Slip-weakening distance in dynamic rupture of in-slab normal-faulting earthquakes

Takeshi Mikumo\textsuperscript{1} and Yuji Yagi\textsuperscript{2}

\textsuperscript{1}Instituto de Geofísica, Universidad Nacional Autónoma de Mexico, Ciudad Universitaria, Mexico 04510 D.F., Mexico
\textsuperscript{2}International Institute for Seismology and Earthquake Engineering, Building Research Institute, Tsukuba, Ibaraki 305-0802, Japan

Accepted 2003 May 21. Received 2003 April 10; in original form 2002 October 9

SUMMARY
We estimate the critical slip-weakening distance on in-slab earthquake faults in a subduction zone, by applying a recent approach proposed by us. This approach is to find a relation between the breakdown time of shear stress $T_b$, the time of peak slip velocity $T_{pv}$, and the slip-weakening distance $D_c$, from the time histories of shear stress, slip and slip velocity at each point on the fault. The previous results show that $D_c$ at $T_b$ can be well approximated by $D'_c$ at $T_{pv}$ for faults even with a heterogeneous stress drop distribution, except at locations near barriers and fault edges.

We apply the above method to three large in-slab, normal-faulting earthquakes in the Mexican subduction zone. To do this, we calculate the spatial distribution of slip-velocity functions and final slip from kinematic waveform inversion of strong-motion and teleseismic records, and the stress history and final stress change from dynamic rupture calculations. By integrating the slip-velocity functions obtained from the inversion, from the rupture arrival time to the time of peak slip velocity, we obtain slip $D'_c$ at $T_{pv}$ and then correct it for $D_c$ at $T_b$ through dynamic calculations. We also estimate the lowest resolvable limit and probable errors of $D_c$ from the slip-velocity functions, and its upper bound from a theoretical constraint between the dynamic stress drop and $D_c$. We found that the slip-weakening distance $D_c$ estimated in the frequency band between 0.05 and 0.5 Hz ranges between 40 and 120 cm on the in-slab fault of the 1999 Oaxaca earthquake ($M_w = 7.5$). The largest $D_c$ is detected in the central fault and in part of the deeper sections, and $D_c$ in the zone around the hypocentre ranges between 50 and 70 cm. The estimated $D_c$ values appear to be less depth-dependent but are rather more dependent on the local maximum slip. This possible slip dependence might be interpreted by the degree of fault roughness, in addition to stress heterogeneities. The fracture energy $G$ in the central section and in the hypocentral zone are roughly estimated to be of the order of 10–15 and 5–8 MJ m\(^{-2}\), respectively. Both of the estimated $D_c$ and $G$ values are somewhat larger than those on the vertical fault of two recent, shallow strike-slip earthquakes in western Japan.

Key words: dynamic rupture, in-slab faults, normal-faulting earthquakes, slip-weakening.

1 INTRODUCTION
It has been demonstrated by both theoretical studies (e.g. Ida 1972; Andrews 1976a,b; Day 1982; Ohnaka & Yamashita 1989; Mats'ura et al. 1992; Fukuyama & Madariaga 1998; Madariaga et al. 1998; Shibazaki & Mats'ura 1998; Campillo et al. 2001) and laboratory experiments (e.g. Dieterich 1981; Okubo & Dieterich 1984; Ohnaka et al. 1987; Ohnaka & Kuwahara 1990; Ohnaka & Shen 1999) that the constitutive frictional relations, particularly the slip-weakening behaviour of shear stress, play a critical role in the dynamic part of the rupture process and hence on strong ground motions during large earthquakes. Accordingly, it is now one of the important issues in rupture dynamics to estimate the critical slip-weakening distance at which the shear stress drops to the final frictional level during natural earthquakes.

For actual earthquakes, several attempts have been made to date to infer the slip-weakening distance $D_c$ by various methods (Papageorgiou & Aki 1983; Ide & Takeo 1997; Olsen et al. 1997; Day et al. 1998; Guatteri & Spudich 2000; Ohnaka 2000; Pulido & Irikura 2000; Peyrat et al. 2001). It seems, however, that some of the $D_c$ values may be overestimated due to limited data resolution, although $D_c$ may be a significant fraction of the maximum slip on the fault.

More recently, Mikumo et al. (2003) estimated the critical slip-weakening distance on earthquake faults, by applying a new approach that is independent of the estimate of fracture energy or...
radiated seismic energy. In this approach, dynamic calculations provided a physically based relation between the breakdown time of shear stress \( T_b \), the time of peak velocity \( T_{pv} \), and the prescribed slip-weakening distance \( D_c \) at each point on the fault. The validity of the method has been justified by Fukuyama et al. (2003) based on the theoretical background for the shear traction on a 2-D fault given by boundary integral equations in Fukuyama & Madariaga (1998).

It has been shown that \( T_b \) is well approximated by \( T_{pv} \) except at points near fault edges and strong barriers, and that \( D_c \) at time \( T_b \) may be estimated from slip \( D_i \) at time \( T_{pv} \) after some corrections. The above method has been applied to two strike-slip earthquakes in western Japan, the 2000 Tottori and the 1995 Kobe earthquakes, by estimating \( T_{pv} \) in the slip-velocity functions on the vertical fault obtained from kinematic waveform inversion of strong-motion and teleseismic records. From these procedures, Mikumo et al. (2003) found that the slip-weakening distance \( D_c \) estimated in the frequency window between 0.05 and 0.5 Hz ranges between 40 and 90 cm in the major part of the two earthquake faults. Referring to this method, Olsen et al. (2003) also estimated the average \( D_c \) on the Tottori earthquake fault from direct measurement of near-fault strong-motion waveforms. However, all of the above estimates may have quite large uncertainties, considering the limited frequency resolution of the observed waveforms, probable time errors in the slip-velocity function and the uncertainty of the slip-weakening behaviour. Nevertheless, it is still interesting to note that the estimated \( D_c \) values appear to be spatially variable and dependent on the local maximum slip.

The findings from this approach as well as several previous attempts were all for strike-slip earthquakes in the shallow part of the continental crust. There still remains a question as to what extent the slip-weakening distance scales with the size of the fault zone and varies with its environments, particularly for large interplate thrust and in-slab normal-faulting earthquakes in subduction zones. In the present paper, we focus our attention on three in-slab events formed from 21 strong-motion records from seven local stations (e.g. Singh et al. 2003; Cocco et al. 1997; Mikumo et al. 1999, 2002). In the present study, we deal mainly with the rupture dynamics of the 1999 Oaxaca earthquake (\( M_w = 7.5 \)) that occurred below the down-dip edge of the 1978 thrust earthquake fault (\( M_w = 7.8 \)). In addition, we also investigate the 1999 Tehuacan earthquake (\( M_w = 7.0 \)), which took place in the unbending portion of the subducted plate, and the 1997 normal-faulting event (\( M_w = 7.1 \)) that occurred beneath the rupture area of the 1985 Michoacan thrust earthquake (\( M_w = 8.1 \)). For these earthquakes, the source mechanism and slip distribution on the fault have been analysed using well-recorded local, regional and teleseismic data. Possible stress interactions have also been suggested between the preceding thrust and the following two normal-faulting earthquakes (Mikumo et al. 1999, 2002).

### 1999 Oaxaca earthquake

The mainshock hypocentre of the 1999 September 30 Oaxaca earthquake was located at 16.00°N and 97.02°W at a depth of 40 km from the local broad-band seismic network (Singh et al. 2000) (see Fig. 1a). The centroid moment tensor solutions given by different institutions provided two nodal planes: I (\( b = 51°–55°, φ = 295°–315°, \text{slip} = -81° \) to \(-83°\)) and II (\( b = 36°–40°, φ = 103°–120°, \text{slip} = -99° \) to \(-102°\)), indicating a normal-faulting mechanism, with the epicentral location and focal depth somewhat deviated from the local observations. The estimated seismic moment ranges between 1.3 and \( 2.0 \times 10^{37} \) dyn cm. The kinematic inversion has been performed from 21 strong-motion records from seven local stations (Hernandez et al. 2001), together with 15 teleseismic \( P \) waveforms (Yagi et al. 2001). The two analyses, both referring to the focal

\[ \begin{align*}
T_{pv} &= \text{peak slip velocity time} \\
D_c &= \text{slip-weakening distance} \\
M_w &= \text{magnitude}
\end{align*} \]

**Figure 1.** (a) Location of the 1999 Oaxaca, Mexico, earthquake (\( M_w = 7.4 \)) with its fault-plane solution, broad-band and strong-motion stations in the surrounding region, which are used in the kinematic waveform inversion. (b) Location of teleseismic stations used for the kinematic waveform inversion.
depth of 40 km, give a smaller misfit between the recorded and synthetic waveforms for a northeastward-dipping nodal plane than for the other. This indicates that the 1999 Oaxaca earthquake was a normal-faulting event with the fault plane dipping at about 50° toward inland in the subducting Cocos plate. The slip distribution on the fault plane given by Hernandez et al. (2001) shows two areas with a maximum slip around 2.5 m at depths between 40 and 55 km and an area with a smaller slip of 2 m around the hypocentre. On the other hand, Yagi et al. (2001) provided the fault extending from 34 to 62 km with a maximum slip of 2.3 m at depths between 47 and 52 km, and with a smaller slip of 1.4 m near the hypocentre.

Slip distribution from kinematic waveform inversion

To discuss the slip-weakening distance from the stress-breakdown time during the dynamic fault rupture, we performed more detailed, kinematic waveform inversion than the previous inversion by Yagi et al. (2001). The data used for our present purpose are 21 strong-motion records from seven local and regional stations (Fig. 1a) and 15 long-period P waveforms from 15 teleseismic stations (Fig. 1b). The strong-motion records were band-pass-filtered between 0.05 and 0.5 Hz and numerically integrated to ground displacement with a sampling interval of 0.25 s. For this procedure, we applied a fourth-order Butterworth filter to ground displacement with a sampling interval of 0.25 s. In the inversion procedure, we put relative weights on the time during the dynamic fault rupture, we performed more detailed, kinematic waveform inversion than the previous inversion by Yagi et al. (2001). The data used for our present purpose are 21 strong-motion records from seven local and regional stations (Fig. 1a) and 15 long-period P waveforms from 15 teleseismic stations (Fig. 1b). The strong-motion records were band-pass-filtered between 0.05 and 0.5 Hz and numerically integrated to ground displacement with a sampling interval of 0.25 s, while the teleseismic data were band-pass-filtered between 0.005 and 1 Hz and integrated to ground displacement with a sampling interval of 0.25 s. For this procedure, we applied a fourth-order Butterworth filter to remove low-frequency noise arising from the integration and high-frequency waves above 0.5–1.0 Hz for which the Green’s function for local minor velocity structures is not well known. For the inversion analysis, the entire fault plane with dimensions of 60 × 40 km² was divided into 384 subfaults each having a size of 2.5 × 2.5 km². The crust and uppermost mantle structures used here are given in Table 1 for the Oaxaca region and in Table 2 for teleseismic receiver stations. The Green’s functions for all subfault to local and regional station pairs were calculated by the discrete wavenumber method developed by Koketsu (1985), while the functions for teleseismic stations were calculated by the method of Kikuchi & Kanamori (1991). The basis source time function on each subfault was expanded into 39 sequential, overlapping triangles, each having a half duration of 0.25 s with a time-shift of 0.25 s, and re-sampled at an interval of 0.05 s. In the inversion procedure, we put relative weights on the recording stations, considering the quality of the records observed at these stations. The weights are taken to be inversely proportional to the standard deviation of the recorded maximum amplitude, which is 50 per cent for the Jami station and 10 per cent for other stations.

The strong-motion and teleseismic displacement waveforms and the corresponding synthetic seismograms obtained from the final inversion are shown in Fig. 2. The observed and synthetic waveforms generally show a very good fit, except for station Jami. The misfit at this station may be due to some site effects. Fig. 3 shows the spatial slip distribution over the fault, which has been revealed from the inversion. It can be seen that there are four large-slip zones; one is located in the central section at depths between 38 and 52 km with a maximum slip of 2.3 m, the second is its downward and rightward continuation at depths between 53 and 60 km, the third one is located around the hypocentre at depths between 38 and 42 km, and the last one is located near the upper right-hand section of the fault.

Recently, Fukahata et al. (2003) performed another inversion analysis, by applying Akaike’s Bayesian information criterion (ABIC) (Akaike 1980) to the same data set as above. Their results using the more rigorous criterion provided a slip distribution that includes a clear global minimum of the ABIC value. The distribution shows a pattern almost similar to that from the above analysis, suggesting high reliability of the present results. The difference may come from taking into account the quality of the observed records and from the smaller size of subfaults and the shorter duration of the basis function adopted in the present study.

Spatial distribution of stress change

Next, we calculate the spatial distribution of static stress change from the slip distribution shown in Fig. 3. The procedure follows the approach by Mikumo & Miyatake (1993) for a 3-D dynamic model incorporating a dipping fault embedded in a horizontally layered velocity structure. In this 3-D model, the shear stress is applied along the dipping fault just above and just below it. We solve elastodynamic equations, incorporating the kinematic slip under appropriate boundary conditions with a second-order finite-difference scheme. The boundary conditions imposed here are: (1) the continuity of the normal stress and normal displacement component across the dipping fault; (2) traction-free vertical stress components at the ground surface; (3) the continuity of all stress and displacement components at each of the layer interfaces (Mikumo et al. 1987); and (4) the absorbing boundary conditions (Clayton & Engquist 1977) at the side and bottom of the model space. First, we estimate approximate static stress changes from the fault slip using Okada’s formulations for a uniform half-space (Okada 1992). Given these stress changes as starting values, we calculate the slip distribution from dynamic rupture propagating with fixed rupture velocities in the layered velocity structure, and then take the ratio between the already obtained kinematic slip and the calculated dynamic slip at each point on the fault. The ratio is multiplied to the previously given static stress drop, and this procedure is repeated until the rms difference between the kinematic and dynamic slips over the fault can be minimized within a reasonably small value.

For numerical calculations, we interpolate the kinematic slip obtained at every 2.5 km into a grid of 1.25 or 0.625 km on the dipping fault. The extent of the 3-D model space is taken as 125 × 85 × 85 km³, and the grid spacings are 1.250, 0.884 and 0.884 km or 0.625, 0.442 and 0.442 km in the strike direction, the downward vertical direction and in the direction perpendicular to the strike.
respectively. Fig. 4 shows the spatial distribution of the calculated static stress changes over the fault. The maximum stress drop in the four large-slip zones ranges between 125 and 110 bar (12.5 and 11.0 MPa). In addition, there are three zones of negative stress drop (a stress increase up to a maximum of 70 bar or 7 MPa) corresponding to smaller-slip zones adjacent to large-slip zones. The spatial distribution of final slip and stress change derived here is used as observational constraints in the next section.

**Estimate of** $D_c$ **from the slip-velocity functions**

In order to estimate the slip-weakening distance during dynamic fault rupture, we follow the approach introduced by Mikumo et al. (2003) using the slip-velocity functions on the fault. Fig. 5(a) illustrates a simple slip-weakening behaviour proposed by Andrews (1976a,b), where the initial stress $\sigma_0$ at each point on a 2-D fault increases up to the yield stress $\sigma_y$ as the rupture front approaches, and then decreases linearly with ongoing slip down to the final stress $\sigma_f$, although there could be different slip-weakening behaviours as suggested by Ida (1972), Ohnaka & Yamashita (1989), Matsu’ura et al. (1992) and Campillo et al. (2001). One of the previous dynamic calculations on a vertical fault (Mikumo et al. 2003), incorporating the above simple slip-weakening law, yield typical time histories of shear stress, slip and slip velocity at a location on the fault, as depicted in Fig. 5(b). The slip velocity starts to evolve as the shear stress increases due to the arrival of the rupture front at

*Figure 2.* Observed waveforms (thick lines) recorded at near-field broad-band and strong-motion stations and teleseismic stations, and the corresponding synthetic seismograms (thin lines) calculated from the inversion. The numerals below the near-field and teleseismic station codes indicate the maximum amplitude in centimetres and micrometres, respectively.
Fig. 3. Spatial distribution of slip obtained from the kinematic waveform inversion. S1–S8 are the selected locations where the calculated slip-velocity functions are shown in Fig. 6. The star indicates the hypocentre location.

Fig. 4. Spatial distribution of stress change calculated from the slip distribution. Dark-shaded zones including contours with minus sign indicate the zone of static stress drop, and light-shaded zones including contours with a plus sign indicate the zones of stress increase.

time $T_r$, and then reaches the peak value at time $T_{pv}$ as the stress drops from its yield value. However, $T_{pv}$ deviates slightly from the stress-breakdown time $T_b$, and hence the slip $D_c'$ at $T_{pv}$ is somewhat different from the slip-weakening distance $D_c$ at $T_b$. This typical behaviour is almost similar for the dipping fault in the present case.

Fig. 6 shows examples of the slip-velocity time functions at eight selected points on the in-slab fault, which have been obtained at every 0.05 s to match the synthetics to the recorded waveforms (Fig. 2) by the kinematic waveform inversion described in the foregoing section. To estimate $D_c'$ from these slip-velocity functions, we numerically integrate the time functions from the time of rupture arrival $T_r$ to the time of the peak slip velocity $T_{pv}$ on each subfault, where $T_r$ has been obtained within an error of 0.25 s in the inversion. The calculations were made only for subfaults having slip greater than 40 cm and slip velocity greater than 30 cm s$^{-1}$ to use reliable slip-velocity functions. The functions contaminated by minor spurious oscillations or having unusual shapes were excluded. The estimated $D_c'$ values are found to be in the range between 40 and 120 cm.

To make corrections on the $D_c'$ values for estimating the actual slip-weakening distance $D_c$, we perform dynamic calculations imposed with the four boundary conditions described in the previous section, incorporating the final slip shown in Fig. 3 and the stress change shown in Fig. 4 as observational constraints. These calculations are performed with the slip-weakening conditions introduced by Andrews (1976a,b),

$$
\sigma(D) = \sigma_f \quad \text{for} \quad D > D_c
$$

$$
\sigma(D) = \sigma_f - (\sigma_f - \sigma_y)D/D_c \quad \text{for} \quad D < D_c
$$

where $D$ is the current slip, which is the difference between the downward oblique slip on the hanging-wall side and the upward

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Typical Behavior of Stress, Slip and Sliprate on the Fault

Figure 5. (a) Linear slip-weakening friction law, modified from Andrews (1976a,b). $\sigma_y$ is the yield stress, $\sigma_0$ is the initial stress, $\sigma_f$ is the final frictional stress. $\sigma_y - \sigma_0$ is defined as the strength excess, $\Delta \sigma_0 = \sigma_y - \sigma_1$ is called the breakdown stress drop, and $\Delta \sigma = \sigma_0 - \sigma_f$ is the dynamic stress drop. In this model without dynamic overshooting, the dynamic stress drop is equal to the static stress drop. $D_c$ is defined as the critical slip-weakening distance, and $G$ is the fracture energy, defined as $G = (\sigma_y - \sigma_f) D_c/2$. (b) A typical behavior of the time history of shear stress, slip and slip velocity on the fault. $T_b$ is the breakdown time of stress, $T_{pv}$ is the time of peak slip velocity, $D_c$ is the slip at time $T_b$, and $D_c'$ is the slip at time $T_{pv}$.

Figure 6. Slip-velocity functions obtained from the kinematic inversion and the estimated $D_c'$ values (cm) at eight selected points (S1–S8) on the in-slab fault shown in Fig. 3.

oblique slip on the footwall side, which occur along the upper interface of the subducting plate. The stress drop $\sigma_0 - \sigma_f$ is taken from Fig. 4, and the strength excess $\sigma_y - \sigma_0$ is roughly estimated during dynamic calculations when the rupture time $T_r$ at each point on the fault is specified (Miyatake 1992) with fixed rupture velocities of 0.7$V_s$.

The above dynamic calculations were made for selected points with large slip velocities, for prescribed values of $D_c = 40$ and 80 cm. Fig. 7 shows examples of the calculated time histories of stress change, slip and slip velocity at four selected points. Fig. 8 gives the $T_b-T_{pv}$ relation on the left-hand side and the $T_{pv}-D_c'$ relation on the right-hand side. It can be seen that $D_c'$ estimated from $T_{pv}$ falls between 33 and 47 cm for $D_c = 40$ cm, and between 63 and 77 cm for $D_c = 80$ cm. This indicates that the correction factors $D_c/D_c'$ to obtain $D_c$ from $D_c'$ are within 22 per cent, which are comparable with the probable errors described in the next section. However, it was found that the time histories of both stress change and slip velocity at points close to the lower edge of the fault are strongly affected by the edge effect rather than by the prescribed slip-weakening distance, as has been pointed out by Fukuyama et al. (2003). For this reason, these points were excluded from the present analysis.
Slip-weakening distance in dynamic rupture

Additional theoretical constraint

There is another theoretical constraint in estimating the slip-weakening distance $D_c$. The breakdown zone length at the rupture front is related to the critical length $L_c$ (see, for example, Ohnaka 2000), which may be approximated by $D_c$, divided by half the strain drop as, $L_c \sim D_c / (\Delta \sigma / \mu)$ (S. Day, personal communication, 2002). Since the stress-breakdown time $T_c$ is given by $L_c / v$, we obtain,

$$D_c = c T_c \Delta \sigma_b / \rho \beta,$$

where $\Delta \sigma_b$ is the breakdown stress drop, $\rho$ is the density of the medium, $v$ and $\beta$ are the rupture velocity and shear wave velocity, respectively, and $c$ is a numerical constant close to unity. The above relation can also be derived from eq. (50) in Ohnaka & Yamashita (1989). Eq. (2) means that the slip-weakening distance may also be roughly estimated from the stress drop if we measure $T_c$. Since $T_c = T_b - T_r$, and the time from the rupture arrival to the time of peak slip velocity $T_p = T_{pv} - T_r$ (see Fig. 5b), then $T_c \sim T_p$ if $T_b$ can be approximated by $T_{pv}$ as shown in the previous section. $T_c$ in the slip-velocity function obtained during the kinematic inversion ranges between 1.25 and 3.10 s. We tentatively replace the breakdown stress drop $\Delta \sigma_b$ by the dynamic stress drop $\Delta \sigma$, because the breakdown drop including the strength excess may not be well resolved. Combining the stress drop shown in Fig. 4 with $T_c$, we obtain the spatial distribution of $D_c$ on the fault except for the numerical constant $c$, because $\rho$ and $\beta$ in the zone of the in-slab fault are already known. In Fig. 9 we plot the previously estimated $D_c$ values versus $D_c$ calculated from eq. (2) using $\Delta \sigma$ and $T_c$. Although these points scatter over a rather wide range, we see that $c$ is in the range between 0.4 and 0.8. These results suggest that the approach taken in the present study provided a better-resolved slip-weakening distance $D_c$ than from the indirect estimate of using the stress drop and $T_c$, and that the theoretical constraint described here gives the upper bound of the real values of $D_c$. The upper bound of $D_c$ in

Figure 7. Time histories of stress change, slip and slip velocity at selected points, obtained from dynamic calculations for two prescribed values of $D_c = 40$ and 80 cm.
Figure 8. Relations between $T_{pv}$ and $T_b$ (left), and between $T_{pv}$ and $D_c'$ (right) for the case shown in Fig. 8, where $T_b$ and $T_{pv}$ are measured from the origin time at the rupture starting point.

Figure 9. $D_c$ values estimated from the slip-velocity functions plotted versus those calculated from a theoretical constraint $D_c = c T_c \Delta \sigma_b / \rho \beta$. In these plots, $c$ is in the range between 0.4 and 0.8.
the large stress drop zone in the central fault section will be about 150–180 cm, if we assume \( c \sim 0.8 \).

Resolution and probable errors of the estimated \( D_c \) values

As a next step, we estimate the resolution and probable errors of the calculated \( D_c \) values, following the procedure taken by Mikumo et al. (2003). The resolution of \( D_c \) depends on the frequency bandwidth of the recorded waveforms (Fig. 2). The minimum resolvable value \( D_{c \text{min}} \) would be \( D_{c \text{min}} = T_{c \text{min}} V_{\text{av}} \sim T_{c \text{min}} V_{\text{max}}/2 \), if we refer to Guatteri & Spudich (2000), where \( T_{c \text{min}} \) is the shortest modelled period involved in the waveform data, and \( V_{\text{av}} \) and \( V_{\text{max}} \) are the average slip velocity over \( T_{c \text{min}} \) and the maximum slip velocity, respectively. For a filtering procedure for the observed waveforms, we applied a fourth-order Butterworth filter with a high-cut frequency of 0.5 Hz to the strong-motion records and of 1.0 Hz to the teleseismic records. The 0.5 Hz high-cut filter still retains amplitude levels of about 70 per cent at 0.5 Hz (or 2.0 s), 50 per cent at 0.67 Hz (or 1.5 s) and 25 per cent at 1.0 Hz (or 1.0 s). For this reason, we assume that the shortest period remaining on the filtered records, \( T_{c \text{min}} \), will be in the range between 1.0 and 1.5 s. \( V_{\text{max}} \) in the obtained slip-velocity function ranges between 30 and 105 cm s\(^{-1}\). For these cases, \( D_{c \text{min}} \) is estimated to be in the range between 15 and 53 cm, depending on the location on the fault. If we assume \( T_{c \text{min}} = 1.0 \) s, most of the estimated \( D_c \) values significantly exceed \( D_{c \text{min}} \). If we assume \( T_{c \text{min}} = 1.5 \) s, instead, 18 out of 125 estimated \( D_c \) values would have to be discarded. However, many others are still above this higher, minimum resolvable limit.

On the other hand, the probable errors in estimating \( D_c \) come mainly from the precision of \( T_c \), as expected from eq. (2).

\[
\Delta D_c / D_c = \Delta T_c / T_c \sim \Delta T'_c / T'_c.
\]

(3)

In the inversion procedure, we estimate a probable error of \( T'_c \) to be less than 0.25 + 0.05 = 0.30 s, where the first value is a time-shift of the basis source time function and the second one is the sampling time interval of the slip-velocity functions. Since \( T'_c \) is in the range between 1.25 and 3.10 s, \( \Delta D_c / D_c \) ranges from 10 to 40 per cent or \( \Delta D_c = 11–18 \) cm, depending on the location on the fault. Each point plotted in Fig. 9 has probable errors both in the vertical and horizontal axes.

We did not construct the stress-slip relations directly from the slip-velocity functions shown in Fig. 6, as has been tested in the previous study (Ide & Takeo 1997; Day et al. 1998; Mikumo et al. 2003). This is because some artefact would be introduced due to the finite grid size and the time increment used in this type of calculation, as has been shown by the resolution analysis made by Ide & Takeo (1997) for the 1995 Kobe earthquake.

Spatial distribution of \( D_c \) and \( G \)

The \( D_c \) values estimated here are plotted with their probable errors versus the local maximum slip in Fig. 10 to see whether there is any correlation between them. A few exceptional cases when \( D_c - \Delta D_c < D_{c \text{min}} \) are excluded from the figure. It can be seen that the median values of \( D_c \) ranging between 40 and 120 cm increase with the local maximum slip. If we exclude several estimates, almost all points appears to fall in the range \( 0.25 < D_c / D_{c \text{max}} < 0.60 \). However, if the product \( T_{c \text{min}} V_{\text{av}} \) is larger by a factor of 3 than assumed here, the minimum resolvable \( D_{c \text{min}} \) would become larger and the above trend would be almost masked, although it seems unlikely.

In order to look at the spatial distribution of \( D_c \), we tentatively divide the estimated values into four ranges, considering uncertainties in kinematic and dynamic calculations. Fig. 11 shows the spatial distribution of \( D_c \) in large-slip zones on the fault, which has been slightly smoothed by a Laplacian operator. For peripheral subfaults, the estimated values are less reliable due to small slip and slip velocities. Subfaults in the negative stress-drop zones are excluded because the slip-velocity functions have not been well resolved and also because our technique cannot be applied. From Fig. 11, we see that \( D_c \) in the central section with a maximum slip of 2.3 m exceeds 100 cm and sometimes reaches 120 cm, and that \( D_c \) in its right-downward section with a local maximum slip of 2.0–2.1 m also reaches 110–120 cm. Around these two zones, somewhat smaller \( D_c \) values of between 60 and 90 cm are distributed. In the zone around the hypocentre with a slip of 1.0–1.4 m and in the upper right-hand fault section, \( D_c \) is in the range of \( \sim 50–70 \) cm. Thus, the distribution does not indicate larger \( D_c \) values in shallow fault sections nor smaller \( D_c \) at deeper sections, but clearly shows a spatially heterogeneous distribution.

On the other hand, the breakdown stress drop \( \Delta \sigma_b = \sigma_y - \sigma_i = (\sigma_y - \sigma_0) + (\sigma_0 - \sigma_i) \) is roughly estimated on the large-slip zones. If we combine the estimated slip-weakening distance \( D_c \) with the breakdown stress drop \( \Delta \sigma_b \), the fracture energy \( G = \Delta \sigma_b D_c / 2 \), which is the effective surface energy to be the work done, may also

![Figure 10](http://gji.oxfordjournals.org/) The estimated \( D_c \) values with their probable errors (thin vertical lines) plotted versus the local maximum slip on subfaults.

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be roughly estimated. The estimated energy $G$ in the central section and in the right-downward section ranges between 10 and 15 MJ m$^{-2}$, while it is about 5–8 MJ m$^{-2}$ around the hypocentral zone.

### 1999 Tehuacan earthquake

The 1999 June 15 Tehuacan earthquake ($M_w = 7.0$) was located at 18.15$^\circ$N and 97.52$^\circ$W at a depth of 60 km from local and regional seismic observations (Singh et al. 1999). The source parameters determined from the moment tensor inversion of regional and teleseismic data by different institutions were: $\delta = 43^\circ$–46$^\circ$, $\phi = 290^\circ$–310$^\circ$, rake $= -82^\circ$ to $-92^\circ$ for the northward-dipping nodal plane, which is more likely to be the fault plane for a normal-faulting-type mechanism, in view of the duration of ground motion at near-field stations and the directivity of the recorded $S$ waveforms (Singh et al. 1999). The estimated seismic moment ranges between 2.0 and $3.4 \times 10^{26}$ dyn cm. A kinematic waveform inversion with simulated annealing algorithms has been made of 21 local strong-motion and regional broad-band records from seven stations (Iglesias 2001). The slip distribution obtained on the fault with a dimension of 60 km$^2$, and a fixed rupture velocity of 2.80 km s$^{-1}$ and a triangular source time function with a duration of 1.20 s was finally adopted, respectively, after many trial calculations. The results (Santoyo et al. 2003) show that a major slip larger than 2 m is concentrated at depths between 28 and 40 km, with a maximum slip exceeding 3 m. Dynamic rupture calculations (Mikumo et al. 2000) incorporating the above slip distribution revealed a stress drop higher than 200 bar, even up to 280 bar in a few localized zones, where a large slip was observed.

Since the kinematic inversion did not use a multitime window analysis, but assumed a single triangular source time function, it is difficult to estimate the time $T_s'$ from the rupture arrival to the time of peak slip velocity at different locations on the fault, as in the case of the 1999 Oaxaca earthquake. However, it might be possible to assume the half duration of the source time function, 1.20/2 = 0.60 s, as $T_s'$. This short time cannot be resolved by the present technique. Instead, if we apply eq. (2) to the present case, assuming $c \sim 1.0$, the upper bound of the slip-weakening distance $D_w$ may be estimated to be in the range between 100 and 120 cm in this high slip zone.

### 1997 Michoacan earthquake

The 1997 January 11 Michoacan earthquake ($M_w = 7.1$) was located at 18.09$^\circ$N, 102.79$^\circ$W at a depth of 35 km from local seismic observations. An inversion of teleseismic body waves (Santoyo et al. 2003), as well as two other moment tensor inversion, gave a nearly vertical faulting mechanism ($\delta = 87^\circ$, $\phi = 105^\circ$, slip $= -110^\circ$), with a seismic moment of $4.5 \times 10^{26}$ dyn cm. The kinematic waveform inversion (Santoyo et al. 2003) of four local strong-motion records together with 12 teleseismic data, provided the slip distribution on the vertical fault extending for a horizontal dimension of 45 km at a depth range between 20 and 50 km. In the inversion analysis, the entire fault was divided into 375 subfaults each with a dimension of $2 \times 2$ km$^2$, and a fixed rupture velocity of 2.80 km s$^{-1}$ and a triangular source time function with a duration of 1.20 s were finally adopted, respectively, after many trial calculations. The results (Santoyo et al. 2003) show that a major slip larger than 2 m is concentrated at depths between 28 and 40 km, with a maximum slip exceeding 3 m. Dynamic rupture calculations (Mikumo et al. 2000) incorporating the above slip distribution revealed a stress drop higher than 200 bar, even up to 280 bar in a few localized zones, where a large slip was observed.

Dynamic calculations incorporating the slip distribution yield a stress drop of about 110 bar in the central high-slip zones. From the above kinematic inversion, on the other hand, we estimate $T_s'$ from the rupture arrival time to the time of peak slip velocity to be 1.3–1.5 s with a probable error of 0.5 s. If we apply eq. (2) to these values, tentatively assuming $c \sim 1.0$, the upper bound of the slip-weakening distance in the present case may be roughly estimated to be in the range between 95 and 110 cm in this high slip zone.

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**Figure 11.** Spatial distribution of the estimated $D_c$ values on the main part of the in-slab fault, which are classified into four ranges.
3 DISCUSSION

In the present study, we estimated a probable range for the slip-weakening distance \( D_c \) on the in-slab fault of the 1999 Oaxaca, Mexico earthquake, following the approach proposed by Mikumo et al. (2003), which has been supported by the theoretical background (Fukuyama et al. 2003). Actually, our dynamic calculations for the in-slab fault with a heterogeneous stress drop distribution indicate that \( D_c \) at the stress breakdown time \( T_b \) can be well approximated in most cases by \( D'_c \) at the peak slip velocity \( T_{p_v} \), as shown in Fig. 8. This relation is badly contaminated, however, at locations near the fault edges and strong barriers, as has been predicted by theoretical relations (Fukuyama et al. 2003). As has also been discussed in our previous paper, actual slip-weakening behaviour could be different from an idealized model proposed by Andrews (1976a,b), in which the shear stress is assumed to decrease linearly with ongoing slip up to the critical distance \( D_c \). Numerical calculations of dynamic rupture propagation with different types of slip-weakening friction laws (Fukuyama et al. 2003) indicate that the deviations of \( D'_c \) from the prescribed value of \( D_c \) are within 20 per cent in most cases but sometimes reaches 50 per cent for the case of a very gentle decrease of shear stress. In addition, there are also observational uncertainties, including the limited frequency resolution of the recorded waveforms, which leads to the lowest resolvable value of \( D_c \), and probable errors in measuring \( T_{p_v} \) in the slip-velocity functions obtained from kinematic waveform inversion. Thus, it is to be mentioned that the slip-weakening distances estimated here should be regarded as those viewed from the frequency range limited between 0.05 and 0.5–1.0 Hz, and those located between the minimum resolvable limit and the upper bound of their real values.

Although our estimates have quite a large uncertainty as mentioned above, it is still interesting to note that the estimated \( D_c \) values appear to be spatially heterogeneous and dependent on the local maximum slip on the fault. These features are found to be similar to the case of shallow, vertical strike-slip earthquakes in Japan (Mikumo et al. 2003). This possible slip dependence of the critical slip-weakening distance might be interpreted by the roughness of fault surfaces (e.g. Okubo & Dieterich 1984; Ohnaka & Shen 1999), which appear to be distributed as a fractal-like structure (Scholz & Aviles 1986; Power et al. 1987), and/or the thickness of fault gouge layers (Marone & Kilgore 1993). Also to be noted here is that somewhat larger \( D_c \) values up to 120 cm are observed on the central section of the Oaxaca in-slab fault compared with those on the shallow fault of the two crustal events.

We also estimate the upper bound of the slip-weakening distance from a theoretical constraint between the breakdown stress drop \( \Delta \sigma \), during the breakdown time \( T_c \), and \( D_c \), as indicated in eq. (2). Comparing the \( D_c \) values estimated from the slip-velocity functions with those from the spatial distribution of \( \Delta \sigma \) and \( T_c \), a numerical constant \( c \) in eq. (2) appears to be in the range between 0.4 and 0.8, although this constant has a large uncertainty. If we tentatively assume \( c \approx 1.0 \) for the other in-slab, normal faulting earthquakes in the subducting Cocos plate, the upper bound of \( D_c \) values in the large stress drop zones would be about 110 cm for the 1999 Tehuacan earthquake and about 120 cm for the 1997 Michoacan earthquake, which are somewhat smaller than about 180 cm for the 1999 Oaxaca event. These estimated upper bounds are still larger than those calculated in the same way for the 2000 Tottori crustal earthquake. This difference might suggest the possible existence of a somewhat larger characteristic wavelength of the fault roughness on the in-slab fault surface due to higher normal stress in the subducting slab. On the other hand, laboratory experiments show that \( D_c \) also tends to increase with increasing temperature above 300 °C (Kato et al. 2003). This effect might also explain slightly larger \( D_c \) values for in-slab faults compared with crustal faults. The fracture energy estimated in the central fault section of the in-slab fault is also found to be larger by a factor of about 2 than that for the shallow crustal fault. For the same reason, the fracture energy needed to break asperities would be larger for the in-slab fault.

Although we should deal with many more earthquakes with different magnitudes, it seems that the above possible difference in the slip-weakening distance and the fracture energy between the in-slab and crustal faults may not be due to their scale effects with earthquake size but might be attributable to environmental effects.

4 CONCLUSIONS

We have estimated the critical slip-weakening distance during dynamic rupture on in-slab faults of normal-faulting earthquakes in the Mexican subduction zone, by applying a recent approach proposed by Mikumo et al. (2003), separately from the fracture energy or seismic radiation energy. For this purpose, we calculated the spatial distribution of slip-velocity functions and the final slip from a kinematic inversion of near-field strong-motion records and teleseismic waveforms in the frequency range between 0.05–0.5–1.0 Hz, and the stress time history and final stress change from dynamic calculations. The main conclusions we have obtained here are as follows.

1. Dynamic calculations for a shear crack propagating on a dipping in-slab fault with a heterogeneous stress drop distribution show that the slip-weakening distance \( D_c \) at the stress-breakdown time \( T_b \) can be well approximated by the slip \( D'_c \) at the time of peak slip velocity \( T_{p_v} \), except at locations near strong barriers and fault edges.

2. We found that the slip-weakening distance \( D_c \) reaches 120 cm in the central fault section and ranges between 50 and 70 cm around the hypocentre on the fault of the 1999 Oaxaca earthquake (\( M_w = 7.5 \)), although these estimates have quite a large uncertainty.

3. The estimated \( D_c \) values appear to be less depth-dependent but are rather more dependent on the local maximum slip. This possible slip dependence might be interpreted by the degree of fault roughness, in addition to stress heterogeneities.

4. The fracture energy \( G \) in the central section and in the hypocentral zone are roughly estimated to be of the order of 10–15 and 5–8 MJ m\(^{-2}\), respectively.

5. Both of the estimated \( D_c \) and \( G \) values are somewhat larger than those on the vertical fault of two recent, shallow strike-slip earthquakes in western Japan. This difference might be due to the possible existence of a larger characteristic wavelength of the fault roughness and higher normal stress and higher temperature in the subducting slab than in the shallow crust.

ACKNOWLEDGMENTS

We are grateful to our colleagues, Kim B. Olsen and Eiichi Fukuyama who have made great scientific contributions, and Paul Spudich and Steve Day for their constructive suggestions to previous study on a similar topic. We wish to thank Steven N. Ward (Associate Editor), Ruth Harris and an anonymous reviewer for their comments that helped to improve the manuscript. The senior author also thanks Luis Quintanar and Arturo Iglesias for telling him about their results. The present work started with a joint research programme headed

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