PAPER

Fixed-Rate Resource Exchange for Multi-Operator Pico eNodeB

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SUMMARY In this paper, we introduce a new multi-operator pico eNodeB (eNB) concept for cellular networks. It is expected that mobile data offloading will be performed effectively after installing the pico eNBs in cellular networks, owing to the rapid increase in mobile traffic. However, when several different operators independently install the pico eNBs, high costs and large amounts of space will be required for the installation. In addition, when several different operators accommodate their own user equipments (UEs) in the pico eNBs, not enough UEs can be accommodated. This is because the UEs are not evenly distributed in the coverage area of the pico eNBs. In this paper, the accommodation of the UEs of different operators in co-sited pico eNB is discussed as one of the solutions to these problems. For the accommodation of the UEs of different operators, wireless resources should be allocated to them. However, when each operator independently controls his wireless resources, the operator is not provided with an incentive to accommodate the UEs of the other operators in his pico eNBs. For this reason, an appropriate rule for appropriate allocation of the wireless resources to the UEs of different operators should be established. In this paper, by using the concepts of game theory and mechanism design, a resource allocation rule where each operator is provided with an incentive to allocate the wireless resources to the UEs of different operators is proposed. With the proposed rule, each operator is not required to disclose the control information like link quality and the number of UEs to the other operators. Furthermore, the results of a throughput performance evaluation confirm that the proposed scheme improves the total throughput as compared with individual resource allocation.
key words: *multi-operator, pico eNodeB, resource allocation, game theory, mechanism design*

1. Introduction

Recently, the 3rd Generation Partnership Project (3GPP) standardized the Long Term Evolution-Advanced (LTE-Advanced) as the next-generation mobile communication system. In the LTE-Advanced system, data rates of up to 1 Gbit/s at 100 MHz bandwidth are supported in the downlink.

For the LTE-Advanced system, a new concept of network sharing has been proposed. This concept is gaining attention because it has the potential to reduce both the infrastructure and environmental costs [1], [2]. Network sharing is a concept of multiple operators sharing the same eNodeB (eNB) or spectrum resources. It was introduced as a topic in the 3GPP Release 10 standard [3]. For example, up to six operators are allowed to share the same eNB [4], and network sharing can be applied to various communication systems. The effective use of limited resources by network sharing is expected to become a more common agenda [5].

In addition, the installation of small-cell pico eNBs in the coverage area of large-cell macro eNBs for mobile data offloading is gaining more attention. By the installation of the pico eNBs, the area spectral efficiency is expected to be improved [6], [7].

However, when each operator individually installs the pico eNBs and accommodates only his own user equipments (UEs), the number of UEs accommodated in the pico eNBs of each operator is reduced and thus the pico eNBs are not always effectively used. This is because the UEs of each operator are not evenly distributed. Furthermore, there is a high increase in the cost and space for the installation in order to enhance the coverage area of the pico eNBs. Particularly, in an area where the space for the installation of the pico eNBs is limited, such as an underground shopping area and an underground railway, the number of pico eNBs is expected to be reduced.

In this paper, we discuss multi-operator pico eNBs by introducing the concept of network sharing. As shown in Fig. 1, by accommodating the UEs of different operators in the pico eNBs, all the operators can use their pico eNBs more effectively.

We discuss the allocation of the wireless resources to the UEs of different operators to accommodate the UEs of different operators in the pico eNBs. When there is no rule

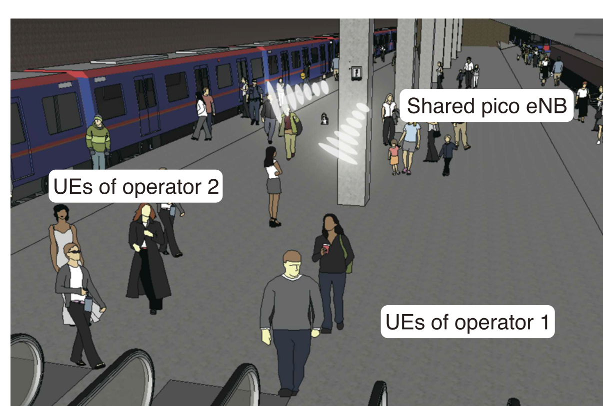

Fig. 1 Multi-operator pico eNB.

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for resource allocation between the operators and when each operator individually controls his resource, each operator is not provided with an incentive to allocate the resources to the UEs of different operators. As a result, the UEs cannot be accommodated in the pico eNBs of different operators and the pico eNBs are not effectively shared.

These problems can be solved by establishing an appropriate rule for resource allocation where each operator is provided with an incentive to accommodate the UEs of different operators. This rule should be informationally decentralized. In other words, it is expected that each operator is not required to disclose the information on the number of UEs or channel quality. In addition, it is expected that the operators will always receive some benefit by joining the system.

In this paper, we propose a fixed-rate resource exchange scheme by introducing the concepts of game theory and mechanism design. Game theory is one of the economic theories which enables the modeling of interactions among multiple rational players. It is applied to the concept of mechanism design. The mechanism design is a theoretical framework to achieve the establishment of rules in a society consisting of rational agents. In addition, it is confirmed that the proposed scheme satisfies the above requirements and improves the total throughput.

The remainder of this paper is organized as follows. In Sect. 2, we describe the concepts of game theory and mechanism design. In Sect. 3, we present the network system model comprising multi-operator pico eNBs and describe the need for establishing a rule for resource allocation between the operators. In Sect. 4, we propose a fixed-rate resource exchange scheme by applying the mechanism design and pure exchange economy model. In Sect. 5, we evaluate the total throughput performance and the amount of exchanged resources by theoretical calculation. In Sect. 6, we present our conclusions.

2. Game Theory and Mechanism Design

In this section, the concepts of game theory and mechanism design are described. First, a strategic form game is introduced as the simplest example of game theory. By using the framework of the strategic form game, the interactions among multiple rational players can be discussed. Further, the concept of mechanism design is described. In the mechanism design, the establishment of the rules in the society consisting of multiple rational players can be discussed. Further, some of the research studies on exchange economy as a branch of mechanism design are introduced.

2.1 Strategic Form Game

A strategic form game consists of a set of players $\mathcal{I} = \{1, \ldots, N\}$, a set of strategies for each player $\mathcal{X}_i$, and the utility of each player f_i. Therefore, the strategic form game is denoted as $(\mathcal{I}, \{\mathcal{X}_i\}_{i\in\mathcal{I}}, \{f_i\}_{i\in\mathcal{I}})$. Each player chooses a strategy $x_i \in \mathcal{X}_i$ in order to maximize his utility f_i. The utility f_i is dependent on the strategies of the other players $\boldsymbol{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N)$. The best response $x_i^\star$ is a strategy that maximizes the utility of player i when the strategies of the other players are $\boldsymbol{x}_{-i}$. $x_i^\star$ satisfies the following condition:

$$f_i(x_i^\star, \boldsymbol{x}_{-i}) \geq f_i(x_i, \boldsymbol{x}_{-i}),\ \forall x_i \in \mathcal{X}_i. \tag{1}$$

In addition, the strategy vector $\boldsymbol{x}^\star = (x_i^\star, \boldsymbol{x}_{-i}^\star)$ that every player chooses the best response is called Nash equilibrium. The Nash equilibrium is well known as a solution concept of a strategic form game. At the Nash equilibrium, every player cannot improve his own utility by changing the strategy as follows:

$$f_i(x_i^\star, \boldsymbol{x}_{-i}^\star) \geq f_i(x_i, \boldsymbol{x}_{-i}^\star),\ \forall x_i \in \mathcal{X}_i, \forall i \in \mathcal{I}. \tag{2}$$

2.2 Mechanism Design

Let a set of players be denoted as $\mathcal{J} = \{1, \ldots, M\}$ and a set of the available resource allocations be denoted as $\mathcal{Y}$. Player $i \in \mathcal{J}$ is assumed to have the preference $\succsim_i$ on $\mathcal{Y}$. Preference $\succsim_i$ is the notation of economics and it denotes the order of priority. For example, $\boldsymbol{a} \succsim_i \boldsymbol{b}$ means that player i prefers $\boldsymbol{a}$ to $\boldsymbol{b}$. Let the preferences of the players be denoted as $\succsim \equiv (\succsim_1, \ldots, \succsim_M)$. When the preferences of the players are $\succsim$, a function $g(\succsim)$ that chooses the only socially appropriate result from $\mathcal{Y}$ is called a social choice function.

By setting the outcome function $h(\boldsymbol{m})$ from the messages of the players $\boldsymbol{m} = (m_1, \ldots, m_M)$, the rule to choose the best distribution can be designed. It can be noted that the message $\boldsymbol{m}$ is transmitted from the players to the manager of the rule and $h(\boldsymbol{m})$ is the function of $\boldsymbol{m}$ and selects the distribution from $\mathcal{X}$. If $\boldsymbol{m} = \succsim$, the social objectives can be easily achieved by setting $h = g$. However, the manager does not know the information on the truthful priorities $\succsim$, and the joined players have the incentive to transmit strategic massages to the manager in order to maximize their own utility. This results in unfairness among the players and system instability. For this reason, appropriate rules should be established where every player is not required to take strategic actions.

For obtaining an appropriate outcome function, we consider the function g which satisfies the following conditions:

$$g(\succsim) \succsim_i g(\succsim_i', \succsim_{-i}), \forall i \in \mathcal{J}, \forall \succsim, \forall \succsim_i', \tag{3}$$

where the false preference of player i is denoted as $\succsim_i'$ and the preferences of the other players are denoted as $\succsim_{-i}$. The social choice function g that satisfies this condition satisfies the strategy-proofness. Transmitting genuine messages to the manager is the best response for all the players, regardless of the messages of the other players. In other words, by setting a social choice function g that satisfies the strategy-proofness, we need not consider the strategic messages of all the players. As a result, the manager can easily establish the rule and the system instability caused by the strategic actions can be overcome.

However, the distribution gained by the social choice function that satisfies the strategy-proofness does not always satisfy the Pareto efficiency and individual rationality. There is a trade-off among the strategy-proofness, Pareto efficiency, and individual rationality.

When there is no distribution $\boldsymbol{b} \in \mathcal{X}$ that satisfies the following conditions:

$$\boldsymbol{b} \succsim_i \boldsymbol{a},\ \forall i \in \mathcal{J}, \tag{4}$$

$$\boldsymbol{b} \succ_i \boldsymbol{a},\ \exists i \in \mathcal{J}, \tag{5}$$

the distribution $\boldsymbol{a}$ satisfies Pareto efficiency. It means that every player cannot improve the utility of his own without decreasing the utility of other players.

When the initial distribution $\boldsymbol{w} = (w_1, \ldots, w_N)$ satisfies the following condition:

$$\boldsymbol{a} \succsim_i \boldsymbol{w},\ \forall i \in \mathcal{J}, \tag{6}$$

the distribution $\boldsymbol{a}$ satisfies the individual rationality. This implies that every player can always improve his own utility by joining the mechanism.

2.3 Mechanism Design for Exchange Economy

By using the concept of mechanism design, the resource allocation problems among the operators can be treated as a type of exchange economy. The exchange economy can be modeled as a problem to establish a social choice function $g(\succsim)$. There are many research studies on the trade-off among the Pareto efficiency, individual rationality, and strategy-proofness. In 1972, Hurwicz confirmed that there is no social choice function that simultaneously satisfies the Pareto efficiency, individual rationality, and strategy-proofness in two-person and two-goods exchange economies [8]. In 2002, it was confirmed that the Pareto efficiency, individual rationality, and strategy-proofness cannot be simultaneously satisfied in more than two-person and more than two-goods case [9]. In 2003, Serizawa confirmed that there is a serious unfairness in the social choice function that satisfies the Pareto efficiency and strategy-proofness [10]. Barbera and Jackson confirmed that the only exchange rule that satisfies the individual rationality and strategy-proofness is the fixed-price trading [11].

2.4 Strategy-Proof Exchange

In [11], the exchange economies with a finite number of players who know the information about their preferences are discussed. It is confirmed that the only social choice function that satisfies the strategy-proofness and individual rationality is the fixed-price trading.

The simplest example of the fixed-price trading can be illustrated by using a two-person two-good exchange economy model. Let us assume that player 1 is endowed with A units of good 1 and player 2 is endowed with B units of good 2, as shown in Fig. 2. Each player has a private utility function $u_i(p, q)$ depending on the combined units of the amount of goods 1 and 2, where p and q are the number of units of goods 1 and 2, respectively, which are finally allocated to player i. The utility functions are continuous and strictly quasi-concave as follows:

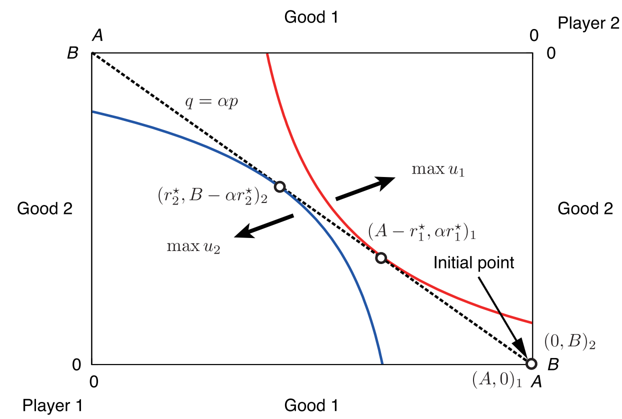

Fig. 2 Fixed-price trading represented by the Edgeworth box.

$$u_i(tp+(1-t)p', tq+(1-q)q') > \min\{u_i(p,q), u_i(p',q')\}, \tag{7}$$

$$\forall(p,q), \forall(p',q'), \forall t, \tag{8}$$

where t is defined as $0 < t < 1$ and (p, q) is a point different from (p', q').

When the pre-specified proportion value is α and the goods are traded between the players corresponding to α, each player has only one distribution that maximizes his utility function on $q = \alpha p$. If the point that maximizes the utility function of player 1 is $(A - r_1^\star, \alpha r_1^\star)$, it implies that player 1 is willing to trade $r_1^\star$ units of good 1 with $\alpha r_1^\star$ units of good 2. If the point that maximizes the utility function of player 2 is $(r_2^\star, B - \alpha r_2^\star)$, it implies that player 2 is willing to trade $\alpha r_2^\star$ units of good 2 with $r_2^\star$ units of good 1. In this case, by exchanging $\min(r_1^\star, r_2^\star)$ units of good 1 and $\alpha \min(r_1^\star, r_2^\star)$ units of good 2 between the players, the strategy-proofness and individual rationality can be satisfied.

It can be noted that this discussion can be extended to the case of more than two goods, and the fixed-price trading is the only social choice function that satisfies both the strategy-proofness and individual rationality, as proven in [11]. In addition, the fixed-price trading can be extended to the case of more than two-person exchange economies. In this paper, for the sake of simplicity, only the two-person two-good exchange economy model is introduced.

3. System Model

In this section, we present the system model for multi-operator pico eNBs. Further, we explain the disadvantage of individual resource control and the necessity of the rule to share the radio resources between the operators.

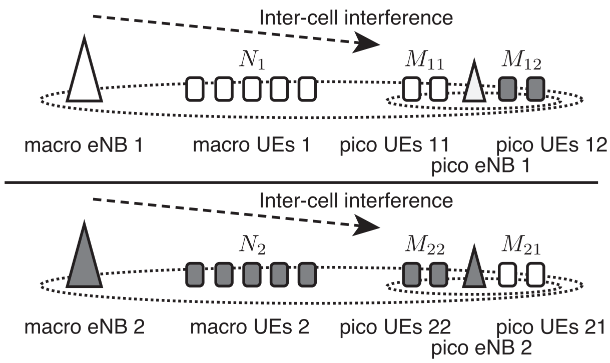

Fig. 3 System model.

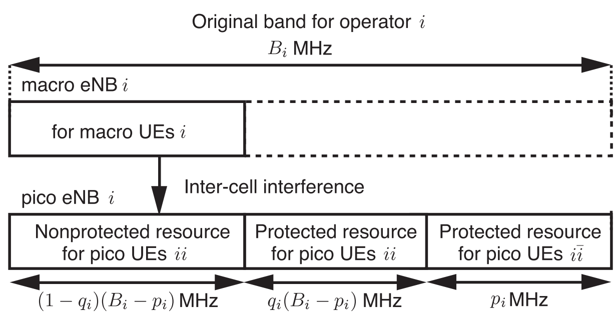

Fig. 4 Original bandwidth of operator i.

3.1 System Model

In this paper, as the simplest model, we discuss the case where there are two operators. Using the system model, as shown in Fig. 3, operators $i \in \{1, 2\}$ individually install macro eNB i and pico eNB i. Let the UEs of operator i accommodated in macro eNB i be denoted as macro UEs i, and let the UEs of operator j accommodated in pico eNB i be denoted as pico UEs ij.

Further, the bandwidth available to each operator is shown in Fig. 4. When one of the operators is operator i, let the other operator be denoted as operator $\bar{i}$. Each operator is assigned a fixed amount of dedicated spectrum B_i MHz and allocates p_i MHz ($p_i \le B_i$) of spectrum to pico UEs $\bar{i}\bar{i}$.

Furthermore, for the reduction in the interference in the transmitted signal from macro eNB i during communications with pico eNB i, an inter-cell interference coordination (ICIC) is introduced [6]. In this paper, for a simple discussion on ICIC, protected and nonprotected resources are introduced [12]. A protected resource is a resource that is used for only pico eNBs-pico UEs communication, and a nonprotected resource is a resource that is used for macro eNBs-macro UEs and pico eNBs-pico UEs communications occurring simultaneously. Operator i can determine the bandwidths of the protected and nonprotected resources. Thus, $q_i(B_i - p_i) + p_i$ MHz is used as a protected resource and $(1 - q_i)(B_i - p_i)$ MHz is used as a nonprotected resource. It can be noted that q_i satisfies $0 \le q_i \le 1$.

3.2 Individual Resource Control

The individual resource allocation and ICIC by each operator can be formulated by introducing a strategic form game. When the operators attempt to maximize the product of the user throughputs u_i in order to satisfy the fairness of macro UEs and pico UEs, such a resource allocation problem can be formulated as follows:

$$\begin{aligned}&\max_{p_1,q_1} u_1(p_1,p_2,q_1)\\&= \max_{p_1,q_1}\left[\frac{r_{\mathrm{m}1}(1-q_1)(B_1-p_1)}{N_1}\right]^{N_1}\left[\frac{r_{\mathrm{p}21}p_2}{M_{21}}\right]^{M_{21}}\\&\times\left[\frac{r_{\mathrm{p}11}q_1(B_1-p_1)+r_{\mathrm{n}11}(1-q_1)(B_1-p_1)}{M_{11}}\right]^{M_{11}}, \qquad (9)\end{aligned}$$

$$\begin{aligned}&\max_{p_2,q_2} u_2(p_1,p_2,q_2)\\&= \max_{p_2,q_2}\left[\frac{r_{\mathrm{m}2}(1-q_2)(B_2-p_2)}{N_2}\right]^{N_2}\left[\frac{r_{\mathrm{p}12}p_1}{M_{12}}\right]^{M_{12}}\\&\times\left[\frac{r_{\mathrm{p}22}q_2(B_2-p_2)+r_{\mathrm{n}22}(1-q_2)(B_2-p_2)}{M_{22}}\right]^{M_{22}}, \qquad (10)\end{aligned}$$

where N_i is the number of macro UEs i and M_{ij} is the number of pico UEs ij. It can be noted that the spectral efficiency of macro eNB i-macro UEs i communication is $r_{\mathrm{m}i}$ and the spectral efficiency of a protected (nonprotected) resource for pico eNB i-pico UEs ij communication is $r_{\mathrm{p}ij}$ ($r_{\mathrm{n}ij}$). Let us assume that $r_{\mathrm{p}ij} > r_{\mathrm{n}ij}$ because there is some interference in a nonprotected resource as compared with a protected resource. Though these maximization problems represent the case where $N_i \ge 1$ and $M_{ij} \ge 1$, the resource allocation problems where $N_i = 0$ or $M_{ij} = 0$ can similarly be treated as the maximization problems of u_i.

Using these conditions,

$$u_1(0, p_2, q_1) > u_1(p_1, p_2, q_1), 0 < p_1 \le B_1, \qquad (11)$$

$$u_2(p_1, 0, q_2) > u_2(p_1, p_2, q_2), 0 < p_2 \le B_2, \qquad (12)$$

are satisfied and strategy $(0, q_i)$ of operator i dominates strategy (p_i, q_i). For this reason, $(p_1^\star, p_2^\star)$ is represented as follows:

$$(p_1^\star, p_2^\star) = (0, 0). \qquad (13)$$

This implies that no resource is allocated to the pico UEs that are accommodated in the pico eNBs of the other operator because each operator is not provided with an incentive to allocate some of the resources to the pico UEs of different operators when they try to maximize the product of the user throughputs. For this reason, a rule where each operator is provided with an incentive to allocate the radio resources to the pico UEs of the other operator should be established.

4. Fixed-Rate Resource Exchange and Its Property

In this section, a fixed-rate resource exchange scheme for multi-operator pico eNBs is presented as the proposed scheme. We confirm that the proposed scheme satisfies the strategy-proofness and individual rationality. In addition, the total throughput performance is confirmed by theoretical calculation.

4.1 Fixed-Rate Resource Exchange

From the viewpoint of the system stability and operator fairness, a fixed-rate resource exchange scheme is proposed as a scheme that satisfies strategy-proofness and individual rationality. In this scheme, we introduce a manager that is independent to both operators, and the manager establishes the allocation rule as shown in Fig. 5. The manager adjusts the amount of resource exchange between the operators.

In the proposed scheme, both the operators report the desired bandwidth m_i MHz to be exchanged. The manager reports $m^\star = \min(m_1, m_2)$ to the operators. Therefore, operator 1 allocates $m^\star$ MHz and operator 2 allocates $\alpha m^\star$ MHz to the UEs of the other operator. It can be noted that α represents the rate of resource exchange, which is predetermined by the manager.

As shown in Fig. 6, the fixed-rate resource exchange scheme is illustrated by using an Edgeworth box. In this figure, point O represents the initial resource allocation. The manager decides on the amount of exchanged resource to improve the utility of each operator u_i on the line $p_2 = \alpha p_1$. When both the operators report m_i to the manager, they calculate the optimal value of m_i to maximize their utilities. However, the optimal amount of resource exchange differs for each operator. The best response for operator 1 is point A and that for operator 2 is point B.

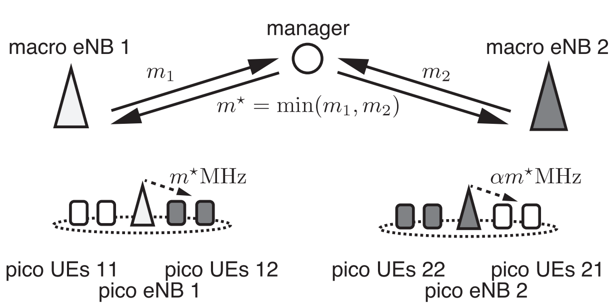

Fig. 5 Fixed rate resource exchange.

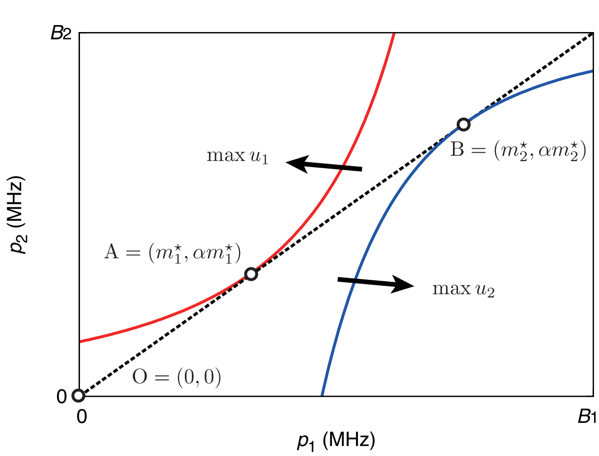

Fig. 6 Fixed rate resource exchange represented by the Edgeworth box.

4.2 Messages of Both Operators

Let us denote the optimal amount of exchanged resource and the value of q_i for operator i as $m_i^\star$ and $q_i^\star$, respectively. When N_i and M_{ij} are greater than 0, we can calculate $m_i^\star$ and $q_i^\star$ as follows:

$$\begin{aligned}
(m_1^\star, q_1^\star) &= \mathop{\arg\max}_{m,q_1} u_1(m, \alpha m, q_1) \\
&= \mathop{\arg\max}_{m,q_1} \left[\frac{r_{\mathrm{m1}}(1-q_1)(B_1-m)}{N_1}\right]^{N_1} \left[\frac{r_{\mathrm{p21}}\alpha m}{M_{21}}\right]^{M_{21}} \\
&\times \left[\frac{r_{\mathrm{p11}}q_1(B_1-m) + r_{\mathrm{n11}}(1-q_1)(B_1-m)}{M_{11}}\right]^{M_{11}} \\
&= \mathop{\arg\max}_{m,q_1} \frac{\left[(B_1-m)^{N_1+M_{11}} m^{M_{21}}\right] r_{\mathrm{m1}}{}^{N_1} (r_{\mathrm{p21}}\alpha)^{M_{21}}}{N_1{}^{N_1} M_{11}{}^{M_{11}} M_{21}{}^{M_{21}}} \\
&\times (1-q_1)^{N_1} \left[(r_{\mathrm{p11}} - r_{\mathrm{n11}})q_1 + r_{\mathrm{n11}}\right]^{M_{11}}, \qquad (14)
\end{aligned}$$

$$\begin{aligned}
(m_2^\star, q_2^\star) &= \mathop{\arg\max}_{m,q_2} u_2(m, \alpha m, q_2) \\
&= \mathop{\arg\max}_{m,q_2} \left[\frac{r_{\mathrm{m2}}(1-q_2)(B_2-\alpha m)}{N_2}\right]^{N_2} \left[\frac{r_{\mathrm{p12}} m}{M_{12}}\right]^{M_{12}} \\
&\times \left[\frac{r_{\mathrm{p22}}q_2(B_2-\alpha m) + r_{\mathrm{n22}}(1-q_2)(B_2-\alpha m)}{M_{22}}\right]^{M_{22}} \\
&= \mathop{\arg\max}_{m,q_2} \frac{\left[(B_2-\alpha m)^{N_2+M_{22}} m^{M_{12}}\right] r_{\mathrm{m2}}{}^{N_2} r_{\mathrm{p12}}{}^{M_{12}}}{N_2{}^{N_2} M_{22}{}^{M_{22}} M_{12}{}^{M_{12}}} \\
&\times (1-q_2)^{N_2} \left[(r_{\mathrm{p22}} - r_{\mathrm{n22}})q_2 + r_{\mathrm{n22}}\right]^{M_{22}}. \qquad (15)
\end{aligned}$$

In order that operator 1 evaluates (14), operator 1 needs being informed the information on the number of pico UEs 21, M_{21}, and the spectral efficiency of a protected resource for pico eNB 2-pico UEs 21 communication, r_{p21}, from operator 2 and vice versa. It means that the proposed fixed rate resource exchange scheme requires additional information compared to the individual resource control. In addition, additional functions between these networks are required so that additional information can be exchanged. Note that even in the proposed scheme, the number of UEs and the spectral efficiency of UEs of the other operator are not required.

By solving these optimization problems, we obtain $m_i^\star$ and $q_i^\star$ as follows:

$$m_1^\star = \frac{M_{21}B_1}{N_1 + M_{11} + M_{21}}, \qquad (16)$$

$$m_2^\star = \frac{M_{12}B_2}{\alpha(N_2 + M_{12} + M_{22})}, \qquad (17)$$

$$q_1^\star = 1 - \frac{N_1 r_{\mathrm{p11}}}{(r_{\mathrm{p11}} - r_{\mathrm{n11}})(N_1 + M_{11})}, \qquad (18)$$

$$q_2^\star = 1 - \frac{N_2 r_{p22}}{(r_{p22} - r_{n22})(N_2 + M_{22})}. \tag{19}$$

Of course, $m_i^\star$ and $q_i^\star$ can be calculated by solving the optimization problem in the case $N_i = 0$ or $M_{ij} = 0$. When $M_{\bar{i}i} = 0$, we obtain

$$m_i^\star = 0, \tag{20}$$

$$q_i^\star = 1 - \frac{N_i r_{pii}}{(r_{pii} - r_{nii})(N_i + M_{ii})}, N_i + M_{ii} \neq 0, \tag{21}$$

where $q_i^\star$ is not defined when $N_i + M_{ii} = 0$. This is because there is no optimization problem for operator i to solve when $N_i + M_{ii} + M_{\bar{i}i} = 0$.

When $M_{\bar{i}i} \neq 0$ and $N_i + M_{ii} = 0$, we obtain

$$m_1^\star = B_1, m_2^\star = \frac{B_2}{\alpha}, \tag{22}$$

where $q_i^\star$ is not defined. This is because operator i has no macro UEs i or pico UEs ij to allocate his radio resource. In the other cases, $m_i^\star$ and $q_i^\star$ can be represented as (16)–(19).

It can be noted that $m_i^\star$ is not the amount of exchanged resource, which is finally determined by the manager. Operator i can maximize utility u_i only when the manager finally determines the amount of exchanged resource $m^\star = m_i^\star$. However, $m_1^\star$ and $m_2^\star$ are different values. For this reason, both the utilities u_1 and u_2 are not always maximized by the decision of the manager.

$q_i^\star$ is constant regardless of $m^\star$. This implies that each operator can set his optimal $q_i^\star$ before the manager sets the amount of exchanged resource $m^\star$. In the following, q_i is assumed to be set as $q_i^\star$. Let us denote $u_i(m, \alpha m, q_i^\star)$ as $v_i(m)$ because $q_i^\star$ can be treated as a constant value.

Theorem 1: $v_i(m)$ has only one local maximum at $m = m_i^\star$.

Proof 1: By the definition of $v_i(m)$, $v_1(m)$ and $v_2(m)$ are represented as follows:

$$v_1(m) = C_1(B_1 - m)^{N_1+M_{11}} m^{M_{21}}, \tag{23}$$

$$v_2(m) = C_2(B_2 - \alpha m)^{N_2+M_{22}} m^{M_{12}}, \tag{24}$$

where C_i is a positive constant value, $M_{\bar{i}i} \neq 0$, and $N_i + N_{ii} \neq 0$.

$\mathrm{d}v_1(m)/\mathrm{d}m$ and $\mathrm{d}v_2(m)/\mathrm{d}m$ are calculated as follows:

$$\frac{\mathrm{d}v_1(m)}{\mathrm{d}m} = C_1 [M_{21}B_1 - m(N_1 + M_{11} + M_{21})] \times (B_1 - m)^{N_1+M_{11}-1} m^{M_{21}-1}, \tag{25}$$

$$\frac{\mathrm{d}v_2(m)}{\mathrm{d}m} = C_2 [M_{12}B_2 - \alpha m(N_2 + M_{22} + M_{12})] \times (B_2 - \alpha m)^{N_2+M_{22}-1} m^{M_{12}-1}. \tag{26}$$

It is confirmed that $\mathrm{d}v_1(m)/\mathrm{d}m = 0$ when $m \in \{0,\ m_1^\star,\ B_1\}$ and $\mathrm{d}v_2(m)/\mathrm{d}m = 0$ when $m \in \{0,\ m_2^\star,\ B_2/\alpha\}$. Furthermore, $\mathrm{d}v_1(m)/\mathrm{d}m > 0$ when $0 < m < m_1^\star$ and $\mathrm{d}v_1(m)/\mathrm{d}m < 0$ when $m_1^\star < m < B_1$ and $B_1 < m$. Similarly, $\mathrm{d}v_2(m)/\mathrm{d}m > 0$ when $0 < m < m_2^\star$ and $\mathrm{d}v_2(m)/\mathrm{d}m < 0$ when $m_2^\star < m < B_2/\alpha$ and $B_2/\alpha < m$. As a result, $v_i(m)$ has the only one local maximum at $m = m_i^\star$.

If $M_{\bar{i}i} \neq 0$ and $N_i + M_{ii} = 0$, $\mathrm{d}v_i(m)/\mathrm{d}m$ is calculated as follows:

$$\frac{\mathrm{d}v_i(m)}{\mathrm{d}m} = C_i M_{\bar{i}i} m^{M_{\bar{i}i}-1}. \tag{27}$$

It is confirmed that $\mathrm{d}v_i(m)/\mathrm{d}m > 0$ if $m \neq 0$. As a result, $v_i(m)$ has the only one local maximum at $m = m^\star$.

If $M_{\bar{i}i} = 0$ and $N_i + M_{ii} \neq 0$, $\mathrm{d}v_1(m)/\mathrm{d}m$ and $\mathrm{d}v_2(m)/\mathrm{d}m$ are calculated as follows:

$$\frac{\mathrm{d}v_1(m)}{\mathrm{d}m} = -C_1(N_1 + M_{11})(B_1 - m)^{N_1+M_{11}-1}, \tag{28}$$

$$\frac{\mathrm{d}v_2(m)}{\mathrm{d}m} = -C_2\alpha(N_2 + M_{22})(B_2 - \alpha m)^{N_2+M_{22}-1}. \tag{29}$$

It is confirmed that $\mathrm{d}v_1(m)/\mathrm{d}m < 0$ if $B_1 \neq m$ and $\mathrm{d}v_2(m)/\mathrm{d}m < 0$ if $B_2 \neq \alpha m$. As a result, $v_i(m)$ has only one local maximum at $m = m^\star = 0$.

For the abovementioned reasons, it is confirmed that $v_i(m)$ has only one local maximum at $m = m_i^\star$. □

This implies that $v_i(m)$ is improved when the final amount of exchanged resource $m^\star$ is close to $m_i^\star$. When the amount of exchanged resource is too small, the throughput performance of the UEs accommodated in the pico eNBs of the other operator is degraded. On the contrary, when the amount of exchanged resource is too large, the macro UEs and pico UEs of the same operator cannot achieve enough throughput.

However, $m_i^\star$ is unknown to the manager, and each operator is provided with an incentive to report a strategic message $m_i \neq m_i^\star$ to maximize his own utility.

4.3 Strategy-Proofness and Individual Rationality

In this section, we confirm that the proposed scheme satisfies both the strategy-proofness and individual rationality.

Theorem 2: The proposed scheme satisfies the strategy-proofness.

Proof 2: As summarized in Table 1, the utility of operator 1 versus the messages of each operator (m_1, m_2) is shown. Let us denote the strategic message m_1 that is lower than $m_1^\star$ as L and that higher than $m_1^\star$ as H. As summarized in Table 1, the pairs of messages are divided into 12 patterns.

When $m_2 \leq L$, operator 1 can achieve a utility of $v_1(m_2)$ regardless of his message m_1. This is because the manager determines $m^\star$ by the calculation of $\min(m_1, m_2)$.

When $L \leq m_2 \leq m_1^\star$, operator 1 can achieve a utility

Table 1 The utility of operator 1 when the messages of both operators are (m_1, m_2).

	$m_2 \leq L$	$L \leq m_2 \leq m_1^\star$	$m_1^\star \leq m_2 \leq H$	$H \leq m_2$
$L = m_1 \leq m_1^\star$	$v_1(m_2)$	$v_1(L)$	$v_1(L)$	$v_1(L)$
$m_1 = m_1^\star$	$v_1(m_2)$	$v_1(m_2)$	$v_1(m_1^\star)$	$v_1(m_1^\star)$
$m_1^\star \leq m_1 = H$	$v_1(m_2)$	$v_1(m_2)$	$v_1(m_2)$	$v_1(H)$

of $v_i(L)$ when he reports $m_1 = L$. However, he can achieve $v_i(m_2)$ when he reports $m_1 = m_1^\star$ or $m_1^\star = H$. It is confirmed that $m_1 = m_1^\star$ or $m_1^\star = H$ is the best response for operator 1 when $L \leq m_2 \leq m_1^\star$. This is because $v_1(m)$ has only one local maximum when $m = m_1^\star$ and $v_1(L) \leq v_{m_2}$.

When $m_1^\star \leq m_2 \leq H$ or $H \leq m_2$, $m_1 = m_1^\star$ is the best response with which the utility of $v_1(m_1^\star)$ can be achieved, and it is the maximum value of $v_1(m)$.

Overall, the reporting message $m_1 = m_1^\star$ is the best response for operator 1 regardless of m_2. Similarly, the reporting message $m_2 = m_2^\star$ is the best response for operator 2. Thus, the proposed scheme satisfies the strategy-proofness. □

Theorem 3: The proposed scheme satisfies the individual rationality.

Proof 3: Operator 1 can achieve the utility of $v_1(m_2)$ when $m_2 \leq m_1^\star$ and $v_1(m_1^\star)$ when $m_1^\star \leq m_2$. This is because the best response of operator 1 is the reporting message $m_1 = m_1^\star$.

In the case where $0 \leq m_2 \leq m_1^\star$, we obtain $v_1(0) \leq v_1(m_2)$ because $v_i(m)$ has only one local maximum when $m = m_1^\star$. In the case where $m_1^\star \leq m_2 \leq 1$, we obtain $v_1(0) \leq v_1(m_1^\star)$. In other words, the utility of operator 1, with the proposed scheme, is always greater than that of the individual resource allocation $v_1(0)$. Similarly, the utility of operator 2 is also greater than $v_2(0)$.

Thus, the proposed scheme satisfies the individual rationality and both operators 1 and 2 always improve their utilities. □

4.4 Theoretical Performance

The amount of exchanged resource $m^\star$ can be determined as follows:

$$p_1 = m^\star, p_2 = \alpha m^\star, m^\star = \min(m_1^\star, m_2^\star). \tag{30}$$

When $m_1^\star \geq m_2^\star$ as shown in Fig. 6, A= $(m_1^\star, \alpha m^\star)$ is the final allocation result.

In addition, we determine the total throughput performance of operator i, t_i, as follows:

$$\begin{aligned} t_1 = {} & r_{\text{p}11} q_1^\star (B_1 - m^\star) + \alpha r_{\text{p}21} m^\star \\ & + (r_{\text{m}1} + r_{\text{n}11})(1 - q_1^\star)(B_1 - m^\star), \end{aligned} \tag{31}$$

$$\begin{aligned} t_2 = {} & r_{\text{p}22} q_2^\star (B_2 - \alpha m^\star) + r_{\text{p}12} m^\star \\ & + (r_{\text{m}2} + r_{\text{n}22})(1 - q_2^\star)(B_2 - \alpha m^\star). \end{aligned} \tag{32}$$

5. Numerical Results

5.1 Parameters

In this section, the total throughput and the amount of exchanged resource are evaluated by using the examples of the parameters as listed in Table 2. The average spectral efficiency in each link is set to a fixed value. Note the effect of the fading is averaged out. The spectrum efficiency of the macro eNB-macro UEs communication is set to 2 bit/s/Hz. The spectrum efficiency of the protected resource for the pico eNB-pico UEs communication is set to 5 bit/s/Hz, and that of the nonprotected resource is set to 0.5 bit/s/Hz. The number of UEs accommodated in the macro eNBs and pico eNBs varies. The amount of dedicated spectrum for each operator B_i is set to 20 MHz, and the rate of resource exchange α is set to 1.

Table 2 Example of parameters.

(N_1, N_2)	(5, 5), (20, 20)
$r_{\text{m}1}, r_{\text{m}2}$	2 bit/s/Hz
$r_{\text{p}11}, r_{\text{p}12}, r_{\text{p}21}, r_{\text{p}22}$	5 bit/s/Hz
$r_{\text{n}11}, r_{\text{n}22}$	0.5 bit/s/Hz
B_1, B_2	20 MHz, 20 MHz

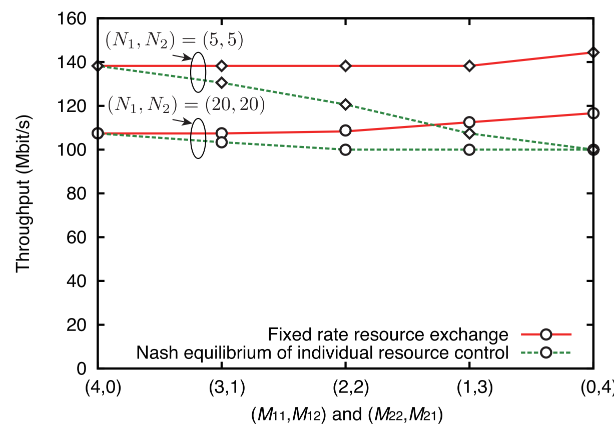

Fig. 7 Total throughput performance of both operators against the number of UEs accommodating in pico eNBs.

5.2 Total Throughput

As shown in Fig. 7, the total throughput performances of the proposed scheme and individual resource control are compared. We obtain the total throughput performance of individual resource control, T, by the calculation of the Nash equilibrium as follows:

$$T = \sum_{i=1}^{2} \left[(r_{\text{m}i} + r_{\text{n}ii})(1 - q_i^\star) + r_{\text{p}ii} q_i^\star \right] B_i. \tag{33}$$

As shown in Fig. 7, as M_{12} and M_{21} increase, the total throughput of the individual resource control decreases. This is because both the operators are not provided with an incentive to allocate the resources to the pico UEs of the other operator. This implies that the pico UEs are not accommodated in the pico eNBs of the other operator when there is no rule for resource allocation between the operators, and both the operators attempt to maximize their utilities without considering the other operator.

With the proposed scheme, the total throughput performance is improved. When (M_{11}, M_{12}) and (M_{22}, M_{21}) are set to $(2, 2)$, the total throughput is improved by from 10 to

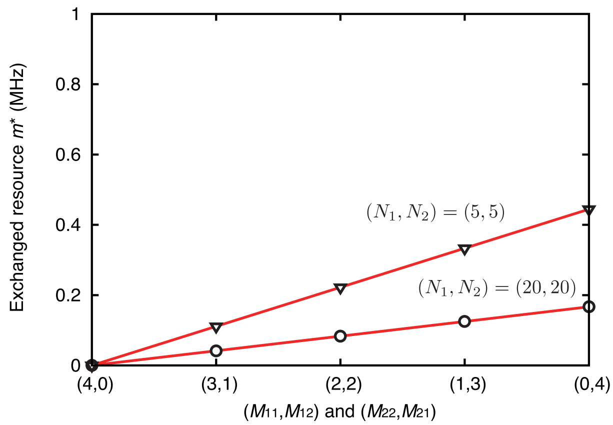

Fig. 8 The amount of exchanged resource by the proposed scheme.

Table 3 Example of parameters in the unbalanced situation.

	Case 1	Case 2
(N_1, N_2)	(5, 5), (20, 20)	(5,20)
r_{m1}, r_{m2}	2 bit/s/Hz	2 bit/s/Hz
$r_{p11}, r_{p12}, r_{p21}, r_{p22}$	5 bit/s/Hz	5 bit/s/Hz
r_{n11}, r_{n22}	0.5 bit/s/Hz	0.5 bit/s/Hz
B_1, B_2	20 MHz, 10 MHz	20 MHz, 20 MHz

20% compared with the individual resource control. Particularly, when (M_{11}, M_{12}) and (M_{22}, M_{21}) are set to $(0, 4)$, the total throughput is improved by from 20 to 40%. This is because more protected resources are allocated to pico UEs 12 and pico UEs 21 by using the proposed scheme.

5.3 Amount of Exchanged Resource

The amount of exchanged resource, with the proposed scheme, $m^\star$, is shown in Fig. 8. It is confirmed that a large amount of resource is exchanged when M_{12} and M_{21} are large, regardless of N_1 and N_2. This is because the both operators simultaneously require a large amount of resource for pico UEs 12 and pico UEs 21.

Furthermore, when N_1 and N_2 are small, it is confirmed that a large amount of resource is exchanged. This is because there are a few macro UEs and both the operators require to allocate a large amount of resource to their pico UEs in order to maximize the product of the user throughputs. On the contrary, when N_1 and N_2 are large, both the operators require to allocate a large amount of resource to the macro UEs. Therefore, the amount of exchanged resource is small.

5.4 Unbalanced Situations

In sections from 5.1 to 5.3, the system performance is evaluated when the number of UEs, the amount of required resources, and the bandwidth of operators are balanced. To confirm the throughput improvement of the proposed scheme in unbalanced situations, we conduct additional evaluations using parameters summarized in Table 3. In case 1, B_1 and B_2 are different from each other and the other parameters are the same as Table 2. In case 2, (N_1, N_2) is set

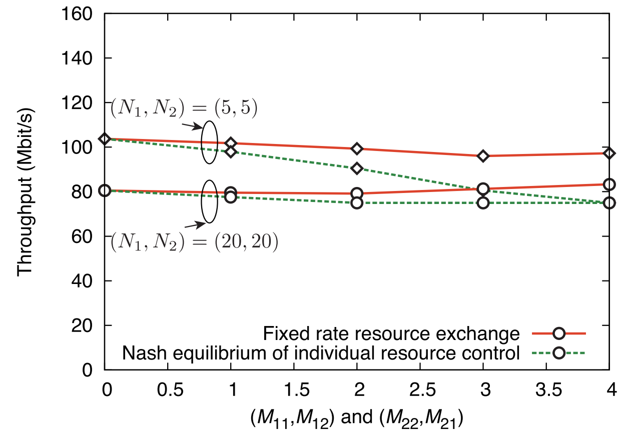

Fig. 9 Total throughput performance of both operators against the number of UEs accommodating in pico eNBs in case 1.

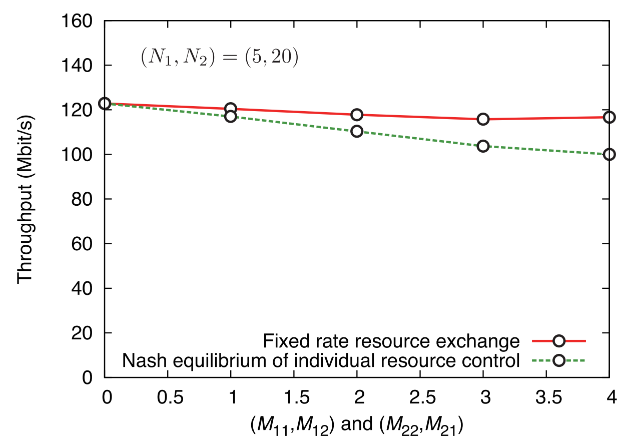

Fig. 10 Total throughput performance of both operators against the number of UEs accommodating in pico eNBs in case 2.

to $(5, 20)$ and the other parameters are the same.

Figure 9 shows the total throughput in case 1, i.e., B_1 and B_2 are different from each other. Figure 10 shows the total throughput in case 2, i.e., N_1 and N_2 are different from each other. In both cases, we confirmed that the total throughput performance of the proposed scheme is improved compared with that of individual resource control. However, the amount of throughput improvement in unbalanced situations is smaller than that in balanced situations because the amount of exchanged resource is determined by the smaller value of desired bandwidths from two operators.

6. Conclusion

In this paper, accommodation of the UEs of different operators in the pico eNBs is discussed. When the UEs of different operators are accommodated in the pico eNBs, the wireless resources for the communication are not appropriately allocated to them. This is because the operator who owns the pico eNBs is not provided with an incentive to allocate the wireless resources to the UEs of the other operators.

To solve this problem, we introduce the concepts of game theory and mechanism design. The resource allocation problem for multi-operator pico eNBs is discussed by using the pure exchange economy model. In this paper, a fixed-rate resource exchange scheme is proposed to satisfy the strategy-proofness and individual rationality in the resource allocation.

It is confirmed from the theoretical performance evaluation results that the total throughput performance can be improved when both the operators allocate the wireless resources to maximize the product of the user throughput performances. Furthermore, when the number of UEs in the macro eNBs is small or the number of UEs in different pico eNBs is large, large amounts of resources are exchanged between the operators. This implies that both operators can exchange the wireless resources when they simultaneously require the resource for the UEs accommodated in the pico eNBs of the other operator.

We would like to emphasize that the objective of this paper is to propose a fixed-rate resource exchange scheme for multi-operator pico eNBs and to confirm the effect of the proposed scheme. We hope that the results presented in this paper will provide insights that are useful for the design of heterogeneous network systems.

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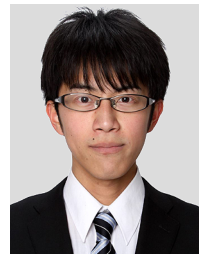

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