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Differential Game-Theoretic Analysis on Information Availability in Decentralized Demand-Side Energy Management Systems*

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SUMMARY Differential games are considered an extension of optimal control problems, which are used to formulate centralized control problems in smart grids. Optimal control theory is used to study systems consisting of one agent with one objective, whereas differential games are used to formulate systems consisting of multiple agents with multiple objectives. Therefore, a differential-game-theoretic approach is appropriate for formulating decentralized demand-side energy management systems where there are multiple decision-making entities interacting with each other. Moreover, in many smart grid applications, we need to obtain information for control via communication systems. To formulate the influence of communication availability, differential game theory is also promising because the availability of communication is considered as part of an information structure (i.e., feedback or open-loop) in differential games. The feedback information structure is adopted when information for control can be obtained, whereas the open-loop information structure is applied when the information cannot be obtained because of communication failure. This paper proposes a comprehensive framework for evaluating the performance of demand-side actors in a demand-side management system using each control scheme according to both communication availability and sampling frequency. Numerical analysis shows that the proposed comprehensive framework allows for an analysis of trade-off for decentralized and centralized control schemes.

key words: *demand-side energy management, smart grids, differential game, optimal control, information availability, decentralized control*

1. Introduction

Smart grids are attracting a great deal of attention recently as key technologies for efficient power management. A smart grid is a power network in which various power sources and consumers are connected and controlled using information and communication technologies such as two-way communication, control technologies, decentralized processing, and sensor technologies. In conventional power grids, the supply side manages power. On the other hand, in future smart grids, distributed power sources such as photovoltaic (PV) power systems will be installed in demand-side actors, and in order to deal with these sources, demand-side actors will be required to participate in power management. This type of management is called "demand-side management."

Many studies on demand-side management have been conducted [1]–[13]. The control of electrical appliances has usually been investigated with respect to demand-side management [1]–[3]. For example, the optimal temperature setting for air conditioning was scheduled according to day-ahead pricing and temperature forecasts [1]. In demand-side energy management systems, demand-side actors act independently to achieve their purposes. To evaluate the comfortability of each demand-side actor, the concept "disutility" was introduced for systems comprising storage and a renewable energy resource in [3]. In demand-side management, keeping the supply-demand balance is important to achieve stability in the power supply [4], [5]. In particular, in [4], demand-side management in a large population regime was focused on energy supply. In [5], efficient matching of supply and demand was achieved using a prediction method for heat demand. Other prediction methods for the output of PV power systems have also been developed [6], [7]. The research [6] outlines existing solar forecasting models and evaluates various forecasting providers.

For analyzing smart grids, game theory has been widely used [4], [13], [14] as well as optimal control theoretic approaches [8]–[12]. In partciular, there are some studies applying differential game theory to demand-side management [4], [14]. Using differential game theory, we can discuss the situation when there are multiple agents acting independently and interacting with each other. In particular, in [14], a differential game was used for analyzing the interaction among different actors in demand-side management through the dynamic model of market price.

By applying a differential game-theoretic approach, this paper proposes a comprehensive framework for decentralized control in smart grids (particularly focusing on information availability). In differential game theory, the concept of "information structure" can directly compare the performance with information and the performance without information. Therefore, differential game theory is suitable for analyzing demand-side management when there are multiple demand-side actors, taking into account communication availability. Introducing the concept of information structure is the difference between the present study and the previous studies using differential game [4], [14]. In [4], [14], information for control is assumed to be available, i.e., only the "feedback" information structure was adopted. By contrast, it is possible that information is unavailable because of communication failures such as electrical equipment failure and packet loss. Even in these cases, demand-side actors do not necessarily stop managing and

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they might utilize other types of control. Thus, to discuss control schemes while taking communication availability into account, we introduce two information structures, "feedback" and "open-loop," and apply these structures to the cases where information is available and unavailable, respectively.

As a system model, we introduce demand-side energy management for PV systems and two decentralized control methods: decentralized feedback control and decentralized open-loop control. Decentralized feedback control is used when information is available through communication systems. On the other hand, decentralized open-loop control is used when information is unavailable. Both decentralized control schemes are modeled using two information structures in differential game theory. For comparison, we also introduce centralized control, which is discussed in terms of optimal control theory. In all control schemes, the power consumption of electrical equipment is managed while taking the actual and predicted PV output into account. The purpose is to minimize electricity rates while maintaining the occupants' comfortability [3] and the overall supply-demand balance [4]. Note that in this case, the sampling frequency is the important factor, i.e., how frequently demand-side actors manage power consumption per unit time. The detailed analysis conducted here is different from those of previous papers [15], [16].

The remainder of this paper is organized as follows. In Sect. 2, a demand-side energy management system and three control schemes based on the communication state are introduced. In Sect. 3, a differential game for decentralized control schemes is introduced. In Sect. 4, the system is formulated as a differential game and two decentralized control methods are derived. In Sect. 5, an optimal control problem is formualted for centralized control. In Sect. 6, numerical analysis is performed and the control schemes using the criteria of information availability and sampling frequency are discussed. Our concluding remarks are provided in Sect. 7.

2. System Model

2.1 Demand-side Energy Management System

The model discussed in this paper is a simplified version of the demand-side energy management system model introduced in [3], [4]. This model comprises n buildings and a power grid. As an example, Fig. 1 shows the model for the case of two buildings. In each building, it is assumed that there are electrical appliances whose power consumption is controllable. Let the total power consumption of the electrical appliances in building i ($i \in \{1, \ldots, n\}$) at time t be denoted by $p_i(t)$. In each building, there is also a photovoltaic power generation system that produces electric power $r_i(t)$. In addition to the PV system, each building is provided with power from the power grid. In some cases, however, buildings provide power to the grid. We define $d_i(t) \overset{\text{def}}{=} p_i(t) - r_i(t)$ as insufficient power ($d_i(t) > 0$) or surplus power ($d_i(t) < 0$) for building i. If $d_i(t) > 0$, building i buys power from the

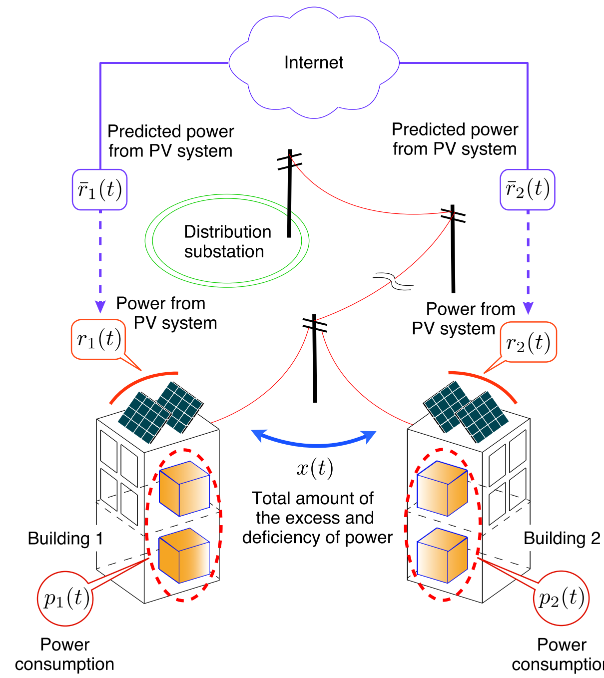

Fig. 1 Model of demand-side energy management system consisting of two buildings.

power grid, and if $d_i(t) < 0$, building i sells power to the power grid at a unit electricity price ϵ. Thus, the electricity rate paid to the grid is written as $\epsilon d_i(t) = \epsilon(p_i(t) - r_i(t))$, where $\epsilon d_i(t) < 0$ represents the income of building i, and $\epsilon d_i(t) > 0$ is the expense of building i. We define $x(t)$ to be the total amount of insufficient power and surplus power for all buildings over time t as follows:

$$\dot{x}(t) \overset{\text{def}}{=} \sum_{i \in \{1,\ldots,n\}} d_i(t),\ x(0) = 0. \tag{1}$$

Next, we discuss how the buildings select their power management strategies. Each building aims to minimize expenses paid to the grid when buying power to compensate for insufficient power, i.e., each building attempts to minimize $\epsilon d_i(t)$. In this case, the minimization of $\epsilon d_i(t)$ results in the minimization of power consumption $p_i(t)$. However, if the electrical appliances are used for heating, this can be difficult, as insufficient heating may be uncomfortable for the occupants. Thus, we introduce a term to prevent insufficient heating. This term indicates the disutility [3] for consumers in building i, which is defined as $D_i(p_i(t)) \overset{\text{def}}{=} \beta_i(P_{i,\text{target}} - p_i(t))^2$, where β_i is a weight parameter and $P_{i,\text{target}}$ represents the target power consumption value for building i. The disutility increases as the actual power consumption $p_i(t)$ deviates from the target value.

Based on [4], we consider the target value of the grid power supply. If the PV power is less than the power consumption of building i, the power grid supplies power to building i. The demand should be similar to the supplied power. We denote the target value of the power supplied by the grid as S, and we introduce a term $\alpha_i(x(T) - S)^2$ to balance the demand $x(t)$ and supply S, where α_i is a weight parameter for building i, and T represents the control end time. Note that $x(T)$ is the total power demand of all buildings at the control end time.

Given these factors, the cost for building i, J_i, can be defined as

$$J_i \stackrel{\text{def}}{=} \int_0^T \left[\epsilon d_i(t) + \beta_i(P_{i,\text{target}} - p_i(t))^2\right] \mathrm{d}t + \alpha_i(x(T) - S)^2,\ i \in \{1, \ldots, n\}. \quad (2)$$

Recall that building i minimizes (2) by controlling $p_i(t)$.

In (2), α_i is a weight parameter that reflects the importance of the entire supply-demand balance for building i. If the value of α_i is large, building i refrains from selfish power consumption and attempts to maintain the supply-demand balance. On the other hand, β_i is a weight parameter that reflects the importance of the comfortability of residents in building i. If the value of β_i is large, building i consumes power to maintain the comfortability of residents.

As shown in (1), in this study, we assume a simple model based on the first-order system, as in the previous study [3], which discussed energy management within the framework of optimal control theory. This assumption is made because there are many studies that discuss energy management using optimal control theory (e.g., [2], [3], [9], [12]); thus, it is reasonable to assume a first-order system. In addition to optimal control theory, in this study, we use differential game theory, which is considered to be an extension of optimal control theory. In particular, we focus on the comparison of control methods with respect to the availability of $x(t)$ within the framework of differential game theory. Differential games are also generally formulated as first-order systems [17]–[19]. Thus, it is reasonable to use this simple model based on a first-order system.

In this model, each building determines its strategy for power consumption before buildings start to control their power. In this case, each building uses weather prediction data instead of the actual weather, i.e., each building determines its future actions based on (2) and

$$\begin{aligned} \dot{x}(t) &\stackrel{\text{def}}{=} \sum_{i\in\{1,\ldots,n\}} d_i(t) \\ &\stackrel{\text{def}}{=} \sum_{i\in\{1,\ldots,n\}} (p_i(t) - \overline{r}_i(t)), \\ x(0) &= 0, \end{aligned} \quad (3)$$

instead of (1), where $\overline{r}_i(t)$ represents the predicted value of $r_i(t)$. We assume that this predicted data are obtained via the Internet. Then, after buildings start to control their power, the actual power consumption is determined according to the actual PV output and the strategy based on the predicted PV output.

Next, we explain the dynamics of the proposed energy management system. First, buildings plan their power consumption to minimize (2) before control starts. After control starts ($t = 0$), they consume power based on the strategy, the actual PV output, and the overall state $x(t)$. At each time, if the power consumption of building i is more than the PV output of building i, building i buys power from the power grid. If power consumption is less than PV output, building i sells power to the grid. In the present model, we assume that the power grid has infinite capacity, i.e., enough power to sell/buy throughout the day. In this case, the state of power in the power grid evolves according to the dynamics (1).

2.2 Communication State and Control Methods

All buildings are assumed to independently control their power consumption, $p_i(t)$, i.e., they can turn their electrical appliances on or off. When controlling $p_i(t)$ to minimize (2), the situation will depend greatly upon whether both buildings obtain information with respect to $x(t)$. Each building controls power consumption according to $x(t)$ and its own power consumption $p_i(t)$, if it obtains $x(t)$ at each time t. We refer to this control as "decentralized feedback control." By contrast, if each building does not obtain $x(t)$ because of communication loss, power consumption would only be controlled based on both the last value of $x(t)$ that has been obtained and $p_i(t)$. We refer to this control as "decentralized open-loop control."

For comparison, "centralized control" based on optimal control theory is introduced. If all buildings act in a coordinated manner, they are said to use "centralized control." In centralized control, the power consumption of all buildings is controlled simultaneously. Similar to decentralized feedback control, centralized control can be used only when buildings obtain information about $x(t)$ at each time t.

3. Decentralized Control

A promising approach to analyzing decentralized feedback control and decentralized open-loop control is to apply feedback and open-loop information structures in a differential game. In this section, we describe the differential game for decentralized control. The following definitions and equations refer to [17]–[19].

3.1 Differential Game

A differential game is a framework for analyzing the actions of multiple decision-making entities, known as "players" in game theory. In a dynamic, continuous-time system, players act independently to achieve their purposes. The action of each player affects the overall state, and the state affects the action taken by each player. Differential games are used as extensions of optimal control problems. The situation in which there is a single player with one objective function is formulated as an optimal control problem, whereas the situation in which multiple players interact with each other for their own purposes is formulated as a differential game.

An n-person linear quadratic differential game is generally written as follows:

$$
\begin{aligned}
\min_{u_i} \quad & J_i(y, u_1, \ldots, u_n),\ \forall i \in \mathcal{N},\ \mathcal{N} = \{1, 2, \ldots, n\}, \\
& J_i(y, u_1, \ldots, u_n) \\
& = \frac{1}{2}\int_0^T \left(q_i y^2 + \sum_{j \in \mathcal{N}} r_{ij} u_j^2 \right) \mathrm{d}t + \frac{1}{2} q_{iT} y^2, \qquad (4) \\
\text{s.t.} \quad & \dot{y}(t) = a y(t) + \sum_{i \in \mathcal{N}} b_i u_i(t) + c(t),\ y(0) = y_0,
\end{aligned}
$$

where $a, b_i, r_{ij} \in \mathbb{R}$, $r_{ii} > 0$, $q_i \geq 0$, and $q_{iT} \geq 0$. $c(t)$ is the given continuous function. $[0, T]$ represents time horizon of the game. $y(t)$ denotes the state of the game at time t, and y_0 represents the initial state of $y(t)$. $u_i(t)$ denotes the action of player i. J_i represents the cost function of player i. In the game (4), each player selects a strategy to minimize his/her cost function.

3.2 Nash Equilibrium

As in general non-cooperative games, Nash equilibria are defined in differential games. An action $u_i^\star$ that satisfies (4) is known as an equilibrium action. A set of equilibrium actions $(u_1^\star, \ldots, u_n^\star)$ is known as Nash equilibrium; it satisfies the following condition:

$$
\begin{aligned}
& J_i(y, u_1^\star, \ldots, u_{i-1}^\star, u_i^\star, u_{i+1}^\star, \ldots, u_n^\star) \\
& \quad \leq J_i(y, u_1^\star, \ldots, u_{i-1}^\star, u_i, u_{i+1}^\star, \ldots, u_n^\star),\ \forall i \in \mathcal{N}. \qquad (5)
\end{aligned}
$$

At Nash equilibrium, the cost increases if players change their actions independently. Therefore, players have no incentive to change their actions.

3.3 Information Structure

In differential games, both the strategy that each player selects and the property of the Nash equilibrium depend on the available information. The configuration of available information is known as an "information structure." Typical information structures are open-loop and feedback.

In an open-loop information structure, each player knows the initial value of a state in a game and determines his/her strategy as a function of the initial state and time t. This is written as $\eta_i(t) = \{y_0\}$, where $\eta_i(t)$ represents the information about the game that the player has.

By contrast, in a feedback information structure, players determine their strategies using the state $y(t)$, i.e., $\eta_i(t) = \{y(t)\}$.

4. Relationship between System Model and Decentralized Control

In this section, we formulate (2) and (3) as a differential game. In addition, we derive open-loop Nash equilibria and feedback Nash equilibria of the game.

4.1 Translation from System Model into Differential Game

The system model described in Sect. 2 can be formulated as a differential game in which each building is a player, the power consumption of its appliances is its strategy, and (2) is a cost function that needs to be minimized. We define the following additional variables:

$$
\begin{aligned}
& y(t) \stackrel{\text{def}}{=} x(t) - S, \\
& u_i(t) \stackrel{\text{def}}{=} p_i(t) - \left(P_{i,\text{target}} - \frac{\epsilon}{2\beta_i} \right), \\
& c(t) \stackrel{\text{def}}{=} \sum_{i \in \mathcal{N}} \left(P_{i,\text{target}} - \frac{\epsilon}{2\beta_i} - \overline{r}_i(t) \right), \\
& J_{i,\text{given}} \stackrel{\text{def}}{=} \beta_i P_{i,\text{target}}^2 T \\
& \qquad - \int_0^T \left[\epsilon \overline{r}_i(t) + \beta_i \left(P_{i,\text{target}} - \frac{\epsilon}{2\beta_i} \right)^2 \right] \mathrm{d}t.
\end{aligned}
$$

Using these variables, (1) and (2) are rewritten as follows:

$$
\begin{aligned}
\min_{u_i} \quad & J_i,\ \forall i \in \mathcal{N}, \\
& J_i = J_{i,\text{given}} + \int_0^T \beta_i u_i(t)^2\, \mathrm{d}t + \alpha_i y(T)^2, \\
\text{s.t.} \quad & \dot{y}(t) = \sum_{i \in \mathcal{N}} u_i(t) + c(t), \\
& y(0) = x(0) - S = -S.
\end{aligned} \qquad (6)
$$

For a general differential game (4), a solution is obtained by solving the corresponding Riccati differential equations. In the following sections, on the other hand, equations and conditions are written based on the differential game (6) that captures our system model. Thus, the equations corresponding to the Riccati differential equations do not have constant terms.

4.2 Open-loop Nash Equilibrium

In this section, we solve a linear quadratic differential game (6) with an open-loop information structure and derive the open-loop Nash equilibrium based on [18].

We assume that the following differential equation based on the Riccati differential game has a set of solutions $M_i(t)$:

$$
\begin{aligned}
& \dot{M}_i(t) - M_i(t) \sum_{j \in \mathcal{N}} \frac{1}{2\beta_j} M_j(t) = 0, \\
& \qquad M_i(T) = 2\alpha_i,\ i \in \mathcal{N}. \qquad (7)
\end{aligned}
$$

Thus, the differential game (6) has an open-loop Nash equilibrium solution as follows:

$$
u_i^\star(t) = -\frac{1}{2\beta_i} [M_i(t) y^\star(t) + m_i(t)],\ i \in \mathcal{N}, \qquad (8)
$$

where $m_i(t)$ represents the solution of the linear differential equation:

$$
\dot{m}_i(t) + M_i(t) c(t) - M_i(t) \sum_{j \in \mathcal{N}} \frac{1}{2\beta_j} m_j(t) = 0,
$$

$$m_i(T) = 0,\ i \in \mathcal{N}, \quad (9)$$

and $y^\star(t)$ represents an equilibrium trajectory generated by

$$\begin{aligned} y^\star(t) &= \Phi(t,0)y_0 + \int_0^t \Phi(t,\sigma)\psi(\sigma)\mathrm{d}\sigma, \\ \dot{\Phi}(t,\sigma) &= F(t)\Phi(t,\sigma);\ \Phi(\sigma,\sigma) = I, \\ F(t) &= -\sum_{i\in\mathcal{N}} \frac{1}{2\beta_i} M_i(t), \\ \psi(t) &= c(t) - \sum_{i\in\mathcal{N}} \frac{1}{2\beta_i} m_i(t). \end{aligned} \quad (10)$$

As shown above, the open-loop Nash equilibrium (8) is a function that depends on the initial state y_0 and time t, as mentioned in Sect. 3.3. We would like to mention that in decentralized open-loop control, building i acts according to (8).

Next, we discuss the existence of the solution of the differential equation (7). We assume that $M_i(t)\sum_{j\in\mathcal{N}} \frac{1}{2\beta_j} M_j(t)$ is continuous and that the partial derivative $\frac{\partial M_i(t)\sum_{j\in\mathcal{N}} \frac{1}{2\beta_j} M_j(t)}{\partial M_i(t)}$ exists and is continuous. In this case, according to the fundamental existence-uniqueness theorem [19], (7) has a solution. Note that the existence of the solutions of the following differential equations (9), (11), (14), (18), and (19) can be determined in the same way.

4.3 Feedback Nash Equilibrium

In this section, we solve the linear quadratic differential game (6) for a feedback information structure and derive the feedback Nash equilibrium using [18].

We assume that the following differential equation has a set of solutions $Z_i(t)$:

$$\begin{aligned} \dot{Z}_i(t) + 2Z_i(t)\widetilde{F}(t) + \frac{1}{2\beta_i} Z_i(t)^2 &= 0, \\ Z_i(T) &= 2\alpha_i,\ i \in \mathcal{N}. \end{aligned} \quad (11)$$

$\widetilde{F}(t)$ is written as follows:

$$\widetilde{F}(t) = -\sum_{i\in\mathcal{N}} \frac{1}{2\beta_i} Z_i(t). \quad (12)$$

Thus, the differential game (6) has a feedback Nash equilibrium solution as follows:

$$u_i^\star(t,y) = -\frac{1}{2\beta_i}[Z_i(t)y(t) + \zeta_i(t)],\ i \in \mathcal{N}, \quad (13)$$

where $\zeta_i(t)$ represents the solution of the linear differential equation:

$$\begin{aligned} \dot{\zeta}_i(t) + \widetilde{F}(t)\zeta_i(t) + Z_i(t)\beta(t) + \frac{1}{2\beta_i} Z_i(t)\zeta_i(t) &= 0, \\ \zeta_i(T) &= 0,\ i \in \mathcal{N}, \end{aligned} \quad (14)$$

where $\beta(t)$ is written as:

$$\beta(t) = c(t) - \sum_{i\in\mathcal{N}} \frac{1}{2\beta_i} \zeta_i(t). \quad (15)$$

As shown above, the feedback Nash equilibrium (13) is a function that depends on the game state $y(t)$ at each time t, as discussed in Sect. 3.3. Note that in decentralized feedback control, building i acts according to (13).

5. Centralized Control

A suitable approach to analyze centralized control is to be formulated as an optimal control problem [18]. Optimal control theory is a framework for analyzing the action of a decision-making entity in a dynamic system that varies according to the control input. Unlike a differential game, this problem involves one decision-making entity with one cost function that needs to be minimized.

Using (6), the system model described in Sect. 2 can be formulated as a linear quadratic optimal control problem in which the cost for all buildings, i.e., $\sum_{i\in\mathcal{N}} J_i$, is a cost function of this optimal control problem. This optimal control problem can be written as follows:

$$\begin{aligned} \min_{\boldsymbol{u}} \quad & J(\boldsymbol{y},\boldsymbol{u}), \\ & J(\boldsymbol{y}(t),\boldsymbol{u}(t)) = \frac{1}{2}\int_0^T \boldsymbol{u}(t)^T \mathbf{R} \boldsymbol{u}(t)\mathrm{d}t \\ & \qquad + \frac{1}{2}\boldsymbol{y}(T)^T \boldsymbol{Q}_\mathrm{f} \boldsymbol{y}(T) \\ & \qquad + \sum_{i\in\mathcal{N}} J_{i,\mathrm{given}}, \\ \text{s.t.} \quad & \boldsymbol{y}(t) = \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{c}(t), \quad \boldsymbol{y}(0) = \begin{bmatrix} -S \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \end{aligned} \quad (16)$$

where

$$\boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_n(t) \end{bmatrix}, \boldsymbol{y}(t) = \begin{bmatrix} y(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\boldsymbol{Q}_\mathrm{f} = \begin{bmatrix} 2(\sum_{i\in\mathcal{N}} \alpha_i) & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \boldsymbol{c}(t) = \begin{bmatrix} c(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & \cdots & 1 \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}, \boldsymbol{R} = \begin{bmatrix} 2\beta_1 & 0 & \cdots & 0 \\ 0 & 2\beta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\beta_n \end{bmatrix},$$

where $\boldsymbol{u}(t)$, $\boldsymbol{y}(t)$, and $\boldsymbol{c}(t) \in \mathbb{R}^n$. $\boldsymbol{Q}_f$, $\boldsymbol{B}$, and $\boldsymbol{R} \in \mathbb{R}^{n\times n}$.

The solution of this optimal control problem is written as follows:

$$\boldsymbol{u}^{\star}(t) = -\boldsymbol{R}^{-1}\boldsymbol{B}^T[\boldsymbol{S}(t)\boldsymbol{y}(t) + \boldsymbol{k}(t)],\ i \in \mathcal{N}, \tag{17}$$

where $\boldsymbol{S}(t) \in \mathbb{R}^{n\times n}$ and $\boldsymbol{k}(t) \in \mathbb{R}^n$ satisfy

$$\dot{\boldsymbol{S}}(t) - \boldsymbol{S}(t)\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{S}(t) = \boldsymbol{0},\quad \boldsymbol{S}(T) = \boldsymbol{Q}_f, \tag{18}$$

$$\dot{\boldsymbol{k}}(t) - (\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{S}(t))^T\boldsymbol{k}(t) + \boldsymbol{S}(t)\boldsymbol{c}(t) = \boldsymbol{0},\quad \boldsymbol{k}(T) = \boldsymbol{0}. \tag{19}$$

As shown above, similar to the case of decentralized feedback control, the optimal control function (17) depends on the game state $y(t)$ at each t.

6. Numerical Analysis

In this section, first, we numerically analyze the performance of both decentralized open-loop control and decentralized feedback control for the model introduced in Sect. 2. This study derives equilibrium strategies by defining the time horizon as one day and the frequency of control as once every half hour. Based on the discussion in Sect. 2.2, when the communication fails at the beginning of the day and is not restored all day, decentralized open-loop control is adopted. On the other hand, when the communication state is normal throughout the day, decentralized feedback control is adopted. Note that when the communication state is normal throughout the day, decentralized open-loop control can also be used with only the information obtained at time $t = 0$. Then, we compare decentralized control with centralized control. When the communication state is normal throughout the day, centralized control is used.

The control procedure has two stages. First, building i makes a plan $u_i^{\star}(t)$ using the predicted data $\overline{r}_i(t)$ before $t = 0$. Then, during $t \in [0, T]$, the actual power consumption is determined according to both $u_i^{\star}(t)$ and actual weather data $r_i(t)$. Therefore, with decentralized open-loop control, building i determines its action using (8). With decentralized feedback control, building i determines its action using (13). With centralized control, building i determines its action using (17).

With respect to Nash equilibrium solutions, we explain how to solve the differential equation, e.g., (7). First, the time horizon $[0, T]$ is divided into L intervals. Then, using a given terminal condition and the differential equation (7), the gradient at $t = T$, $\dot{M}_i(T)$, is calculated. Next, using $M_i(T)$ and $\dot{M}_i(T)$, the value at $t = (1-1/L)T$, $M_i((1-1/L)T)$, is calculated. Finally, in the same way, $M_i((1 - l/L)T)$ and $\dot{M}_i((1 - l/L)T)(l \in \{0, \ldots, L\})$ are determined. Note that other differential equations can be solved in the same way.

6.1 Parameters

The parameters used in the system model are summarized in Table 1. We set the time horizon $[0, T]$ as one day, i.e., $T = 24$ h. The interval of control is set as 30 min, i.e., $L = 48$. In addition, we set the number of buildings to two, for the sake of simplicity. Note, however, that this model is scalable in terms of the number of buildings. Figure 2 shows three patterns of power obtained from a PV power generator depending on the weather. These datasets were obtained from a PV power generator in a building at Kyoto University. For the sake of simplicity, the conditions are assumed to be the same for both buildings, i.e., the performances of both buildings are the same. The reason for this simplification is that this study aims to provide a framework for discussing centralized/decentralized control for demand-side energy management systems and to present a trade-off analysis between them, not a detailed performance analysis of them.

Table 1 Parameters.

α_i	0.80 JPY/(kWh)^2
$P_{i,\text{target}}$	1.40 kW
S	10 kWh
ϵ	22 JPY/kWh
T	24 h
L	48
n	2

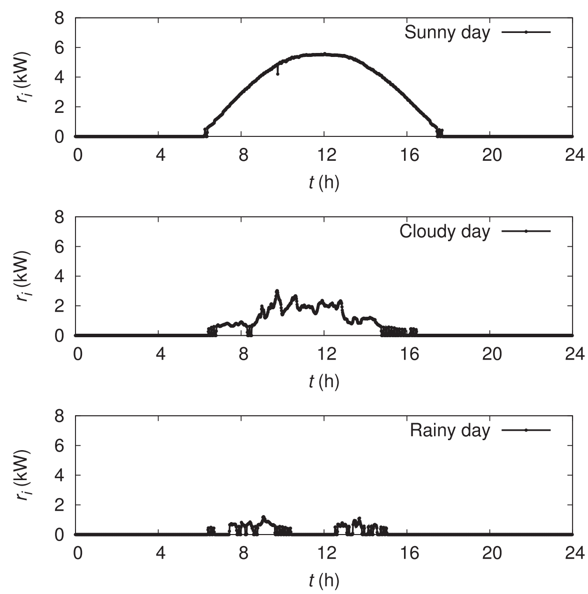

Fig. 2 Amount of power obtained from a photovoltaic power system depending on weather.

6.2 Performance with Decentralized Control

We first compare the performance of decentralized open-loop control and decentralized feedback control. The performance depends on both the predicted weather and the actual weather.

Table 2 summarizes the difference between the costs of decentralized open-loop control C_{OL} and those of decentralized feedback control C_{FB}, i.e., $C_{\text{OL}} - C_{\text{FB}}$. These costs correspond to J_i in (2). The parameter β_i is set to

Table 2 Difference in the costs $C_{\mathrm{OL}} - C_{\mathrm{FB}}$ (JPY) ($\beta_i = 30\,\mathrm{JPY} \cdot \mathrm{h/(kWh)^2}$).

		Predicted weather		
		Sunny	Cloudy	Rainy
Actual weather	Sunny	−135.97	253.42	440.44
	Cloudy	310.91	1.33	0.28
	Rainy	555.24	40.30	−15.87

Table 3 Difference in the costs $C_{\mathrm{OL}} - C_{\mathrm{FB}}$ (JPY) ($\beta_i = 40\,\mathrm{JPY} \cdot \mathrm{h/(kWh)^2}$).

		Predicted weather		
		Sunny	Cloudy	Rainy
Actual weather	Sunny	−110.48	174.05	309.89
	Cloudy	249.25	1.65	−6.41
	Rainy	441.93	37.90	−12.35

$30\,\mathrm{JPY} \cdot \mathrm{h/(kWh)^2}$.

We can see from Table 2 that C_{FB} is lower than C_{OL} in most cases, whereas C_{OL} is small when the predicted weather corresponds to the actual weather, especially when the predicted weather and actual weather are both sunny (or rainy). This is because of "the time consistency" [19] as follows. With decentralized open-loop control, buildings determine all their strategies before they start to control power consumption. Therefore, when the actual weather corresponds to the predicted data, the control is drastic and efficient without serious loss. However, determining all actions before the start of control means that decentralized open-loop control is vulnerable to unexpected changes.

As a result, decentralized open-loop control does not work well, and the cost increases if the predicted weather does not correspond to the actual weather. By contrast, with decentralized feedback control, buildings determine their actions at each time; therefore, its control is resistant to unexpected changes. As a result, even if the predicted weather is different from the actual weather, decentralized feedback control works well and the cost decreases.

To discuss the impact of the parameter β_i, we set β_i to $40\,\mathrm{JPY} \cdot \mathrm{h/(kWh)^2}$. The results are summarized in Table 3. This table shows that the difference $C_{\mathrm{OL}} - C_{\mathrm{FB}}$ changes to some extent, but the overall trend does not change. Thus, the parameter β_i does not seriously affect the trade-off between these two control schemes.

6.3 Comparison of Decentralized Control and Centralized Control

Next, we numerically analyze the performance of the three control schemes. In this section, the two buildings are assumed to obtain information about $x(t)$ via communication systems at all times.

6.3.1 Performance when Predicted Weather Condition Corresponds to Actual Weather Condition

We analyze the performance of both buildings within the accurately predicted data, i.e., $r_i(t)$ in (1) is assumed to be equal to $\overline{r}_i(t)$ in (3); for example, both the predicted and actual weather conditions are sunny.

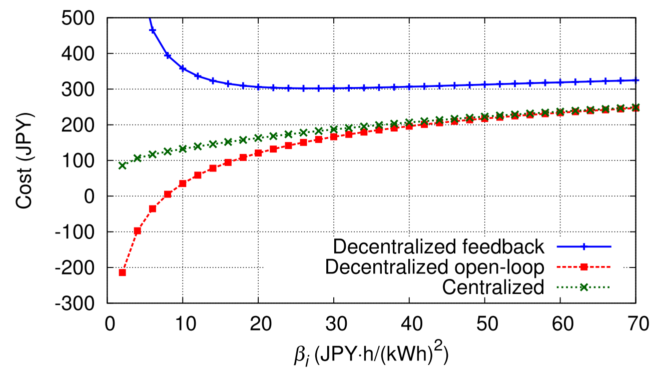

Fig. 3 Costs of each building as a function of the parameters (predicted weather: sunny, actual weather: sunny) (decentralized control: cost = J_i, centralized control: cost = $(J_1 + J_2)/2$).

Figure 3 shows the cost based on β_i when $r_i(t) = \overline{r}_i(t)$, where the data are those for the sunny day shown in Fig. 2. The value shown for centralized control is half that of $J(\boldsymbol{y}, \boldsymbol{u})$ in (16), i.e., $(J_1 + J_2)/2$, and equal to the value for one building. Thus, the power consumption and costs shown in all figures and tables in this study are for one building.

In Fig. 3, within the range of β_i, the cost is always the highest when using decentralized feedback control and lowest when using decentralized open-loop control. Therefore, when the predicted weather condition corresponds to the actual weather condition, decentralized open-loop control works more efficiently.

In addition, note that the values of C_{OL} are negative when β_i is small. As shown in (2), the negative cost implies that the value of the electricity rates $\epsilon d_i(t) = \epsilon(p_i(t) - r_i(t))$ are negative and their absolute values are larger than that of the disutility $\beta_i(P_{i,\mathrm{target}} - p_i(t))^2$ and the supply-demand balance $\alpha_i(x(T) - S)^2$. In other words, while keeping their residents' comfortability and the overall supply-demand balance, buildings suppress electricity consumption and sell a great deal of PV power.

6.3.2 Performance when Predicted Weather Condition is Different from Actual Weather Condition

In this section, we discuss the situation wherein the predicted weather condition is different from the actual weather condition. In this situation, it is estimated that decentralized feedback control and centralized control outperform decentralized open-loop control because the former two control schemes are based on actual weather.

Figure 4 shows the cost based on β_i where $r_i(t)$ is the data for a rainy day and $\overline{r}_i(t)$ is the data for the sunny day shown in Fig. 2. Figure 4 shows that within the range of β_i, the cost is always higher when using decentralized open-loop control than when using the other schemes. The cost is high for decentralized feedback control if β_i is small. By contrast, decentralized feedback control manages the power consumption well otherwise.

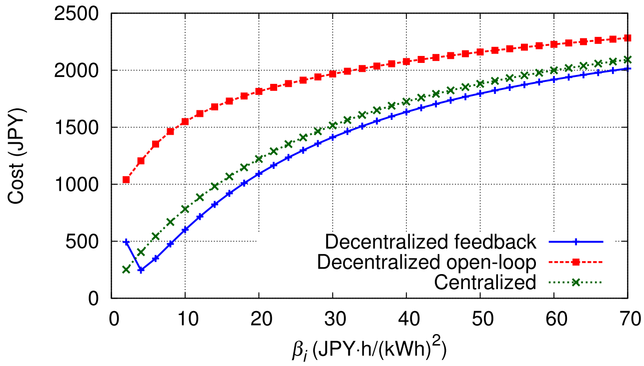

Fig. 4 Costs for each building as a function of different parameters (predicted weather: sunny, actual weather: rainy) (decentralized control: cost = J_i, centralized control: cost = $(J_1 + J_2)/2$).

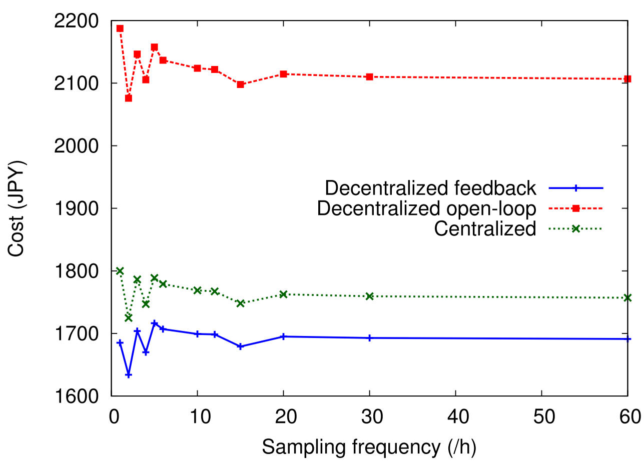

Fig. 5 Costs as a function of sampling frequency (predicted weather: sunny, actual weather: rainy).

Note that the performance of these schemes highly depends on parameters particularly when the actual weather is different from the predicted weather. In addition, decentralized feedback control is a solution of the optimization problem (6), whereas centralized control is a solution of the different optimization problem (16). Thus, it is not clear which control scheme more effectively suppresses the cost, decentralized feedback control or centralized control.

6.4 Impact of Sampling Frequency on Performance

In previous results, houses determine power consumption once every half hour. In this section, we discuss the sampling frequency, i.e., how many times buildings determine and adjust power consumption per hour. The impact of the sampling frequency on the performance depends on the frequency of the weather data. In Fig. 2, the maximum frequency on a rainy day is larger than that of other days. As an example, we show the impact when the data for a sunny day are used as the predicted weather and the data for a rainy day are used as the actual weather in Fig. 5. The horizontal axis represents the sampling frequency. The vertical axis represents the cost for buildings.

In Fig. 5, the cost changes randomly (particularly if the sampling frequency is low) because of the lack of sampling frequency compared to the frequency of weather data. In this case, houses cannot compensate for drastic changes in the PV output on a rainy day. On the basis of these figures, adjusting power consumption approximately 10–20 times per hour is sufficient.

7. Conclusion

In this study, we proposed a game-theoretic framework for analyzing control schemes in a demand-side management system on the basis of communication availability and sampling frequency. In our system, we discussed power consumption control in buildings with PV power generators. We introduced differential game theory and optimal control theory for both decentralized and centralized control schemes. Our numerical analysis reveals that when information for control is available, decentralized feedback control is more robust, i.e., decentralized feedback control works well even if the predicted data are not accurate. When information is unavailable, only decentralized open-loop control can be used. This control scheme works well if the predicted data is highly accurate. Moreover, control becomes more stable as sampling frequency increases; however, it is not necessary to increase the sampling frequency to a great extent.

Although we have analyzed the case of two demand-side actors and some fixed parameters for simplicity, we would like to emphasize that the proposed framework using differential game theory and optimal control theory is scalable with respect to the number of buildings and time-dependent parameters. We hope that the results presented in this paper will provide a useful insight into the design of decentralized demand-side energy management systems.

Acknowledgments

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