MEMOIRS OF THE COLLEGE OF SCIENCE, University of Kyoto, Series A,
Vol. XXVIII, No. 2, Article 4, 1956

PRECISE SEIMOMETRIC OBSERVATIONS IN THE EPICENTRAL REGION OF LOCAL SHOCKS

BY

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(*Received January 30, 1957*)

ABSTRACT

The precise observations of local earthquakes were carried out in Wakayama District with many sets of high magnification seismometer, for the purpose of studying the crustal structure and the nature of local shocks.

The travel-time anomalies were detected on the time-distance curves when the focus was determined in either case, both from the arrival times and from the P-S times at each station. The anomalous travel-time curves were separable into two groups which seemed to be related with the recorded directions of initial motion of the local shocks at their respective stations. These anomalies may safely be regarded as being attributed to the nature of hypocentre and mechanism of earthquake occurrence.

With regard to the structure of the crust in the district concerned under the assumption of horizontal parallel layers, the velocities in the surface layer, second, and third layers were estimated to be 4.3, 5.4~5.6 and 6.0 km/sec for the P-waves and 2.5, 3.2 and 3.4 km/sec for the S-waves, and the thickness of surface layer about 4 km, respectively.

1. Introduction

The seismometric study on minor earthquakes, as the local shocks and the aftershocks, has successively been made by many researchers in our country since the observation (**1**) of aftershocks accompanied by the Tango Earthquake in 1929. And a great many results of investigations were already published as regards the aftershocks following great earthquakes, for example, the Tango, Formosa, Niijima, Ogashima, Tottori, Tonankai, Mikawa, Nankai, Fukui and Imaichi Earthquake, and the earthquake-swarms in North-Idu and Wakayama Districts.

But, in spite of the gradual amelioration of observation methods with the considerable advance of instrumental functions: for example, for the seismograph, the recording apparatus and the time marking, most studies thus far made were prone to remain within the field of estimating the positions of hypocentre of these micro-earthquakes by the P-S duration times and discussing the time variation of seismic activities, the spatial distribution of foci, the frequency distribution of P-S times and of maximum amplitudes and so forth, as pointed out by S. Miyamura (**2**).

While in recent years, noticeable results have been obtained in new fields of research with improved accuracies of observation. For instance, S. Omote (**3**) tried to determine the focus from the measurement of arrival times at five different stations, in his observation of an aftershock of the Fukui Earthquake. He also compared the position of hypocentres respectively determined by these methods using the commencement times of the P- and S-waves and P-S times, and discussed the travel time curves. But the time accuracy of the above observation did not seem to be sufficiently high. In the case of aftershocks of the Fukui and Imaichi Earthquake, T. Asada and Z. Suzuki (**4**, **5**) estimated the apparent velocities of longitudinal waves by the tripartite station method and presumed that there was a simple crustal structure from the relations between their velocities and the P-S times. This method is excellent, but the actual velocity in the crust cannot be obtained favourably on account of the unknown focal depths. Omote (**6**) furthermore made a trial, in the Imaichi aftershocks, for determination of the underground structure by means of incident angles of the initial waves.

Of the same case, R. Ikegami (**7**) discovered two prominent phases between the P- and S-waves on the seismograms obtained with a displacement seismograph of low magnification, and estimated the depth of the discontinuity surface, considering these phases to be the seismic waves reflected from the bottom of a granitic layer. It is said, however, according to another report of Asada and Suzuki (**8**), that many phases could be picked out between the P-S interval in the observation of aftershocks at Fukui and Imaichi with sensitive seismometers, and no regularity in these distributions was recognized.

Omote (**3**, **6**) constructed a map of hypothetical distribution of 'push' and 'pull' fro mmany data taken from a few observation stations, under the assumption that all aftershocks at Fukui or Imaichi occurred at the same depth with the same mechanism. But the results of this study suggest the invalidity of the foregoing assumption.

The observations of minor earthquakes, of the same kind as stated above, have also been made at Witwatersrand District in South-Africa (**9**). H. J. Logie (**10**) determined the velocities of the P- and S-waves from many minor shocks under the assumption of uniform medium. Accurately observing a host of tremors which generated in Witwatersrand mine, P. G. Gane and others (**11**) compared the actual position of tremors with the hypocentre determined from the times of occurrence of shock at six stations, and discussed the relationship between the geological structure and tremor-occurrence in reference to the distribution of initial directions of surface movement. P. L. Willmore and others (**12**) successfully determined the structure of the crust in the district concerned, not only from the data of those stations but by many seismograms obtained in distant stations.

Presumption of local crustal structure merely from very near shocks of small magnitude seems to be much more difficult than from distant earthquakes, in spite of the various fore-mentioned attempts.

In recent years, however, artificial explosions were utilized for the investigation of the structure of crust, and explosion seismic experiments on a large scale were successfully executed several times in Japan (**13**). It cannot be denied that the artificial explosion method is more accurate in determining the underground structure than natural earthquakes on account of the previously known position and time of origin. However, in explosion the mechanism of fracture at the origin and the nature of seismic waves are regarded as being considerably different from the natural earthquake. In the sense stated above, the precise observation of minor earthquakes seems to be of great interest not only from the standpoint of the study of their nature, but, moreover, far the study of the nature of great earthquakes by extension of results obtained from these observation of minor ones. And in this case, the fact that a great many shocks will be observed in the very vicinity of epicenter in some suitably selected area is expected to offer an effective clue to the mechanism of earthquake occurrence, the nature of various seismic waves, and other problems, as well as indicating crustal structures.

It is well known that in Wakayama District local earthquake have frequently occurred since 1920 with a diversity to some extent in the location and frequency of their generation, although these shocks were of very slight magnitude, and their range which can be observed even with a sensitive seismograph was only a score of kilometres at most. Moreover, several destructive earthquakes, for example, the Nankaido Earthquake in 1946, occurred during this period, and these great earthquakes and the crustal deformation in the Kii-Peninsula may be considered to have some effects on the vicissitudes of local shocks in this region.

A. Imamura and others (**14, 15**) had previously studied the earthquake-swarms in Wakayama District, but recently the Earthquake Research Institute of Tokyo University in co-operation with the Geophysical Institute of Kyoto University began more detailed studies on these shocks. A. Kamitsuki and myself, both of the Geophysical Institute, made preliminary observations with a seismometer of high magnification at Yura in Wakayama Prefecture twice in 1952 (**16**). From these observations the later phases which may be regarded as the seismic waves reflected from several discontinuity surfaces in the crust could be learned, but conclusive presumptions were avoided because the observations were made at only one station. The Earthquake Research Institute also made successive seismic observations in this region during every year since 1952 (**17**).

The present observations were executed with many seismographs of high magnification for the purpose of studying in detail the unsolved problems of minor local earthquakes and the underground structure in Wakayama District from the precise observation of these shocks in their epicentral region.

In this paper the results obtained from the observations in 1954 and 1956 will be reported and discussed.

2. Seismometric Observation

(1) *Observation stations*

The present seismometric observations were carried out from Nov. 24th to Dec. 2nd in 1954 (**18**) and from April 10th to April 22nd in 1956.

Temporary observation stations were installed so as to be situated in the epicentral area at a limited region of 10 km in radius with Kainan City as its centre, where minor shocks frequently occur. For the geological structure of the area concerned, the ground rocks of the superficial layer consist of metamorphic rocks, being the crystalline shist and phyllite, belonging to the Sanbagawa system and the Mikabu series. This area covers the region which Imamura (**14**) had already observed. All of our station sites were selected within this region of metamorphic rocks, so that the superficial condition of ground rock on which the observation instruments were installed should be identical. The number of our stations were 4 in 1954 and 6 in 1956, and the three of these stations were fully equipped with three components of seismometers and the others with only vertical component. The united observations were carried out in close connection with the Earthquake Research Institute. The locations of all stations are shown in Tables 1 and 2 and Fig. 1. These co-ordinates were determined from triangulations and aerial photographs furnished by the Earthquake Research Institute (abbr. E. R. I.) and theirs errors are considered to be approximately 10 m. These

Table 1. Observation stations in Wakayama District

Station	Location		Elev.	Seismometer	Observers	Obs. Period
	λ	φ	h			
Wakanoura (W)	135°10′04″.4 E	34°11′23″.1 N	35 m	V1	Y. Okamoto	1956
Kimiidera (Km)	11′29″.6	10′44″.5	25	V1, H2	T. Mikumo, Y. Itô	1954
Idakiso (I)	15′14″.3	11′53″.4	35	V1	Y. Tanaka	1954 1956
Fuyuno (F)	12′59″.6	10′49″.8	40	V1	K. Hosoyama, Y. Itô	1956
Kainan (K)	13′31″.4	08′42″.0	50	V1, H2	T. Mikumo, Y. Yamamoto	1956
Nokami (N)	17′50″.0	09′46″.8	80	V1, H2 H1	M. Ōtsuka, K. Zako M. Ōtsuka	1954 1956
Yoro (Y)	09′40″.4	06′54″.0	15	V1	Y. Kishimoto A. Kamitsuki I. Nakagawa	1954 1956

Table 2. Co-ordinates of Observation Stations

Stations		X_i	Y_i
		km	km
Wakanoura	(W)	0	0
Fuyuno	(F)	1.04	4.45
Idakiso	(I)	−0.91	7.90
Nokami	(N)	2.99	11.76
Kainan	(K)	4.94	5.15
Yoro	(Y)	8.10	−0.69
Kimiidera	(Km)	1.27	2.20
Todoroki	(R_2)	3.19	12.88
Shimotsu	(R_3)	8.53	−4.04
Kanaya	(L_1)	15.58	8.79
Idakiso	(L_2)	−0.91	7.90
Kainan T.	(Kh)	4.55	3.72
Kotonoura	(Ko)	3.82	2.08
Taisei H. S.	(T)	3.44	12.87
Dejima	(D)	0.49	−0.33
Sanda	(S)	−0.80	3.62
Niitakamachi	(Ni)	−1.00	0.32
Kotonoura	(Ko′)	3.84	2.08
Kanan H. S.	(Ka)	4.34	5.72
Taisei H. S.	(T′)	3.41	12.87
Minoshima	(M)	11.67	−4.47
Yuasa	(Yu)	17.49	1.49

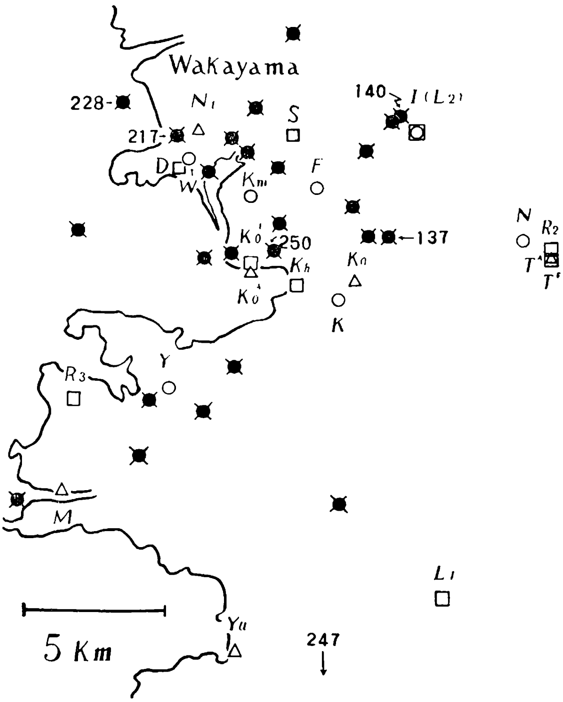

Fig. 1. Positions of the observation stations and the epicentres of the earthquakes mentioned.
□; observation stations of the E. R. I. with electromagnetic seismometers.
△ ; observation stations of the E.R.I. with acceleration seismographs.
○ ; observation stations of the Kyoto Univ. with electromagnetic seismometers.
✱ ; epicentres.

determined co-ordinates were used in our analysis by the courtesy of the E.R.I.. The origin point of the co-ordinate was selected at Wakanoura, taking the X-axis southwards and the Y-axis eastwards.

(2) *Instruments*

The instruments used in the present observations were the electromagnetic seismometers of the variable reluctance type with short period and high magnification, which were designed and constructed especially for the purpose of observing small earthquakes and artificial explosions. The detailed properties of these seismometers named R_2-type will be described in the last paragraph of this paper as an appendix.

Both the vertical and horizontal seismometers were used in the conditions of 0.45 sec in natural period and 1.79 volt/kine in voltage-sensitivity. Maximum magnification of these seismometers in direct connection with the galvanometer of 0.35 sec in free period and 3.20×10^{-8} amp/mm in direct current sensitivity is approximately 80,000 at the optical distance of 1 metre.

The time marks, at all stations, were recorded on the oscillogram by interruption of a light beam with a relay-apparatus for receiving the time signals of JJY standard short wave (4 or 8 Mc) of every second kept by the Tokyo Astronomical Observatory. Reception conditions of the short wave were excellent during about 3/4 of the day-time. The rotation speed of the oscillogram winding around a recording cylinder was nearly 5.0 mm/sec. The time error accompanied with precise readings on seismograms is less than 1/20 sec.

Most stations of the E.R.I. carried out an observation with electromagnetic seismometers using a line carrier telerecording (**19**) or a radio telerecording (**20**), and a part of them with acceleration seismographs. The time signals were marked by chronometer (**21**) at all stations of the E.R.I., and paper speed was about 4 mm/sec. The time marks resulting from this chronometer system were compared with those resulting from JJY standard time at our stations, chiefly by the commencement times of initial shock recorded with two kinds of seismometer which were installed at Idakiso, and partly by a close examination several times each day.

(3) *Observed results*

Our observations were made during a day-light hours (08h00m–20h00m) only. It is by reason that there is no need for this study to observe the total number of shocks during a day. The number of earthquakes recorded at each station was from only two or three to ten or more. Those shocks observed at more than three of the four stations first established amounted to 56 in the primary observation, and those at more than five of six stations were 44 in the second observation. The maximum amplitude in the greater part of these shocks is estimated to be approximately $1/10\mu$.

The P–S duration times recorded, for the most part, are within the range from 0.5 to 4.0 sec. Constructing the frequency spectrum of the P–S times observed at each station, the highest frequency lies in the period of 0.5～1.0 sec at Kimiidera and Fuyuno, 0.5～1.5 at Wakanoura, 1.0～1.5 at Idakiso and Yoro, 1.0～2.0 at Kainan and 1.5～2.0 at Nokami, respectively. Therefore, the majority of shocks mentioned here are considered, as a result of this examination, to have occurred within a very short range from these stations.

With respect to the correlation between the maximum amplitudes and the number of shocks accompanying them, the Ishimoto-Iida's formula does not seem to hold always, evidencing a deviation especially in larger amplitudes, for a large percentage of these minor quakes recorded at each station, as pointed out also by Miyamura (**23**).

The especially well-recorded earthquakes which seem to have occurred inside or near our observation network were selected from the shocks observed at more than five stations and used in the analysis. In the ensuing discussions the data obtained by the E.R.I. will synthetically be used with those of our own.

3. Anomalous Time-Distance Curves

In determining the hypocentres of the previously mentioned earthquakes, the crust in this region was assumed to be of a homogeneous medium as a first approximation. And since the difference in the elevation of these stations is less than 50 meters we may safely say that they are of the same elevation, as such a slight discrepancy, in comparison with the mutual distances between the respective stations, could hardly effect the final outcome.

First a trial was made to estimate the position of foci from the arrival times of the initial wave observed at each station, but the hypocentres, for the most part, could not analytically be determined by this means.

(1) *Determination of hypocentres by the P–S times and the time-distance curves*

The next trial of determining the hypocentre was made by a graphical method or the method of least squares from the P–S times clearly recorded at more than four stations, as was the general case in the observation of aftershocks. In the case of ours the P–S times recorded with seismographs of the same type were utillized for the most part, and the horizontal directions of initial motion at the stations with three components of seismometers were taken into account. When this method was adopted (**18**), the epicentre could be determined without much conflicting evidence and the errors in this determination are estimated to be less than several hundred metres. Omori's constants calculated are from 5 to 8.

When the arrival times of the P– and S–waves at each station are plotted against the hypocentral distance on a time-distance graph with regard to each 17 of the shocks whose focus was determined by the above method, the plotted points excessively diverged in a wide range. It is to be remarked that the hypocentral distance, instead of the epicental, was taken as abscissa for convenience sake in this case, because the focal depths calculated are comparable with the epicentral distances.

If the time-distance curve is provisionally represented as a mean straight line the gradient of this line, in other words the reciprocal of the propagation velocity of waves, has a great diversity in every shock, and as is the case even in several shocks which occurred at approximately the same depth in the same area. In these cases, the values of velocity of the P–waves are from 2 to 8 km/sec, and the maximum deviation of travel times at each station from the mean straight line reaches to 0.3 sec, being beyond the limit of observational errors. However, the time-distance curves could successfully be represented as straight lines only in the two of 17 shocks and their velocities of the P-waves are 5.6, 5.0 km/sec, respectively.

For this reason it is undesirable to represent the time-distance curve for most of the shocks as a straight line. Considering, then, the travel-time curve to be a straight band line with a certain time range, or to consist of several minor groups of straight lines, from another point of view, it can naturally and favourably be divided into two systems of straight line, with respect to 15 of the shocks discussed here. The respective groups of the time-distance curve can plausibly be represented as straight lines approximately paralleling each other with the travel-time difference of about 1/4 sec, having small deviation of observed values at each station from either line. This fact can be found for the S-waves as well as for the P-waves. The velocity differences estimated from these two straight lines are within 0.1 km/sec, and the propagation velocities themselves obtained from the above 15 shocks are 4.3～5.3 km/sec for the P-waves and 2.3～2.8 km/sec for the S-waves, respectively. These values are not so diverse in every earthquake as in our previously findings and are considered to be reasonable ones for the velocities in the crustal upper layers.

Furthermore, even when the uncertainty of the positions of hypocentre due to measuring errors of the time at each station was taken into consideration, the general tendency that the time-distance curves are separable into two straight lines is not altered, even though the values of velocities and of time difference would be changed in small percentage.

Comparing the relation between the commencement times and the P-S times at two stations, the P-S time recorded at the first station, in some cases, was of longer duration in spite of the earlier commencement time than at the other. These conflicting cases are not always unreasonable, if consideration is permitted that the travel-time curves consist of two groups.

Then, the correlations between the travel times at the stations in each group of time-distance curves and the following items were next examined throughout all the 15 shocks, provided that the time-distance curve is tentatively divided into two groups:

i) the geographical locations in which the observation points are situated.

ii) the positions of epicentres; the focal distance.

iii) the directions from the epicentre.

iv) the directions of the initial movement recorded at each station.

Regarding points i) and ii), these correlations were not so distinctly recognized. That is, neither station nor group of stations in a fixed region have a tendency of always belonging to a fixed group of the travel-time curve, even in the case of similar positions of hypocentres. Moreover, the stations at shorter distance do not always belong to the group with shorter travel time.

Furthermore, the relation between the direction of initial movement recorded at each station and the travel-time curves was then investigated. In the earthquakes here studied, the distribution of 'push' and 'pull', or of an initial anaseismic and kataseismic movement on a map indicated that it is of the four quadrant type, if the nodal curves were assumed to be a pair of lines intersecting at the epicentre, and the recorded first amplitudes at various stations were taken into account. But these lines did not always intersect at right angle. The 'push' and 'pull' in the initial motion at any station, mereover, was not always the same throughout all shocks.

Returning to the subject, it seems in every shock that the arrival times at the respective stations in two groups of the time-distance curves are closely related to the recorded direction of the first movement at each observatory. Namely, the travel times at the stations in the initial anaseismic area have a tendency to belong to either group or the straight lines of either side on the time-distance graph, and the other group consists of travel times at the stations in the kataseismic region. In other words, these groups of travel time have a relation to the direction from the epicentre. The examples of time-distance graphs are shown in Figs. 2 and 3. At the stations near the nodal lines, however, some exceptional cases were found that undoubtly belong to the other group.

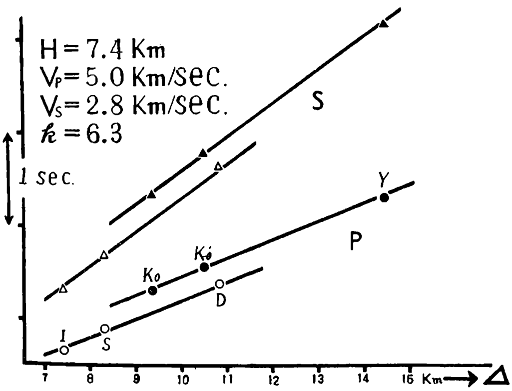

Fig. 2. Time-distance curves. (No. 140)
○ ; stations with the initial anaseismic motion.
● ; stations with the initial kataseismic motion.
△ ; hypocentral distance.

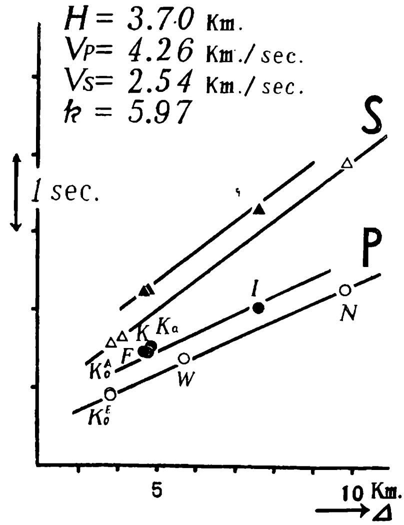

Fig. 3. Time-distance curves. (No. 250)

In two of 17 shocks, the time-distance curve could be represented as a straight line, as shown in Fig. 4, but the recorded directions of the initial movement are anaseismic at all stations.

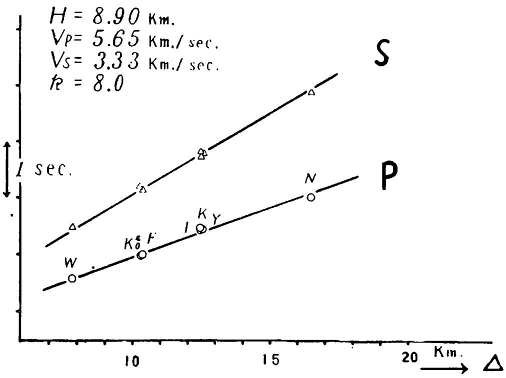

Fig. 4. Time-distance curves. (No. 228)

Taking these points into consideration the group of travel time curves may probably be defined by the initial direction at each station. Of the 15 shocks discussed above, the stations with the initial anaseismic motion belong to the group of shorter travel time and the initial kataseismic stations belong to that of the longer in some cases, and in other cases these relations were found to be reversed.

(2) *Determination of hypocentre from the arrival times and the time-distance curves*

In determination of a hypocentre from the arrival times at each station, the solution of the simultanaeous linear equations of four dimensions, or the graphical method was used in the case of five observation stations, and a least square method in the case of more than five stations. When the arrival times were made use of, the focus, for most of the shocks, could not be determined with some exceptions, as already mentioned. That is, their focal depths resulted in imaginary values or the velocities were calculated as unreasonable values. Miyamura also indicates the same findings (**22**, **23**).

Hence, the problem in which case it is possible or not to determine focus from the arrival times, were investigated in every earthquake, with reference to analysis by means of the P–S times. The following results were obtained.

a) When the directions of first mevement recorded at all stations are the same, in other words, only in the anaseismic or kataseismic sense, the focus could be determined with good accuracy, and the time-distance curves could be represented as straight lines with small probable errors. And the reasonable velocities were obtained for the P– and S–waves in this case. Fig. 5 shows an example of the time-distance curves in these 6 cases. Seismograms of the presently mentioned shock recorded at six stations are shown in Fig. 11.

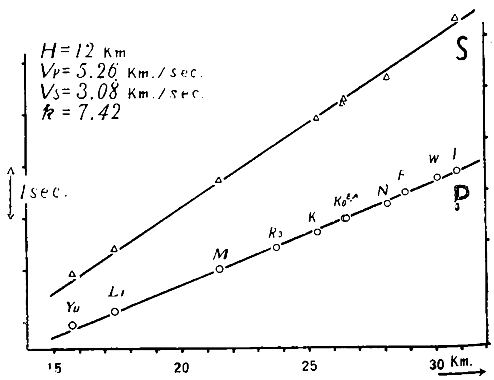

Fig. 5. Time-distance curves. (No. 247)

b) When both the anaseismic and kataseismic first movements were recorded at various stations, the hypocentre was correctly determined if only the arrival times of the P-waves at the stations with the same initial direction were used in calculation. On a time-distance graph, the travel times at these stations successfully lie on a straight line. But the travel times at other stations with the another initial direction, which were out of this calculation, deviate from the above line, and the straight line suitably connecting these plotted points, if possible, approximately parallels the former with a time difference of 0.2~0.4 sec. The reasonable velocities were also obtained. These two examples are shown in Figs. 6 and 7.

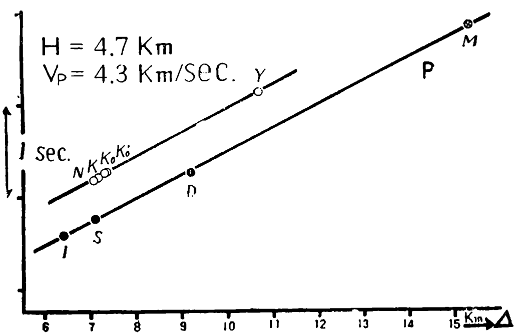

Fig. 6. Time-distance curves. (No. 137)

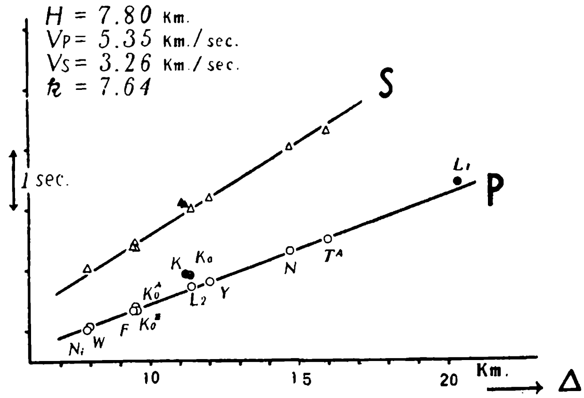

Fig. 7. Time-distance cuaves. (No. 217)

In the present observations, however, there was no earthquake which could be observed at respective more than five stations; both with the initial ana seismic and with kataseismic directions, and so the foci respectively determined from two combinations of the stations with the same direction could not be compared with each other.

c) Again the commencement times at all stations were used in calculation by a least square method without reference to the respective initial directions. If the stations with either initial direction is considerably more than those with another direction, the focus was determinable in a few cases, but the values of velocities became unreasonable or the probable errors in the travel-time curves were found to be very large. When the numbers, both for the former and the latter stations are nearly equal, the hypocentre could not generally be determined.

It was in 6 earthquakes by means of method a) and in 7 shocks by method b) that the focus could be determined from the arrival times of the initial wave. Also, in these methods, as well as the results of the P-S times, the travel time curves are separable into two groups, or parallel straight lines, relating to the direction of initial movement. But such a fact for the S-waves seems not so clear as in the case of using the P-S times. There were two instances, also, where the earthquakes' focus was determined by these methods. That is, the stations with the initial anaseismic movement belonging to the group of the shorter travel time in one case and the initial kataseismic stations belong to that group in the other.

With regard to the presently discussed methods, Miyamura pointed out the following exceptional cases. Namely, he says in his private communication that the hypocentre of a few earthquakes was not determined by the analysis of arrival times at a certain set of five stations with the same initial direction, but this problem should be solved by a further observations.

(3) *Synthesized results*

As described in detail in this paragraph, the time-distance curve has an indication both for the P- and for the S-waves that the travel times are separated into two groups in connection with the direction of initial motion of shocks at each station, when the arrival times as well as the P-S times were made use of in determination of the focus. In other words, the travel times and the 'push' and 'pull' of first motion at the respective stations may be considered to be related each other.

Table 3. List of number of shocks in reference with the direction of initial motion and group of travel time at each station.

W

	U	D	T
1	8	4	12
2	5	1	6
N.G.	5	0	5
T	18	5	23

F

	U	D	T
1	3	2	5
2	3	2	5
N.G.	4	0	4
T	10	4	14

K

	U	D	T
1	1	2	3
2	3	4	7
N.G.	4	0	4
T	8	6	14

I

	U	D	T
1	7	3	10
2	6	5	11
N.G.	6	0	6
T	19	8	27

N

	U	D	T
1	4	6	10
2	4	5	9
N.G.	6	0	6
T	14	11	25

Y

	U	D	T
1	8	3	11
2	3	1	4
N.G.	5	0	5
T	16	4	20

Ko

	U	D	T
1	4	3	7
2	5	8	13
N.G.	6	0	6
T	15	11	26

U; Up, D; Down, T; total number of shocks, 1; first group of travel time, 2; second group of travel time, N.G.; no group in travel time.

Table 4. Analyzed results.

Shock No.	X (km)	Y (km)	Z (km)	Anal. Method.	Vp (km/sec.)	Vs (km/sec.)	Vp/Vs	k	Time-Distance Curves	δT
121	12.17	5.20	11.20	P-S	4.83	2.58	1.87	5.54	(1) Y, N(D) ; (2) Km, D, I(U)	0.20sec.
122	0.47	3.15	3.70	P-S	4.36	2.40	1.82	5.34	(1) Ko, Ko′, Y*, N(D) ; (2) Km, I(U)	0.20
123	2.77	6.20	3.30	P-S	4.25	2.43	1.75	5.68	(1) I, Km*, Y(U) ; (2) Ko, N(D)	0.35
124	−0.28	6.20	4.40	P-S	5.04	2.52	2.00	4.78	(1) Y(U) ; (2) I, Ko, N(D)	0.25
135	3.57	0.60	4.60	P-S	4.64	2.47	1.87	5.30	(1) D, Y, S(U) ; (2) Ko, I, N(D)	0.20
136	7.37	1.50	4.70	P-S	4.62	2.54	1.82	5.65	(1) Y, D*, I(U) ; (2) Ko, Km, N(D)	0.25
137	2.77	6.95	4.75	P	4.40	—	—	—	(1) I, S, D, M*(D) ; (2) Km, N, Ko, Ko′, Y(U)	0.45
140	−1.23	7.35	7.40	P-S	5.07	2.81	1.81	6.31	(1) I, S, D*(U) ; (2) Ko, Ko′, Y(D)	0.25
151	1.72	5.40	5.80	P-S	4.73	2.70	1.85	5.88	(1) Km, I, S, D(U) ; (2) Ko, N, Y*(D)	0.20
201	25.65	5.67	10.30	P	5.11	2.92	1.75	6.84	(1) L_1, Y, R_3, K, Kh, Ko, N, I(U)	—
202	2.45	−3.85	6.20	P	5.10	(3.38)	—	—	(1) F, K*, I, N*(U) ; (2) Ni, Ko, Ko′, R^*_3, Kh, L_1, (D)	0.20
206	11.97	−5.98	9.25	P	5.43	3.13	1.74	7.35	(1) M, R_3, Y, Ko, Ko′, Kh, K, W, Ka, L_1, L_2, T′(D) ; (2) Yu (U)	0.15
207	2.30	3.05	3.60	P-S	4.31	2.67	1.62	(7.01)	(1) Y, N (D) ; (2) Ko, Ko′, K, W, I(U)	0.40
217	−0.98	−0.36	7.90	P	5.35	(3.56)	(1.58)	7.64	(1) Ni, W, F, Ko, Ko′, L_2, Y, N, T′(U) ; (2) K, Ka, L_1(D)	0.20
220	−1.20	7.25	5.10	P	4.66	2.85	1.64	7.35	(1) W(D) ; (2) I, F, N, K, Ko′, L_1(U)	0.35
222	8.35	−1.25	4.40	P	4.72	2.96	1.60	5.94	(1) Y, Ko, K, W, F*, I, N(U)	—
223	3.35	1.59	5.15	P-S	4.72	2.72	1.73	6.41	(1) Ko, W, Y, I, N(U) ; (2) F(D)	0.20
224	−0.70	0.65	8.90	P-S	5.38	2.91	1.85	6.35	(1) F, Ko (D) ; (2) W, I, N(U)	0.25
228	−2.05	−2.30	7.25	P	5.65	3.54	1.60	—	(1) W, Kc, F, I, Y, K, N(U)	—
230	0.45	0.65	8.40	P	5.65	(2.60)	—	—	(1) F, I(D) ; (2) W, Ko, K, N, Y, (U)	0.20
236	10.50	−1.75	6.30	P	4.83	2.93	1.65	7.45	(1) L_1, N, R_2 (D) ; (2) Y, Ko, Ko′, K*, Ka, W, F, I(U)	0.25
247	27.30	4.15	12.00	P	5.26	3.08	1.71	7.42	(1) Yu, L_1, M, R_3, K, Ko, Ko′, N, F, W, I(U)	—
248	−0.17	2.00	4.55	P-S	4.93	2.66	1.87	5.63	(1) W, Y(U) ; (2) F, Ko, K, I(D)	0.40
249	−1.87	2.30	8.80	P-S	5.33	3.20	1.68	(8.00)	(1) W, F, Ko, N(U) ; (2) I(D)	0.20
250	3.20	2.80	3.70	P-S	4.26	2.54	1.73	5.97	(1) Ko, Ko′, W, N, R_2*, (U) ; (2) F*, K, Ka, I(D)	0.30
251	8.96	0.46	5.30	P	4.86	(3.23)	—	—	(1) Ko, Ko′, F, W, I, N(U)	—
253	−4.10	3.60	6.60	P-S	5.00	2.88	1.80	6.78	(1) I, W, Ko, Ko′, K, N, R_2, Y(U)	—

X, Y ; location of epicentre, Z ; focal depth, Analytical method : P-S ; using the P-S times, P ; using the arrival times of the P-waves, Vp ; velocity of the longitudinal waves, Vs ; velocity of the transversal waves, k ; Omori-constant, (1), (2) ;groups of travel time, U ; initial 'Upward'-motion, D ; initial 'Downward'-motion. δT ; difference of the travel times between two groups. The shoulder mark * indicates that the direction of inital motion at the station recorded the opposite one or it was not clearly recorded.

The observed data (the arrival times of the P- and S-waves and the recorded directions of initial motion) at the stations of the E.R.I. : R_2, R_3, L_1, L_2, Kh, Ko, Ko′, T, T′, Ni, Ka, M and Yu, were used in the present analysis by the courtesy of the Institute.

In several shocks the positions of hypocentre determined from the above two methods were compared, but the discrepancy between the two positions seems to be within the range of the observational errors.

Putting together all of 27 shocks analysed here, the time-distance curve could be represented as a straight line in 6 shocks and could be separated into two groups in 21 shocks. In the former cases the initial direction at the respective stations are all identical, and in the latter the stations with the initial anaseismic direction belong to the group of the shorter travel time, and the initial kataseismic stations form that of the longer one in 12 shocks, and contradictory cases to these are found in 9 shocks. The travel time difference is in the range from 0.2 to 0.4 sec.

The directions of initial movement and the group in travel times in the main observation stations are shown for every shock in Table 3. The initial motion recorded at any station were not always of the same kind and the travel times did not always belong to the same group.

The location of epicentre, focal depth, velocities of the P- and S-waves, Omori's constant, the travel-time difference in the two groups and the abbreviations of observation stations constructing the time-distance curves, in every shock, are listed on Table 4.

4. Crustal Structure in Wakayama District

The analysis was thus made assuming that the crust as a first approximation was of a uniform medium. The problem whether this assumption may be used as a first

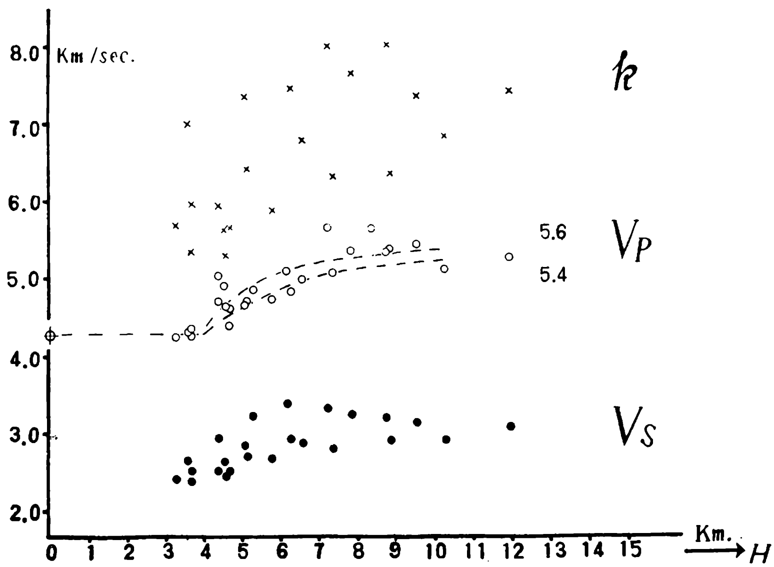

Fig. 8. Velocity-focal depth graph.

approximation or not is certainly an essential one. And next, in this section, a simple model of horizontally layered structure will be treated as a second approximation.

The trials for determining the fine regional structure from the aftershocks or local shocks have been made by various methods, as stated in the first paragraph. In the present study the following method was tentatively adopted, making use of a great number of minor shocks which occurred within a considerably limited region.

The velocity-focal depth graph in many shocks obtained from calculations under the uniform medium assumption is shown in Fig. 8, where the velocities in this case are the calculated ones from either method of the P–S times or the arrival times. In the ensuing discussions the validity of the values of velocity obtained by the above method will be premised.

(1) *Surface layer*

As easily seen in Fig. 8, a clear relationship between the velocities and focal depths seems to be shown by the P–waves, but the relationship of this kind is not so clear for the S–waves and the values of k. It is perhaps disturbed by the observational errors in the case of the S–waves.

The velocity of the P–waves tend to increase with increasing focal depth. According to the results obtained, on the other hand, in the seismic prospecting carried out by A. Kubotera and T. Yoshizawa (**24**) at Shimotsu in the area concerned, the velocity of the longitudinal waves in the upper surface layer of a metamorphic rock, named serpentine, was estimated to be 4.27 km/sec, as plotted at zero-depth in Fig. 8. Of the shocks, moreover, whose focal depth were less than 4 km, the calculated velocity for the P–wave is approximately 4.3 km/sec, and therefore, the surface layer consisting of the metamorphic rock with the longitudinal velocity of 4.3 km/sec is estimated to have a layer thickness of 4 km. The mean velocity of the S–waves in this layer is estimated to be about 2.5 km/sec. In addition, the velocity of the elastic waves from the quarry blasts at Kotonoura was measured by the E.R.I., but the observed value seems to be too large for the velocity in the superficial layer.

(2) *Second layer*

The structure of the horizontal parallel layers will then be assumed to be a second approximation, considering that the velocity of the P–waves in the case of uniform medium tend to gradually increase with increasing focal depth of more than 4 km. However, the focal depths in that case are not, of course, regarded as the actual depths because of the existence of a surface layer with a lower velocity. Secondly, the trial for determination of the longitudinal velocity in the second layer was made so as to be adapted to the distribution of the uniformly assumed velocities against the depths, on the grounds of previous knowledge about the thickness and velocity in the surface layer. In the present method, the travel times to the respective observation stations from the hypothetical foci which were assumed to be at various depths in the second layer were calculated, taking the velocity in that layer as a parameter. Regarding, then, these travel times as the arrival times at the respective stations, the theoretical relation between the velocities and focal depths was learned from their respectively calculated values under the postulation of a homogeneous medium. Needless to say, this relation is variable with the geographical distribution of the observation stations used in these analyses, but ought not to change in its general appearance. The relations indicated by two broken curves in Fig. 8 were derived from the calculation with the uniform medium-assumption by means of the hypothetical arrival times at five stations, namely, W, F, K, Y and N (abbreviated), when the velocity of the P-waves in the second layer was assumed to be 5.4 and 5.6 km/sec, respectively. These curves show fairly good agreement with the actual distribution of the observed values, and therefore the velocity of the P-waves in the second layer was estimated to be in the range from 5.4 to 5.6 km/sec. The suitable velocity of the S-waves was estimated to be 3.2 km/sec, provided that the Poisson's ratio is 0.25.

In order to estimate the thickness of the second layer we tried to make use of the seismic waves which were emitted from a shock in this layer and reflected at the bottom of the layer mentioned. That is, several prominent phases were chosen from the seismograms recorded at the respective stations, and the duration times between the initial P-wave and the later phases concerned were plotted against the epicentral distance. While the theoretical curves indicating this relation were calculated and drawn on this graph with the variously assumed thickness of the second layer. The later phases seeming to be reflected waves were detected in a few examples. Some phases may probably be regarded as the transformed waves PS or SP, and from the duration times of these waves the already ascertained thickness of surface layer is reliable.

(3) *Third layer*

During the present observation of local shocks, a distant earthquake of small magnitude which occurred in the vicinity of Kyoto was recorded by chance at five temporary stations of ours in Wakayama District. The location and focal depth of hypocentre in this earthquake was roughly estimated to be about 135°32′. 5 E, 35°01′. 5 N and 11 km from the P-S times and the horizontal directions of initial motion recorded at the Kamigamo and Abuyama Seismological Observatory of the Kyoto University and the Kyoto Meteorological Observatory. On constructing the time-distance curves merely from the above five stations, the mutual differences of their epicentral distances and the directions from the epicentre become the subject of discussion, and the absolute values of epicentral distances are only supplementarily treated. Even if the epicentre was not so accurately determined in this case, the effects on the travel-time curves are considered to be very slight, because the distance from Kyoto is much larger than the mutual differences. Actually in this case, the recorded directions of first motion were 'push' at all stations. The time-distance curves could be represented as a straight line with a good accuracy for the P- and S-waves as shown in Fig. 9. The apparent velocities of the longitudinal and transversal waves are 6.00 and 3.40 km/sec, respectively. A example of seismograms of the earthquake is shown in Fig. 12.

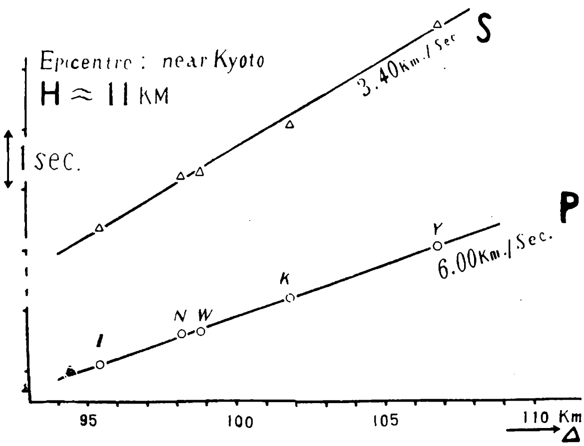

Fig. 9. Time-distance curves. (No. 240)
△ ; epicentral distance.

The tripartite station method for estimating the apparent velocity and the propagating direction of waves by the arrival times at three stations in a small area was thus far adopted in some observations of aftershocks, but the method here used may be regarded as an extension of the prior one.

If the thickness of the second layer was now assumed to be about 6 km, being equivalent to that of the granitic layer in a model of the horizontal structure in Kinki District as derived by Y. Kishimoto (**25**), the depth of the bottom of the second layer is found to be 10 km, which is nearly equal to the focal depth in this case. As the epicentral distances are about 100 km, the initial waves observed at the above five stations are presumed to be the longitudinal waves which horizontally propagated in the main along and just under the interface between the second and third layers. Hence, it may safely be considered that the apparent velocities observed should represent the actual ones in the third layer.

	P	S
4 km	4.3	2.5
(6 km)	5.4 ~5.6	(3.2)
	(6.0)	(3.4)

Fig. 10. Horizontal structure in Wakayama District.

The estimated velocities and thickness in the respective layers are as shown in Fig. 10, when the horizontal parallel structure of the crust was assumed in Wakayama District.

5. Discussions

In the present observations the unanticipated results stated above were obtained.

The various problems in the analysis will be mentioned and discussed under this heading, and in addition, some presumable interpretations for these results tentatively will be given.

1) *Accuracy of time*

The cause of diversity of travel times in the seismometric observations hitherto made was attributed, for the most part, to the unsatisfactory time-keepings, and really the time accuracy in the past observations is considered to be inadequately low for observation of minor earthquakes.

From this point of view, the common times from the JJY standard waves were marked every second on seismograms at our six stations by means of the same type of time-marker in the present observations. The rotation of a recording drum was actuated by a synchronus motor and the irregularity of rotation owing to a voltage fluctuation is less than 0.2 mm/sec. At the stations of the E.R.I. the common times transmitted from one chronometer were placed on seismograms, and two systems of the time mark were checked by a close examination, as stated in the second paragraph. The time errors accompanied by a precise reading of seismograms by a comparator are considered to be less than 1/50 sec in especially good records and within 1/20 sec in poorer ones, judging from the clearness of the initial P- and S-waves in most of the shocks analysed here. Therefore, the large deviations in the travel times as stated in the third paragraph are not regarded as resulting from time errors because of the high accurary in the present observations.

2) *Discrimination of initial wave*

In the present observations, the stations with a low ground noise-level were selected, but it seems undesirable to consider that there are no mis-discrimination of the initial wave, when it is of smaller amplitude than that of the second movements. Namely, the question may arise whether the separation of the time-distance curves into two groups is due to a mis-discrimination of the initial wave in 1/2 cycle or one cycle, on account of their small amplitudes recorded at the respective stations belonging to the group with apparently longer travel time. As the wave frequency in the initial part of seismic motions is more than 5 cycles, the time error resulted from a mis-discrimination of 1/2 cycle is less than 0.1 sec being insufficient to the travel time difference from 0.2 to 0.4 sec between two groups. And seeing also that the distribution of the 'push' and 'pull' in the initial movements shows that of four quadrant type, a possibility of mis-discrimination of 1/2 cycle is difficult to be anticipated. The following facts, moreover, may be regarded as the counter-evidences to the above questions. That is, in the group of the longer travel time many stations with the clearly recorded initial wave of large amplitude are included. And the time-distance curve could reasonably be represented as a straight line in the case of only the stations with a same initial direction, and otherwise it was in many shocks separable into two groups.

3) *Accuracy in determination of the hypocentres*

On an estimation for the errors of the focal position due to the observational time errors, some studies have been made. N. Nasu (**1**) investigated them using the P–S times at four stations in the observation of aftershocks of the Tango Earthquake. Y. Miyayama (**26**) examined these results in detail and stated that the inevitable errors in determination of a hypocentre is about 1 km. But it is possible to become larger when the P–S times at more than four stations are used in combination by a graphical method. The accuracy of hypocentre's determination in artificial explosions was investigated by S. Asano (**27**), when a least square method from the arrival times of initial wave at many stations were used in the analysis. When the calculated focus was compared with the actual position of a shot point, the epicentre could be determined with small discrepancy from the latter of about one fiftieth of a span of observation network, and with the probable errors of the same order. And it is stated that the accuracy of the velocity was considerably good, but the origin time and focal depth were not determinable with such a good accuracy as was possible in locating the epicentre.

The theoretical estimate of the probable errors in determination of focus is very troublesome, because it is concomitant to the geographical distribution of stations, the arrival times and their probable reading errors at the respective stations, and differs in every shock.

At any rate, the uncertainties from the time errors in the present observations are presumed to be less than several hundred metres in the positions of epicentre and 1 km or so in the focal depths by either method of the P–S times or of the arrival times under the uniform medium assumption. However, the general appearance on time-distance curves is not changed within those ranges.

4) *Effect of crustal structure*

In this section the amount of error in the focal position and the degree of effect on the travel-time curves will be investigated, when the analysis is made referring to a horizontally layered structure instead of a homogeneous medium. Namely, for such a purpose, it is needed to compare the respective positions of focus determined from the arrival times at each station assuming the structure of horizontal parallel layers, and of the uniform medium, and examining the incidence of travel-time curves in both cases. But calculation in the former case is difficult, and so the following method was used. The already-presumed horizontal structure was here made use of as an example. Now the hypothetical focus was assumed to be in the second layer, and the travel times of the P-waves from it to each station were calculated. The discrepancy between the position of hypocentre which was calculated from these travel times and the previously assumed one was 1 km at most. The appearance of time-distance curves are not greatly influenced by the errors of the above order as to the position of focus.

Applying the analysis of this kind to the actual earthquake No. 217 or 250, when only the stations with the same direction of initial motion were used in calculation, and or when all stations were used, the focus was well determined, indicating a slight discrepancy of a few hundred metres from the hypothetical one. In both cases, the derived time distance curves can be represented as a straight line with only small probable errors being less than 5/100 sec.

Such large diviations in the travel time that the time-distance curves were separable into two groups cannot be explained from the effects of such a simple model of the horizontal structure. If we interpret this fact merely from the effects of crustal structure, the more complicated structures, as the dipped discontinuity surfaces, many discontinuities in the lateral directions or fault planes need consideration. Even in these cases, a similar systematic deviation on the travel times is expected to be shown for the shocks in similar positions, when the time-distance graphs are constructed under the uniform medium supposition. But no regularity as above described was recognized, and the travel times at any station did not always belong to a fixed group.

From the various reasons already stated, it may safely be said that the cause of large anomalies in travel times is attributable to the effects of focal conditions, such as the mechanism of earthquake occurrence, or the form of the hypocentral region.

5) *On the origin of local shocks*

Logically, it is not until the underground structure can be determined in detail that the problems of the mechanism of earthquake occurrence should be investigated. In the natural earthquakes, in comparison with the artificial explosion, the condition of the hypocentre will, to some extent, influennce the travel times, as well as the properties of the medium of wave propagation. In this sense, the determination of the more detailed structure of crust by means of the artificial explosion is desirable in the district concerned in future. In the present case, however, it is unreasonable, as already discussed, to attribute the travel-time anomalies observed in local shocks in this area to the effect of crustal structure. Consequently, some simple interpretations for the mechanism or condition of earthquake occurrence will tentatively be given.

In the analysis already made, the seismic focus was provisionally regarded as a point source, but, in reality, the hypocentre is considered to have a certain extent named focal region.

On this dimension, Y. Inouye (**28**) considered in relation to his theoretical study on the origin of the earthquake that its diameter is of comparable order to the emitted wave length, if the focal region is assumed to be a sphere. Furthermore, K. E. Bullen (**29**) investigated theoretically the relation between the linear dimension and the times of the first P and S half-oscillation, under the same assumption. According to these studies, the dimension of focus of the shocks here discussed seems to be less than several hundred metres, and not distinguished from the uncertainty of the position of hypocentre in the analysis.

On the other hand, R. Yoshiyama (**30, 31**) is the first who made a practical trial for estimating the size of a hypocentral region from the travel-time analysis. He presumed it from the difference between the apparent origin time at the centre of focus calculated from the time-distance curves of the P-waves and the origin time at the surface of focal region by means of the relation of arrival time–P–S duration. Moreover, S. Takagi (**32**) estimated a dimension of focal region comparing the observed $\triangle$-P–S curve with that calculated from the assumed structure of the crust. Their studies, however, are such that the dimension was estimated by the data at distant stations in considerably large earthquakes, supposing the focal region to be a sphere, and no investigation on such a case of the minor earthquakes as the present one was made.

Provided that the cause of the travel time anomalies in this study is attributed to the dimension of hypocentre, there seems to be no help for it but to consider that the hypocentral region is not a sphere, but an ellipsoid or a line source. Namely, if the elastic waves were simultanaeously emitted from the surface of them, the travel-time difference between two groups as referred to the third paragraph is approximately explainable. Here, the difference between the longer and shorter dimension of focal region seems to be from 1 to 2 km as deduced from the point of the travel-time gap.

B. Gutenberg and C. F. Richter (**33**) tried to estimate a dimension of focus by an entirely different hypothesis from the above ones. That is, supposing that the earthquake occurs with generation of faulting, they pointed out, the existence of the region in which the S-waves emitted with the travel of fracture arrive faster than the first S-waves from the centre of fracture, when the travel speed of fracture is slower than the velocity of the P-waves and faster than that of S-waves in the medium. From such an idea, the linear dimension of fault and the travel speed of fracture were tentatively estimated by the mean travel-time difference of the two types of S-waves at the whole observed area. But S. Takagi denied this postulation from the results of studies on Japanese earthquakes.

If we try to interpret the observed results here studied with the similar supposition in relation to the mechanism of earthquake occurrence, we cannot but consider that the travel speed of fracture or shock wave is faster than that of the P-waves, though it is a question whether this hypothesis is mechanically possible or not. When the fracture travels emitting the elastic waves from a position (a centre of fracture) for some distance along a straight line with the velocity faster than that of the P-waves, the region in which the P-waves propagate with the ordinary velocity and that in which the P-waves propagate with an apparently enlarged velocity effected by the travelling of fracture, are divided into four quadrants by a pair of 'nodal curves' differentiated only by their travel times. The travel-time difference of the first P-waves in two kinds of regions is maximum at any two points on the intersecting straight lines at right angle, dividing these areas into two equal parts. Hence, from the observation of the above travel-time difference and the intersecting angle of 'nodal curves', the linear dimension and travel speed of the fracture is expected to be known.

Inversely speaking, the observed maximum difference of travel times in two groups in the present case seems not to be totally unexplainable, provided the travel speed of fracture can be considered to be twice the order of the longitudinal wave velocity, and the linear dimension to be about 2 km. If the above assumption on the fracture is possible and we are permitted to consider that the 'push-pull' regions in the directions of initial motion agree with two kinds of regions defined by the travel times, it seems possible in some part to interpret our observed results in light of the present observations.

In summary, it is too premature to deduce a definite conclusion from the present observed results, but some possible interpretations were tentatively given.

The anomalous fact of separation of travel times into two groups in the present observation of local shocks in their epicentral region was considered to be intimately related with the nature of hypocentre and mechanism of earthquake occurrence. With a view to clarify the effects on the origin of earthquakes, the more precise seismometric observations of natural earthquakes in their epicentral region with dense nets of high magnification seismometers and high accuracy of time determination are recommended at various places of seismically active areas.

Acknowledgement

The writer wishes to express his cordial thanks to Prof. E. Nishimura for his kind guidance and encouragement throughout the present study. The author also expresses his heart-felt gratitude to Assist. Prof. S. Miyamura, Mr. M. Hori and others, members of the Earthquake Research Institute of Tokyo University, for their generous permission in analysing the data obtained at their observation stations. The writer is greatly indebted to the kind co-operation during these observations by the following members of our institute: Messrs. K. Hosoyama, Y. Kishimoto, A. Kamitsuki, Y. Itô, I. Nakagawa,

Y. Tanaka, M. Ōtsuka, K. Zako and Miss. Y. Okamoto. The writer is also thankful for the kind help during the observation to Messrs. Y. Inouye and Y. Yamamoto of the Wakayama University, and Mr. K. Shinya, Mr. H. Imamura and many other persons in Wakayama District. Finally, the author again expresses his sincere thanks to Mr. M. Ōtsuka for his valuable advice and earnest discussion on this study.

The expense for the present observations was partly defrayed by a grant-in-aid for Scientific Research of the Ministry of Education.

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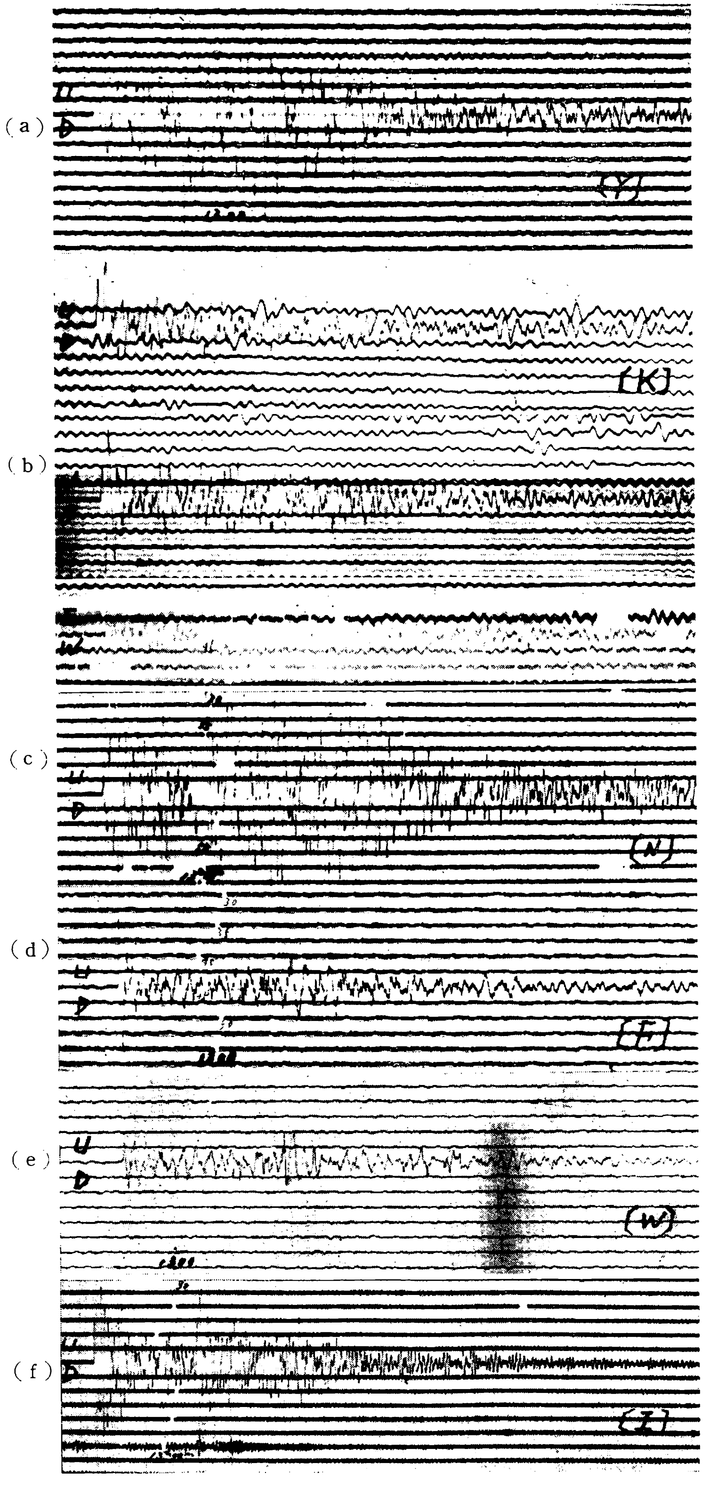


Fig. 11. Examples of seismograms obtained at six stations ; No. 247, Apri 21, 1956, 12h 45m. (a) Yoro, (b) Kainan, (c) Nokami, (d) Fuyuno, (e) Wakanoura, (f) Idakiso.

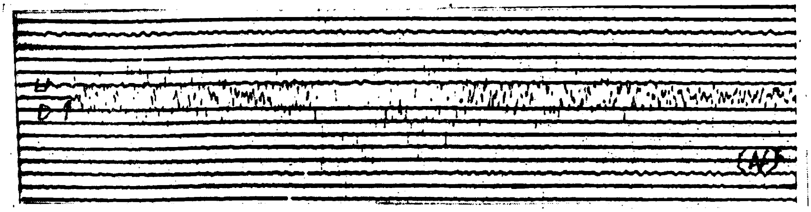


Fig. 12. Seismogram recorded at Nokami-station ; No. 240, April 21, 1956, 19h 26m, near Kyoto.

APPENDIX

Notes on Seismometers of Variable Reluctance Type

The electromechanical transducer of variable reluctance type or changing-flux type has the remarkable feature that the magnetic flux density varies with the change in the length of working air gaps between two magnetic poles due to a pendulum motion, while in the transducer of moving-coil type a coil moves within a constant magnetic field. In the former type a high magnification can be obtained, and it is suitable for measurement of vibrations of short period. The first seismographs with variable reluctance type transducer were designed by H. Benioff (**2**) in 1932. His vertical seismographs and the Bullard's seismometer are representative of transducers of this type.

In the study on the crustal structure from the seismometric observation of local shocks and artificial explosions, the sensitive seismometer without amplifier has gradually been required with a view to investigate, in particular, the seismic waveforms, and therefore variable reluctance seismometers with short period and high magnification were designed and constructed in the present case. The properties of these seismometers designated R_2-type will be described in an appendix in this paper through the theoretical calculations regarding the transducer and the determination of constants by experimental measurements.

1. Electromechanical transducer

The theory of this type of electromechanical transducer has been developed by E.C. Bullard (**5**), T.T. Devlin (**6**) and others since its first study by Benioff, and recently seems to have been completed for the greater part by K. Tajime (**13～18**).

Of this type of transducer, however, there are some limitations to efficiency. For stability of a pendulum motion, the suspension must be controlled to overcome the negative stiffness due to magnetic force with an adequate factor of safety. And it is said (**12**) that the linearity between the displacement of the pendulum and the voltage output does not always remain constant, and the theoretical calculation of various constants of the transducer is more complex than with a moving-coil type transducer.

In the present study, several types of variable reluctance transducers were designed taking the foregoing disadvantages into account. The complete calculations on these types seem not to have been previously made, and hence, the theoretical and experimental considerations on the detailed properties, comparing each with the other, will be made in this paper.

(A)-type

A-1: This type is the "moving armature geophone" designed by S.A. Scherbatskoy and J. Neufeld (**3**). It consists of two parmanent magnets elastically supported by two springs attached to the case, and of an armature placed in the air gap between the magnets and rigidly connected to the case. Two connected magnets move vertically as a suspended mass.

A-2: With this type the armature is connected to a pendulum and moves horizontally with a vertical or horizontal motion of the pendulum in the air gap between two fixed magnets opposite each other.

A-3: The armature moves vertically with a vertical pendulum motion in the air gap between two fixed magnets placed one upon the other.

(B)-type In this new type, two magnets are placed facing each other with an adequate interval, and the armature moves vertically in the air gaps between two poles of the respective magnets, with a vertical motion of the dendulum.

The rough sketches of these types of transducer are shown in Fig. 1.

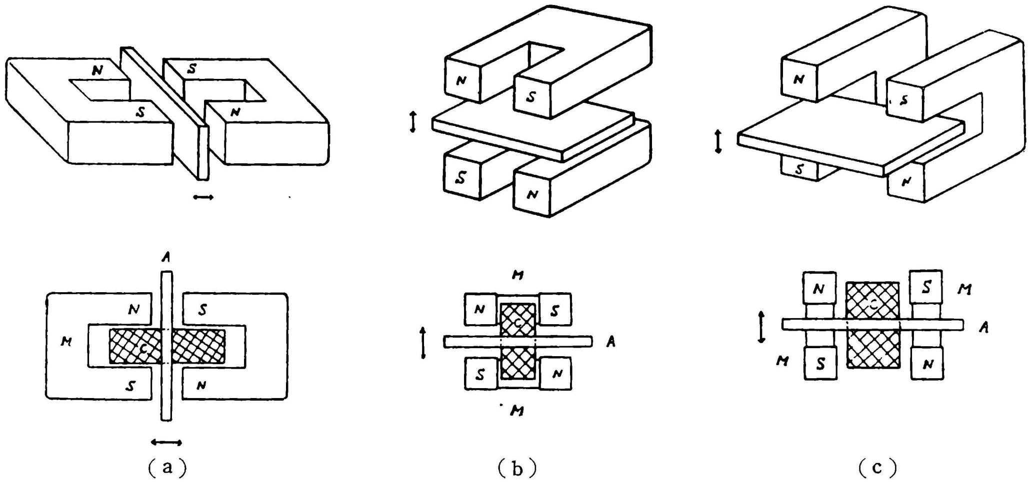


Fig. 1. Sketches of three types of variable-reluctance transducer.
(a) A-1, A-2, (b) A-3, (c) B
M; magnet, A; armature, C; coil.

(1) *Electromotive force and voltage sensitivity*

The displacement of the armature caused by movements of the inertia reactor relative to the fixed system of the seismometer varies the length of working air-gaps and hence reluctance of them, and this in turn varies the magnetic flux through the gaps. The changes in flux thus induce the electromotive force in the coil surrounding the armature and the resulting currents are lead to the circuits.

The variation of flux through the moving armature due to the pendulum motion and the electromotive force induced in the coil were here calculated. In this case the effects such as magnetic hysteresis, eddy currents and viscosity inside, or in the vicinity of the magnetic poles were neglected. The changes in magnetic flux caused by displacement of the reactor are found with a phase lag to the displacement. This is considered to be owing to the eddy currents and other effects (**15**).

Now, let the length of air-gap when the armature is in the position of equilibrium be x_0, let each length of both sides in the case of displacement of x_0 from the position be x_1 and x_2, let the reluctances corresponding to the respective lengths $R_A/2$, $R_1/2$ and $R_2/2$, respectively; and let the cross section of magnet A.

Then, $$R_A = 2x_0/A, \quad R_1 = 2x_1/A, \quad R_2 = 2x_2/A$$

and $$x_1 = x_0 - x, \quad x_2 = x_0 + x \qquad (x_2 > x_1) \tag{2.1}$$

The idealized magnetic circuits on the (A)- and (B)-types of transducers are schematically represented as shown in Fig. 2.

Here, let:

F be the magnetomotive force of each magnet.
R be the reluctance of armature. $R = l/\mu A$
R_M be the reluctance of each magnet. $R_M = L/\mu' A$
R_1, R_2 be the reluctance of each working air gap.
R_S, R_L be the reluctance of leakage circuits.
l be the length of armature.
L be the length of magnet.
μ be the permeability of armature consisting of iron piece.
μ' be the apparent permeability of magnet.

Let the flux through the armature caused by its displacement be ϕ, and ϕ_i be the fluxes respectively indicated in Fig. 2.

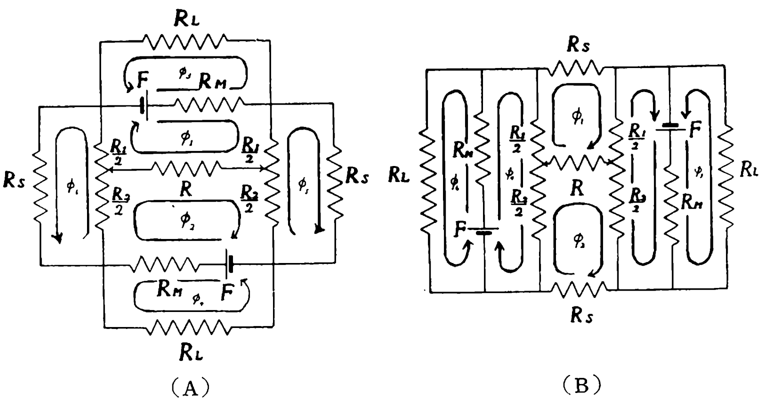


Fig. 2. Schematic representations of the magnetic circuits of variable reluctance transducers.
(A), (B): Magnetic circuits of (A)- and (B)-type respectively.

And then applying the Kirchhoff's law to the above magnetic circuits, we have,

(A)

$$\left.\begin{array}{ll}
\phi=\phi_1-\phi_2 & \text{i)}\\
\phi_1(R_1+R_M+R)-\phi_2R+\phi_3R_M-\phi_5R_1/2+\phi_6R_1/2=F & \text{ii)}\\
\phi_2(R_2+R_M+R)-\phi_1R+\phi_4R_M-\phi_5R_2/2+\phi_6R_2/2=F & \text{iii)}\\
\phi_3(R_M+R_L)+\phi_1R=F & \text{iv)}\\
\phi_4(R_M+R_L)+\phi_2R=F & \text{v)}\\
\phi_5\{(R_1+R_2)/2+R_S\}-\phi_1R_1/2-\phi_2R_2/2=0 & \text{vi)}\\
\phi_6\{(R_1+R_2)/2+R_S\}+\phi_1R_1/2+\phi_2R_2/2=0 & \text{vii)}\\
R_1+R_2=4x_0/A=2R_A & \text{viii)}
\end{array}\right\} \quad (2.2.\text{a})$$

Solving these simultanaeous equations;

$$D\phi=4FR_SR_L(R_A+R_S)(R_M+R_L)(R_2-R_1)$$

where $D=[2(R_1+R+R_M)(R_M+R_L)(R_A+R_S)-2R_M^2(R_A+R_S)-R_1^2(R_M+R_L)]$
$\times[2(R_2+R+R_M)(R_M+R_L)(R_A+R_S)-2R_M^2(R_A+R_S)-R_2^2(R_M+R_L)]$
$-(R_M+R_L)^2[2R(R_A+R_S)+R_1R_2]^2$

Now, let the flux through each magnet when the armature is in equilibrium, ie. when $x=0$, be ϕ_0, that is; $(\phi_1)_{x=0}=(\phi_2)_{x=0}\equiv\phi_0$

we get; $D_0\phi_0=4FR_L(R_A+R_S)^2\{(R_A+2R)(R_M+R_L)+R_MR_L\}$

where $D_0\equiv(D)_{x=0}=4(R_A+R_S)\{(R_A+2R)(R_M+R_L)+R_MR_L\}$
$\times\{R_AR_S(R_M+R_L)+R_MR_L(R_A+R_S)\}$

then; $$\frac{D\phi}{D_0\phi_0}=\frac{R_S(R_M+R_L)(R_2-R_1)}{(R_A+R_S)\{(R_A+2R)(R_M+R_L)+R_MR_L\}}$$

Therefore, the flux ϕ can be written in the form;

$$\left.\begin{array}{l}
\phi=2c_a\dfrac{\phi_0}{x_0}x[1+\alpha_a(x/x_0)^2+\cdots]\\
\text{where}\quad c_a=\dfrac{R_AR_S}{(R_A+R_S)(R_A+2R+R'_M)}<1\\
\alpha_a=\dfrac{R_A{}^2(R_S+R'_M)}{(R_A+2R+R'_M)\{R_AR_S+R'_M(R_A+R_S)\}}<1,\\
\dfrac{1}{R'_M}=\dfrac{1}{R_M}+\dfrac{1}{R_L}
\end{array}\right\} \quad (2.3.\text{a})$$

(B)

$$\left.\begin{array}{ll}
\phi=\phi_2-\phi_1 & \text{i)}\\
\phi_1(R_1+R+R_S)-\phi_2R-\phi_3R_1/2-\phi_4R_1/2=0 & \text{ii)}\\
\phi_2(R_2+R+R_S)-\phi_1R-\phi_3R_2/2-\phi_4R_2/2=0 & \text{iii)}\\
\phi_3\{(R_1+R_2)/2+R_M\}-\phi_1R_1/2-\phi_2R_2/2+\phi_5R_M=F & \text{iv)}\\
\phi_4\{(R_1+R_2)/2+R_M\}-\phi_1R_1/2-\phi_2R_2/2+\phi_6R_M=F & \text{v)}\\
\phi_5(R_M+R_L)+\phi_3R_M=F & \text{vi)}\\
\phi_6(R_M+R_L)+\phi_4R_M=F & \text{vii)}
\end{array}\right\} \quad (2.2.\text{b})$$

Solving the above equations by the same approach in the case of (A)-type,

we have;
$$\phi = 2c_b \frac{\phi_0}{x_0} x[1+\alpha_b(x/x_0)^2+\cdots]$$

where,
$$\left.\begin{aligned}
\phi_0 &= (\phi_3)_{x=0} = (\phi_4)_{x=0} \\
&= \frac{FR_L(R_A+R_S)}{R_AR_S(R_M+R_L)+R_MR_L(R_A+R_S)} \\
c_b &= \frac{R_AR_S}{(R_A+R_S)(R_A+2R+R_S)} < 1 \\
\alpha_b &= \frac{R_A{}^2(R_S+R'_M)}{(R_A+2R+R_S)\{R_AR_S+R'_M(R_A+R_S)\}} < 1
\end{aligned}\right\} \quad (2.3.\mathrm{b})$$

In both cases, ϕ_0, c and α are related with the gap width x_0, but are the constants having nothing to do with the displacement x, so far as the gap width is constant.

The variation of ϕ with gap-width change can be found from $c\phi_0/x_0$.

$$\begin{aligned}
c\phi_0/x_0 &= \frac{2F}{A}\cdot\frac{R_SR'_M/R_M}{(R_A+2R+R'_M)[R_AR_S+R'_M(R_A+R_S)]} \quad (A) \\
&= \frac{2F}{A}\cdot\frac{R_SR'_M/R_M}{(R_A+2R+R_S)[R_AR_S+R'_M(R_A+R_S)]} \quad (B)
\end{aligned} \qquad (2.3')$$

If the reluctance R_A of air gaps are much larger than that of circuits, it is in nearly inverse proportion to the square of x_0. That is, the obtained flux increases rapidly with decreasing x_0, but there is a limitation on decreasing x_0. It is by reason that the pendulum motion becomes unstable owing to the increasing of magnetic stiffness as subsequently mentioned, and the linearity of flux in relation to x results unfavourably due to the increase of x/x_0. In the (B)-type ϕ has a maximum value when $R_S{}^2 = \frac{R_AR'_M(R_A+2R)}{R_A+R'_M}$, which are found out by a reluctance matching. Namely the maximum value of ϕ can be obtained by an adequate adjustment of the interval between two magnets.

The flux through the armature, in both cases, can formally be represented as follows:

$$\phi = 2c\frac{\phi_0}{x_0}x[1+(x/x_0)^2+\cdots] \qquad (2.3)$$

when the phase lag to displacement is omitted.

Hence, the electromotive force or open circuit voltage induced in the coil is,

$$\begin{aligned}
E_0 &= -N\frac{d\phi}{dt} = -N(d\phi/dx)(dx/dt) \\
&= -2cN\frac{\phi_0}{x_0}[1+3\alpha(x/x_0)^2+\cdots]\frac{dx}{dt}
\end{aligned} \qquad (2.4)$$

where N is the number of turns of the coil.

If $x/x_0 < 1/10$, the second and its following terms in bracket in the above equation can be disregarded, because $\alpha < 1$, and we get;

$$\phi = 2c\phi_0 x/x_0, \quad E_0 = -Gx, \quad G = 2cN\phi_0/x_0 \tag{2.5}$$

where G is designated the voltage sensitivity.

In the types of transducer designed here, it was also ascertained that the induced flux is approximately proportional to the armature's displacement, and the electromotive force to the velocity of the armature's motion.

(2) *Secondarily induced magnetic flux*

When the electric current i is induced into closed circuits including the coil due to the armature's displacement, a secondary magnetic flux is caused by this induction current in the sense to prevent the motion of armature. This flux has a phase lag to the current.

The counter-magnetomotive force is expressed as $F' = 4\pi N\ i$. Provisionally separating the total flux Φ induced by the displacement of armature into two parts; ϕ directly due to the displacement, and ϕ' due to the induction current, and then Φ can be written; $\Phi = \phi + \phi' = \Phi_1 - \Phi_2$. When the magnetomotive force F' is induced into the circuit R indicated in Fig. 2(A), ϕ_1' streams in the same sense with ϕ_1, and ϕ_2' in the opposite one with ϕ_2, because $R_1 < R_2$ and $\phi_1 > \phi_2$.

Therefore,

$$\left.\begin{aligned} &\Phi_1 = \phi_1 + \phi_1', \quad \Phi_2 = \phi_2 - \phi_2' \\ &\Phi = \Phi_1 - \Phi_2 = (\phi_1 - \phi_2) + (\phi_1' + \phi_2') = \phi + \phi' \\ &\phi' = \phi_1' + \phi_2' \end{aligned}\right\} \tag{2.6}$$

ϕ' can be calculated from the ensuing equations. A dash in the shoulder denotes the quantities relating to the induction current.

(A)

$$\left.\begin{aligned} &\phi' = \phi_1' + \phi_2' && \text{i)} \\ &\phi_1'(R_1 + R_M + R) + \phi_2' R + \phi_3' R_M - \phi_5' R_1/2 + \phi_6' R_1/2 = F' && \text{ii)} \\ &\phi_2'(R_2 + R_M + R) + \phi_1' R - \phi_4' R_M + \phi_5' R_2/2 - \phi_6' R_2/2 = F' && \text{iii)} \\ &\phi_3'(R_M + R_L) + \phi_1' R_M = 0 && \text{iv)} \\ &\phi_4'(R_M + R_L) - \phi_2' R_M = 0 && \text{v)} \\ &\phi_5'\{(R_1 + R_2)/2 + R_S\} - \phi_1' R_1/2 + \phi_2' R_2/2 = 0 && \text{vi)} \\ &\phi_6'\{(R_1 + R_2)/2 + R_S\} + \phi_1' R_1/2 - \phi_2' R_2/2 = 0 && \text{vii)} \end{aligned}\right\} \tag{2.6.a}$$

From these equations,

$$\left.\begin{aligned} &\phi' = \frac{8\pi N i}{R_A + 2R + R'_M}[1 + \beta_a (x/x_0)^2 + \cdots] \\ &\text{where} \quad \beta_a = \frac{R_A{}^2(R_S + R'_M)}{(R_A + 2R + R'_M)\{R_A R_S + R'_M(R_A + R_S)\}} = \alpha_a < 1 \end{aligned}\right\} \tag{2.7.a}$$

(B) In this case, $\phi_2 > \phi_1$ when $R_1 < R_2$, and then

$$\left.\begin{aligned} &\phi' = \phi_1' + \phi_2' && \text{i)} \\ &\phi_1'(R_1 + R + R_S) + \phi_2' R + \phi_3' R_1/2 + \phi_4' R_1/2 = F' && \text{ii)} \\ &\phi_2'(R_2 + R + R_S) + \phi_1' R - \phi_3' R_2/2 - \phi_4' R_2/2 = F' && \text{iii)} \\ &\phi_3'\{(R_1 + R_2)/2 + R'_M\} + \phi_1' R_1/2 - \phi_2' R_2/2 = 0 && \text{iv)} \\ &\phi_4'\{(R_1 + R_2)/2 + R'_M\} + \phi_1' R_1/2 - \phi_2' R_2/2 = 0 && \text{v)} \end{aligned}\right\} \tag{2.6.b}$$

$$\left.\begin{aligned} 1/R'_M &= 1/R_M + 1/R_L \\ \phi' &= \frac{8\pi Ni}{R_A+2R+R_S}[1+\beta_b(x/x_0)^2+\cdots] \\ \text{where} \quad \beta_b &= \alpha_b < 1 \end{aligned}\right\} \tag{2.7}$$

Hence, the secondarily induced flux ϕ' caused by the induction current can be expressed in the following form, in both cases of (A) and (B),

$$\left.\begin{aligned} \phi' &= \frac{F'}{R_0'}[1+\alpha(x/x_0)^2+\cdots] \\ \text{where} \quad F' &= 4\pi Ni, \quad R_0' = \frac{1}{2}(R_A+2R+R'_M) \text{ (A)}, \quad R_0' = \frac{1}{2}(R_A+2R+R_S) \text{ (B)}, \end{aligned}\right\} \tag{2.7}$$

The induced electromotive force E is expressed as follows, neglecting the higher order terms following $(x/x_0)^2$ in comparison with unity and omitting the phase lags.

$$\begin{aligned} E &= -N(d\Phi/dt) = -N(d\phi/dx)(dx/dt) - N(d\phi'/dt) \\ &= -Gx - \frac{4\pi N^2}{R_0'}\frac{di}{dt} \end{aligned}$$

where $4\pi N^2/R_0' \equiv L$ is equivalent to the self-inductance of coil.
Hence, Φ and E can be written,

$$\left.\begin{aligned} \Phi &= G\,x/N + L\,i/N \\ E &= -G\,x - L\,di/dt \end{aligned}\right\} \tag{2.8}$$

(3) *Magnetic stiffness*

In the variable reluctance type transducer, the restitutive force of the moving system must overcome the negative stiffness due to magnetic force, because the magnetic force pulls the armature to an unbalanced position. The magnetic force acting between two poles at intervals in the air gap is expressed in the following formula in general, provided that the flux density B through the air gap is uniform, $f = B^2A/8\pi$, where A is the cross section of the air gap.

In our case, the armature is placed between two magnets with an equal length of air gap on both sides, and therefore, the magnetic pulls from both sides are balanced so far as the armature does not displace from the position of equilibrium.

When the armature moves x from the original position, the magnetic force is

$$f = f_1 - f_2 = \frac{\Phi_1^2 - \Phi_2^2}{4\pi A} \tag{2.9}$$

And then it becomes, taking the relations (2.6) into consideration, $f = \frac{[(\phi_1+\phi_2)+(\phi_1'-\phi_2')]\,\Phi}{4\pi A}$. Substituting the calculated values from the afore mentioned circuits for $(\phi_1+\phi_2)$, $(\phi_1'-\phi_2')$ and Φ, assuming that

$$R'_M\left(\frac{1}{R_A}+\frac{1}{R_S}\right)\ll 1,\quad \frac{x/2\phi_0 x_0}{R_A R_S+R'_M(R_A+R_S)}\ll 1,\quad \frac{R'_M/2+R}{R_A}\ll 1 \quad \text{and} \quad (x/x_0)^2\ll 1,$$

it becomes

$$f=\frac{c^2\phi_0^2}{\pi x_0 A}x+\frac{2cN\phi_0}{x_0}i=s''x+Gi$$

where

$$s''=c^2\phi_0^2/\pi x_0 A \tag{2.10}$$

The first term represents the negative stiffness which results from the distribution of the flux in the air gaps even when no current is flowing in the seismometer's coil and lengthens the natural period of the pendulum. The second term represents the force resulting from the change in the flux due to the current. It acts as an electro-magnetic damping.

2. Seismometers

The electromagnetic seismometers designed here have a variable reluctance type transducer. The armature connected with the pendulum moves in the air gap between two magnets with vertical or horizontal surface movements. The vertical and horizontal seismometers were constructed so as to be convertible from one to the other.

In the following discussions the under-mentioned notations were used.

- M: the mass of the moving system in the seismometer.
- K: the moment of inertia of pendulum about the rotation axis.
- H: the distance between the centre of gravity of the pendulum and its rotation axis.
- L_1: the distance between the centre of the armature and the rotation axis.
- L_2: the distance between the supporting point of the helical spring and the rotation axis.
- l: the equivalent length of the pendulum.
- c: the restitutive force of the leaf spring for a unit rotation angle.
- f: the magnetic stiffness.
- D: the coefficient of the fluid friction.
- $\bar{R}$: the total resistance of the circuit.
- L: the self-inductance of the seismometer's coil.
- θ: the angular displacement.
- x: the displacement of the armature's centre from a position of equilibrium.
- ξ: the displacement of the ground surface.

(1) *Horizontal seismometer*

This consists of a vertical pendulum with the inertia mass and the armature, supported by two sets of crossed leaf springs in the upper part. Fig. 3 shows a horizontal seismometer. The equation of motion of the horizontal motion pendulum treated here is,

$$K\ddot{\theta} = -D\dot{\theta} - c\theta - MgH\sin\theta - L_1 f - MH\ddot{\xi}$$

Provided θ is not too great, $\sin\theta \approx \theta$ and $x \approx L_1\theta$

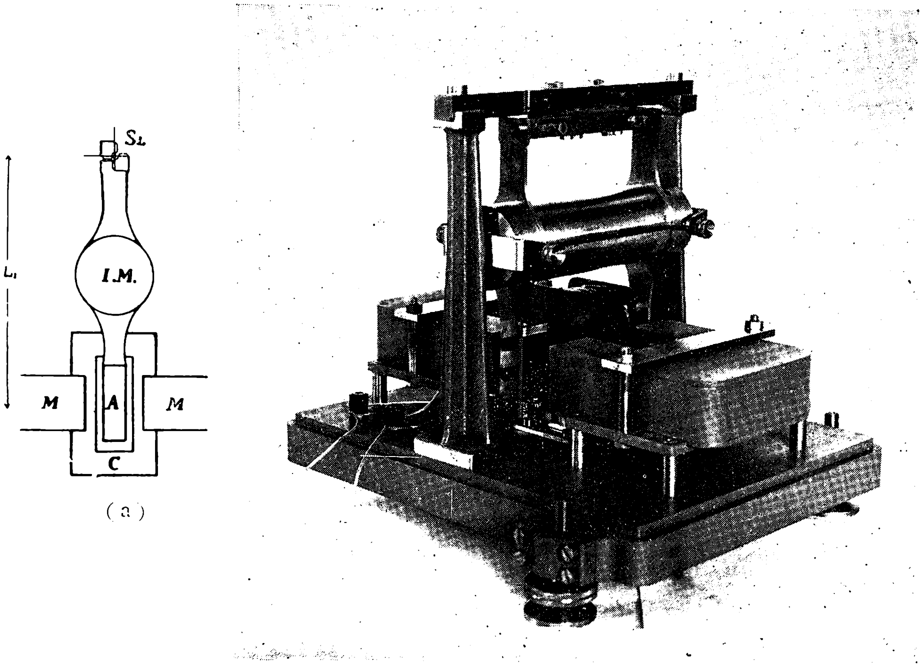
Hence, the equation about the armature's displacement x is as follows, considering $f = -s''x + Gi$,

$$K\ddot{x} + D\dot{x} + (c + MgH - L_1^2 s'')x = -ML_1H\ddot{\xi} - L_1^2 Gi \qquad (3.1)$$

The natural period without damping in an open circuit is,

$$T_H = 2\pi\sqrt{K/(c + MgH - L_1^2 s'')} \qquad (3.2)$$

where $c/2 = Eab^3/9r$, E is the Young's modulus of the leaf spring, r the length, a the width and b the thickness.



(a)

(b)

Fig. 3. (a), (b) Horizontal seismometer of A-2 (R_2)-type.
I.M.; inertia mass, *M*; magnet, *A*; armature, *C*; coil, S_L; leaf springs, S_H; helical spring.

(2) *Vertical seismometers*

i) A-1: Two connected magnets supported by a helical spring and two leaf springs move themselves as a suspended mass. The armature is fixed to the case. (Fig. 6)

ii) A-2: The inertia mass is supported by a close-coiled helical spring, and two sets of crossed leaf springs, and the mass and the armature being rigidly connected with a L-shaped frame form the pendulum. The armature moves horizontally with a vertical motion of mass. In this type, however, the seismometer was constructed so that the moment of inertia of the armature about the rotation axis may be far smaller than that of the mass. (Fig. 4)

iii) A-3: The inertia mass and the armature are connected with a frame corresponding the horizontal seismometer, and supported by a helical spring and crossed springs. (Fig. 5)

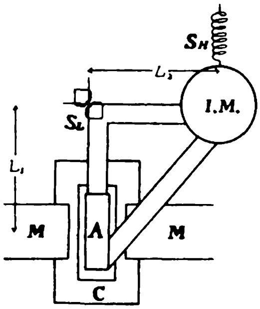


Fig. 4 (a)

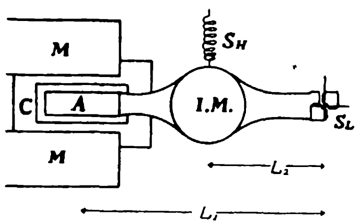


Fig. 5

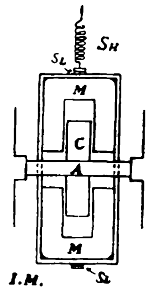


Fig. 6

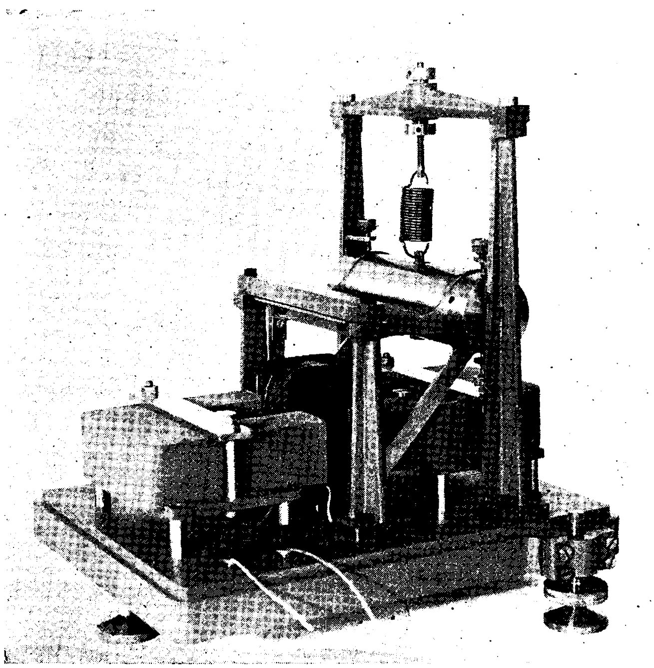


Fig. 4 (b)

Fig. 4. (a), (b) Vertical seismometer of A-2 (R_1)-type.
Fig. 5. Vertical seismometer of A-3 (R_2)-type.
Fig. 6. Vertical seismometer of A-1 type.

The equation of motion of all types of vertical seismometers mentioned here can be expressed in the ensuing symbol, provided that θ is negligibly small,

$$K\ddot{\theta}+D\dot{\theta}+(c+L_2^2k)\,\theta=-MH\ddot{\xi}-L_1f$$

or

$$K\ddot{x}+D\dot{x}+(c+L_2^2k-L_1^2s'')\,x=-ML_1H\ddot{\xi}+L_1^2Gi \tag{3.3}$$

The free period is, $$T_V=2\pi\sqrt{K/(c+L_2^2k-L_1^2s'')} \tag{3.4}$$

where k is the spring constant of the helical spring or the restitutive force for unit elongation and can be written as follows from the balance condition in equilibrium, $k=MgH/L_2\Delta l$. Then, the equation of motion both on the vertical and horizontal seismometers of the variable reluctance type designed here can be written in the general form:

$$K\ddot{x}+D\dot{x}+(S-S'')\,x=-ML_1H\ddot{\xi}-L_1^2Gi$$

or

$$\ddot{x}+2\varepsilon_f\dot{x}+n^2x=L_1^2Gi/K-L_1\ddot{\xi}/l \tag{3.5}$$

where

$$\left.\begin{aligned} &2\varepsilon_f=D/K,\quad n^2=(S-S'')/K,\quad l=K/MH\\ &S=c+L_2^2k\ (\text{Vert.}),\quad S=c+MgH\ (\text{Hor.})\\ &S''=L_1^2s'',\quad s''=c^2\phi_0^2/\pi x_0A \end{aligned}\right\} \tag{3.6}$$

The free period is related with the negative magnetic stiffness. It becomes longer than in the case without a magnet. And the period varies with the change in length of the air gaps on account of the close relation between the negative stiffness and the gap width.

The various constants on our seismometers are shown in Table 1.

Table 1. Constants of seismometers.

	H	V	
	A-2 (R_2)	A-2 (R_1)	A-3 (R_2)
M	3.66×10^3 gr.	3.68×10^3 gr.	3.68×10^3 gr.
K	3.51×10^5 c.g.s.	2.49×10^5 c.g.s.	6.12×10^5 c.g.c.
H	8.81 cm	6.20 cm	12.0 cm
l	10.8 cm	9.18 cm	13.8 cm
L_1	15.5 cm	7.90 cm	19.3 cm
L_2	—	8.40 cm	10.7 cm
c	3.96×10^8 c.g.s.	1.17×10^8 c.g.s.	3.96×10^8 c.g.s.
k	—	2.76×10^5 c.g.s.	2.76×10^5 c.g.s.
T	0.447 sec.	0.455 sec.	0.45 sec.
(X_0)	15.0 mm	14.0 mm	7.0 mm

(3) *Electromagnetic damping and the effect of self-inductance*

In the seismometers designed here the damping relies only upon an electromagnetic one. Matsuzawa (**10**) and Tajime (**14**) had already studied the effects of self-inductance on the electromagnetic damping, and Sparks, Hawleg (**8**) and Silverman (**9**) investigated them especially in conjunction with the reluctance type seismometer.

In this paper the more detailed study of the conditions for critical damping in free oscillation of reluctance seismometers will be made, taking the effects of self-inductance into consideration.

When the effects of self-inductance are taken into account, the voltage difference $\bar{E}$ in both terminals of the seismometer's coil induced by the motion of pendulum is; $\bar{E}=G\dot{x}+Ldi/dt+i\bar{R}$. In a closed circuit, $\bar{E}=0$, and hence $-G\dot{x}=Ldi/dt+i\bar{R}$. Now combining the preceding expression on $\dot{x}$ with the equation (3.6), we obtain the linear differential equations of third order with respect to x. That is,

$$\dddot{x}+(2\mathcal{E}_f+\nu)\ddot{x}+(2\nu\mathcal{E}_e+2\nu\mathcal{E}_f+n^2)\dot{x}+\nu n^2x=-\nu L_1\ddot{\xi}/l$$

where $$1/\nu=L/\bar{R},\quad 2\mathcal{E}_e=L_1^2G^2/K\bar{R} \tag{3.7}$$

This is equivalent to the equation derived by Washburn (4) and Matsuzawa. When the damping depends only upon the electromagnetic one, the above equation in free oscillation becomes,

$$\dddot{x}+\nu\ddot{x}+(2\nu\mathcal{E}_e+n^2)+\dot{x}\nu n^2x=0 \tag{3.8}$$

According to Sparks' method (8), assuming the general solution of equation (3.8) to be the form $x=x_0e^{pt}$, we get the third order equation of p. This equation at least has a real root. Let the other two roots of conjugate and complex be $p=h\pm ik$, and the two equations $f(h, k)=0$, and $g(h, k)=0$ are obtained. With a view to find out the values of h and k to realize the state of maximum damping by adjustment of the terminal resistance, the following procedures are pursued. Eliminating k from the above two equations, we obtain $F(h)=0$, and the condition for maximum damping is $dF(h)/d\nu=0$. From these two equations we get the values to give a maximum damping, that is,

$$\left.\begin{aligned} h_m &= -\tau^2/2n \\ k_m &= n[1+(\tau^2/n^2)-(\tau^2/2n^2)^2]^{1/2} \\ \nu_m &= \bar{R}_m/L=(n^2+\tau^2)/n \\ \text{where}\quad \tau^2 &= \nu\varepsilon_e \end{aligned}\right\} \tag{3.9}$$

For critical damping which is greater than a maximum one, the motion must be non-oscillatory, ie. $k_m=0$, that is, $1+(\tau^2/n^2)-(\tau^2/2n^2)^2=0$

therefore, $$\tau^2/n^2=2(\sqrt{2}+1)\equiv C_{\tau n} \tag{3.10}$$

While, from the following formula, the equation (3.11) is obtained;

that is, $G=2cN\phi_0/x_0$, $L=4\pi N^2/R_0$, $R_0\approx x_0/A$ and $s''=c^2\phi_0^2/\pi x_0A$

therefore, $$G^2=Ls'' \quad\text{and}\quad L_1G^2=LS'' \tag{3.11}$$

Hence, $\tau^2=S''/2K$, $n^2=(S-S'')/K$, and we get,

$$S''/S=4(3-\sqrt{2})/7=0.906\equiv C_s \tag{3.10'}$$

That is to say, Sparks and others suggested that damping depends on the ratio S''/S and the coefficient of negative stiffness must be greater than 91% of the constant of the total restitutive force before the reluctance seismometer can be critically damped only owing to electromagnetic damping (**8**).

However, the condition for critical damping suggested by them is not complete, because the relation between the resistance and self-inductance was omitted from consideration. This relation can be expressed in the ensuing form by means of the constants $C_{\tau n}$ or C_s in critical damping.

$$\nu_m = \bar{R}_m/L = (1+C_{\tau n})n = (2-C_s)n/2(1-C_s) \qquad (3.12)$$

because $\tau^2 = \nu\mathcal{E}_e = \nu_m n - n^2 = C_{\tau n} n^2$.

Substituting the values indicated by (3.10) or (3.11) for the above expression,

$$\bar{R}_m/nL = 3+2\sqrt{2} \qquad (3.13)$$

This is one of the conditions for critical damping.
Namely, in order to obtain critical damping, total resistance in the circuit must be adjusted to satisfy the equation (3.13) for the known value of self-inductance, and S''/S must be selected so as to be $0.91 < S''/S < 1$. The condition derived by the above authors is one of the necessary conditions for critical damping, but not complete by itself.

In addition, when S''/S tends to unity in (3.12), $\bar{R}_m$ tends to increase to infinitive, that is, it has no objective however much one may select the values of resistance for critical damping. In this case, however, the pendulum motion becomes unstable and the armature finally sticks to the magnet. S''/S is required to be as small as possible only for stability sake.

Moreover, the damping factor can be written with the constants $C_{\tau n}$ or C_s,

$$\mathcal{E}_e/n = C_{\tau n}/(1+C_{\tau n}) = C_s/(2-C_s) \qquad (3.14)$$

Substituting $C_{\tau n} = 2(\sqrt{2}+1)$ for the above expression, we get,

$$(\mathcal{E}_e/n)_m = 2(\sqrt{2}-1) = 0.83 < 1 \qquad (3.15)$$

$\mathcal{E}_e/n$ is formally equivalent to the damping factor h in the case of no self-inductance. The condition for critical damping is, of course, $h=1$ when the effect of self-inductance is not taken into account, and so it can be said that the self-inductance increases the damping. That is, it acts favourably to damping in the range $0 < nL/\bar{R} < 3-2\sqrt{2}$, but when this condition does not hold even by adjustment of $\bar{R}$, owing to the largeness of L, it acts disadvantageously. This result suggests the same sense with that derived by Matsuzawa (**10**).

The general effect of self-inductance on the electromagnetic damping in the case of external forced-vibrations were investigated in detail by K. Tajime (**14**).
According to his study, the damping factor with self-inductance can be expressed,

$$h_L = h/[(1+\alpha^2)\sqrt{1-\{2\lambda/(1+\alpha^2)\}}] \tag{3.16}$$

where $\lambda = \mathcal{E}_e/\nu = hnL/\bar{R}$, and $\alpha = \omega/\nu = \omega L/\bar{R}$

ω is the angular frequency of external force.

The above formula indicates that the larger the natural frequency of the seismometer's pendulum in comparison with the external one is, the larger the effect of self-inductance on the admping. In the case of no external force or free oscillation, $\alpha = 0$, and then, $(h_L)_{\alpha=0} = h/\sqrt{1-2\lambda}$. Especially in the critical damping, $h_L = 1$. Combining the condition for critical damping $nL/\bar{R} = 3-2\sqrt{2}$ and the above expression, h agrees with the values shown in (3.15). In our seismometers, $L = 1.62$ henries, $\bar{R} =$ 350 ohms, $n = 14$, $\omega < 60$, and hence $\alpha = 0.25 < 1$, $\lambda = 0.06 < 1$, impedance $Z = \bar{R} + j\omega L \approx \bar{R}$, $0 < nL/\bar{R} = 0.06 < 3-2\sqrt{2}$. From the above conditions, S''/S must be larger than 0.97 in order that our seismometer may be critically damped only by electromagnetic damping. As mentioned above, this is difficult to realize in respect to stability.

The equation of motion is as follows, when $1/\nu = L/\bar{R} = 0$, that is, the self-inductance may be ignored when compared with resistance.

$$\ddot{x} + 2\mathcal{E}_f\dot{x} + n^2x = L_1^2Gi/K - L_1\ddot{\xi}/l \quad \text{and} \quad i = E/\bar{R} = -G\dot{x}/\bar{R}$$

or
$$\ddot{x} + 2\mathcal{E}_e\dot{x} + n^2x = -L_1\ddot{\xi}/l$$

where
$$2\mathcal{E} = 2\mathcal{E}_f + 2\mathcal{E}_e, \quad 2\mathcal{E}_f = D/K, \quad 2\mathcal{E}_e = L_1^2G^2/K\bar{R} \tag{3.18}$$

If another damper is equipped, $2\mathcal{E} = 2(\mathcal{E}_f + \mathcal{E}_e + \mathcal{E}')$.

It is usual for the purpose of increasing the electromagnetic damping to equip a shunt-coil. In the present case it is wound around the outside of an ordinary coil.

When the armature moves, the electromotive force $E' = -G'\dot{x}$, $(G' = 2cN'\phi_0/x_0)$ is induced and the current $i' = E'/R'$ streams in the shunt-coil. The counter-magnetomotive force $F' = 4\pi N'i'$ and hence the secondary flux are caused by this induction current. Therefore, the magnetic stiffness increases to $f' = s''x + Gi + G'i'$. Hence, the new electromagnetic damping whose factor is expressed with $2\mathcal{E}' = L_1^2G'^2/KR'$ is added. The method for using the conductive bobbin with coil for increasing the damping is the same in its principle as that of the shunt-coil. The vertical seismometer of A-1 type is equipped with a copper bobbin. But this method is not so good on account of many reasons as pointed out by Tajime (**14**).

3. Experimental determination of constants

In this paragraph, the voltage sensitivity will be determined from an experimental measurement and the linearity between the voltage output and the displacement will be examined. And the variations of the voltage sensitivity, period, magnetic stiffness, and damping factor with the changes in the length of air gap are measured, and the magnification was calculated here.

(1) *Determination of voltage sensitivity*

The voltage sensitivity was determined from the damping factor h by means of galvanometric registration of free oscillation of the seismometer's pendulum.

The damping factor can be written,

$$h = \frac{\varepsilon}{n} = \frac{T_0}{2\pi}\cdot\frac{1}{2K}\left(D+f_d+\frac{L_1^2G^2}{\bar{R}}\right) \tag{4.1}$$

where $2\varepsilon' = f_d/K$, and f_d is the coefficient of damping force other than the electromagnetic damping. As D is usually far smaller than the coefficient of electromagnetic damping, G can immediately be obtained from h on the record of free oscillation, omitting D, if there is no damping force besides the electromagnetic one (**13**).

Otherwise, the damping factor must be measured in each case of varied values of the external resistance r_i. That is,

$$h_i = \frac{T_0}{2\pi}\cdot\frac{1}{2K}\left(D+f_d+\frac{L_1^2G^2}{\bar{R}_i}\right),\quad \bar{R}_i = R+\frac{rS_i}{r+S_i}+r_i$$

Hence,

$$\left.\begin{aligned} G &= \frac{2}{L_1}\sqrt{\frac{\pi K}{T_0}\cdot\frac{R_1R_2}{R_2-R_1}(h_1-h_2)} \\ D+f_d &= \frac{4\pi K}{T_0}\cdot\frac{R_1h_2-R_2h_1}{R_1-R_2} \end{aligned}\right\} \tag{4.2}$$

G can also be obtained from the following relationship. Combining the equation (3.13), $L_1G^2 = LS''$, and the free period $T_0 = 2\pi\sqrt{K/(S-S'')}$, we get, $G=\sqrt{\frac{L}{L_1}\left(S+\frac{4\pi^2K}{T_0^2}\right)}$.
That is, G can be calculated by the value of S'' from measurement of the natural period.

From the experimental measurements in the present case, G was calculated as follows;

$$G = \begin{matrix}1.71\\1.79\end{matrix}\times 10^8 \text{ e.m.u. when } \begin{matrix}x_0 = 13.0\text{ mm (Vert.)}\\ x_0 = 15.0\text{ mm (Hor.)}\end{matrix}$$

The generalized voltage sensitivity is defined as $\bar{G} = GL_1/l$. The value of $\bar{G}$ of horizontal seismometer is 2.57 volt/kine. Comparing with the theory described in the first section, the value of c was estimated to be 0.342 at $x_0 = 15.0$ mm, because $N=$ 4,000 turns and $\phi_0 = 115{,}000$ maxwells. And for $D+f_d$, it was found $D+f_d = 1.18\times 10^6 < \frac{L_1^2G^2}{\bar{R}} = 1.74\times10^7$. The coefficient f_d of damping force due to a supplementary damper (copper plate) takes charge of about 10% of total damping. Moreover, it is found from the equation (4.1) that $\bar{R}$ must be adjusted to be 240 ohms in vertical and 700 ohms in horizontal seismometer, in order to realize the critical damping.

In these types of variable reluctance seismometers the voltage output is proportional to the velocity of the pendulum's motion, if the oscillation amplitudes are not much larger than the length of the air gap, consequently expressed, $(x/x_0)^2 \ll 1$. Whether or not this linearity holds even in larger amplitudes was examined by the ensuing method. The equation of motion on the seismometer's pendulum in free oscillation is expressed in the following form, if the coupling motion with the galvanometer's coil can be ignored.

$$\ddot{x}+2\mathcal{E}_f x+n^2\dot{x} = L_1^2 Gi/K \qquad \text{i)}$$

or

$$\ddot{x}+2\mathcal{E}x+n^2\dot{x} = 0 \qquad \text{ii)}$$

When a displacement x_1 is given to the pendulum by sending a weak current i_1 to the seismometer's coil, x is estimated from the above equation i),

$$n^2 x_1 = L_1^2 G i_1/K \tag{4.3}$$

Then, making the pendulum free oscillation by cutting off the current, the motion can be expressed in the formula derived from solving the equation ii) with the initial conditions: $x = x_1$ and $\dot{x} = 0$, when $t = 0$,

$$x = x_1 e^{-\varepsilon t}\left(\cos \gamma t + \frac{\mathcal{E}}{\gamma} \sin \gamma t\right), \quad \text{where} \quad \gamma^2 = n^2 - \mathcal{E}^2 \tag{4.4}$$

Hence, the equation of motion on a galvanometer's coil is,

$$\ddot{\varphi}+2\mathcal{E}_g\dot{\varphi}+n_g^2\varphi = \beta x = -(\beta \dot{x}_1 n^2 e^{-\varepsilon t}/\gamma) \sin \gamma t \tag{4.5}$$

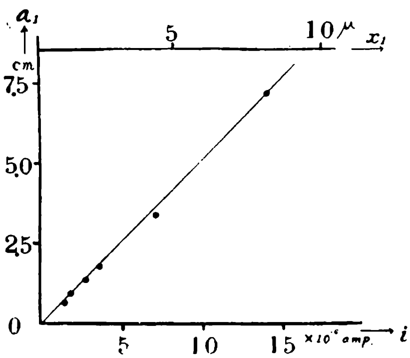


Fig. 7. Linearity between displacement and voltage output.

a_1 ; initial maximum amplitude on record,

x_1 ; initial given displacement of armature,

i_1 ; initial current to give the displacement x_1.

where β is the transmission constant or "die Übertragungsfaktor".

The particular solution of (4.5) is as follows when $\mathcal{E}_g = n_g$, neglecting the free oscillation of the galvanometer's coil by means of strong damping.

$$\left.\begin{aligned} \varphi &= \frac{-\beta x_1 n^2/\gamma}{\sqrt{(\mathcal{E}-n^g)^2+\gamma^2}} e^{-\varepsilon t} \sin(\gamma t-\delta), \\ \text{where} \quad \tan\delta &= \frac{2(\mathcal{E}-n_g)\gamma}{\gamma^2-(\mathcal{E}-n_g)^2} \end{aligned}\right\} \tag{4.6}$$

The deflection of the galvanometer's coil is proportional to the initial given displacement of the seismometer's pendulum. Let the first maximum amplitude on the record be a_1, and then a_1 can be estimated from the equation

(4.6) and from the condition for its maximum or $d\varphi/dt=0$. Substituting the equation for x_1 in the modified equation of (4.6), we get,

$$a_1 = 2A\varphi = \frac{2\beta AL_1^2Gi/K}{n\sqrt{(\mathcal{E}-n_g)^2+\gamma^2}} \exp\left[-\frac{\mathcal{E}}{\gamma}\left\{\tan^{-1}\left(\frac{\gamma}{\mathcal{E}}\right)+\delta\right\}\right] \tag{4.7}$$

That is, a_1 is found to be proportional to i_1, and therefore, the linearity can be examined when a_1 is plotted against various values of i_1 and against x/x_0. The relation between a_1 and i_1 or x_1 is shown in Fig. 7. Generally speaking, the linearity holds good in the range of $x<10\mu$.

(2) *Variation of voltage sensitivity, period, magnetic stiffness, and damping factor with the change in the gap width*

As described from the afore-mentioned theoretical calculation,

$$G=2cN\phi_0/x_0,\quad T=2\pi\sqrt{K/(S-S'')},\quad h\approx L_1^2G^2T_0/4\pi K\bar{R} \quad \text{and} \quad S''=L_1^2c^2\phi_0^2/\pi x_0A \tag{4.8}$$

All of these factors vary with the change in the gap width. C also changes with the change of the air gap because it is a function of x_0, but has no relation to displacement. It is an advantageous in these types of variable reluctance seismometers that the above factors can be adjusted by variation of the gap width, but unfavourable because they are not independent of each other. From the records of free oscillation of the seismometer's pendulum, the damping factor $h(x_0)$, and the natural period without damping $T(x_0)$ can easily be obtained. The coefficient of negative stiffness $S(x_0)$ is then calculated after the spring constant S was known from the period T in the case of $x=\infty$. The voltage sensitivity $G(x_0)$ can be estimated from $h(x_0)$ and $T(x_0)$ by the above described method. The general appearance of variation of G with the gap-width change will also be learned from the recorded initial amplitudes in free oscillation of pendulum. These factors were plotted against the length of air-gap as shown in Fig. 8.

The voltage sensitivity increases, theoretically, in inverse proportion to the square of the gap width and the negative stiffness in inverse proportion to its cube, because the factor c has an order of $1/x_0^{\frac{2}{3}}$ provided that the reluctance of the air gap is much larger than that of other magnetic circuits. The measured values of these factor in experiment shows a comparatively good accordance with this theory. For comparison with the theory of electromagnetic damping and self-inductance, the relationship between h and S''/S was supplementarily investigated.

(3) *Magnification*

The equations of motion of the seismometer's pendulum and the galvanometer's coil, when directly connected, are usually expressed in the following form, when the

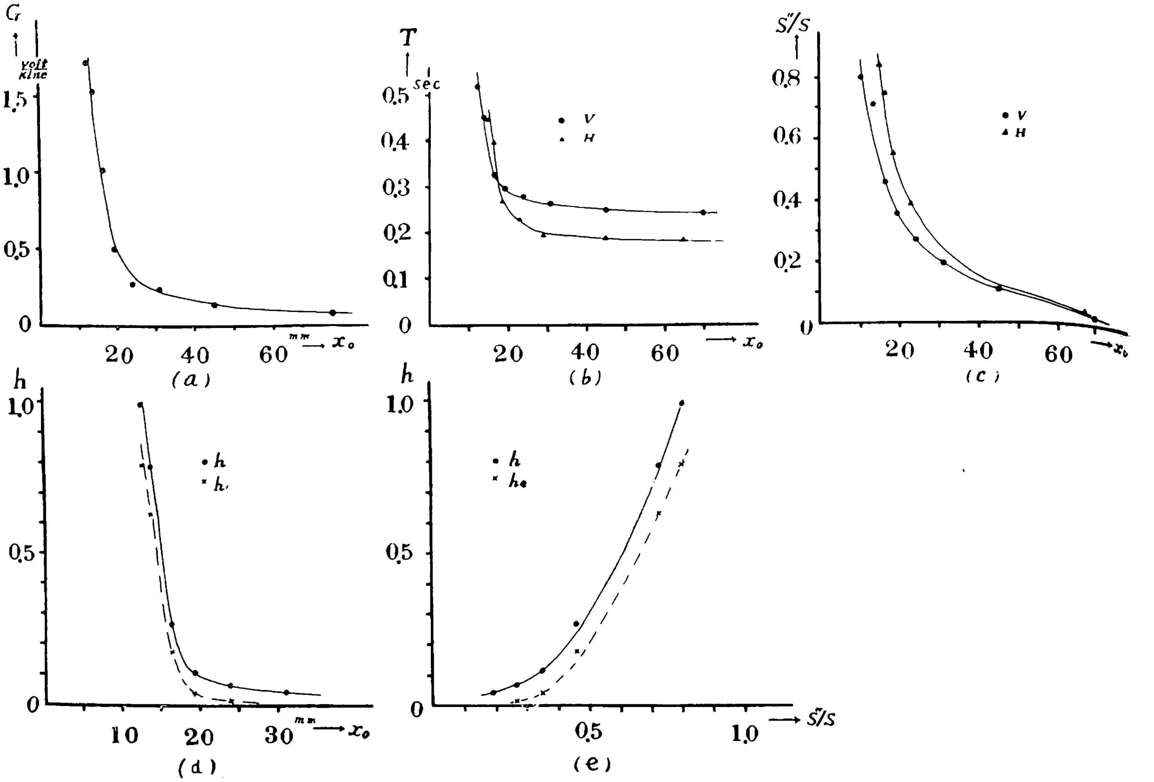


Fig. 8. Variation of various factors with the change in the gap-width.
(a) voltage sensitivity, (b) period, (c) magnetic stiffness,
(d) damping factor, (e) damping factor-magnetic stiffness relation.

self-inductance is omitted in comparison with the resistance. That is,

$$\left.\begin{aligned} &\ddot{x}+2\varepsilon\dot{x}+n^2x=-L_1\ddot{\xi}/l+\gamma\dot{\varphi} \\ &\ddot{\varphi}+2\varepsilon_g\dot{\varphi}+n_g^2\varphi=\beta\dot{x} \end{aligned}\right\} \tag{4.9}$$

where $2\varepsilon_g=(d+g^2/\bar{R})/k$, $n^2=u/k$, $\gamma=L_1^2Gg/K\bar{R}$ and $\beta=Gg/k\bar{R}$

Now, if the effect of counter-electromotive force caused by the reaction of galvanometer's coil, as represented by the second term on the right-hand side of the former equation, is negligible, the motion of the seismometer's pendulum can be expressed with a linear differential equation of second order. According to the Tajime's study (**17**), it was found that the coupling effect between the two systems is avoidable without respect to the motional characteristics of them, if an adequate combination of values of resistances has only to be obtained by means of a shunt circuit. In the present case, $R=360$ ohms, $r=35$ ohms and the shunt resistance $S=40$ ohms, and hence σ

in his paper becomes 0.05. The coupling effect, therefore, seems permissible to neglect.

The magnification can be expressed as follows, from the steady part of the solution for the above equations of motion, assuming the ground motion to be a simple harmonic motion (**1**).

$$V = \frac{2AL_1}{l} \cdot \frac{Gg}{k\bar{R}} \cdot \frac{1}{p} \frac{1}{\sqrt{(u_1^2-1)^2+4h_1^2u_1^2}} \cdot \frac{1}{\sqrt{(u_2^2-1)^2+4h_2^2u_2^2}} \tag{4.10}$$

$$= \frac{2\pi\bar{G}}{S_g T_2^2 \bar{R}} \cdot \frac{1}{\sqrt{(u_1^2-1)^2+4h_1^2u_1^2}} \cdot \frac{1}{\sqrt{(u_2^2-1)^2+4h_2^2u_2^2}} \tag{4.11}$$

where $\bar{G} = GL_1/l, \quad l = K/MH, \quad S_g = u/2Ag, \quad \bar{R} = R+r+Rr/S$

$$u_1 = T_p/T_1, \quad u_2 = T_p/T_2$$

S_g is the sensitivity of the galvanometer and T_p, T_1 and T_2 are the periods of ground motion, seismometer's pendulum and galvanometer's coil, respectively. In this special case $h_1 = h_2 = 1$, that is, when the two moving systems are critically damped,

$$V = \frac{2\pi\bar{G}}{S_g T_2 \bar{R}} \frac{u_2}{(1+u_1^2)(1+u_2^2)} \tag{4.12}$$

Recently, a new formula on the magnification with simple expression was derived by Tajime (**18**). According to his study,

$$V = \frac{a}{\xi} = \frac{2A}{l} \sigma^{1/2} \left(\frac{K}{k}\right)^{1/2} \sqrt{\frac{2h_1u_1}{(u_1^2-1)^2+4h_1^2u_1^2}} \sqrt{\frac{2h_2u_2}{(u_2^2-1)^2+4h_2^2u_2^2}} \tag{4.13}$$

That is, the static magnication depends only upon the ratio of the moments of inertia involving two moving systems, the geometrical magnification, and the combination of resistances. The upper limit of magnification can be estimated from $V < 2AK^{1/2}/lk^{1/2}$ without determining the voltage sensitivity. But the voltage sensitivity is implicitly included in the damping factor. Namely, the formula (4.13) is reduced to the equation (4.10), when h_1 and h_2 are re-written. The two formulae imply the same result in spite of the apparent difference of the motional characteristics and other factors.

In our seismometers the various constants related to magnification are,

	(V)	(H)	
G	1.71	1.79	volt/kine
K	2.49×10^5	3.51×10^5	c.g.s.
L	7.90	15.5	cm
l	9.18	10.8	cm
$\bar{R}$	386 $(R=353)$	736 $(R=335)$	ohms
	$(r=33)$	$(r=33)$	
		$(S=33)$	
σ	1	0.05	
T_1	0.455	0.447	sec.

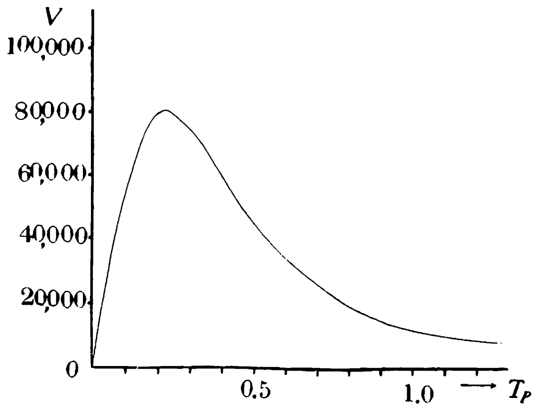


Fig. 9. Magnification curve.

and $A=1$ m, $S_g=3.11\times10^{-8}$ amp/mm, $T_2=0.35$ sec., $k=7.64\times10^{-4}$ c.g.s. and $h_1=h_2=1$.

The maximum magnification is 80,300 when $T_p=0.227$ sec.

Fig. 9 shows the calculated magnification curve.

To recapitulate, the properties of variable reluctance seismometers designed were investigated. There are some defects in respect of linearity, stability and frequency characteristics, but other charac-

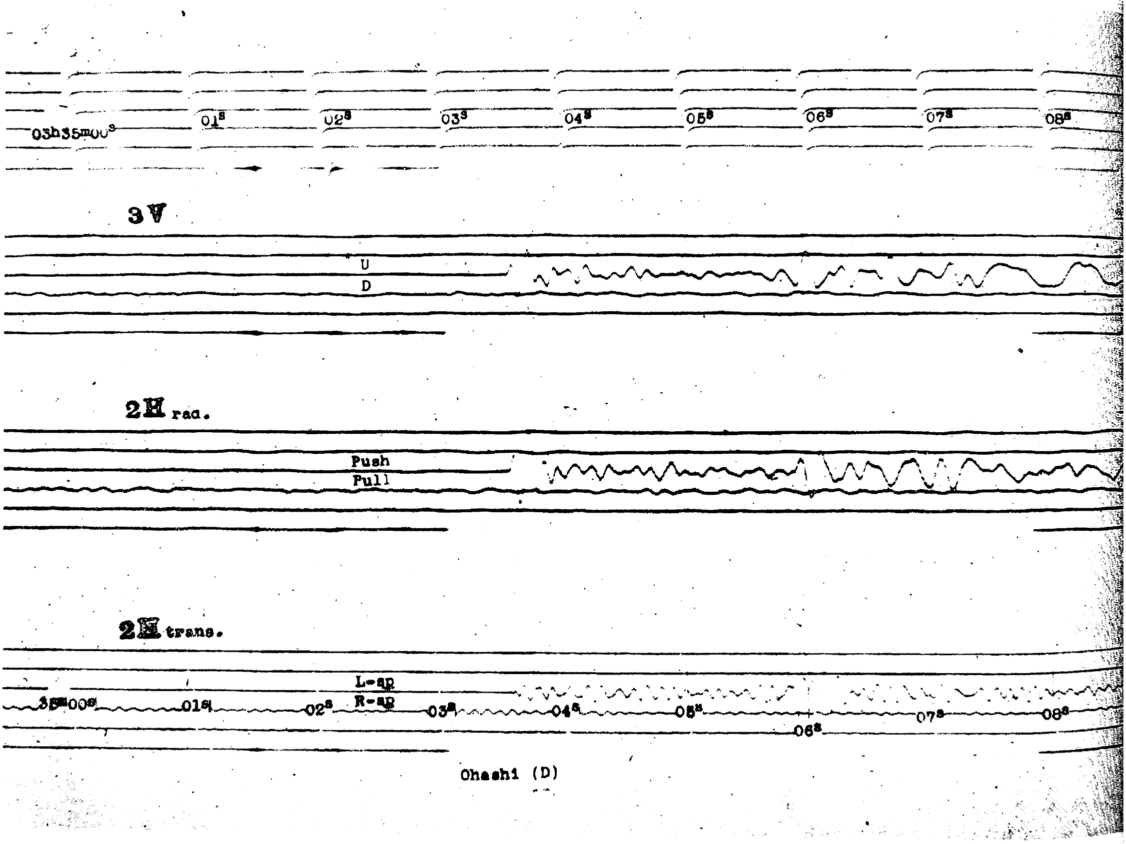


Fig. 10. Seismogram recorded by the R_2- seismometers in an explosion seismic observation. (at Ohashi-station, May, 3rd, 1954, 03h35m, Kamaishi-explosion.)

teristics seem to be satisfactory for the greater part. These seismometers were already successfully used in the observations of local earthquakes and artificial explosions. The examples of seismograms obtained are shown in Fig. 10. Moreover, a grouping observation with four vertical seismometers of this type was made in explosion for the purpose of recording the reflected waves.

The writer expresses his heartfelt thanks to Dr. E. Nishimura and Mr. A. Kamitsuki for their kind advices and to Messrs. M. Ōtsuka and Y. Tanaka for their considerate help in these experimental works and for their valuable suggestions. He is also indebted to Mr. Y. Satō for his laborious calculations.

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