\right\}, \tag{7}$$

$$\text{s.t. } c = w(z,\mu,\sigma_{\epsilon_x})x\overline{h} + [1+r(z,\mu,\sigma_{\epsilon_x})]k - k', k' \geq \overline{k}, c \geq 0, \mu' = \Gamma(z,\mu,\sigma_{\epsilon_x}),$$

where β is the discount factor, E is the conditional expectation, r is the rental rate of capital, and Γ is the law of motion for μ. The value of nonemployment $V^N(k,x;z,\mu,\sigma_{\epsilon_x})$ is

$$V^N(k,x;z,\mu,\sigma_{\epsilon_x}) = \max_{c,k'}\left\{u(c,0) + \beta E\left[V(k',x';z',\mu',\sigma'_{\epsilon_x})|x,z,\mu,\sigma_{\epsilon_x}\right]\right\}, \tag{8}$$

$$\text{s.t. } c = [1+r(z,\mu,\sigma_{\epsilon_x})]k - k', k' \geq \overline{k}, c \geq 0, \mu' = \Gamma(z,\mu,\sigma_{\epsilon_x}).$$

3.2. Representative Firm

A representative firm produces the final good Y using capital K and labor L. The production function is $Y = zF(K,L)$ and it exhibits constant returns to scale. Given r and w, the firm chooses K and L, maximizing static profits. The first-order conditions are

$$r(z,\mu,\sigma_{\epsilon_x}) = zF_K(K(z,\mu,\sigma_{\epsilon_x}), L(z,\mu,\sigma_{\epsilon_x})) - \delta, \tag{9}$$

and

$$w(z,\mu,\sigma_{\epsilon_x}) = zF_L(K(z,\mu,\sigma_{\epsilon_x}), L(z,\mu,\sigma_{\epsilon_x})), \tag{10}$$

where δ is the capital depreciation rate.

3.3. General Equilibrium

A recursive competitive equilibrium is a set of functions $\left(w, r, V^E, V^N, V, c, k', h, K, L, \Gamma\right)$ satisfying the following conditions.

1. Individuals' Optimization:

   The value functions $V(k,x;z,\mu,\sigma_{\epsilon_x}), V^E(k,x;z,\mu,\sigma_{\epsilon_x})$, and $V^N(k,x;z,\mu,\sigma_{\epsilon_x})$ satisfy (6), (7), and (8), while $c(k,x;z,\mu,\sigma_{\epsilon_x}), k^{'}(k,x;z,\mu,\sigma_{\epsilon_x})$, and $h(k,x;z,\mu,\sigma_{\epsilon_x})$ are the associated policy functions.

2. Firms' Optimization:

   The representative firm chooses $K(z,\mu,\sigma_{\epsilon_x})$ and $L(z,\mu,\sigma_{\epsilon_x})$ to satisfy (9) and (10).

3. Labor Market Clearing:

   $L(z,\mu,\sigma_{\epsilon_x}) = \int xh(k,x;z,\mu,\sigma_{\epsilon_x})\mu([dk \times dx])$

4. Capital Market Clearing:

   $K(z,\mu,\sigma_{\epsilon_x}) = \int k\mu([dk \times dx])$

5. Goods Market Clearing:

   $\int \left\{k^{'}(k,x;z,\mu,\sigma_{\epsilon_x}) + c(k,x;z,\mu,\sigma_{\epsilon_x})\right\} \mu([dk \times dx])$

   $= zF\left(K(z,\mu,\sigma_{\epsilon_x}), L(z,\mu,\sigma_{\epsilon_x})\right) + (1-\delta)\int k\mu([dk \times dx])$

6. Evolution of Individual Distribution:

   $\Gamma(z,\mu,\sigma_{\epsilon_x})$ is consistent with individual choices and the laws of motion for $(x,\, z, \sigma_{\epsilon_x})$. Let $\pi_x(x^{'}|x,\sigma_{\epsilon_x})$ be the transition probability from x to $x^{'}$ under σ_{ϵ_x}. For all $D \subseteq K$, $\mu^{'}(D,x^{'}) = \int_{\left\{(k,x)|k^{'}(k,x;z,\mu,\sigma_{\epsilon_x})\in D\right\}} \pi_x(x^{'}|x,\sigma_{\epsilon_x})\mu([dk^{'} \times dx^{'}])$.

4. Calibration and the Steady State

I determine the parameter values on idiosyncratic productivity by matching moments of the model's individual wages with those of the PSID wages. I choose the other parameter values so that the model's steady state replicates several features of the U.S. economy. The end of this section presents the steady-state distributions of wealth and labor income.

4.1. Parameters Other Than Idiosyncratic Productivity

Table 1 lists the parameters other than those on idiosyncratic productivity. One period is one quarter. The discount factor β is 0.9829, implying a one percent rental rate of capital. The momentary utility is $u(c,h) = \ln c - Bh$ with $B = 3.061$. The employment rate is 60% as in Chang and Kim (2007) and it is close to the average U.S. employment-population ratio for 1948Q1–2009Q3. Individuals use one third of their time when working ($\overline{h} = 1/3$). The borrowing limit is $\overline{k} = -2.0$ and individuals can borrow up to 44% of the average annual income. This is similar to the limit set by Krusell and Smith (1998).[13]

The production function is $Y = zK^{1-\alpha}L^{\alpha}$ and labor's share α is 0.64. The capital depreciation rate δ is 0.025. Aggregate TFP z follows $\ln z' = \rho_z \ln z + \epsilon_z'$, where $\epsilon_z' \sim N(0, \sigma_{\epsilon_z}^2)$. As in Cooley and Prescott (1995), $\rho_z = 0.95$, and $\sigma_{\epsilon_z} = 0.007$.[14]

4.2. Parameters on Idiosyncratic Productivity

Four parameters concern idiosyncratic productivity x. The first is the persistence in $\ln x$, or ρ_x. The other three parameters concern fluctuations in idiosyncratic wage risk σ_{ε_x}. The analysis in Section 2 finds no strong cyclicality in the estimated annual idiosyncratic wage risk $\widehat{\sigma_{\varepsilon_x}^a}$. Hence, the benchmark is the independent risk model, where σ_{ε_x} evolves independently of aggregate TFP z. A Markov chain with three states, high (H), middle (M), and low (L), is assumed. Motivated by the symmetry of risk variation discussed in Section 2, $\sigma_{\varepsilon_x,H} = (1+\lambda)\bar{\sigma}_{\varepsilon_x}$, $\sigma_{\varepsilon_x,M} = \bar{\sigma}_{\varepsilon_x}$, and $\sigma_{\varepsilon_x,L} = (1-\lambda)\bar{\sigma}_{\varepsilon_x}$, where $\bar{\sigma}_{\varepsilon_x}$ is the steady-state risk and $\lambda > 0$ is the size of risk variation. With a probability of $\rho_{\sigma_{\varepsilon_x}}$, σ_{ε_x} stays unchanged, while it transitions to each of the other two states with a probability of $(1-\rho_{\sigma_{\varepsilon_x}})/2$.

The values of the four parameters $(\rho_x, \bar{\sigma}_{\varepsilon_x}, \lambda, \rho_{\sigma_{\varepsilon_x}})$ are determined as follows. I simulate the model, which is quarterly, with 60,000 individuals for 1,500 periods (discarding the first

[13] The business cycle results with $\overline{k} = -4.0$ and 0.0 do not substantially differ from that with $\overline{k} = -2.0$.

[14] The model here includes uncertainty shocks and using the estimate of Cooley and Prescott (1995), who only include aggregate TFP shocks, might overstate the volatility of aggregate TFP. Assuming a lower volatility of aggregate TFP shocks ($\sigma_{\epsilon_z} = 0.005$) does not change the present paper's main results substantially.

500 periods) and generate the panel data on *annual* wages, which is comparable to the PSID wage data. I then choose the model parameter values so that four moments match between the PSID and model annual wages. The first two moments are the persistence in individual wages $\widehat{\rho_x^a}$ and the long-run idiosyncratic wage risk $\widehat{\sigma_{\varepsilon_x}^a}$. I compute them by estimating (3) with year dummies using the pooled OLS. The other two moments are the standard deviation of idiosyncratic wage risk relative to the output volatility, $std(\widehat{\sigma_{\varepsilon_x}^a})$, and the first-order autocorrelation, $corr(\widehat{\sigma_{\varepsilon_x}^a}, \widehat{\sigma_{\varepsilon_x,-1}^a})$. I compute $\widehat{\sigma_{\varepsilon_x}^a}$ by estimating (3) each year with OLS and then remove trend using the Hodrick-Prescott filter with a smoothing parameter of 10.

The PSID moments in the second column of Table 2A pin down the parameter values for the independent risk model as in the fourth column of Table 2B. The persistence in idiosyncratic productivity is $\rho_x = 0.930$, while the steady-state risk is $\bar{\sigma}_{\varepsilon_x} = 0.223$. These are close to Chang and Kim (2007)'s values. As for risk variation, the persistence $\rho_{\sigma_{\varepsilon_x}}$ is 0.90 and the size λ is 0.067. As shown, the model's moments match the PSID ones well.

The moments of the constant risk model, where idiosyncratic wage risk is constant at the steady state, are presented in the third column of Table 2A. Because of endogenous employment choice, the annual idiosyncratic wage risk estimated using the model data exhibits some variation. However, the volatility is much smaller than the PSID one. This finding provides further evidence for fluctuations in idiosyncratic wage risk.

4.3. Steady State

The inequality of wealth and labor income in the present model is comparable to that in the U.S. The Gini coefficient of annual labor income is 0.54 at the steady state and 0.65 in the 1991 PSID.[15] The Gini coefficient for wealth is 0.63 in the model. Since it is difficult to define individual wealth in the actual economy, I compare this individual-level wealth inequality with the household-level inequality in the U.S. According to Díaz-Giménez et al.

[15] Appendix B1 explains the solution method for the steady state. I generate the distribution of annual labor income through the model simulation with 60,000 individuals. Appendix A2 explains the PSID data.

(1997), the Gini coefficient is 0.78 in the 1992 Survey of Consumer Finances.

Further, the present model generates a weakly positive correlation between wealth and labor income of 0.29, which is close to 0.23 in the U.S. (Díaz-Giménez et al. (1997)).[16] Since idiosyncratic productivity is persistent, individuals with higher current productivity tend to hold larger wealth. Further, individuals are more likely to work when they have higher current productivity and smaller wealth. These two factors generate the weakly positive correlation between wealth and labor earnings in the model.

5. Business Cycle Results

This section analyzes how time-varying idiosyncratic wage risk affects business cycles.

5.1. Time-Varying Idiosyncratic Wage Risk and Business Cycles

Table 3 lists the business cycle moments of the U.S. and model economies.[17] Introducing uncertainty shocks substantially improves two labor market statistics in the models. One is the correlation between total hours worked and average labor productivity.[18] The independent risk model generates a weakly negative correlation (–0.17) close to the U.S. value (–0.32), while the constant risk model produces a strong positive correlation (0.83).

The other is the volatility in the labor wedge. The labor wedge is the ratio of average labor productivity to a representative individual's marginal rate of substitution of leisure for consumption. As in Chang and Kim (2007), it is computed by $\ln wedge = \ln Y/H - \ln H^{1/\gamma}C$ with $\gamma = 1.5$. The labor wedge is volatile in the U.S. The independent risk model reproduces the feature reasonably well: The volatility is about 70% of that in the U.S. In contrast, the

[16] The steady state is similar to that of Chang and Kim (2007). Table 2 of Chang and Kim (2007) provides more evidence that the distributions of wealth and income are comparable to the U.S. counterparts.

[17] Appendix A3 explains the source of the U.S. data. Appendix B2 explains the solution method for the business cycle, which is based on Krusell and Smith (1997, 1998) and Takahashi (2014). The same sequence of aggregate TFP is used for all the simulations.

[18] Total hours worked is $H \equiv \int h(k, x; z, \mu, \sigma_{\epsilon_x})\mu([dk \times dx])$.

constant risk model accounts for only 17% of the empirical volatility.

Figure 2 shows how these improvements depend on risk variation. Even under almost no persistence ($\rho_{\sigma_{\varepsilon_x}} = 0.4$), the hours-productivity correlation is 0.06 and the volatility of the labor wedge is 56% of that in the U.S. Even when the size of risk variation λ decreases by almost 40% ($\lambda = 0.04$), the independent risk model generates a low hours-productivity correlation of 0.28 and accounts for 44% of the volatility in the labor wedge seen in the U.S.

Introducing risk variation also increases the volatility of hours worked and reduces the output-productivity correlation, moving their values closer to the U.S. data. One problem in the independent risk model is the acyclical labor wedge, while the wedge is countercyclical in the U.S. Section 5.3 shows that this problem is fixed by introducing a negative correlation between aggregate TFP growth and idiosyncratic wage risk.

Lastly, the independent and constant risk models generate similar volatilities and comovements of output, consumption, and investment. Thus, introducing variation in idiosyncratic wage risk strengthens the model's ability to explain labor market fluctuations, without weakening its ability to account for other business cycle moments.

5.2. Responses to a Rise in Idiosyncratic Wage Risk

To clarify the mechanism behind the above results, I analyze the response of the independent risk model to a rise in idiosyncratic wage risk σ_{ε_x}. The simulation starts from the steady state. For initialization, σ_{ε_x} is in the middle state (i.e., at the steady state) for 150 periods and then it moves into the high state, rising by 6.7% for one period. I normalize the period of this risk rise to period 0, as in the upper-left panel of Figure 3. The timing assumption implies that the dispersion of idiosyncratic productivity shocks in period 1 increases, while individuals learn of it in period 0. Aggregate TFP is fixed at its steady-state level.

The remaining panels of Figure 3 show the responses of other variables. Output increases and then returns to the pre-shock level. Consumption drops initially and then recovers.

Hours increases in period 0, drops below the pre-shock level in period 1, and then recovers.[19] Average labor productivity and the labor wedge move in a direction exactly opposite to that of hours. These results explain why introducing variation in idiosyncratic wage risk weakens the hours-productivity comovement and increases the volatility of the labor wedge.

I turn to micro-level responses underlying these aggregate responses. Indivisible labor implies a threshold rule for employment: Given current productivity, individuals whose wealth is below a certain level choose to work. Further, those wealth thresholds increase with productivity. Figure 4 shows the determination of the employment rate for the low and high productivity groups.[20] The upward sloping curve is the cumulative wealth distribution, while the vertical line is the wealth threshold for employment. Hence, the employment rate is determined by the intersection of the two. Below, I explain how the rise in risk shifts the threshold for employment, the wealth distribution, and thereby the employment rate.

The timing assumption implies two effects of the rise in idiosyncratic wage risk. The first is an *uncertainty effect*. In period 0, individuals become more uncertain about their future wage and change their current labor supply. This is described in Figure 4 by a shift in the wealth threshold for employment without a shift in the wealth distribution. The second is a *distribution effect*. When the volatility of idiosyncratic productivity shocks actually increases in period 1, the dispersion of idiosyncratic productivity increases. As explained below, this also shifts the wealth distribution *given productivity*. In contrast, the uncertainty effect disappears because uncertainty returns to the pre-shock level. Hence, both the threshold for employment and the wealth distribution conditional on productivity shift in period 1.

In period 0, the low-productivity employment increases and to a lesser extent the high-productivity employment decreases. As for the low productivity, their wealth threshold for

[19] Although total hours worked shows a sawtooth response, its first-order autocorrelation is 0.68 in the independent risk model and it is comparable to that in the U.S. data (0.75).

[20] I consider 17 levels of idiosyncratic productivity: $x_1 < ... < x_{17}$. The low productivity group is those with x_6 and between the 10-20 percentiles in the idiosyncratic productivity distribution, whereas the high productivity are those with x_{12} and between the 80-90 percentiles.

employment shifts to the right, increasing their employment rate. This occurs because those around the threshold have small wealth and increase labor supply to insure against greater risk. In contrast, the threshold for the high productivity shifts to the left, decreasing their employment slightly. Those around the threshold have large wealth and are well insured. Further, the wage rate falls because the low-productivity employment increases. Total hours worked increases. Output increases less than hours, not only because aggregate TFP and capital remain unchanged, but also because employment disproportionately increases among the low productivity. Average labor productivity decreases. Consumption drops slightly because precautionary saving increases. The labor wedge decreases.

In period 1, the low-productivity employment decreases and to a lesser degree the high-productivity employment increases. Because of the increased volatility in idiosyncratic productivity shocks, the mobility between different productivity increases. This greater mobility reduces the positive productivity-wealth correlation, which is generated by the persistence in idiosyncratic productivity. Specifically, the wealth distribution of the low productivity shifts to larger wealth, decreasing their employment. The opposite occurs for the high productivity and their wealth distribution shifts towards smaller wealth. However, since the wealth threshold for employment is substantially high, the number of individuals below the threshold increases only slightly, so does their employment.

Further, relative to the pre-shock one, the wealth threshold for employment shifts to the left for both the low and high productivity: Wage uncertainty returns to the pre-shock level, while the wage rate is a bit lower due to a slight increase in the high-productivity employment. However, the shifts in the wealth distributions have a dominant effect on employment, especially for the low-productivity one. I quantify their role by the following thought experiment. In period 1, the dispersion in idiosyncratic productivity increases and the wealth thresholds for employment also shift as described in Figure 4. However, the wealth distributions given productivity are unchanged from period 0. Hence, the employment rate is determined by the intersection of the period 0 wealth distribution and the period 1 threshold. In such a case, total hours worked in period 1 is lower than the pre-shock level only by 0.11% in contrast to 0.75% when the wealth distributions also shift. Hence, while the increased dispersion in idiosyncratic productivity itself reduces employment, the impact is substantially amplified by the shifts in the wealth distributions conditional on productivity.[21]

In contrast, output and consumption increase only slightly in period 1 because efficiency-weighted labor increases little. Hence, the labor wedge rises. Crucially, even though idiosyncratic wage risk increases only for one period, it takes long for the wealth-productivity distribution to return to the pre-shock one. Thus, the distribution effect has a persistent impact on total hours worked, average labor productivity, and the labor wedge.

Following Bachmann and Bayer (2013), I analyze the psych risk model, which shuts down the distribution effect: Individuals receive information on changes in idiosyncratic wage risk σ_{ε_x} and respond, but those changes in σ_{ε_x} never realize ex post.[22]

As seen above, the uncertainty effect alone lowers the hours-productivity correlation and increases the volatility of the labor wedge. However, the impact is small. As shown in Table 3, the psych risk model generates a positive hours-productivity correlation of 0.69 and accounts for only 22% of the empirical volatility of the labor wedge. Figure 3 shows the response of the psych risk model to the one-period increase in idiosyncratic wage risk considered above. The response is qualitatively similar to that of the independent risk model.[23] However, the negative hours-productivity comovement disappears much more quickly and the labor wedge fluctuates much more modestly in the psych risk model. Hence, the main impact of changes

[21] The increase in saving in period 0 also shifts the wealth distributions to larger wealth. However, the effect is quantitatively small. See footnote 23.

[22] In the model solution described in Appendix B2, the value function iteration (Step 4) is the same as in the independent risk model. However, in the simulation (Step 5), while individuals respond to changes in idiosyncratic wage risk σ_{ε_x} (the sequence of σ_{ε_x} is the same as that fed into the independent risk model), the steady-state value of σ_{ε_x} is used to update the idiosyncratic productivity distribution, leaving the distribution unchanged from the steady state.

[23] Total hours worked in period 1 is slightly lower than the pre-shock level even in the psych risk model. Individuals increase saving in period 0 due to the elevated uncertainty. See Marcet et al. (2007) on the ex-post wealth effect under constant idiosyncratic income risk.

in idiosyncratic wage risk arises from the distribution effect.

5.3. Countercyclical Risk Model

This subsection analyzes the countercyclical risk model, where idiosyncratic wage risk is negatively correlated with aggregate TFP growth. The independent risk model is rationalized based on the result in Section 2 that the estimated annual idiosyncratic wage risk is acyclical. As shown above, however, a rise in idiosyncratic wage risk increases output slightly in the independent risk model. Hence, when estimated using the model data, a weakly positive correlation appears between annual idiosyncratic wage risk and output (0.35). In contrast, the countercyclical risk model presented below generates a negative correlation (–0.51). Hence, the reality should lie between the two polar cases.

In the countercyclical risk model, σ_{ε_x} is high (low) when aggregate TFP z fell (rose) by more than 1.67% (i.e., one grid point) from the previous to current periods.[24] Otherwise, σ_{ε_x} is in the middle state. As before, $\rho_x = 0.930$ and $\bar{\sigma}_{\varepsilon_x} = 0.223$. Changes in z govern the transition probabilities of σ_{ε_x}. I set $\lambda = 0.112$, targeting the volatility of annual idiosyncratic wage risk relative to the output volatility in the U.S., as is done for the independent risk model and as shown in Table 2.

Four points are worth mentioning for the business cycle results in Table 3. First and most importantly, the main results of the present paper survive: Introducing countercyclical risk substantially reduces the hours-productivity correlation and increases the volatility of the labor wedge. Second, the countercyclical risk model generates a countercyclical labor wedge, fixing the problem in the independent risk model. In the countercyclical risk model, the distribution effect of a rise in idiosyncratic risk raises the labor wedge, while at the same time a fall in aggregate TFP decreases output. Third, the output-productivity correlation becomes closer to the U.S. data. Fourth, output is smoothed even relative to the constant

[24] The business cycle results do not change substantially under the assumption that aggregate TFP growth from the current to next periods is negatively correlated with σ_{ε_x}.

risk model because a rise in risk raises output, mitigating the output decline caused by a drop in aggregate TFP.

To summarize, the cyclicality of idiosyncratic wage risk is not very relevant to the main results of the present paper. However, it affects the movement of output and other labor market statistics. The reality should be a weakly negative correlation between aggregate TFP growth and idiosyncratic wage risk, and even if included such a feature, the present model would still account for the U.S. labor market fluctuations reasonably well.

5.4. Quantifying Composition Effect

The uncertainty and distribution effects change the composition of workers. They move aggregate employment and the share of low-productivity workers in the same direction, generating negative comovement between total hours worked and average labor productivity.

To examine whether the composition effects in the independent and countercyclical risk models are comparable to the U.S. counterpart, I conduct a regression in Solon et al. (1994) using the model data and compare the results with the U.S. one. Specifically, I estimate the following equation with OLS:

$$\Delta \ln w_t = \beta_1 + \beta_2 t + \beta_3 \Delta CI_t + \nu_t, \tag{11}$$

where w_t is the economy-wide average wage in year t, CI_t is a cyclical indicator, and ν_t is an error term. I then estimate (11), replacing $\ln w_t$ with the average of log of wages from the balanced panel data. Comparing the coefficient β_3 from the two regressions reveals the composition effect. While the economy-wide average wage is affected by changes in the composition of workers, the average wage computed from the balanced panel data is free from such a composition bias and β_3 reveals the true cyclicality of individual wages.

Table 4 presents the result. As in Solon et al. (1994), three cyclical indicators are considered: the unemployment rate (U), log of GDP (lnY), and log of per capita hours worked

(lnH).[25] The U.S. result, which is taken from Solon et al. (1994), shows a strong composition effect. Individual wages are much more procyclical than the average wage. The result indicates that the share of low-productivity workers increases during expansions.

The independent and countercyclical risk models show a composition effect similar to the U.S. one.[26] Individual wages increase more strongly with labor input than the average wage. Hence, the share of low-productivity workers increases with labor input. A deviation from the U.S. result is observed when GDP is a cyclical indicator in the independent risk model. In that model, the distribution effect of an increase in risk increases output and also the share of high-productivity workers. As a result, the average wage moves with GDP more strongly than individual wages. In contrast, the result in the countercyclical risk model is in line with the U.S. result, and as argued above, the reality should lie between the two models.

The constant risk model shows essentially no composition effect and the average wage and individual wages are almost equally procyclical. In the model, aggregate TFP shocks are the solo driver for aggregate fluctuations. A fall in aggregate TFP reduces labor demand, without significantly affecting labor supply, as in the prototype equilibrium business cycle model. The wage rate falls, and employment decreases across all productivity groups almost uniformly, leaving the composition of workers largely unchanged.[27]

The analysis so far has focused on the composition of workers. Next, I examine the composition of individuals switching from employment to nonemployment. If unproductive workers disproportionately become nonemployed during recessions, then the quality of the remaining workforce improves. Berger (2015) argues that countercyclical restructuring generates such a pattern in the U.S.

I categorize employed individuals in year $t-1$ into four equal groups according to their

[25] In the models, the nonemployment rate is used as the unemployment rate, while the per capita hours worked is equal to total hours worked.

[26] I use balanced panel data for 20 years, which is the same as the U.S. data used by Solon et al. (1994).

[27] Appendix F of Supplementary Materials shows the response of the independent risk model to aggregate TFP shocks.

average wage in that year. I then compute the transition rate from employment to nonemployment, the EN rate, between years $t-1$ and t for each group. The result is compared between the PSID and model data.[28]

Table 5 shows the result. The overall EN rate is countercyclical in the U.S. The EN rate tends to be countercyclical for the low wage, but not so strongly for the highest wage, as is in line with the argument by Berger (2015). The constant risk model implies the opposite. The EN rate is more countercyclical for the high wage because they are highly productive and their employment decisions are more sensitive to changes in the aggregate wage rate. The independent risk model fails to account for the countercyclicality of the overall EN rate. As seen above, the model implies a weak correlation between output and labor input, and it also implies a low correlation between output and the EN rate. In contrast, the countercyclical risk model generates a result much closer to the U.S. data. However, the countercyclicality of the EN rate is still too weak for the low wage and too strong for the high wage. Introducing risk variation also raises the volatility in the EN rate, moving it closer to the U.S. data, but the models still understate the empirical one.

The results in this subsection suggest that introducing risk variation generally improves the model's ability in accounting for the cyclicality of the composition of workers in the U.S. However, there remains a substantial gap on the EN rate between the model and U.S. economies. Accounting for the strong countercyclicality of the low-wage EN rate and the weaker countercyclicality for the high wage would require additional features, such as countercyclical restructuring as in Berger (2015) and unemployment benefits. Such modifications are likely to amplify the composition effect already present in the current varying risk models, i.e., the pattern that the share of low-productivity workers increases with aggregate employment, and hence they would strengthen the main results of the present paper.

[28] For the model data, employed are those with positive labor hours in a year. For the PSID data, employed are those with annual labor hours larger than 240 hours as in Chang et al. (2014).

6. Implications for Labor Earnings Risk

This section shows that the independent and countercyclical risk models, which are calibrated to the variation in idiosyncratic wage risk in the U.S., generate cyclical variation in idiosyncratic *earnings* risk generally in line with that documented by Guvenen et al. (2014).[29] I then examine whether introducing changes in the *skewness* of idiosyncratic productivity shocks narrows a remaining gap between the U.S. and model results.

Table 6 summarizes the result. Let's start with the standard deviation of annual earnings growth (i.e., a log earnings growth rate), $\sigma_{earnings}$, which is the conventional measure for earnings risk. It is weakly countercyclical, showing a correlation with output of –0.38.[30] The risk volatility relative to the output volatility is 1.22. As highlighted by Guvenen et al. (2014), the risk variation is much smaller than the finding by Storesletten et al. (2004).

The independent and countercyclical risk models account for the risk volatility in the U.S. well. In contrast, the constant risk model substantially understates the empirical volatility. Further, the risk-output correlation suggests that the reality should be somewhere between the independent and countercyclical risk models, i.e., a weakly negative correlation between aggregate TFP growth and idiosyncratic wage risk. These findings provide further support for the earlier calibration using the cyclical variation in idiosyncratic wage risk in the U.S.

As for statistics related to the skewness, Guvenen et al. (2014) find that the differential between the 90th and 50th percentiles of annual earnings growth, L90–50, is procyclical in the U.S. and it is positively correlated with GDP growth.[31] In contrast, the differential between the 50th and 10th percentiles, L50–10, is countercyclical. Hence, Kelly's skewness of earnings growth is procyclical.[32]

[29] I thank Guvenen et al. (2014) for making their result available. The model statistics are computed using the panel data with 60,000 individuals for 1,500 periods used for calibration in Section 4.2.

[30] I compute the percent deviation from trend using the Hodrick-Prescott filter with a smoothing parameter of 10, as is done to the estimate for idiosyncratic wage risk.

[31] As discussed in footnote 24 of Guvenen et al. (2014), these statistics are detrended with linear trend and then their average is added. The same is applied to the model statistics.

[32] Kelly's skewness is ((L90–50)–(L50–10))/(L90–10), where L90–10 is the differential between the 90th

The present paper's models generate a procyclical skewness of earnings growth, even though the skewness of idiosyncratic productivity shocks is constant at zero. In the constant risk model, L50–10 is countercyclical. A decline in aggregate TFP lowers the aggregate wage rate, increasing a flow from employment to nonemployment. Hence, getting very low income growth is more likely during recessions and L50–10 rises. Getting high earnings growth is not very much affected by changes in the aggregate wage rate and L90–50 is acyclical. In the independent risk model, a rise in idiosyncratic risk increases L90–50, L50–10, and output. Hence, L90–50 and L50–10 become more procyclical than in the constant risk model. In the countercyclical risk model, a rise in idiosyncratic risk raises L90–50 and L50–10, while a fall in aggregate TFP reduces output simultaneously. Thus, L90–50 and L50–10 are countercyclical.

Given that the reality should lie between the independent and countercyclical risk models, the biggest problem in the model results is the weak procyclicality of L90–50. Making L90–50 more procyclical is not likely to weaken the main results of the present paper significantly.[33] Nontheless, I examine here whether simply introducing the procyclical skewness of idiosyncratic productivity shocks mitigates the gap between the U.S. and model results. I employ a specification similar to Guvenen et al. (2014) and include it into the independent and countercyclical risk models. Below, I present the countercyclical risk model with the procyclical skewness of idiosyncratic productivity shocks, while Appendix D in Supplementary Materials describes the independent risk model with the procyclical skewness.

There are three states for the standard deviation and skewness of idiosyncratic productivity shocks ε_x'.[34] If aggregate TFP changed from the previous to current periods by less than

and 10th percentiles. The measure is more robust to outliers than the third central moment.

[33] In the independent risk model, a rise in idiosyncratic wage risk increases both output and L90–50. Hence, increasing the procyclicality of L90–50 means enhancing the composition change arising from an increase in the high-productivity employment. In the countercyclical risk model, a rise in wage risk increases L90–50, but a fall in aggregate TFP lowers output simultaneously. Hence, increasing the procyclicality of L90–50 means weakening the composition change arising from an increase in the high-productivity employment. However, as the analysis in Section 5.2 suggests, the composition effect mainly arises from a decrease in the low-productivity employment. Hence, the main results of the present paper are likely to survive under more procyclical L90–50.

[34] Guvenen et al. (2014) consider two states instead of three states herein.

$\pm 1.67\%$ (i.e., one grid point), then the standard deviation-skewness state is Middle-Zero and $\varepsilon_x^{'} \sim N(0, \overline{\sigma}_{\epsilon_x}^2)$ with $\overline{\sigma}_{\epsilon_x} = 0.223$.[35] If aggregate TFP fell by more than 1.67%, then the state is High-Negative:

$$\begin{aligned} \varepsilon_x^{'} \quad \sim \quad & N(\mu_{1,N}, (1+\lambda_H)^2\sigma_1^2) \text{ with probability } p \\ & N(\mu_{2,N}, (1+\lambda_H)^2\sigma_2^2) \text{ with probability } 1-p. \end{aligned} \tag{12}$$

If aggregate TFP rose by more than 1.67%, then the state is Low-Positive:

$$\begin{aligned} \varepsilon_x^{'} \quad \sim \quad & N(\mu_{1,P}, (1-\lambda_L)^2\sigma_1^2) \text{ with probability } p \\ & N(\mu_{2,P}, (1-\lambda_L)^2\sigma_2^2) \text{ with probability } 1-p. \end{aligned} \tag{13}$$

There are four restrictions:

$$p\mu_{1,N} + (1-p)\mu_{2,N} = 0 \tag{14}$$

$$p\mu_{1,P} + (1-p)\mu_{2,P} = 0 \tag{15}$$

$$p[\mu_{1,N}^2 + (1+\lambda_H)^2\sigma_1^2] + (1-p)[\mu_{2,N}^2 + (1+\lambda_H)^2\sigma_2^2] = (1+\lambda)^2\overline{\sigma}_{\epsilon_x}^2 \tag{16}$$

$$p[\mu_{1,P}^2 + (1-\lambda_L)^2\sigma_1^2] + (1-p)[\mu_{2,P}^2 + (1-\lambda_L)^2\sigma_2^2] = (1-\lambda)^2\overline{\sigma}_{\epsilon_x}^2, \tag{17}$$

with $\lambda = 0.112$. The first two imply that the mean of $\varepsilon_x^{'}$ is zero. The rest implies that the standard deviation is unchanged from the original countercyclical risk model.

I introduce modest changes in the skewness of idiosyncratic productivity shocks. The other parameter values are inherited from the original independent and countercyclical risk models and those values are chosen so that the models are consistent with the short-run and long-run features of the U.S. economy, including the distributions of wealth and labor

[35] The business cycle results do not change substantially under the assumption that aggregate TFP growth from the current to next periods is positively correlated with the skewness in idiosyncratic productivity shocks in the next period $\varepsilon_x^{'}$.

income. Introducing too large variation in the skewness of idiosyncratic productivity shocks may break these successes and inferences obtained in such a model would not be reliable.[36] Hence, the present paper focuses on how much a small departure from the original models can mitigate the remaining gap between the U.S. and model results.

Given these considerations, I set $p = 0.5, \sigma_1 = 0.239, \sigma_2 = 0.150, \mu_{1,N} = -0.1, \mu_{1,P} = 0.1, \mu_{2,N} = 0.1$, and $\mu_{2,P} = -0.1$.[37] The above conditions imply $\lambda_H = 0.139$ and $\lambda_L = 0.142$. As in the original countercyclical risk model, the standard deviation of idiosyncratic productivity shocks varies by 11.2% and it is negatively correlated with aggregate TFP growth. In addition, the skewness of idiosyncratic productivity shocks is positively correlated with aggregate TFP growth. The skewness (i.e., the third central moment) is 0.489 in Low-Positive and –0.439 in High-Negative. The same values of $(p, \sigma_1, \sigma_2, \mu_{1,N}, \mu_{1,P}, \mu_{2,N}, \mu_{2,P})$ are used for the independent risk model with the procyclical skewness.

As shown in Table 6, introducing the procyclical skewness of idiosyncratic productivity shocks makes L90–50 more procyclical, moving closer to the Guvenen et al. (2014)'s result, although a substantial gap remains. The finding suggests that explaining Guvenen et al. (2014)'s finding more closely requires additional modifications to the current setting. The main results of the present paper survive: The models with uncertainty shocks can account for the low hours-productivity correlation and the volatile labor wedge seen in the U.S. data.

[36] Making the skewness of idiosyncratic productivity shocks strongly procyclical under the present specification brings about large changes in the kurtosis and the strongly countercyclical median. McKay (2015) proposes a specification where the skewness of idiosyncratic earnings risk changes continuously as opposed to discretely here and uses a mixture of three normals as opposed to two normals herein. His analysis suggests that the specification is flexible enough to account for Guvenen et al. (2014)'s moments well. I choose the present specification mainly for computational reasons discussed in footnote 37. An interesting exercise is to include McKay (2015)'s specification into the present setting. I would like to leave it to future research.

[37] One way to choose these parameter values is to use a simulated method of moments, targeting Guvenen et al. (2014)'s moments, as is done by McKay (2015). However, it is too computationally intensive. I use the method similar to Takahashi (2014), which is based on Krusell and Smith (1997, 1998) algorithm. It takes several hours to solve the model one time even though I use a computer including Intel Core i7 Processor with 6 cores, 12 threads, and 3.3GHz and solve the model with Fortran. Further, the model frequency is quarterly, while Guvenen et al. (2014)'s moments are annual. Hence, additional simulation is needed to generate the model data in the annual frequency.

7. Conclusion

Is cyclical variation in idiosyncratic wage risk relevant to business cycles? The present paper has investigated the question quantitatively using a heterogeneous-agent model with uninsured idiosyncratic wage risk. I have introduced risk variation using uncertainty shocks and calibrated those shocks to micro-level evidence. The analysis suggests that cyclical variation in idiosyncratic wage risk would be an important driver for labor market fluctuations.

One remaining task is to account for the finding of Guvenen et al. (2014) more closely. While the present paper's models are partly consistent with their finding, it is far from perfect. Two modifications would be needed. One is labor supply. Introducing the intensive margin adjustments as in Chang et al. (2014) and unemployment as in Krusell et al. (2015) would be important. The other is the wage process. The present paper has assumed an AR(1) process for idiosyncratic productivity and introduced risk variation in a simple way. Future work should allow richer dynamics in idiosyncratic productivity and uncertainty. The present paper's finding indicates that it would be interesting to evaluate aggregate implications of cyclical variation in idiosyncratic wage risk in those extended models.

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Parameter	Description	Value
β	Discount factor	0.9829
B	Disutility of labor	3.061
$\overline{h}$	Working hours	1/3
$\overline{k}$	Borrowing limit	-2.0
α	Labor share	0.64
δ	Capital depreciation rate	0.025
ρ_z	Persistence in aggregate TFP	0.95
σ_{ϵ_z}	Volatility of aggregate TFP shocks	0.007

Table 1: Parameters other than those on idiosyncratic productivity.

	U.S.	Constant	Independent	Countercyclical
A. Calibration moments				
$\widehat{\rho_x^a}$	0.854	0.855	0.855	0.855
$\widehat{\bar{\sigma}_{\varepsilon_x}^a}$	0.282	0.279	0.282	0.280
$std(\widehat{\sigma_{\varepsilon_x}^a})$	1.729	0.373	1.745	1.707
$corr(\widehat{\sigma_{\varepsilon_x}^a}, \widehat{\sigma_{\varepsilon_x,-1}^a})$	0.198	-0.299	0.265	–0.331
B. Parameters				
ρ_x	-	0.930	0.930	0.930
$\overline{\sigma}_{\epsilon_x}$	-	0.223	0.223	0.223
$\rho_{\sigma_{\epsilon_x}}$	-	-	0.900	-
λ	-	-	0.067	0.112

Table 2: Calibration moments and the parameter values on idiosyncratic productivity. Panel A lists the moments of annual individual wages used for calibration. *std* is the standard deviation relative to the output volatility. *corr* is a correlation. I take logs of $\widehat{\sigma_{\varepsilon_x}^a}$ and remove trend using the Hodrick-Prescott filter with a smoothing parameter of 10. Panel B shows the parameter values.

	U.S.	Constant	Independent	Psych	Countercyclical
$std(Y)$	1.69	1.37	1.40	1.37	1.22
$std(C)$	0.54	0.32	0.34	0.33	0.34
$std(I)$	2.85	3.10	3.12	3.19	3.02
$std(H)$	1.00	0.57	0.72	0.59	0.90
$std(Y/H)$	0.63	0.48	0.83	0.50	0.54
$std(wedge)$	1.40	0.23	0.98	0.31	0.95
$corr(Y, C)$	0.78	0.90	0.85	0.89	0.92
$corr(Y, I)$	0.80	0.99	0.98	0.99	0.99
$corr(Y, H)$	0.80	0.96	0.58	0.93	0.84
$corr(Y, Y/H)$	0.31	0.95	0.70	0.91	0.45
$corr(H, Y/H)$	-0.32	0.83	-0.17	0.69	–0.10
$corr(H, wedge)$	–0.94	–0.96	–0.80	–0.86	–0.94
$corr(Y, wedge)$	–0.67	–0.86	0.01	–0.65	–0.61

Table 3: Business cycle statistics. I take logs of all of the series and remove trend using the Hodrick-Prescott filter with a smoothing parameter of 1,600. *std* is a standard deviation. The volatility of output is multiplied by 100. Other volatilities are their ratio with respect to the output volatility. *corr* is a correlation.

		U.S.	Constant	Independent	Countercyclical
Economy-wide	U	–0.006	–0.017	0.000	–0.003
		(0.002)	(0.003)	(0.046)	(0.002)
	lnY	0.293	0.467	0.559	0.172
		(0.077)	(0.036)	(0.115)	(0.116)
	lnH	0.373	0.743	0.053	–0.096
		(0.101)	(0.117)	(0.274)	(0.127)
Balanced panel	U	–0.014	–0.014	–0.012	–0.012
		(0.004)	(0.006)	(0.030)	(0.003)
	lnY	0.617	0.326	0.429	0.531
		(0.165)	(0.141)	(0.122)	(0.184)
	lnH	0.699	0.636	0.829	0.404
		(0.223)	(0.248)	(0.144)	(0.214)

Table 4: Composition effect. The table shows the estimated coefficient β_3 in (11). The values in parentheses are standard errors. The U.S. result is taken from Solon et al. (1994).

	Wage percentile	U.S.	Constant	Independent	Countercyclical
$corr(EN, Y)$	all	–0.66	–0.74	–0.19	–0.64
	0–25th	–0.53	–0.05	0.14	–0.37
	25-50th	–0.44	–0.42	0.07	–0.53
	50–75th	–0.66	–0.60	–0.27	–0.56
	75–100th	–0.24	–0.70	–0.38	–0.64
$std(EN)$	all	8.87	2.18	3.30	3.99
	0–25th	12.15	6.30	9.57	9.06
	25-50th	15.07	2.92	6.14	4.30
	50–75th	13.08	3.25	4.47	4.25
	75–100th	10.57	2.89	3.68	4.33

Table 5: Probability of moving from employment to nonemployment (EN rate). I take logs of all of the series and remove trend using the Hodrick-Prescott filter with a smoothing parameter of 10. *std* is the standard deviation relative to the output volatility. *corr* is a correlation.

	U.S.					
standard deviation		Constant	Independent	Counteryclical	Independent	Countercyclical
skewness		Zero	Zero	Zero	Procyclical	Procyclical
Labor market dynamics						
$std(wedge)$	1.40	0.23	0.98	0.95	0.98	0.76
$corr(H, Y/H)$	−0.32	0.83	−0.17	–0.10	–0.18	0.16
Std of earnings growth						
$std(\sigma_{earnings})$	1.22	0.37	0.95	1.16	0.95	1.08
$corr(\sigma_{earnings}, Y)$	–0.38	–0.50	0.17	–0.60	0.25	–0.59
3rd moment of earnings growth						
$corr(Skewness, \Delta lnY)$	0.74	0.56	0.36	0.71	0.46	0.75
$corr(L90-50, \Delta lnY)$	0.73	0.08	0.29	–0.46	0.35	–0.25
$corr(L50-10, \Delta lnY)$	–0.67	–0.59	–0.02	–0.85	–0.02	–0.82

Table 6: Cyclical variation in the labor market and earnings risk. For the standard deviation of earnings growth, $\sigma_{earnings}$, I take its log and remove trend using the Hodrick-Prescott filter with a smoothing parameter of 10. std is the standard deviation relative to the output volatility. corr is a correlation.

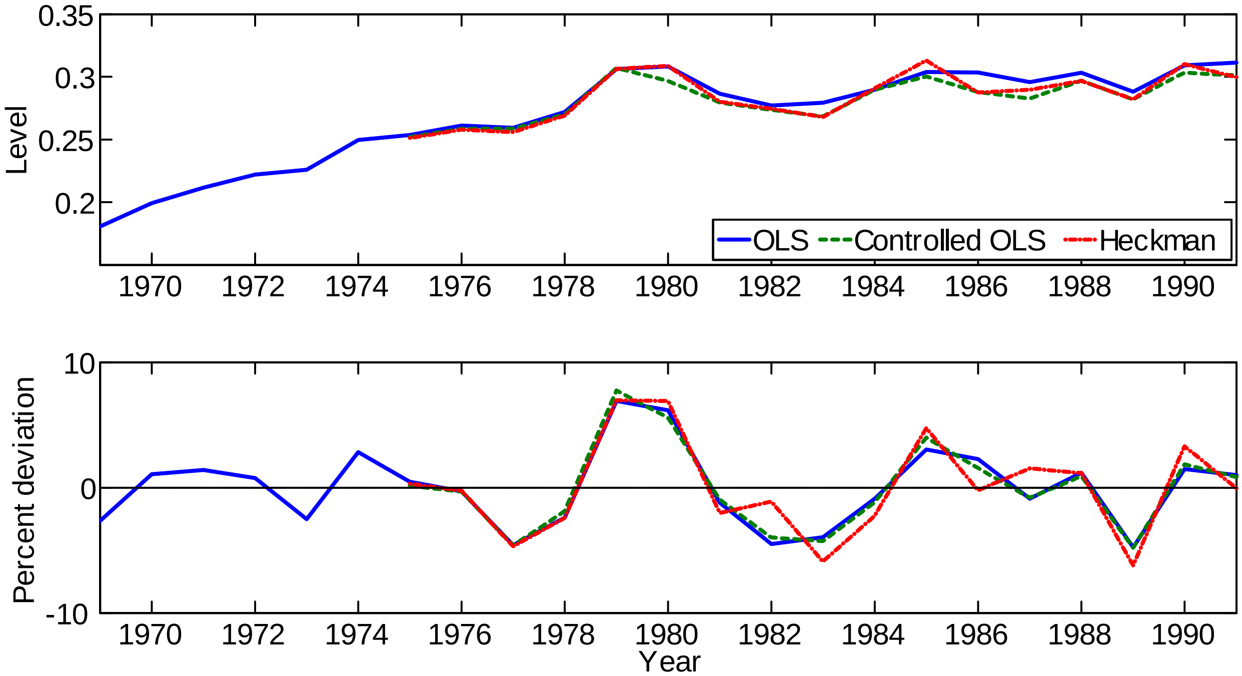

Figure 1: U.S. idiosyncratic wage risk. Upper panel: level. Lower panel: percent deviation from trend.

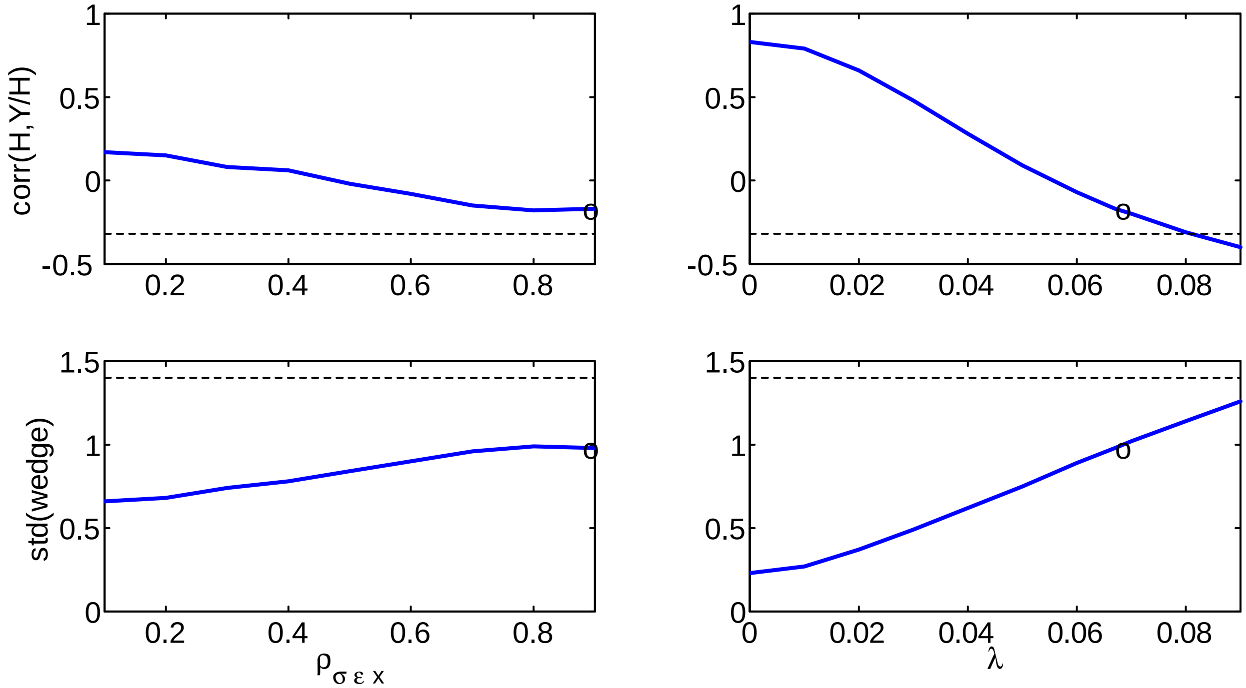

Figure 2: Risk variation and labor market statistics. Dotted line: U.S. data. Circle: benchmark independent risk model.

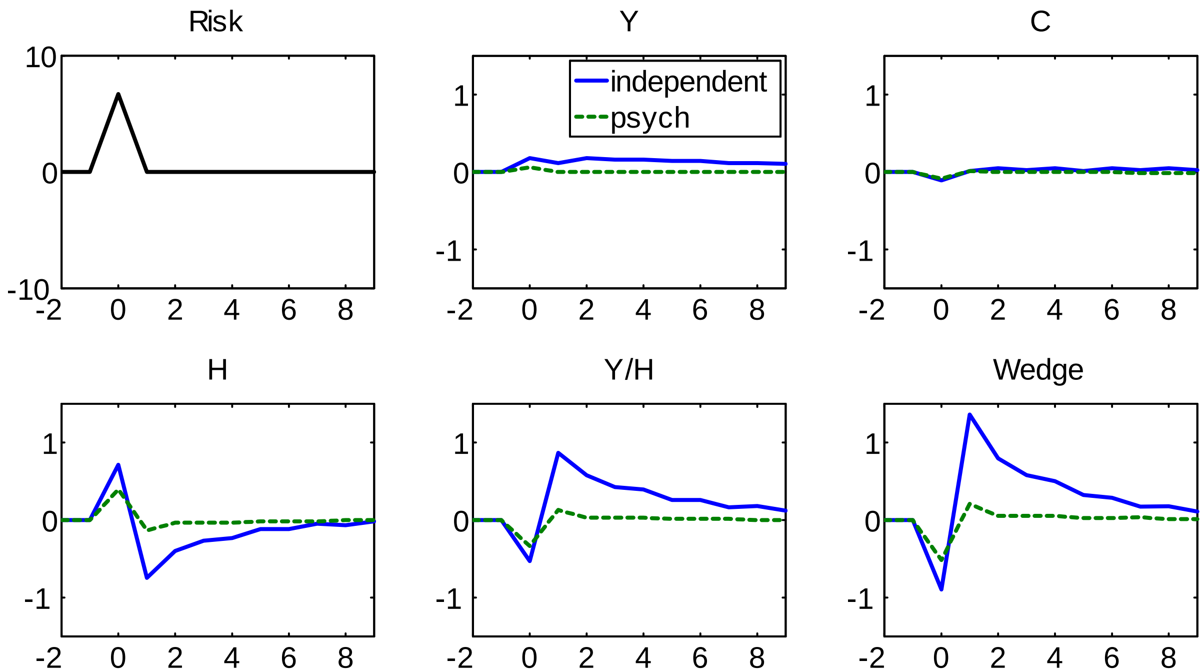

Figure 3: Impulse response to an increase in idiosyncratic wage risk. Horizontal axis: period. Vertical axis: percent deviation from the pre-shock level.

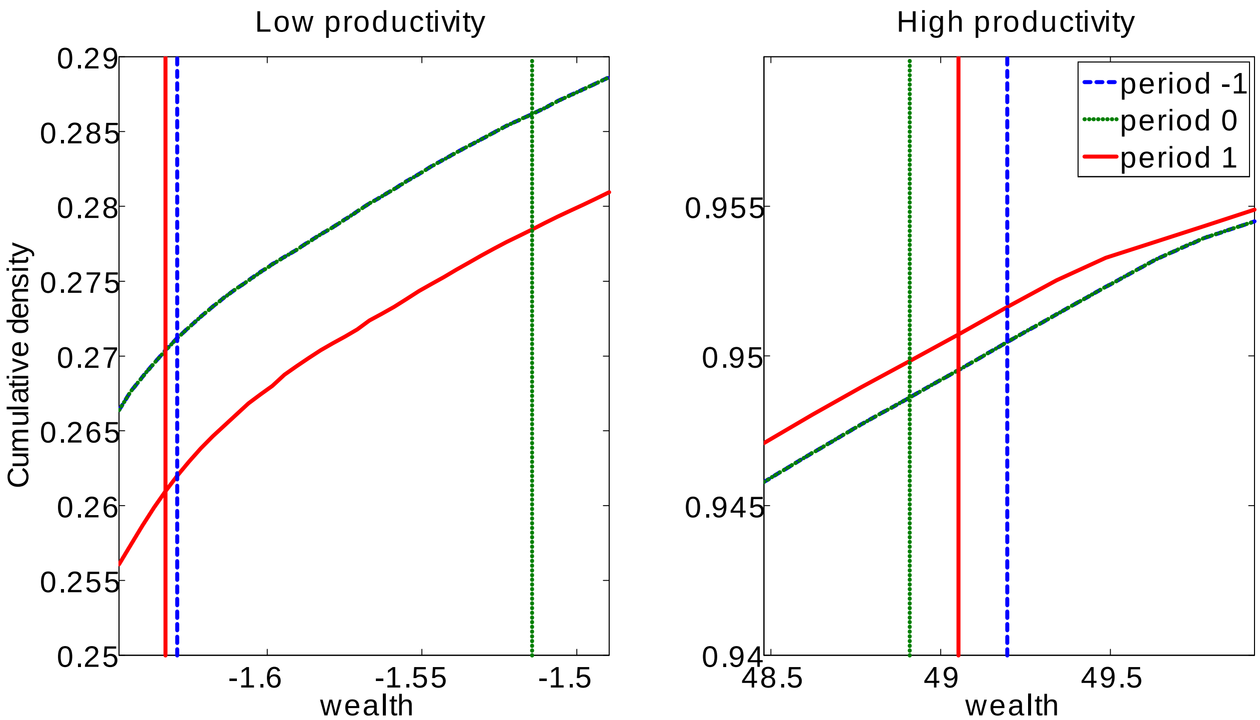

Figure 4: Effect of a rise in idiosyncratic wage risk on the employment rate. Left: individuals between the 10-20th pecentiles in the productivtiy distribution. Right: individuals between the 80-90th percentiles.

Supplementary Materials

Appendix A: Data

This section explains the source of the U.S. data analyzed in the present paper.

A1: Individual Wage Data

I take data for heads of households from the family-level file of the PSID. Individual wages are the ratios of annual labor income (1969: V514–1992: V21484) to annual hours worked (1969: V465–1992: V20344).[1] They are converted to real wages in terms of 1983 dollars using the CPI.[2]

I exclude the following observations.

- Observations whose heads change in the year (1969: V791–1992: V21388).
- Observations with major assignments assigned to the labor income and/or hours.[3]
- Observations with wages of less than one dollar (in 1983 dollars) or higher than 500 dollars (in 1983 dollars).[4]
- The most recent Latino sample and the Survey of Economic Opportunity sample.
- Observations with fewer than 100 annual hours.

[1] Numbers in parentheses are variable labels of the PSID. I exclude the interview year of 1993 because data on major assignment for labor income are not available.

[2] I take "Consumer Price Index for All Urban Consumers: All Items" from the FRED database at the Federal Reserve Bank of St. Louis. Using the PCE deflator instead of the CPI does not substantially change the moments used for calibration in Table 2A.

[3] See Swanson (2007) for details. For labor hours, I use the total hour accuracy (1969: V466–1984: V10038) until 1984, and after 1984, the main job hour accuracy (1985: V11141–1992: V20339), the overtime hour accuracy (1985: V11143–1992: V20341), and the extra job accuracy (1985: V11145–1992: V20343). For labor income, I use the accuracy code for wages and salaries (1970: V1192–1992: V20430) and the accuracy code for labor income except wages and salaries (1970: V1197–1992: V20435). For 1969, the accuracy code for total income is used (V515).

[4] Eliminating observations with wages of less than half of the legal minimum wage does not substantially change the cyclical variation in idiosyncratic wage risk.

- Self-employment observations (1969: V641–1992: V20696).

- Observations in the agricultural sectors (1969: V640–1992: V20701).

- Top-coded observations for income.

Other variables are age (1969: V1008–1992: V20651), sex (1969: V1010–1992: V20652), marital status (1969: V607–1992: V21522), and the number of children (1969: V550–1992: V20654). For years of education, I select "Grade Completed" (1975: ER30169–1992: ER30748) from the individual-level file and define experience as age minus education minus six.

A2: Individual Labor Income Data

The 1992 PSID individual-level file provides annual labor income data in 1991 for individuals, including those other than heads of households. I take total labor income (ER30750), excluding individuals younger than 16 (ER30736) and individuals with major assignments on their income and/or hours worked (ER30751, ER30755).

A3: Macroeconomic Data

The data period is from 1947Q3 to 2009Q3. Output is "Real Gross Domestic Product (billions of chained 2005 dollars)" taken from Table 1.1.6 of the Bureau of Economic Analysis (BEA). Consumption is "Personal Consumption Expenditures (PCE)" less durable goods obtained from Table 2.3.5 of the BEA. Investment is the sum of durable goods consumption in Table 2.3.5 and private fixed investment (including residential investment) in Table 5.3.5. I compute the real values of consumption and investment using the price index for Gross Domestic Product in Table 1.1.4. The data on total labor hours are the data constructed by Cociuba et al. (2009).[5]

[5] I am grateful to the authors for making the data available.

Appendix B: Solution Methods

This section describes the solution method for the steady state and the business cycle.

B1: Steady State

The solution method for the steady state is similar to that of Chang and Kim (2007).

1. Discretize the idiosyncratic state (k, x). Set 100 log-spaced points over [–2, 250] for k. For x, set 17 evenly spaced points over $[-3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}, 3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}]$ and compute the transition matrix using the method of Tauchen (1986).

2. Set a guess for the discount factor β.

3. Solve the individual optimization problem and obtain the beginning-of-period value function $V(k, x)$. The aggregate state $(z, \mu, \sigma_{\varepsilon_x})$ is constant at the steady state and hence omitted.

   (a) Compute the steady-state wage rate $\bar{w} = \alpha\bar{z}((1-\alpha)\bar{z}/(\bar{r}+\delta))^{(1-\alpha)/\alpha}$ with the target steady-state rental rate of capital $\bar{r} = 0.01$ and the steady-state aggregate TFP $\bar{z} = 1.0$.

   (b) Set a guess for the beginning-of-period value function $V_0(k, x)$.

   (c) Solve the consumption-saving problem for each employment choice:

   $$V_1^E(k, x) = \max_{k' \geq \bar{k}} \{u(w\bar{h}x + (1+r)k - k', \bar{h}) + \beta \sum_{x'} \pi_x(x'|x) V_0(k', x')\}$$

   and

   $$V_1^N(k, x) = \max_{k' \geq \bar{k}} \{u((1+r)k - k', 0) + \beta \sum_{x'} \pi_x(x'|x) V_0(k', x')\},$$

   where $\pi_x(x'|x)$ is the transition probability from x to x'. Use cubic spline interpolation to approximate the conditional expectation at k' off the grid points. If

$V_1^E(k,x) \geq V_1^N(k,x)$, then individuals with k and x choose to work. Otherwise, they do not work. Set $V_1(k,x) = \max\left\{V_1^E(k,x), V_1^N(k,x)\right\}$.

(d) If $V_1(k,x)$ is sufficiently close to $V_0(k,x)$, then set $V(k,x) = V_1(k,x)$ and proceed to the next step. Otherwise, update the value function as $V_0(k,x) = V_1(k,x)$ and return to (c).

4. Compute the steady-state individual distribution over wealth and productivity $\bar{\mu}(k,x)$.

   (a) Choose points used for approximating the distribution. Use 2,000 log-spaced points over $[-2, 250]$ for k and the points chosen in Step 1 for x.

   (b) Replace $V_0(k,x)$ in Step 3 (c) with $V(k,x)$ obtained in Step 3 (d). Solve the problems this time for $2,000 \times 17$ pairs of (k,x) and find their optimal wealth holding $k^{'}(k,x)$ and employment $h(k,x)$.

   (c) Suppose $k_m \leq k^{'}(k,x) < k_{m+1}$, where k_m and k_{m+1} are two sequential wealth points. Starting from an initial guess, keep updating the distribution until the distribution converges as follows: Individuals with (k,x) move to $(k_m, x^{'})$ with probability $\omega\pi_x(x^{'}|x)$ and to $(k_{m+1}, x^{'})$ with probability $(1-\omega)\pi_x(x^{'}|x)$, where $\omega = (k_{m+1} - k^{'})/(k_{m+1} - k_m)$. The result is the steady-state distribution $\bar{\mu}(k,x)$.

5. Compute the steady-state aggregate capital $\bar{K} = \displaystyle\int k\bar{\mu}([dk \times dx])$ and aggregate efficiency-weighted labor $\bar{L} = \displaystyle\int xh(k,x)\bar{\mu}([dk \times dx])$. Calculate the implied steady-state rental rate of capital $\bar{r} = (1-\alpha)\bar{z}\bar{K}^{-\alpha}\bar{L}^{\alpha} - \delta$. If $\bar{r}$ is sufficiently close to the target rate, then stop. Otherwise, set a different value for β and repeat Steps 3–5.

B2: Business Cycles

I analyze the model's business cycle generalizing the Krusell and Smith (1997, 1998) algorithm. The method is similar to that used in Takahashi (2014). I show here the method for the independent risk model as an example.

1. Discretize the aggregate state $(z, \mu, \sigma_{\epsilon_x})$. For aggregate TFP z, set nine evenly spaced points over $[-3\bar{\sigma}_{\varepsilon_z}/\sqrt{1-\rho_z^2}, 3\bar{\sigma}_{\varepsilon_z}/\sqrt{1-\rho_z^2}]$, and compute the transition matrix using the method of Tauchen (1986). Replace the individual distribution μ with aggregate capital K. Use seven evenly spaced points over $[0.80\bar{K}, 1.20\bar{K}]$, where $\bar{K}$ is the steady-state aggregate capital. For σ_{ϵ_x}, use the three risk states.

2. Discretize the individual state (k, x). For k, use the 100 points chosen in the steady-state solution. For x, use 17 evenly spaced points over $[-3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}, 3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}]$ for all of the risk states. The transition probabilities vary with the risk states. Compute these probabilities using the method of Tauchen (1986).

3. Individuals forecast $K^{'}$ and w using the following rules:

$$\ln \hat{K}^{'} = a_{0,i} + a_{1,i} \ln K + a_{2,i} \ln z \tag{1}$$

and

$$\ln \hat{w} = b_{0,i} + b_{1,i} \ln K + b_{2,i} \ln z, \tag{2}$$

for each risk state ($i = H, M, L$). Individuals compute $\hat{r} = z(1-\alpha)(\hat{w}/(\alpha z))^{-\alpha/(1-\alpha)}$.

4. Solve the individual optimization problem and obtain the beginning-of-period value function $V(k, x; z, K, \sigma_{\epsilon_x})$.

   (a) Set a guess for the beginning-of-period value function $V_0(k, x; z, K, \sigma_{\epsilon_x})$.

   (b) Solve the consumption-saving problem for each employment choice:

$$V_1^E(k, x; z, K, \sigma_{\epsilon_x}) = \max_{k^{'} \geq \bar{k}} \{u(\hat{w}\bar{h}x + (1+\hat{r})k - k^{'}, \bar{h})$$
$$+ \beta \sum_{x^{'}} \sum_{z^{'}} \sum_{\sigma^{'}_{\epsilon_x}} \pi_x(x^{'}|x, \sigma_{\epsilon_x}) \pi_z(z^{'}|z) \pi_{\sigma_{\epsilon_x}}(\sigma^{'}_{\epsilon_x}|\sigma_{\epsilon_x}) V_0(k^{'}, x^{'}; z^{'}, \hat{K}^{'}, \sigma^{'}_{\epsilon_x})$$

and

$$V_1^N(k,x;z,K,\sigma_{\epsilon_x}) = \max_{k' \geq \bar{k}} \{u((1+\hat{r})k - k', 0)$$
$$+ \beta \sum_{x'} \sum_{z'} \sum_{\sigma'_{\epsilon_x}} \pi_x(x'|x,\sigma_{\epsilon_x})\pi_z(z'|z)\pi_{\sigma_{\epsilon_x}}(\sigma'_{\epsilon_x}|\sigma_{\epsilon_x})V_0(k',x';z',\hat{K}',\sigma'_{\epsilon_x}),$$

where $\pi_x(x'|x,\sigma_{\epsilon_x})$ is the transition probability from x to x' under σ_{ϵ_x}, $\pi_z(z'|z)$ is the transition probability from z to z', and $\pi_{\sigma_{\epsilon_x}}(\sigma'_{\epsilon_x}|\sigma_{\epsilon_x})$ is the transition probability from σ_{ϵ_x} to σ'_{ϵ_x}. Use bivariate cubic spline interpolation in (K,k) to approximate the conditional expectation at $(\hat{K}',k')$ off their grid points. If $V_1^E(k,x;z,K,\sigma_{\epsilon_x}) \geq V_1^N(k,x;z,K,\sigma_{\epsilon_x})$, then individuals with k and x work. Otherwise, they do not. Set $V_1(k,x;z,K,\sigma_{\epsilon_x}) = \max\{V_1^E(k,x;z,K,\sigma_{\epsilon_x}), V_1^N(k,x;z,K,\sigma_{\epsilon_x})\}$.

(c) If $V_1(k,x;z,K,\sigma_{\epsilon_x})$ is sufficiently close to $V_0(k,x;z,K,\sigma_{\epsilon_x})$, then proceed to the next step, setting $V(k,x;z,K,\sigma_{\epsilon_x}) = V_1(k,x;z,K,\sigma_{\epsilon_x})$. Otherwise, update the value function as $V_0(k,x;z,K,\sigma_{\epsilon_x}) = V_1(k,x;z,K,\sigma_{\epsilon_x})$ and return to (b).

5. Generate 3,500-period data using the beginning-of-period value function $V(k,x;z,K,\sigma_{\epsilon_x})$.

(a) Set the initial conditions: $z_1 = \bar{z}, \sigma_{\epsilon_x 1} = \sigma_{\epsilon_x,M}, \mu_1(k,x) = \bar{\mu}(k,x)$, and $K_1 = \int k\mu_1([dk \times dx])$.

(b) Set $\tilde{w}_1$, as a guess for w_1. Then, $\tilde{r}_1 = (1-\alpha)z_1(\tilde{w}_1/\alpha z_1)^{-\alpha/(1-\alpha)} - \delta$. The forecasting rule gives the individuals' forecast of the next period approximate aggregate capital $\hat{K}_2$. Replacing $V_0(k,x;z,K,\sigma_{\epsilon_x})$ with $V(k,x;z,K,\sigma_{\epsilon_x})$, solve the individual problems shown in Step 4 (b) under $w = \tilde{w}_1$, $r = \tilde{r}_1$, and $K' = \hat{K}_2$, this time for $2,000 \times 17$ pairs of (k,x). Record the optimal wealth holding $k_2(k,x)$ and employment $h_1(k,x)$.

(c) Check labor market clearing: $\tilde{L}_1 \equiv (\alpha z_1/\tilde{w}_1)^{1/(1-\alpha)}K_1 = \int xh_1(k,x)\mu_1([dk \times dx])$. If the labor market clears, then proceed to the next step. Otherwise, reset $\tilde{w}_1$ and return to (b).[6]

(d) Compute aggregate variables: $L_1 = \int x h_1(k,x)\mu_1([dk \times dx]), K_2 = \int k_2(k,x)\mu_1([dk \times dx]), H_1 = \int h_1(k,x)\mu_1([dk \times dx]), Y_1 = z_1 K_1^{1-\alpha} L_1^{\alpha}$,

$I_1 = K_2 - (1-\delta)K_1, C_1 = Y_1 - I_1$, and $r_1 = (1-\alpha)z_1 K_1^{-\alpha} L_1^{\alpha} - \delta$.

(e) Obtain the next period distribution $\mu_2(k,x)$ using the method described in Step 4 (c) of the steady-state solution.

(f) Repeat (b)–(e) for 3,500 periods.

6. Using the simulated data (disregarding the first 500 periods), update the coefficients of the forecasting rules by ordinary least squares. If these coefficients converge, then proceed to the next step. Otherwise, repeat Steps 4 and 5 using the new forecasting rules.

7. Check whether the converged forecasting rules are sufficiently accurate. If not, assume different functional forms and repeat Steps 3–6. The forecasting rules of (11) and (12) are quite accurate, as reported in Appendix C.

Appendix C: Forecasting Rules

Table A1 lists the coefficients of the forecasting rules ($\ln \hat{K}' = a_0 + a_1 \ln K + a_2 \ln z$ and $\ln \hat{w} = b_0 + b_1 \ln K + b_2 \ln z$) and the accuracy of the rules for the constant, benchmark independent, and countercyclical risk models.[7] Two accuracy measures are the coefficient of determination R^2 and the standard deviation of the forecasting error $\hat{\sigma}$. Separate rules are used for each of the risk states.

[6] Ensuring market clearing is an essential step of the Krusell and Smith (1998) algorithm and included for the bond market by Krusell and Smith (1997) and Pijoan-Mas (2007), for the goods market by Khan and Thomas (2003, 2007, 2008), and for the labor market by Takahashi (2014).

[7] The results for other models are available upon request.

Appendix D: Procyclical Skewness in Idiosyncratic Productivity Shocks

The main text describes the countercyclical risk model with the procyclical skewness of idiosyncratic productivity shocks, where the standard deviation of idiosyncratic productivity shocks is negatively and the skewness is positively correlated with aggregate TFP growth. This section describes the independent risk model with the procyclical skewness of idiosyncratic shocks, where the standard deviation moves independently of aggregate TFP and the skewness is positively correlated with aggregate TFP growth. There are $3 \times 3 = 9$ states.

1. High standard deviation of idiosyncratic productivity shocks

   (a) If aggregate TFP increased from the previous to current periods by more than 1.67%, then the skewness of idiosyncratic productivity shocks is positive:

   $$\begin{aligned} \varepsilon_x^{'} \;\; \sim \;\; & N(\mu_{1,P}, (1+\lambda_H)^2\sigma_1^2) \text{ with probability } p \\ & N(\mu_{2,P}, (1+\lambda_H)^2\sigma_2^2) \text{ with probability } 1-p. \end{aligned}$$

   (b) If aggregate TFP changed by less than $\pm 1.67\%$, then the skewness is zero:

   $$\varepsilon_x^{'} \sim N(0, (1+\lambda)^2\overline{\sigma}_{\epsilon_x}^2).$$

   (c) If aggregate TFP decreased by more than 1.67%, then the skewness is negative:

   $$\begin{aligned} \varepsilon_x^{'} \;\; \sim \;\; & N(\mu_{1,N}, (1+\lambda_H)^2\sigma_1^2) \text{ with probability } p \\ & N(\mu_{2,N}, (1+\lambda_H)^2\sigma_2^2) \text{ with probability } 1-p. \end{aligned}$$

2. Middle standard deviation of idiosyncratic productivity shocks

(a) If aggregate TFP increased from the previous to current periods by more than 1.67%, then the skewness of idiosyncratic productivity shocks is positive:

$$\begin{aligned} \varepsilon_x^{'} \;\sim\; & N(\mu_{1,P}, \sigma_1^2) \text{ with probability } p \\ & N(\mu_{2,P}, \sigma_2^2) \text{ with probability } 1-p. \end{aligned}$$

(b) If aggregate TFP changed by less than $\pm 1.67\%$, then the skewness is zero:

$$\varepsilon_x^{'} \sim N(0, \overline{\sigma}_{\epsilon_x}^2).$$

(c) If aggregate TFP decreased by more than 1.67%, then the skewness is negative:

$$\begin{aligned} \varepsilon_x^{'} \;\sim\; & N(\mu_{1,N}, \sigma_1^2) \text{ with probability } p \\ & N(\mu_{2,N}, \sigma_2^2) \text{ with probability } 1-p. \end{aligned}$$

3. Low standard deviation of idiosyncratic productivity shocks

(a) If aggregate TFP increased from the previous to current periods by more than 1.67%, then the skewness of idiosyncratic productivity shocks is positive:

$$\begin{aligned} \varepsilon_x^{'} \;\sim\; & N(\mu_{1,P}, (1-\lambda_L)^2\sigma_1^2) \text{ with probability } p \\ & N(\mu_{2,P}, (1-\lambda_L)^2\sigma_2^2) \text{ with probability } 1-p. \end{aligned}$$

(b) If aggregate TFP changed by less than $\pm 1.67\%$, then the skewness is zero:

$$\varepsilon_x^{'} \sim N(0, (1-\lambda)^2\overline{\sigma}_{\epsilon_x}^2).$$

(c) If aggregate TFP decreased by more than 1.67%, then the skewness is negative:

$$\begin{aligned}\varepsilon_x^{'} \sim \; & N(\mu_{1,N}, (1-\lambda_L)^2\sigma_1^2) \text{ with probability } p \\ & N(\mu_{2,N}, (1-\lambda_L)^2\sigma_2^2) \text{ with probability } 1-p.\end{aligned}$$

I set $\lambda = 0.067$ and $\overline{\sigma}^2_{\epsilon_x} = 0.223$ and the standard deviation of idiosyncratic productivity shocks changes as in the original independent risk model. Other parameters are the same as those of the countercyclical risk model with the procyclical skewness described in the main text: $p = 0.5, \sigma_1 = 0.239, \sigma_2 = 0.150, \mu_{1,H} = -0.1, \mu_{1,L} = 0.1, \mu_{2,H} = 0.1, \mu_{2,L} = -0.1$. This implies $\lambda_H = 0.084$ and $\lambda_L = 0.084$. The implied moments are listed in Table A2. As in the original independent risk model, the standard deviation of idiosyncratic productivity shocks varies by 6.7% independently of aggregate TFP. In addition, the skewness of idiosyncratic productivity shocks is positively correlated with aggregate TFP growth. The magnitude is similar to that of the countercyclical risk model with the procyclical skewness of idiosyncratic productivity shocks.

Appendix E: Model Implied Labor Wedge

I feed the estimated idiosyncratic wage risk into the independent risk model and compare the model implied labor wedge with the U.S. wedge. Since the model is quarterly and the estimated risk is annual, I obtain the approximated quarterly series for idiosyncratic wage risk as follows. If the detrended annual risk is above 2.0% in a year, then idiosyncratic wage risk is in the high state for all the quarters in that year. If the detrended annual risk is below 2.0% in the year, then idiosyncratic wage risk is in the low state for all the quarters in that year. In other years, idiosyncratic wage risk is in the middle state throughout the year. The upper panel of Figure A1 shows the result. While not perfect, the approximated series tracks the original series reasonably well. I then choose the aggregate TFP series so that the (detrended) model output matches the (detrended) U.S. output. As shown in the lower panel, the model output successfully keeps track of the U.S. output.[8] The correlation between the two is 0.94.

Figure A2 presents the labor wedge. With only changes in aggregate TFP, the model cannot generate the substantial volatility in the labor wedge seen in the U.S. Adding changes in idiosyncratic wage uncertainty greatly improves the model's ability in generating the volatile labor wedge. While not perfect, the correlation between the U.S. and model wedges is 0.45.[9] The result supports the present paper's argument that introducing uncertainty shocks improves the model's ability in accounting for the movement in the labor wedge.

Appendix F: Impulse Responses to Aggregate TFP Shocks

This section shows the response of the independent risk model to an exogenous decline in aggregate TFP z. The simulation starts from the steady state. For initialization, z is at the steady state for 150 periods and then it declines by 1.67% (i.e., one grid point). I normalize the period to period 0, as in the upper-left panel of Figure A3. Idiosyncratic wage risk σ_{ε_x} is fixed at its steady-state level throughout.

The green dotted lines in the other panels show the responses of other variables. As shown, output, total hours worked, and average labor productivity all decrease following the decrease in aggregate TFP. As in the prototype equilibrium business cycle model, a decline in aggregate TFP reduces labor demand, without significantly affecting labor supply. In equilibrium, the wage rate falls, and employment decreases across all productivity groups largely uniformly. Since aggregate TFP decreases, output decreases more substantially than hours, lowering average labor productivity. Furthermore, since the wealth-productivity distribution hardly shifts, output, hours, and productivity recover quickly.

I also analyze the model's response when idiosyncratic wage risk σ_{ε_x} rises simultaneously

[8] The simulation starts from the steady state and runs for 150 periods for initialization. The U.S. uncertainty and aggregate TFP series are then fed into the model.

[9] The results do not change substantially from that of the 2.0% cutoff. The correlation between the U.S. and model wedges is 0.41 for the 2.5% cutoff and 0.45 for the 1.5% cutoff.

with a fall in aggregate TFP z . After being in the middle state (i.e., at the steady state) for 150 periods, σ_{ε_x} moves into the high state, increasing by 6.7% for one period in period 0, as shown in the upper-left panel of Figure A3. The blue lines in the other panels show the responses of other variables to these two shocks. The recovery shows a feature of a jobless recovery in that output recovers much more quickly than total hours worked. In contrast, output and hours recover together when only aggregate TFP falls. Hence, an increase in idiosyncratic wage risk during a recession generates a jobless recovery.

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		Constant	Independent (H / M / L)	Countercyclical (H / M / L)
$\hat{K}'$	a_0	0.115	0.087 / 0.091 / 0.091	0.092 / 0.095 / 0.096
	a_1	0.953	0.965 / 0.963 / 0.963	0.962 / 0.961 / 0.961
	a_2	0.101	0.089 / 0.090 / 0.096	0.085 / 0.087 / 0.088
	R^2	1.000	1.000 / 1.000 / 1.000	1.000 / 1.000 / 1.000
	$\hat{\sigma}$	0.0079%	0.0880% / 0.0737% / 0.0752%	0.0151% / 0.0142% / 0.0135%
$\hat{w}$	b_0	–0.209	–0.050 / –0.082 / –0.081	–0.084 / –0.086 / –0.081
	b_1	0.438	0.372 / 0.387 / 0.388	0.388 / 0.388 / 0.386
	b_2	0.818	0.890 / 0.872 / 0.842	0.906 / 0.907 / 0.908
	R^2	1.000	0.983 / 0.988 / 0.983	1.000 / 1.000 / 1.000
	$\hat{\sigma}$	0.0407%	0.4810% / 0.3915% / 0.4214%	0.0500% / 0.0555% / 0.0690%

Table A1: Forecasting rules.

standard deviation	High			Middle			Low		
skewness	Positive	Zero	Negative	Positive	Zero	Negative	Positive	Zero	Negative
standard deviation	0.238	0.238	0.238	0.223	0.223	0.223	0.208	0.208	0.208
skewness	0.450	0.0	–0.450	0.466	0.0	–0.466	0.480	0.0	–0.480

Table A2: Standard deviation and skewness (i.e., the third central moment) of idiosyncratic productivity shocks for the independent risk model with the procyclical skewness in idiosyncratic productivity shocks.

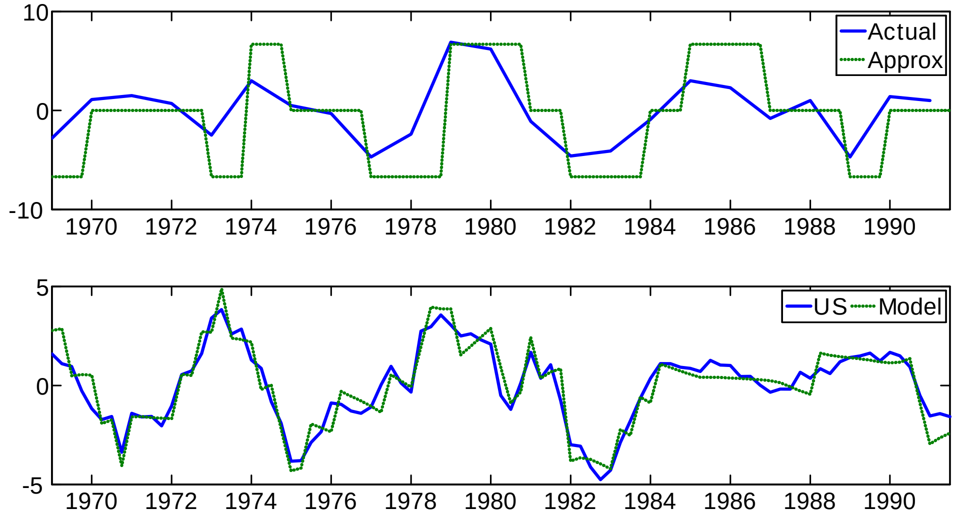

Figure A1: Upper panel: actual and approximated idiosyncratic wage risk. Lower panel: U.S. and model output. Horizontal axis: year. Vertical axis: percent deviation from trend.

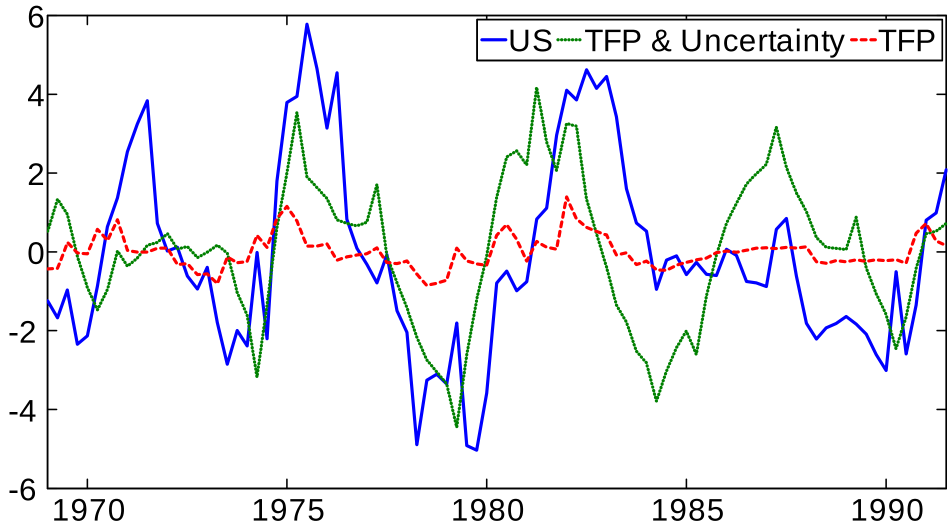

Figure A2: U.S. and model labor wedges. Horizontal axis: year. Vertical axis: percent deviation from trend.

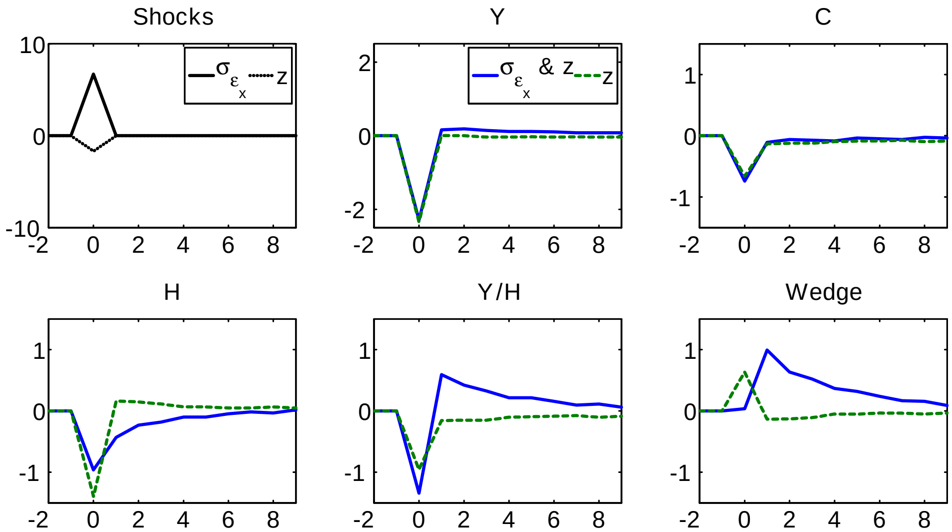

Figure A3: Impulse response to a fall in aggregate TFP and an increase in idiosyncratic wage risk. Horizontal axis: period. Vertical axis: percent deviation from the pre-shock level.