Sector dominance ratio analysis of financial markets

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Abstract

In this paper we present a new measure to investigate the functional structure of financial markets, the Sector Dominance Ratio (SDR). We study the information embedded in raw and partial correlations using random matrix theory (RMT) and examine the evolution of economic sectoral makeup on a yearly and monthly basis for four stock markets, those of the U.S., U.K., Germany and Japan, during the period from January 2000 to December 2010. We investigate the information contained in raw and partial correlations using the sector dominance ratio and its variation over time. The evolution of economic sectoral activities can be discerned through the largest eigenvectors of both raw correlation and partial correlation matrices. We find a characteristic change of the largest eigenvalue from raw and partial correlations and the SDR that coincides with sharp breaks in asset valuations. Finally, we propose the SDR as an indicator for changes in VIX indexes.

Keywords: Financial markets, sector dominance ratio, market structure

1. Introduction

Financial markets are highly complex adaptive systems, resulting from multi-scale interactions amongst individuals, institutions, companies and countries [1]. To better understand the dynamics and structure of financial markets, one can draw on the tools developed in the discipline of complexity science, which has focused on the extraction of useful information for understanding and controlling dynamic interacting systems such as economic, biological, and other complex systems [2, 3, 4, 5, 6, 7, 8]. Physics-based approaches also have been proposed for avoiding and controlling systemic risks and crises in financial markets [9, 10]. While complex phenomena such as chaos generally lead to unpredictability[11], classical dynamics and quantum statistical mechanics in physics have been applied to many fields of science [12]. Such methods have been successful in analyzing information and conservation laws, and have contributed to the understanding of nonlinear and complex dynamical interacting systems that are not in a state of equilibrium[13, 14, 15].

The network approach also has been used to analyze connections between world financial markets, especially since the coupling between markets has strengthened in recent years [16]. The evolution of dependencies in the global market can be quantified by constructing the dependency network for each market [17]. At a smaller scale, the network structure of markets and the interactions and dependencies among economic sectors within national markets also can be modeled using such a structure [18]. The structure of economic sectors within each market is relevant not only to intramarket properties, but also affects the relation across markets. In addition, each market has a unique structure of economic sectors; for example, the energy and financial sectors are of relatively high importance in the U.S. stock market. Uncovering which economic sectors play a dominant role in each market is fundamental to understanding their structure.

Many methods have been applied to studying information embedded in the interactions in stock markets. One key statistically based approach is the use of empirical correlation analysis, and the analysis of empirical correlations between different financial assets [19, 20, 21]. A major contribution of statistical physics to these efforts has been the use of Random Matrix Theory (RMT) to uncover latent information embedded in the observed empirical correlations [22, 23, 24, 25, 26, 27, 28, 29, 30]. RMT is a methodology to evaluate the eigenvalues of empirical correlation matrices, originally developed in the field of nuclear physics by Wigner and Dirac to explain energy levels of complex quantum systems.

In their seminal work, Plerou *et al.* [22] tested the eigenvalue statistics of the empirically measured correlation matrix C, of historical returns from the U.S. stock market against the null hypothesis of a random correlation matrix. This allowed them to distinguish genuine correlations from spurious correlations that are present even in random matrices. They found that the bulk of the eigenvalue spectrum of C shares universal properties with the Gaussian orthogonal ensemble of random matrices. Further, by analyzing deviations from RMT, they showed that the largest eigenvalue and its corresponding eigenvector represent the influence of the entire market on all stocks; using the remaining deviating eigenvectors, they were able to partition stocks into distinct subsets whose identity corresponds to conventionally identified economic sectors. Finally, they introduced an approach which utilizes these results for the construction of portfolios that have a stable ratio of risk to return.

Recently, Kinlaw *et al.* [31] introduced a method to measure systemic importance using the absorption ratio and variance of eigenvectors introduced by [32], which is equal to the fraction of a market's total variance explained by a subset of important factors. This method provides the possibility to assess whether a market is fragile or resilient to shocks or external effects by examining the value of the absorption ratio. Furthermore, this tool was extended to measure the centrality of an economic sector in a given market, by the use of a centrality score.

In this paper we propose to extend previous work by introducing a new approach to uncover the functional dominance of economic sectors, using RMT. We propose a new indicator, the Sector Dominance Ratio (SDR), to examine economic sectoral makeup at a certain reference time interval using both raw correlation and partial correlation matrices. Here we term standard Pearson correlation coefficients [33] as raw correlations. Partial correlations are the correlation between two variables after removing the mediating effect of a third variable (see Methods section), and provides the means to uncover the nature of the hidden embedded relationships between different sectors of the market. We will introduce the SDR methodology and study the dynamic changes of SDR employing eigenvectors obtained from both raw and partial correlation matrices. The SDR uses RMT to identify the informative components of the empirical correlation matrices, and thus, unlike other factor models or principal components-based indicators, does not require making assumptions on where the meaningful system information is embedded. As such, the SDR can shed additional light into the functional structure of financial markets.

We examine the SDR for both yearly and monthly bases for raw correlation and partial correlation matrices. We apply the SDR methodology to study the structure of four different stock markets, those of the U.S., U.K., Germany, and Japan, and investigate whether the economic sectoral makeup is indeed apparent in the observed prices and their evolution over time. The information obtained from the model of SDR provides important insights into the underlying driving forces in the dynamics of real stock markets: not only the importance of each sector, but also the state of the sector and whether its activity or growth rate is increasing or decreasing. Finally, we show the SDR is useful for predicting the behavior of VIX indexes using a Granger causality and cross correlation tests for both raw and partial correlation.

The paper is organized as follows. Section 2 refers to methods for the analysis of stock market data, correlation coefficients and the introduction of the RMT approach as well as the relation between RMT and the factor model. In section 3, we present the formal model of the SDR using components of eigenvectors for raw and partial correlations. In section 4, we examine the distribution of eigenvalues and the components of the largest and second largest eigenvector. We present the results obtained from the SDR analysis in section 5 focusing on the sectoral content of the different markets in different time periods. We show that the method of raw and partial correlations and the model of SDR provide important information on the underlying structure of financial markets and their dynamics. Finally, we discuss the main results and conclusions in section 7.

2. Materials and methods

2.1. Data

We employ the daily adjusted closing price from four major stock markets (see [16]) downloaded from Thomson Reuters Datastream. For the U.S., the U.K. and Japan, we include stocks belonging to each country's most important stock indices, the S&P 500, FTSE 350, and Nikkei 500. For Germany, we start with the entire set of DAX composite shares since the DAX index has only 30 members. After filtering out stocks that did not actively trade over the entire period, we are left with 89 stocks, mainly DAX members and some stocks from the MDAX and SDAX. The number of stocks finally used for the analysis shrinks significantly when we select only stocks that trade actively from January 2000 to December 2010. We also use volume data to filter for very illiquid stocks.

Economic sectors and their market share in each country are shown in Table 1. Each market has unique makeup of economic sectors; for example, the energy sector is substantial in the U.S. and Japan, but not in the U.K. and Germany. The services and health care sectors have a large share in the U.K. and German markets, but not in the U.S. and Japan. The communications sector does not appear in the German stock market. We consider the percentage of representative economic sectors in Table 1 as benchmark values to compare with random data, and employ the data of Table 1 for the analysis of SDR for normalization.

2.2. Stock correlation metrics

The stock return $r_i(t)$ is defined as

$$r_i(t) = \log[p_i(t + \Delta t)] - \log[p_i(t)], \qquad (1)$$

Table 1. The table presents the number and percentage (in parentheses) of representative economic sectors in the U.S., U.K., German and Japanese stock markets. Each sector is indexed by s. The total number of stocks is 403 for U.S. stock market, 116 for the U.K., 89 for Germany and 315 for Japan.

s	Sector	U.S.	U.K.	Germany	Japan
1	Basic materials	21 (5.21%)	9 (7.76%)	4 (4.49%)	36 (11.4%)
2	Communications	29 (7.2%)	1 (0.86%)	0 (0%)	12 (3.81%)
3	Consumer goods	136 (33.8%)	12 (10.34%)	24 (27.0%)	113 (35.9%)
4	Energy	32 (7.94%)	0 (0%)	0 (0%)	2 (0.63%)
5	Financial	62 (15.4%)	33 (28.5%)	10 (11.2%)	39 (12.4%)
6	Health care	0 (0%)	4 (3.45%)	9 (10.1%)	0 (0%)
7	Industrial goods	52 (12.9%)	12 (10.3%)	11 (12.4%)	89 (28.3%)
8	Services	0 (0%)	32 (27.6%)	14 (15.7%)	0 (0%)
9	Technology	45 (11.2%)	6 (5.17%)	13 (14.6%)	19 (6.03%)
10	Utilities	26 (6.45%)	7 (6.03%)	4 (4.49%)	5 (1.59%)

where $p_i(t)$ is the daily adjusted closing price of stock i at time t, and Δt is the time interval which is taken as $\Delta t = 1$ (1 trading day) for this analysis. The raw correlation (Pearson's correlation, [33]) coefficient between two stocks i and j is defined as

$$C(i,j) = \frac{\langle (r_i - \mu_i) \cdot (r_j - \mu_j) \rangle}{\sigma_i \sigma_j}, \tag{2}$$

where r_i and r_j are returns of stocks i and j, μ_i and μ_j are respective means, σ_i and σ_j are standard deviations of corresponding stocks, and the bracket $\langle\rangle$ denotes the average over time. Note that $C(i,j)$ is a symmetric square matrix and $C(i,i) = 1$ for all i.

Recently, a new method to study relationships of influence, or dependency, by using partial correlations to construct a new type of network was introduced in [34, 35]. In their study, they applied this approach to the analysis of stock relationships, and were able to uncover important information regarding the underlying dependency relationships between stocks traded on the New York Stock Exchange (NYSE). This methodology is also capable of providing important information on the evolution of the network [36] and recently, on the investigation of the immune system [37] and semantic networks [38], validating the applicability of the methodology to different types of complex systems.

Partial correlation is another useful tool to investigate correlations between two stocks. The partial correlation measures correlation between two variables after accounting for any common dependence on a third mediating variable. For stocks, we wish to measure correlation after removing the mutual dependence on a systematic economy-wide factor such as the market index. The residual, or partial, correlation between stocks i and j, after accounting for the mediating effect of the market index, m, is defined by [23, 39, 40] as:

$$\rho(i,j|m) = \frac{C(i,j) - C(i,m)C(j,m)}{\sqrt{(1 - C^2(i,m))(1 - C^2(j,m))}}, \tag{3}$$

where $C(i,j)$ is the raw correlation between stock i and j, $C(i,m)$ is the pairwise correlation between stock i and the mediating variable m, and $C(j,m)$ is the pairwise correlation between stock j and the mediating variable m.

While much work has made use of RMT to study empirical correlation matrices, there is very little work using RMT to investigate partial correlation matrices [23]. An important question is how similar the leading eigenvectors of the correlation matrix and the partial correlation matrix are, and what additional information is provided by the eigenvectors of the partial correlation matrix.

2.3. Overview of Random Matrix Theory (RMT)

Considering a portfolio of N stocks with time T records, the distribution of eigenvalue λ of random matrix $\rho(\lambda)$ is given as,

$$\rho(\lambda) = \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda}, \tag{4}$$

where $\lambda_{max} \geq \lambda \geq \lambda_{min}$. The distribution is bounded with $n \to \infty$, $T \to \infty$, and the condition that $Q = T/n \geq 1$ is fixed,

$$\lambda_{min}^{max} = \sigma^2\left(1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}\right). \tag{5}$$

The value of σ^2 is equal to the variance of the elements and equal to one ($\sigma^2 = 1$) for stock correlation matrices Eq. (4). We examine whether there exist eigenvalues obtained from empirical correlation matrices that deviate from the range of the distribution of eigenvalues, λ_{max}, corresponding to random matrices using Eq. (4) and Eq. (5). The eigenvalues of the random matrix represent zero-embedded information; therefore, the deviation from the range, λ_{max}, defined by (5) provides a measure for non-spurious information about behaviors of stock markets that are embedded in the correlation matrices

One important application of RMT in Finance is its use in determining the number of factors to use in a factor model. In order to assess stock returns, the factor model developed by Fama *et al.* [41] is widely used for a multivariate system [42, 43]. The general multifactor model for $x_i(t)$ is defined as

$$x_i(t) = \left\{ \sum_{j=1}^{K} \gamma_i^{(j)} f_j(t) \right\} + \gamma_i^{(0)} \varepsilon_i(t), \tag{6}$$

where $x_i(t)$ is the return from stock i and $\gamma_i^{(j)}$ is a constant describing the weight of factor j in the dynamics of the variable $x_i(t)$. The maximum number of factors, which are described by the time series $f_j(t)$ is K, and the last term $\varepsilon_i(t)$ is a zero mean noise with unit variance.

The factors can be selected on theoretical ground such as interest rates for bonds, industrial production for stocks, or alternately on empirical grounds. In our approach, a factor can be also associated with each relevant eigenvalue and eigenvector; the multifactor model with eigenvalue and eigenvector is given as follows

$$x_i(t) = \sum_{h=1}^{K} \gamma_i^{(h)} \sqrt{\lambda_h}\, f^{(h)}(t) + \sqrt{1 - \sum_{h=1}^{K} \gamma_i^{(h)^2} \lambda_h}\ \ \varepsilon_i(t). \tag{7}$$

In this multifactor model, K is the maximum number of relevant eigenvalues and $\gamma_i^{(h)}$ is the i-th component of the h-th eigenvector of correlation matrix C. The λ_h is h-th eigenvalue and $f^{(h)}$ is defined as h-th factor. The term $\varepsilon_i(t)$ is idiosyncratic firm-specific component of returnterm. In this multifactor model, the eigenvalues which have "meaningful" information on factor $f^{(h)}$ should be included. Using RMT, we can select meaningful eigenvalues for the multifactor model by examining the number of eigenvalues deviating from λ_{max} given by Equation (5).

3. Sector dominance ratio (SDR)

We propose the following Sector Dominance Ratio (SDR) to study evolution of sectoral makeup using eigenvectors derived from empirical correlation matrices and benchmark values in Table 1. After ordering the eigenvalues obtained from an empirical correlation matrix as $\lambda_1 > \lambda_2 \cdots > \lambda_k > \lambda_{k+1} \cdots > \lambda_{max}$, we assess the components of the k-th eigenvector $\boldsymbol{v}_k = (v_{1k}, v_{2k}, \ldots, v_{Nk})$ corresponding to k-th largest eigenvalue deviating from λ_{max}, using SDR. The SDR, Φ_s, is given as follows

$$\Phi_s = N_\theta \sum_{i=1}^{N} \delta_{si}\theta(v_{ik} - \tau) - \varepsilon_s, \tag{8}$$

where s is the sector number in Table 1 and N is the number of stocks for each market. The N_θ is a normalization term defined as $N_\theta = 1/\sum_{i=1}^{N} \theta(v_{ik} - \tau)$. Note that threshold τ is fixed under the maximum value of component of $\boldsymbol{v}_k$, therefore, N_θ always takes finite and positive values. The second term ε_s is a normalization term defined as $\varepsilon_s = n_s/N$ where n_s is the number of stocks belonging to sector s, and values listed in Table 1 are applied to ε_s. The function δ_{si} is Kronecker delta function; if stock i belongs to sector s, it returns $\delta_{si} = 1$ and $\delta_{si} = 0$ otherwise. Each stock i is classified into 10 economic sector (basic materials (s=1), communications (s=2), consumer goods (s=3), energy (s=4), financial (s=5), health care (s=6), industrial goods (s=7), services (s=8), technology (s=9) and utilities (s=10)) as listed in Table 1.

The function $\theta(v_{ik} - \tau)$ represents a step function defined as

$$\theta(v_{ik} - \tau) = \begin{cases} 1, & (v_{ik} \geq \tau), \\ 0, & (v_{ik} < \tau), \end{cases} \tag{9}$$

where τ is a threshold. The function $\theta(v_{ik} - \tau)$ determines whether or not stock i has an active role in its stock market during a certain reference time. The threshold τ can be interpreted as an average value of activity of a financial market obtained from an empirical correlation matrix. If $\theta(v_{ik} - \tau)$ returns 1, the stock i is considered to play an active role at the given reference time, reflecting that the weight of the given stock in the corresponding eigenvector is larger than the activity threshold, τ. The range of SDR is $-1 \leq \Phi_s \leq 1$, and if a certain sector s plays a dominant role at a given reference time, Φ_s takes a positive value close to $\Phi_s \simeq 1$. If SDR takes a negative value close to $\Phi_s \simeq -1$, it indicates that the activity of sector s is low or declining during the given time interval. For the case of random data, it can be shown that $\Phi_s \simeq 0$ (see Appendix A). Here we define the threshold to be $\tau = 1/\sqrt{N}$ (for additional information, see Appendix B).

Defined this way, the proposed SDR measure provides new information, which is not present in either standard factor models, or RMT, or both. The SDR does not require any assumptions regarding the number of factors, rather only the number of stocks in each sector. The SDR provides a quantitative, empirically based approach to study how different sectors dominate the behavior of other sectors, as reflected by their relative size in the empirical eigenvectors. The larger their size, the larger is their effect on the empirical correlations, which reflects how they influence changes in the price changes of other sectors in the market. Thus, for a given sector classification system, this measure provides the means to quantitatively monitor the difference in the contributions of the individual sectors in the real behavior of the market. This information can be used to monitor changes in individual sectors, shifts in market structure, and if needed, reconsideration of the classification scheme.

4. Spectral properties of similarity metrics

For each market (U.S., U.K., Germany and Japan), we calculate the empirical correlation matrix from the stock time series, and derive the eigenvalue distribution of each correlation matrix, for both raw and partial correlation. We study the numbers and values of eigenvalues deviating from λ_{max} compared to the distributions obtained from random matrices. The values of the parameters, Q, λ_{max} and λ_{min} for all markets are listed in Table 2.

Table 2. Value of Q, λ_{min} and λ_{max} for distribution of random matrices (4) derived from data for each stock market, and the number and percentage (value in parenthesis) of empirical eigenvalues which deviate from λ_{max} for raw and partial correlation matrices.

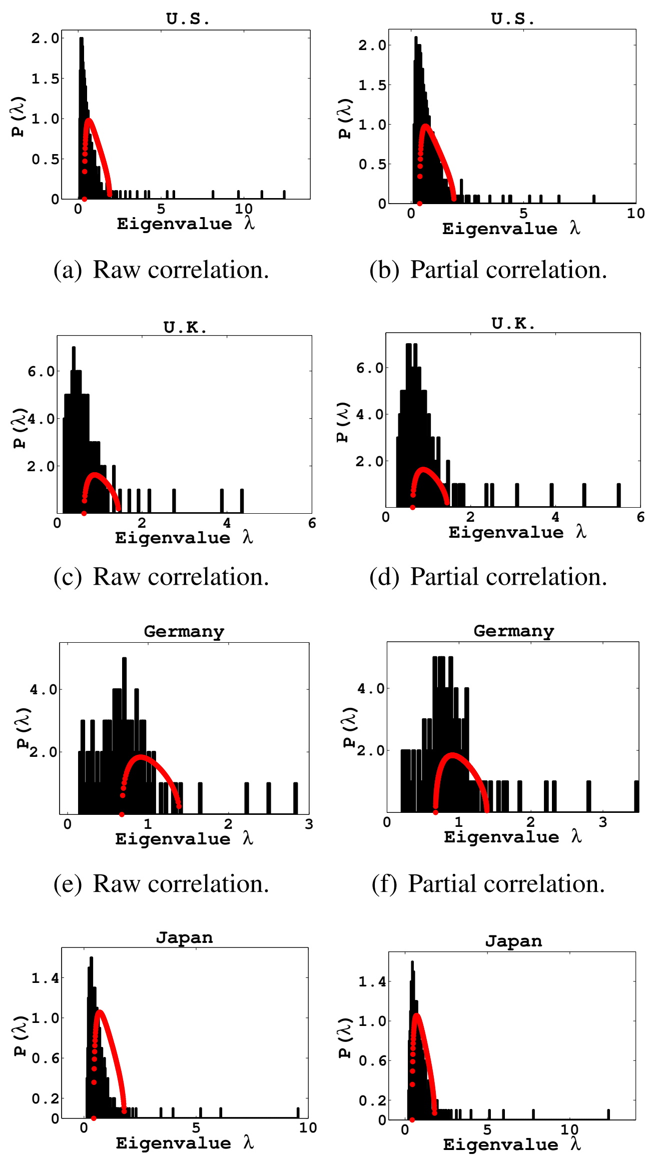
	Value of parameters			Eigenvalues deviating from λ_{max}	
Market	Q	λ_{min}	λ_{max}	Raw correlation	Partial correlation
U.S.	6.86	0.38	1.91	17 (4.22%)	26 (6.45%)
U.K.	24.0	0.63	1.45	8 (6.90%)	13 (11.2%)
Germany	31.4	0.68	1.39	6 (6.74%)	10 (11.2%)
Japan	8.57	0.43	1.80	11 (3.49%)	23 (7.30%)

The distributions of eigenvalues of the similarity measures from the U.S., U.K., German and Japanese stock markets are presented using raw and partial correlations in Fig. 1. The number and percentage of empirical eigenvalues that exceed the theoretical maximal eigenvalue, λ_{max}, for the raw and partial correlations and value of parameters for distributions of a random matrix are listed in

Table 2. Eigenvalues for the case of partial correlations for each market are more prone to exceed λ_{max}, which implies that there is more information present in these matrices, after removing the mediating effect of the market index. In the case of raw correlation, the U.K. stock market has the largest percentage of deviating eigenvalues and the Japanese stock market has the lowest percentage of deviating eigenvalues.

In factor models and financial RMT analysis, the largest eigenvalue is considered the principal eigenvalue, and its corresponding eigenvector is associated with the market mode, specifically, the general movement of the market. As such, an important question is the extent of additional information provided by the partial correlations, after removing the mediating effect of the market index. For example, one can ask whether the largest eigenvalue of the partial correlation matrix is similar to the second largest eigenvalue of the raw correlation matrix. We observe that the largest eigenvalue of raw correlation is generally speaking $3 \sim 5$ times larger than that of the partial correlation matrix, testifying to the importance of the common market factor in driving raw correlations among stocks. The value of the principal eigenvalue is 116.0 for the U.S. stock market, 36.4 for the U.K. stock market, 23.9 for the German stock market and 105.6 for the Japanese stock market. By contrast, the value of the largest eigenvalue for partial correlation is only 35.5 for the U.S., 8.2 for the U.K., 8.9 for Germany, and 20.2 for Japan.

To further understand the additional embedded information present in the partial correlations, we examine the values of the components of the largest eigenvector for both raw correlation and partial correlation for each market. The scatter plots of weights (magnitudes) of components of eigenvectors $\boldsymbol{v}_1$ from raw and partial correlations are given in Fig. 2. We plot each component of eigenvector $\boldsymbol{v}_1$ for both raw and partial correlations (gray circles in Fig. 2) and compare to comparable values obtained from random data (white circles in Fig. 2, see Appendix A for more information). Using raw correlations, all components of eigenvector $\boldsymbol{v}_1$ have positive values; however, the components of the largest eigenvector derived from partial correlations contain negative values in all stock markets. Moreover, in the case of partial correlation for the U.S. and Japanese markets, there are several negative values in the components of eigenvector $\boldsymbol{v}_1$. For Germany negative value in the partial correlation principal eigenvector are also observed; however, Germany has the smallest number of such negative values. Compared to the U.S., U.K. and Japanese stock markets, Germany's stock market is not significantly influenced by its index. All components of $\boldsymbol{v}_1$ derived from the random matrix have positive values for all stocks for both raw and partial correlation.



(a) Raw correlation. (b) Partial correlation.

(c) Raw correlation. (d) Partial correlation.

(e) Raw correlation. (f) Partial correlation.

(g) Raw correlation. (h) Partial correlation.

Fig. 1. Probability distribution of the eigenvalues of the raw and partial correlation matrices of U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets. The distributions are presented using black straight lines and the distributions of random matrices are presented using red circles.

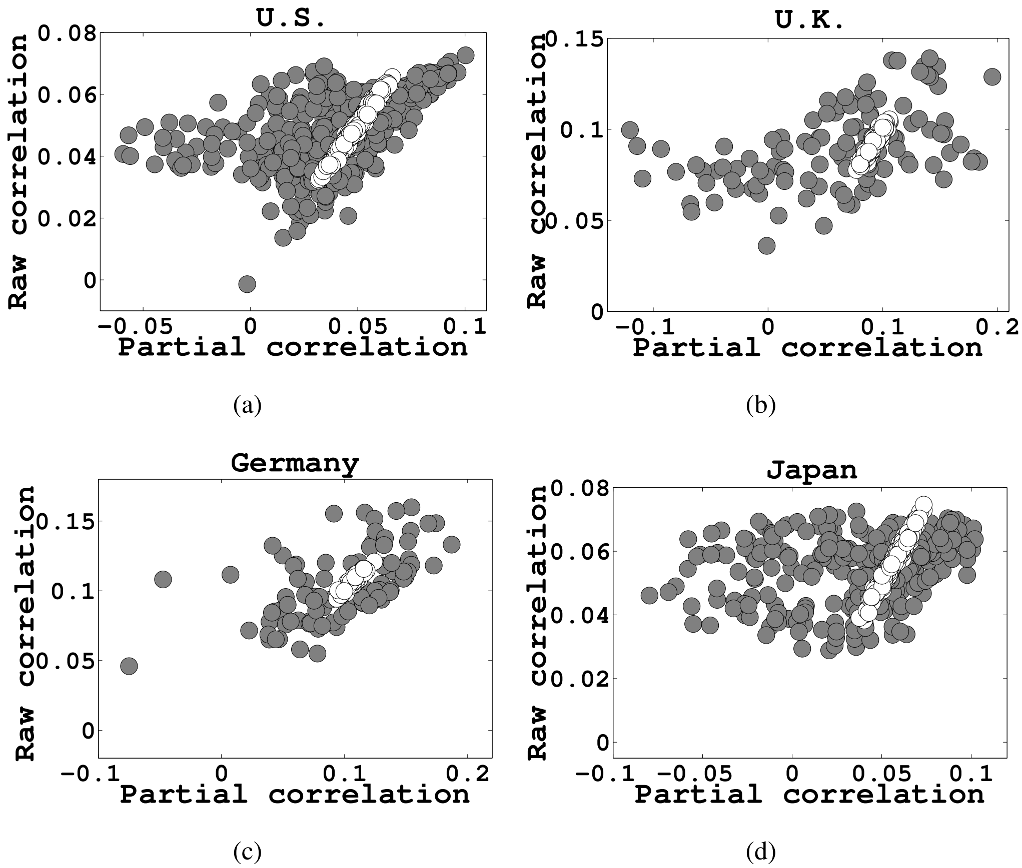


Fig. 2. The weight of components of eigenvector $\boldsymbol{v}_1$ from the U.S. (a), U.K. (b), Germany (c) and Japanese (d) stock markets. The x-axis indicates the value of components of $\boldsymbol{v}_1$ derived from partial correlation and y-axis indicates the value of components for raw correlation for corresponding stocks. Gray circles indicate components of $\boldsymbol{v}_1$ from each stock, and white circles are components of $\boldsymbol{v}_1$ from random data.

5. Uncovering the sectoral makeup of financial markets

To investigate and monitor the dynamical evolution of the structure of financial markets, we apply the SDR methodology. First, we study the SDR for the entire time period. The values of the SDR, Φ_s ($s = 1, \ldots, 10$), using $\boldsymbol{v}_1$ from raw and partial correlations during the period from January 2000 to December 2010 are presented in Table 3. The threshold τ is given as $\tau = 1/\sqrt{N}$ where N is the number of stocks for each stock market (see also Appendix B and Appendix C).

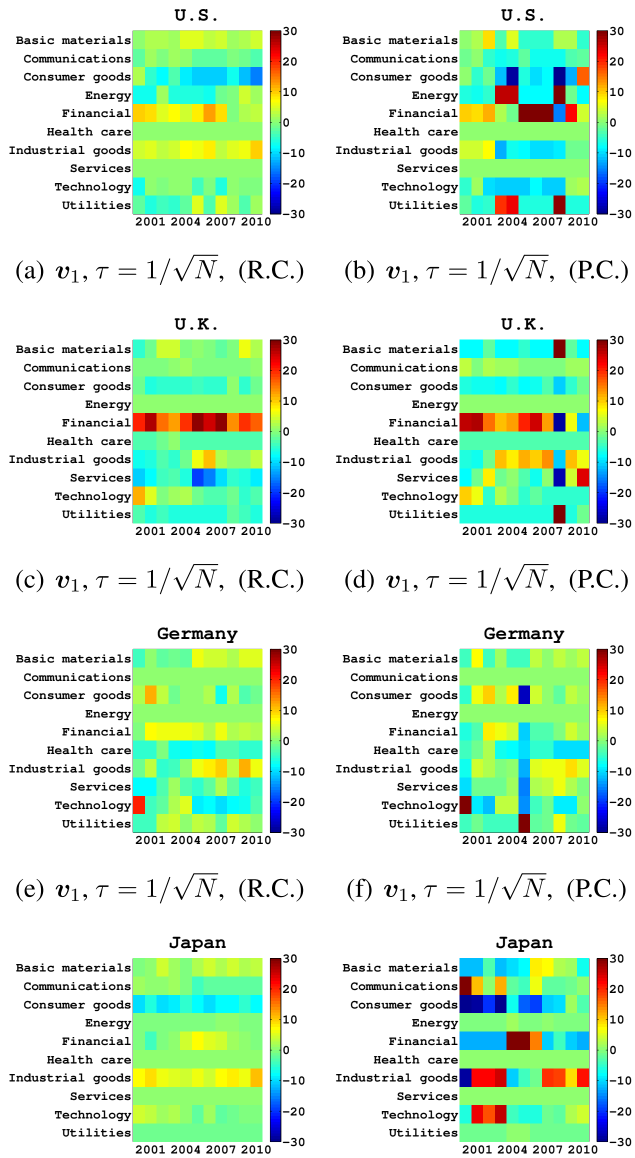
Table 3. The SDR from the largest eigenvector $\boldsymbol{v}_1$ using raw correlation (R.C.) and partial correlation (P.C.) from January 2000 to December 2010. The threshold is $\tau = 1/\sqrt{N}$, where N is the number of stocks for each market.

	U.S.		U.K.		Germany		Japan	
Sector	R.C.	P.C.	R.C.	P.C.	R.C.	P.C.	R.C.	P.C.
Basic materials	4.84%	3.55%	2.44%	-7.76%	5.51%	5.51%	4.61%	9.62%
Communications	-2.96%	-0.86%	-0.86%	-0.86%	0%	0%	-2.74%	-3.81%
Consumer goods	-11.5%	-7.47%	-4.22%	-5.08%	-1.97%	5.53%	-8.6%	-2.04%
Energy	2.64%	0.09%	0%	0%	0%	0%	-0.63%	0.87 %
Financial	11.6%	18.9%	26.7%	16.3%	3.76%	3.76%	2.59%	4.16%
Health care	0%	0%	-3.45%	-3.45%	-7.61%	-7.61%	0%	0%
Industrial goods	6.14%	5.34%	-2.18%	10.7%	7.64%	7.64%	6.51%	-3.44%
Services	0%	0%	-7.18%	1.36%	-5.73%	1.77%	0%	0%
Technology	-8.52%	-10.4%	-5.17%	-5.17%	-4.61%	-14.6%	-0.15%	-6.03%
Utilities	-2.22%	-5.72%	-6.03%	-6.03%	3.01%	-1.99%	-1.59%	0.67%

Table 3 demonstrates that implied sectoral importance can be meaningfully inferred from raw versus partial correlation. This emphasizes the influence of the different sectors with and without the mediating effect of the market index. For example, the SDR using $\boldsymbol{v}_1$ for the U.S. stock market exhibits a large value in the financial sector using both raw and partial correlations, while the basic materials sector is lower in the case of $\boldsymbol{v}_1$ using partial rather than raw correlation. This shows that the influence of the basic materials sectors to a large extent results from the market index. In the case of the SDR for the U.K. stock market, the basic material sector and financial sector is lower in the case of $\boldsymbol{v}_1$ for partial correlation. This again shows that the influence of both these sectors, which is large when studying the raw correlations, is significantly decreased once the effect of the index is removed and the underlying structure of the market is uncovered. The financial sector dominates in both the U.S. and U.K. stock markets, but has very little influence in the case of Germany and Japan, which emphasizes the difference in the structure and makeup of these four markets. Another esample is the Energy sector, which is found to be influential only for the U.S. and Japanese stock markets, emphasizing their strong dependence on energy resources.

Next, we examine dynamical changes and the evolution of SDR on both a yearly and monthly basis by using $\boldsymbol{v}_1$ for both raw correlation and partial correlation. We divide the stock return data for each stock market into yearly and monthly periods, and calculate raw and partial correlation matrices. The SDR was calculated for both raw and correlation matrices.

The transitions of SDR on a yearly basis for the U.S., U.K., German, and Japanese stock markets using $\boldsymbol{v}_1$ for raw correlation and partial correlation with threshold $\tau = 1/\sqrt{N}$ are presented in Fig. 3. We observe that the financial sector exhibited negative values in 2008, which correspond to the shocks in this sector resulting from the 2008 financial crisis in U.S. stock market. The footprints of the financial crisis can also be found when observing the negative or low values of SDR in the U.K. stock market, which are not observed in the case of Germany and Japan. In contrast, it can be observed, especially after removing the mediating effect of the index, that both Germany and Japan exhibit strong variations in the dominance of the different sectors.



(a) $\boldsymbol{v}_1, \tau = 1/\sqrt{N}$, (R.C.) (b) $\boldsymbol{v}_1, \tau = 1/\sqrt{N}$, (P.C.)

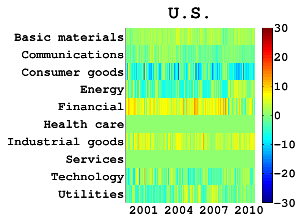
(c) $\boldsymbol{v}_1, \tau = 1/\sqrt{N}$, (R.C.) (d) $\boldsymbol{v}_1, \tau = 1/\sqrt{N}$, (P.C.)

(e) $\boldsymbol{v}_1, \tau = 1/\sqrt{N}$, (R.C.) (f) $\boldsymbol{v}_1, \tau = 1/\sqrt{N}$, (P.C.)

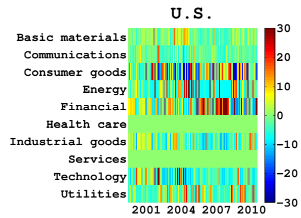
(g) $\boldsymbol{v}_1, \tau = 1/\sqrt{N}$, (R.C.) (h) $\boldsymbol{v}_1, \tau = 1/\sqrt{N}$, (P.C.)

Fig. 3. Yearly based SDR analysis obtained from the largest eigenvector $\boldsymbol{v}_1$ for the raws and partial correlations of U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets. The threshold is given as $\tau = 1/\sqrt{N}$.

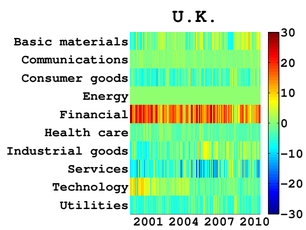
Next, we study the SDR on a monthly basis (Fig. 4). For the U.S. stock market, the monthly based SDR analysis provides important information on the change in the structure of the market. First, we observe that the raw correlations do not provide sufficient information. However, when removing the effect of the index, the changes in the underlying structure of the market become apparent. For example, the growing influence of the Financial sector is clearly visible, together with the weakening influence of the technology and industrial sectors. This provides new information into the evolution of this market leading up to the 2008 financial crisis. For the other three markets, similar results are observed to those found for the yearly based SDR analysis, emphasizing the dominance of the Financial sector in the U.K. stock market, and the changes in market structure observed for Germany and Japan.



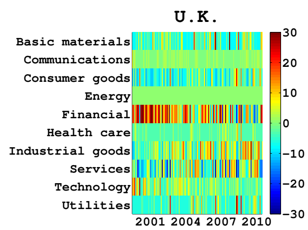
(a) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (R.C.)



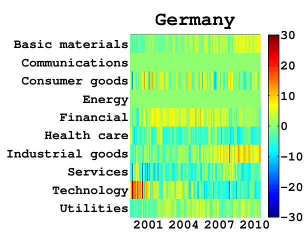
(b) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (P.C.)



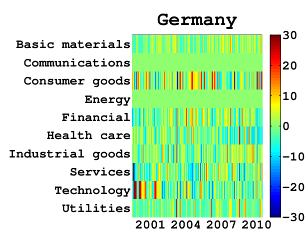
(c) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (R.C.)



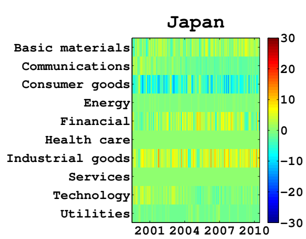
(d) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (P.C.)



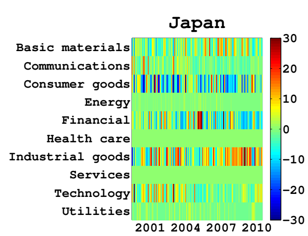
(e) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (R.C.)



(f) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (P.C.)



(g) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (R.C.)



(h) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (P.C.)

Fig. 4. Monthly based SDR analysis for the raw and partial correlations of the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets. The dominance of the Financial sector is observed in both U.S. and U.K. stock market, but not in the German and Japanese stock markets. Furthermore, the monthly based SDR analysis highlights changes in market structure, emphasizing changes that led up to the 2008 financial crisis.

6. SDR Investigation of the dominance of the financial sector

As an example of the application of the introduced methodology, we compare the SDR indicator to the index of implied volatility (VIX) for all stock markets on both a yearly and monthly basis (see Fig. 5), for the period of January 2006 to December 2010. High values for the VIX are interpreted as a forecast of high volatility of returns in the near future. In this example, we focus on the SDR values calculated for the Financial sector, for both yearly and monthly time horizons. The SDR derived for this sector is denoted as Φ_5.

We compare the CBOE VIX index for the period January 2006 until December 2010 to $-\Phi_5$, for the U.S. and U.K. stock markets, the VDAX index to $-\Phi_5$ for the German stock market and compare the Nikkei Stock Average Volatility Index for $-\Phi_5$ for the Japanese stock market. Since we define the SDR with a positive value as an expression of the dominance of a sector in a given time period, the negative of the value of SDR for financial sector, $-\Phi_5$, corresponds to the VIX for each market. When we compared $-\Phi_5$ to VIX on a yearly basis, the peak of VIX and $-\Phi_5$ are consistent in the case of the U.S. (Fig. 5(a)) and U.K. (Fig. 5(b)) stock markets for both raw and partial correlation cases. However, in the case of Germany and Japan, the peaks of $-\Phi_5$ with partial correlation do not agree with those of VIX. This indicates that the financial sector plays a more dominant role in the U.S. and U.K. stock markets, and we find that $-\Phi_5$ is consistent with the VIX index.

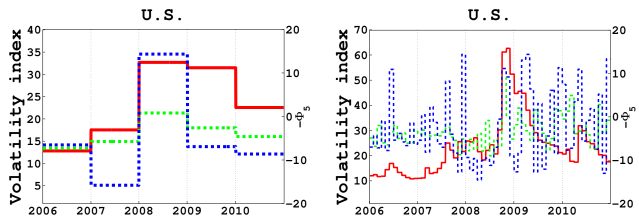
To quantitatively compare the SDR to the VIX, we make use of Granger Causality Analysis (GCA) to analyze the extent to which the SDR predicts changes in the VIX indexes, or vice versa. Granger causality uses temporal precedence to identify the direction of causality from information in the data [19, 44, 45, 46]. Thus, given the two time series $-\Phi_5$ and $I_{vix,k}$ (VIX index for country k), we can independently identify both the influence from $-\Phi_5$ to $I_{vix,k}$, and influence in the reverse direction with suitable models.

Let “y” and “x” be stationary time series. To test the null hypothesis that “x” does not Granger-cause “y”, one first finds the proper lagged values of “y” to include in a univariate autoregression of “y”:

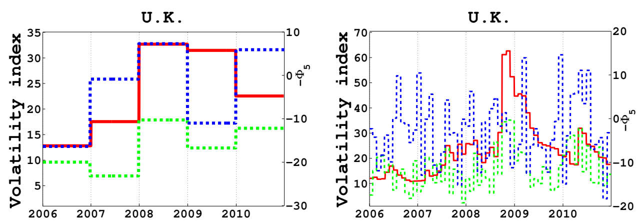
$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \cdots + a_m y_{t-m} + \text{residual}. \quad (10)$$

Next, the autoregression is augmented by including lagged values of “x”:

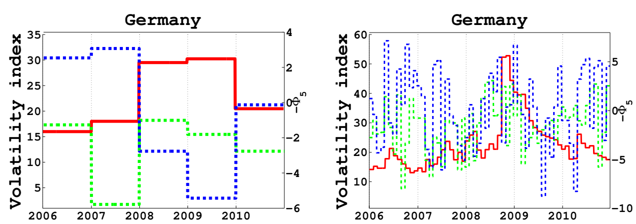
$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \cdots + a_m y_{t-m} + b_p x_{t-p} + \cdots + b_q x_{t-q} + \text{residual} \quad (11)$$



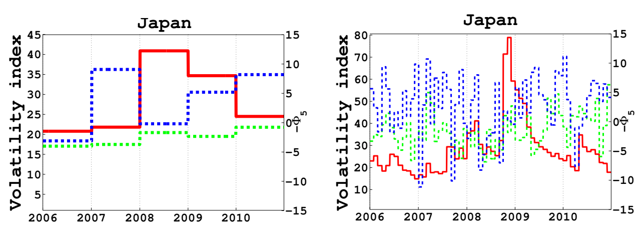
(a) Yearly basis SDR. (b) Monthly basis SDR.



(c) Yearly basis SDR. (d) Monthly basis SDR.



(e) Yearly basis SDR. (f) Monthly basis SDR.



(g) Yearly basis SDR. (h) Monthly basis SDR.

Fig. 5. The comparison of Financial sector based SDR and VIX for the period of 2006-2010, on a yearly and monthly based time horizon for the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets. The red line corresponds to the VIX index on either yearly or monthly time horizon, and green and blue dashed lines correspond to dynamical change of $-\Phi_5$, which expresses deactivation of SDR for financial sector using $\boldsymbol{v}_1$ for raw and partial correlation, respectively. The left y-axis displays the value of volatility index and the right y-axis displays the value of $-\Phi_5$.

One retains in this regression all lagged values of “x” that are individually significant according to their t-statistics, provided that collectively they add explanatory power to the regression according to a standard F-test (whose null hypothesis is no explanatory power jointly added by the “x”’s). In the notation of the above augmented regression, “p” is the shortest, and “q” is the longest, lag length for which the lagged value of “x” is significant. The null hypothesis that “x” does not Granger-cause “y” is not rejected if and only if no lagged values of “x” are retained in the regression. A measure of linear dependence $F_{-\Phi_5, I_{VIX,k}}$, between $-\Phi_5$ and $I_{VIX,k}$, which implements Granger causality in terms of vector autoregressive models, has been proposed by Geweke [47]. $F_{-\Phi_5, I_{VIX,k}}$ is the sum of three components

$$F_{-\Phi_5, I_{VIX,k}} = F_{-\Phi_5 \to I_{VIX,k}} + F_{I_{VIX,k} \to -\Phi_5} + F_{-\Phi_5 \cdot I_{VIX,k}}. \tag{12}$$

$F_{-\Phi_5 \cdot I_{VIX,k}}$ is a measure of the total linear dependence between the series $-\Phi_5$ and $I_{VIX,k}$. If nothing of the value at a given instant of one can be explained by a linear combination of all the values (past, present, and future) of the other, $F_{-\Phi_5 \cdot I_{VIX,k}}$ will evaluate to zero. This term will then contain no directional information, and implies residual correlations in the data that cannot be assigned to causally directed influence. $F_{-\Phi_5 \to I_{VIX,k}}$ is a measure of linear directed influence from $-\Phi_5$ to $I_{VIX,k}$. If past values of $-\Phi_5$ improve the prediction of the current value of $I_{VIX,k}$, then $F_{-\Phi_5 \to I_{VIX,k}} > 0$. The results of the GCA are presented in Table 4. We find that the SDR $-\Phi_5$ Granger causes the VIX index for both the U.S. and U.K., when calculated from raw correlations.

Furthermore, to study the similarity between $-\Phi_5$ and the VIX indexes, we perform a cross-correlation analysis between the two. Cross correlation is a standard method in signal processing for estimating the degree to which two series are correlated at different time lags. The discrete cross-correlation function between two time series X and Y is given by [48]

$$XCF(d) = \frac{\sum_{i=1}^{N-d} \left[(X(i) - \langle X \rangle) \cdot (Y(i-d) - \langle Y \rangle) \right]}{\sqrt{\sum_{i=1}^{N-d} (X(i) - \langle X \rangle)^2} \cdot \sqrt{\sum_{i=1}^{N-d} (Y(i-d) - \langle Y \rangle)^2}} \tag{13}$$

$$d = \pm 1, \pm 2, ..., \pm N - 1 \tag{14}$$

where *d* is the lag. We consider values of $d = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6$. We investigate the monthly values of $-\Phi_5$ and $I_{VIX,k}$ for each year separately, for both raw correlations and partial correlation cases (Fig. 6). In this analysis, the

Table 4. The table represents the results of Granger causality tests between $-\Phi_5$ and VIX for raw correlation and partial correlation for all stock markets. If the value of F-statistic is larger than the value of critical value, $-\Phi_5$ Granger cause VIX. The significance level is 0.05 for all Granger causality tests.

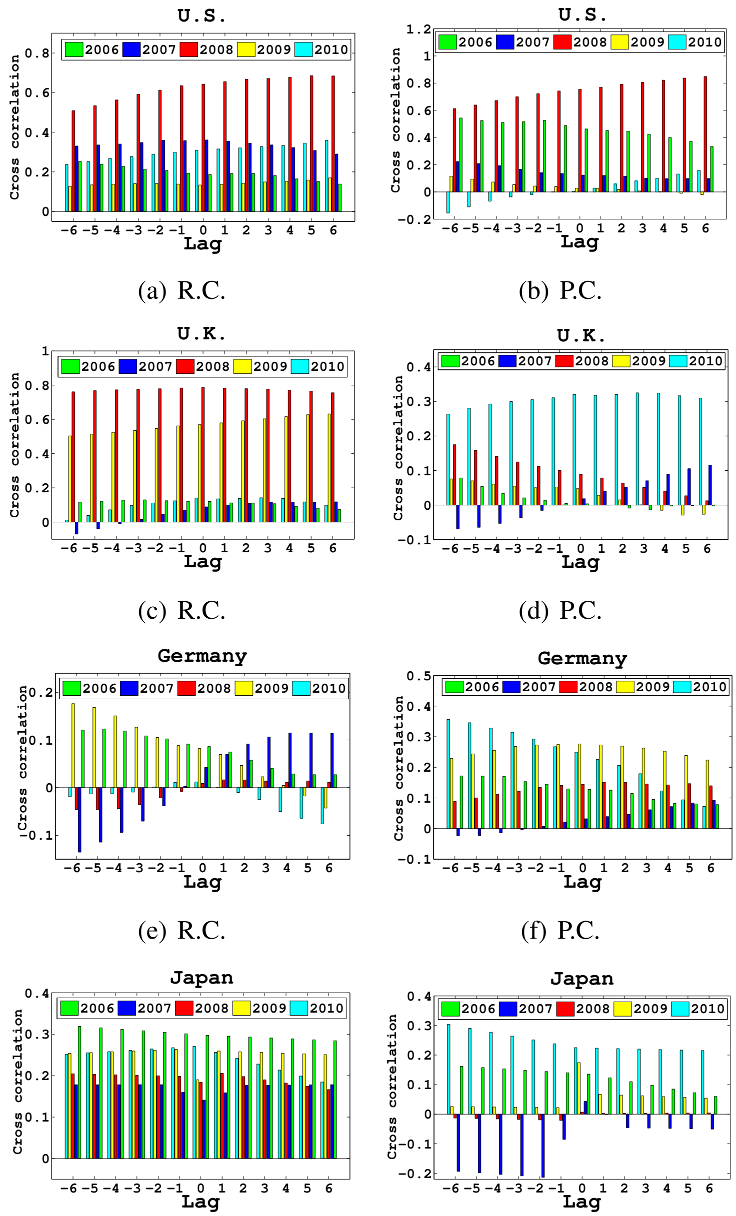
Market	Granger causality test	F-statistic	Critical value
U.S. (R.C.)	$-\Phi_5$ Granger VIX	7.48	3.85
U.S. (P.C.)	$-\Phi_5$ does not Granger VIX	3.32	3.85
U.K. (R.C.)	$-\Phi_5$ Granger VIX	12.5	3.85
U.K. (P.C.)	$-\Phi_5$ does not Granger VIX	0.56	3.85
Germany (R.C.)	$-\Phi_5$ does not Granger VIX	0.71	3.85
Germany (P.C.)	$-\Phi_5$ does not Granger VIX	1.27	3.85
Japan (R.C.)	$-\Phi_5$ does not Granger VIX	0.21	3.85
Japan (P.C.)	$-\Phi_5$ does not Granger VIX	2.78	3.85

large value of $d = +1$ means that the VIX would have similarity to previous month's value of $-\Phi_5$, and a large value of $d = -1$ means $-\Phi_5$ would have similarity to previous month's value of VIX.

7. Summary and discussion

In this paper we present a new measure to investigate the functional micro structure of financial markets, the Sector Dominance Ratio (SDR). To demonstrate the capabilities of this measure, we analyze data from the U.S., U.K., German, and Japanese stock markets from January 2000 to December 2010, a period in which these markets went through different structural changes. Using this data, our aim is threefold: 1) introduce the SDR measure to study the micro structure of financial markets; 2) use the SDR to emphasize the structural differences between the investigated markets; and 3) to present and emphasize the additional information embedded in the stock partial correlations, after removing the mediating effect of the market index.

Significant patterns for the financial sector are evident from the SDR calculated for the raw and partial correlations, especially in the case of the U.S. and U.K. stock markets. Compared to Germany and Japan, these two markets exhibit a strong dominance of the financial sector, as is captured by the SDR analysis. This is further emphasized when we observe that for the U.S. market, the largest eigenvector from partial correlations and the second largest eigenvector from raw



(a) R.C. (b) P.C.

(c) R.C. (d) P.C.

(e) R.C. (f) P.C.

(g) R.C. (h) P.C.

Fig. 6. Cross correlation function (XCF) between the SDR calculated for the financial sector, $-\Phi_5$, and VIX by monthly basis for each year using raw and partial correlation, for the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock market.

correlations are similar and exhibit similar values as the SDR of the financial sector. This finding emphasizes and highlights the extent of the dominance of the financial sector in this market. With regard to the time period surrounding the 2008 financial crisis, the SDR in the financial sectors in the U.S. and U.K. showed negative values in 2008; this tendency was also seen in Germany, but the Japanese stock market did not show such large negative values. This demon- strates the differences between the structural makeup of these four markets, and the extent of the damage of the 2008 financial crisis.

To further study the impact of the financial sector in these four markets, and as an example of the application of the SDR methodology, we compare the SDR measure to the VIX in all stock markets using both yearly and monthly averages. We compare the VIX index with the SDR for the financial sector, $-\Phi_5$, on a yearly basis and find that it is consistent with VIX in the U.S. and U.K. stock markets, especially in the partial correlation case. However, peaks of $-\Phi_5$ do not coincide with those of VIX for the German or Japanese stock markets. This is consistent with the fact that the financial sector plays a more dominant role in U.S. and U.K. stock market; in those markets, therefore, $-\Phi_5$ would be relevant to VIX. The Granger causality tests indicates that $-\Phi_5$ using raw correlations Granger-causes VIX in the U.S. and U.K. stock markets. We further examined the cross correlations between $-\Phi_5$ with the largest eigenvector from raw correlation and partial correlation matrices; therefore, the $-\Phi_5$ would be an indicator for VIX indexes for both U.S. and U.K. stock markets. We conclude that $-\Phi_5$ does not Granger cause VIX for the German and Japanese stock markets, because while the financial sector plays a dominant role in U.S. and U.K. stock markets, it does not do so in the German and Japanese stock markets. For the cross correlation analysis, we examine the cross correlation between $-\Phi_5$ and VIX for each year for all stock markets. The similarities between $-\Phi_5$ and VIX at $d = -1, \ldots, -6$ are observed especially for partial correlations case for all stocks and in the raw correlation case for the German stock market. This indicates that the $-\Phi_5$ using partial correlation can be useful for predicting the behavior of VIX indexes.

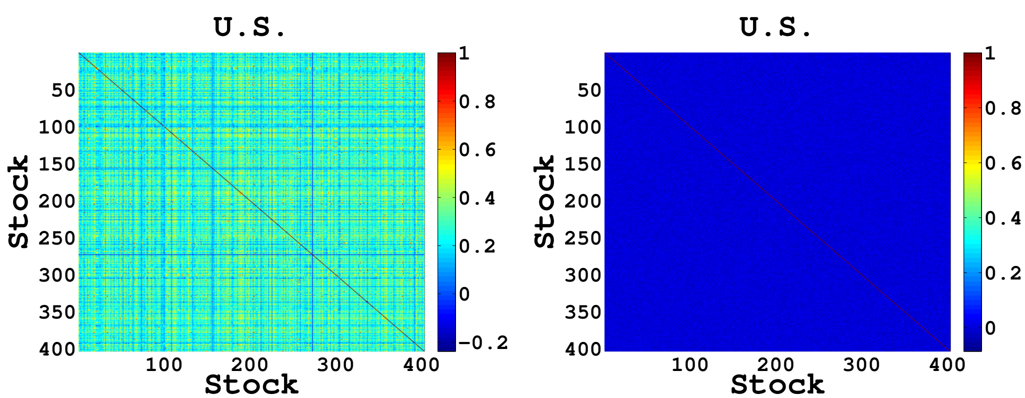
In summary, we present a new measure to quantify the evolution of activity of economic sectors reflected in financial markets. The SDR provides a quantitative measure to study structural versus functional activity in financial markets. As such, it provides a means to identify economic sectors with increasing dominance, which could indicate an increase in systemic risk. Further, the SDR also provides a means to identify markets in which the functional makeup is extremely different from the structural makeup, which could indicate structural reforms. As such, the SDR parameter provides both practitioners and policy makers an important tool useful for understanding the functional structure of financial markets, and its dynamics, and provides valuable information for monitoring and managing systemic risk.

Acknowledgments

We thank Shlomo Havlin for his insightful comments on this work. LU and TA acknowledge financial support by JSPS International Training Program (ITP). DYK and HES acknowledge financial support by DTRA and ONR.

Appendix A. Results for shuffled data

To validate the empirical values observed for the SDR, we make use of a shuffling analysis procedure. For each stock, we first shuffle its price time series in time, thus breaking the temporal order. We then recalculate the returns, the stock correlation matrices, and from them the shuffled based SDR values. For example, in Figure A.7 we compare the raw correlations calculated for the U.S. market empirical data versus those calculated from the randomly shuffled data. Using the same color code to represent the correlation value, it is clear that the correlations for the random case are significantly different, and are very close to zero.



(a) R.C. matrix

(b) R.C. matrix for Shuffled data.

Fig. A.7. Stock raw correlation matrix for the U.S. market, calculated from empirical data (a) and from the random ally shuffled time series (b), using the same color code.

Appendix B. Comparing the effect of the different thresholds

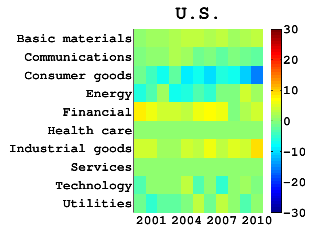
We compare the results of the SDR calculated using the $\tau = 1/\sqrt{n}$ threshold with other threshold values. The SDR, Φ_s ($s = 1, \ldots, 10$), using $\boldsymbol{v}_1$ from raw and partial correlations during the period from January 2000 to December 2010 are presented in Table B.5. The threshold τ is $\tau = \mu$ for $\boldsymbol{v}_1$, where μ is the mean of components of eigenvectors $\boldsymbol{v}_1$ for each market. The components of the eigenvectors $\boldsymbol{v}_1$ for raw correlation are always positive; on the other hand, the components of the eigenvectors $\boldsymbol{v}_2$ for raw correlation and partial correlation take both positive and negative values; therefore, we use the value of τ for $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

Table B.5 shows the SDR for basic materials sector decreased in the German, U.K. and U.S. stock markets in the case of partial correlation. The SDR for the communications sector showed larger percentage in the U.K. stock market and the SDR for consumer goods showed a larger percentage in the German, U.S., and Japanese stock markets in the case of partial correlation. The SDR for energy sector increased in the U.S. and Japanese stock markets and the SDR for the financial sector decreased for all stock markets in the case of partial correlation.

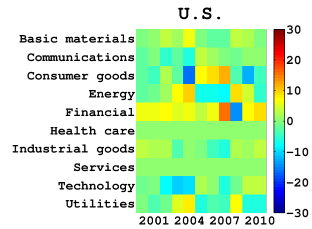
Table B.5. The SDR obtained from the largest eigenvector $\boldsymbol{v}_1$ for raw correlation (R.C.) and partial correlation (P.C.). The threshold is $\tau = \mu$, where μ is the mean of components of eigenvector $\boldsymbol{v}_1$ for each market from January 2000 to December 2010.

	U.S.		U.K.		Germany		Japan	
Sector	R.C.	P.C.	R.C.	P.C.	R.C.	P.C.	R.C.	P.C.
Basic materials	4.06%	1.62%	5.20%	-3.41%	5.26%	4.39%	4.27%	5.06%
Communications	-2.81%	-2.80%	-0.86%	0.59%	0%	0%	-2.76%	-3.28%
Consumer goods	-9.85%	-4.96%	-4.78%	-5.99%	-0.14%	6.36%	-8.12%	1.89%
Energy	2.79%	3.77%	0%	0%	0%	0%	-0.63%	0.43 %
Financial	3.39%	2.09%	25.3%	15.0%	10.47%	9.98%	2.80%	1.98%
Health care	0%	0%	-3.45%	-3.45%	-7.67%	-5.66%	0%	0%
Industrial goods	4.66%	1.73%	-2.94%	4.15%	7.15%	7.64%	6.30%	-1.13%
Services	0%	0%	-7.22%	4.30%	-5.97%	2.04%	0%	0%
Technology	-7.76%	-5.80%	-5.17%	-5.17%	-4.85%	-14.6%	-0.27%	-6.03%
Utilities	-1.57%	-3.52%	-6.03%	-6.03%	2.82%	-2.26%	-1.59%	1.07%

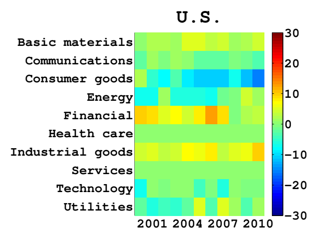
The yearly based SDR derived from $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ using raw and partial correlations for the U.S. stock market are presented in Fig. B.8. We calculate the yearly based SDR for the U.S. stock market using $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, and study the effect of



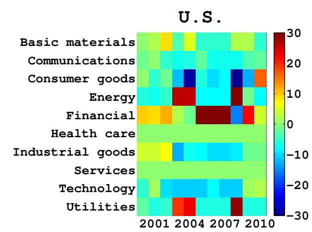
(a) $\boldsymbol{v}_1, \tau = \mu,$ (R.C.)



(b) $\boldsymbol{v}_1, \tau = \mu,$ (P.C.)



(c) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (R.C.)

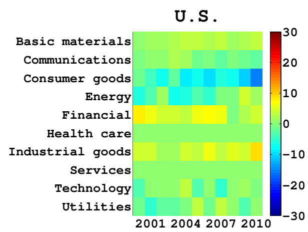


(d) $\boldsymbol{v}_1, \tau = 1/\sqrt{N},$ (P.C.)

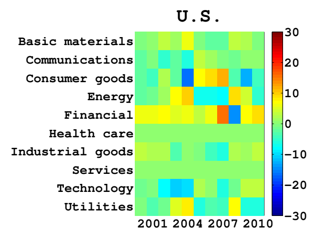
Fig. B.8. The yearly based SDR calculated from $\boldsymbol{v}_1$ for the U.S. stock market for different thresholds and for the first and second largest eigenvectors: (a) SDR calculated from $\boldsymbol{v}_1$ and $\tau = \mu$ for raw correlations; (b) SDR calculated from $\boldsymbol{v}_1$ and $\tau = \mu$ for partial correlations; (c) SDR calculated from $\boldsymbol{v}_1$ and $\tau = 1/\sqrt{N}$ for raw correlations; and (d) SDR calculated from $\boldsymbol{v}_1$ and $\tau = 1/\sqrt{N}$ for partial correlations.

different threshold values. In the case of the SDR using $\boldsymbol{v}_1$ for raw correlation and partial correlation with the threshold $\tau = \mu$, the consumer goods, energy and utilities sector show a larger SDR value using partial correlation. The changes in SDR of the financial sector are also greater using partial correlation. Low SDR values for the financial sector are observed for both raw correlation and partial correlation in 2008. In the case of the SDR for $\boldsymbol{v}_2$ for raw and partial correlations with threshold $\tau = 0$, the technology sector shows a higher value of SDR in raw correlation and partial correlation, compared to that of the SDR obtained from $\boldsymbol{v}_1$ with threshold $\tau = \mu$. Low values of the SDR for the financial sector were also observed during 2008.

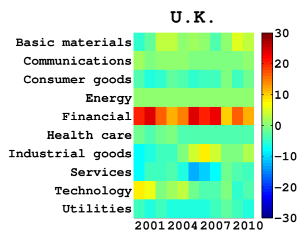
The yearly based and monthly based SDR calculated from $\boldsymbol{v}_1$ with threshold $\tau = \mu$ for the U.S., U.K., German, and Japanese stock markets are presented in Fig. B.9 and Fig. B.10, respectively, for both raw correlation and partial correlation.



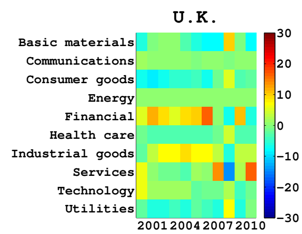
(a) $\boldsymbol{v}_1$, $\tau = \mu$, (R.C.)



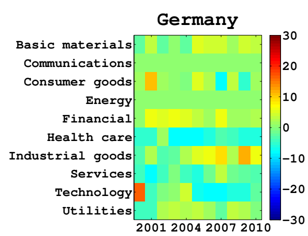
(b) $\boldsymbol{v}_1$, $\tau = \mu$, (P.C.)



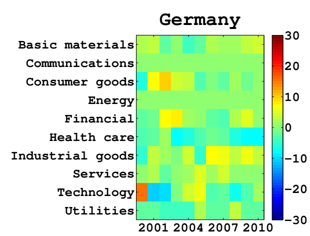
(c) $\boldsymbol{v}_1$, $\tau = \mu$, (R.C.)



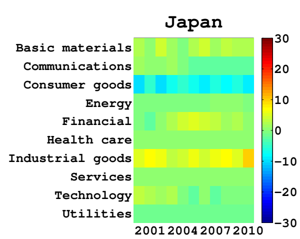
(d) $\boldsymbol{v}_1$, $\tau = \mu$, (P.C.)



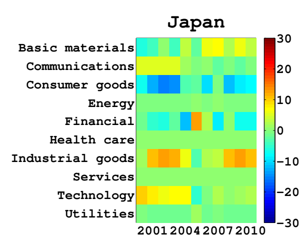
(e) $\boldsymbol{v}_1$, $\tau = \mu$, (R.C.)



(f) $\boldsymbol{v}_1$, $\tau = \mu$, (P.C.)

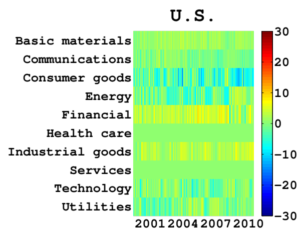


(g) $\boldsymbol{v}_1$, $\tau = \mu$, (R.C.)

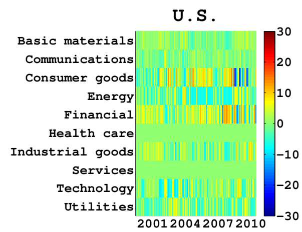


(h) $\boldsymbol{v}_1$, $\tau = \mu$, (P.C.)

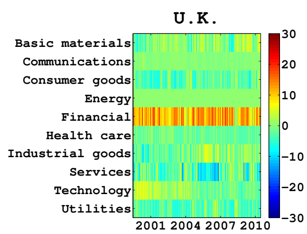
Fig. B.9. The yearly based SDR calculated from $\boldsymbol{v}_1$ with $\tau = \mu$ for raw and partial correlations, for the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets.



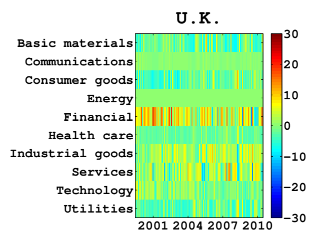
(a) $\boldsymbol{v}_1$, $\tau = \mu$, (R.C.)



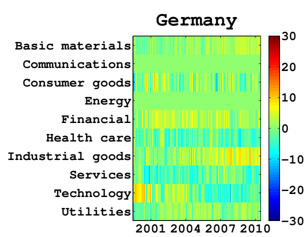
(b) $\boldsymbol{v}_1$, $\tau = \mu$, (P.C.)



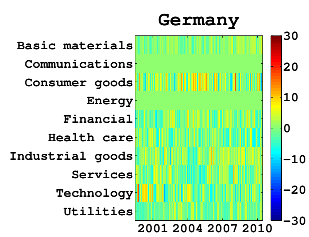
(c) $\boldsymbol{v}_1$, $\tau = \mu$, (R.C.)



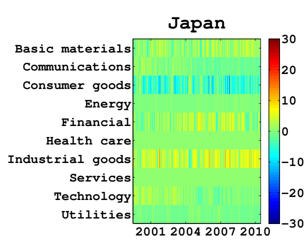
(d) $\boldsymbol{v}_1$, $\tau = \mu$, (P.C.)



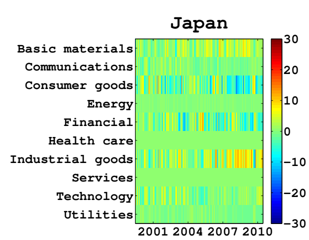
(e) $\boldsymbol{v}_1$, $\tau = \mu$, (R.C.)



(f) $\boldsymbol{v}_1$, $\tau = \mu$, (P.C.)



(g) $\boldsymbol{v}_1$, $\tau = \mu$, (R.C.)



(h) $\boldsymbol{v}_1$, $\tau = \mu$, (P.C.)

Fig. B.10. The monthly based SDR calculated from $\boldsymbol{v}_1$ with $\tau = \mu$ for raw and partial correlations, for the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets.

Appendix C. Results for the second-largest eigenvector

Table C.6 and Table C.7 present the values of the SDR, Φ_s $(s = 1, \ldots, 10)$, using $\boldsymbol{v}_2$ from raw and partial correlations during the period January 2000 to December 2010. The threshold used for the calculation is $\tau = 0$ for $\boldsymbol{v}_2$ in Table C.6 and $\tau = 1/\sqrt{N}$ for $\boldsymbol{v}_2$ in Table C.7. While the components of eigenvectors $\boldsymbol{v}_1$ for raw correlation are always positive, the components of eigenvectors $\boldsymbol{v}_2$ for raw correlation and partial correlation take both positive and negative values; therefore, we changed the value of τ for $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

Table C.6 implies that the SDR for basic materials sector decreased in all stock markets; however, the SDR for the financial sector increased in all markets. The similarities are not observed among SDR for $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ for both raw and partial correlations.

The SDR obtained for a different threshold τ using $\boldsymbol{v}_2$ for raw and partial correlations during the period January 2000 to December 2010 are presented in Table C.7; the threshold τ is given as $\tau = 1/\sqrt{N}$ where N is the number of stocks for each stock market. The SDR listed in Table C.7 shows larger values, especially in the finance, industrial goods and technology sectors compared to the results using thresholds $\tau = \mu$ or $\tau = 0$ which are listed in Table B.5 and Table C.6.

Table C.6. The SDR obtained from the second largest eigenvector $\boldsymbol{v}_2$ for raw correlation (R.C.) and partial correlation (P.C.) from January 2000 to December 2010. The threshold is $\tau = 0$.

	U.S.		U.K.		Germany		Japan	
Sector	R.C.	P.C.	R.C.	P.C.	R.C.	P.C.	R.C.	P.C.
Basic materials	-2.07%	-2.78%	-4.86%	-7.76%	3.51%	2.82%	-1.62%	-7.26%
Communications	2.66%	2.51%	0.59%	1.70%	0%	0%	3.38%	1.75%
Consumer goods	-2.81%	-0.74%	-6.00%	-7.78%	5.03%	4.73%	-10.4%	9.27%
Energy	-7.49%	-7.45%	0%	0%	0%	0%	-0.63%	0.75%
Financial	12.0%	13.7%	7.78%	12.6%	-3.24%	-1.48%	-5.19%	9.15%
Health care	-6.11%	-2.79%	-3.45%	-0.88%	0%	0%	0%	0%
Industrial goods	-1.69%	-5.13%	2.70%	-7.78%	5.64%	7.15%	10.3%	-13.7%
Services	0%	0%	7.20%	5.74%	6.27%	1.34%	0%	0%
Technology	5.87%	6.31%	2.07%	10.2%	-14.6%	-14.6%	5.73%	-1.17%
Utilities	-6.45%	-6.45%	-6.03%	-6.03%	3.51%	2.82%	-1.59%	1.19%

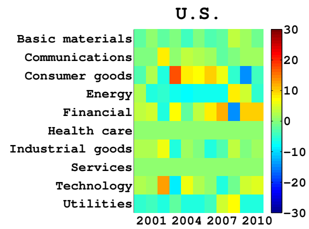
Table C.7. The SDR from the second largest eigenvector $\boldsymbol{v}_2$ using raw correlation (R.C.) and partial correlation (P.C.) from January 2000 to December 2010. The threshold is $\tau = 1/\sqrt{N}$, where N is the number of stocks for each market.

	U.S.		U.K.		Germany		Japan	
Sector	R.C.	P.C.	R.C.	P.C.	R.C.	P.C.	R.C.	P.C.
Basic materials	-5.21%	-5.21%	-7.76%	-7.76%	15.5%	6.62%	-9.35%	-9.51%
Communications	-4.97%	-7.20%	6.28%	6.28%	0%	0%	6.61%	-1.89%
Consumer goods	-7.08%	-9.94%	-10.3%	-3.20%	-6.97%	6.37%	-21.3%	25.7%
Energy	-7.94%	-7.94%	0%	0%	0%	0%	-0.63%	-0.63%
Financial	31.3%	58.4%	-21.3%	-14.2%	-1.24%	-11.2%	-10.3%	6.85%
Health care	0%	0%	-3.45%	-3.45%	-10.1%	1.0%	0%	0%
Industrial goods	-10.7%	-10.5%	32.5%	-10.3%	37.6%	-1.25%	19.7%	-20.6%
Services	0%	0%	-6.16%	8.13%	-15.7%	-15.7%	0%	0%
Technology	11.1%	-11.2%	16.3%	30.5%	-14.6%	-14.6%	16.9%	-6.03%
Utilities	-6.45%	-6.45%	-6.03%	-6.03%	-4.49%	28.8%	-1.59%	6.11%

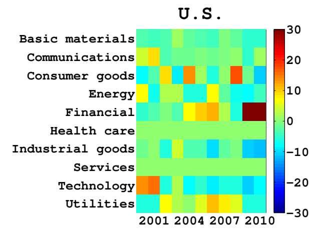
Table C.7 shows that the SDR using $\boldsymbol{v}_2$ for the U.S. stock market has a large value for the financial sector for both raw and partial correlations. The value of SDR using $\boldsymbol{v}_2$ and partial correlation is more than 50% in the case of the financial sector. The basic materials sector is not detected in the case of $\boldsymbol{v}_2$ using raw correlation. The basic material and financial sector SDRs in the U.K. fall in the case of $\boldsymbol{v}_2$ for both raw and partial correlations. In contrast, the SDR for the communications sector increased in the case of $\boldsymbol{v}_2$ for both raw and partial correlations. The technology sector also exhibits larger values in the case of SDR using $\boldsymbol{v}_2$ for both raw and partial correlations. The SDR of the industrial goods sector using $\boldsymbol{v}_2$ for raw correlation also shows a large value. The SDR for the German stock market shows a large value in the basic materials and industrial goods sectors for $\boldsymbol{v}_2$ and raw correlation case, and in the utilities sector for $\boldsymbol{v}_2$ and the partial correlation case. The services and technology sectors are not detected in the case of $\boldsymbol{v}_2$ for either raw or partial correlations. Further, we find that the SDR for the Japanese stock market has a large value in the industrial goods and technology sectors for $\boldsymbol{v}_2$ and the raw correlation case, and in consumer goods sector for $\boldsymbol{v}_2$ for the partial correlation case.

Comparing the values of SDR using different thresholds (Table B.5, Table C.6, Table C.7, and Table 3), one can observe that changing the threshold τ increases the difference in the SDR between the second largest eigenvector $\boldsymbol{v}_2$ and the largest eigenvector $\boldsymbol{v}_1$ for both raw and partial correlations. It is assumed that the SDR derived from components of the second largest eigenvector $\boldsymbol{v}_2$ are relatively more sensitive to noise than the SDR derived from the largest eigenvector $\boldsymbol{v}_1$. It is expected that the second largest eigenvalue for raw correlation and largest eigenvalue for partial correlation are similar to each other; however, the similarity of the second largest eigenvalue for raw correlation and the largest eigenvalue for partial correlation is not found in the analysis of SDR.

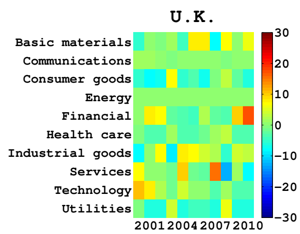
The yearly based and monthly based SDR from $\boldsymbol{v}_2$ with the threshold $\tau = 0$ for the U.S., U.K., German and Japanese stock markets are presented in Fig. C.11. and Fig. C.12, respectively, for both raw and partial correlations. The transitions of SDR on a yearly basis for the U.S., U.K., German, and Japanese stock markets using $\boldsymbol{v}_2$ for raw correlation and partial correlation with threshold $\tau = 1/\sqrt{N}$ are presented in Fig. C.13. Unlike the SDR using $\boldsymbol{v}_1$ for raw and partial correlation, it is difficult to discern tendencies for the different sectors with respect to time. The transitions of SDR on a monthly basis for the U.S., U.K., German, and Japanese stock markets using $\boldsymbol{v}_2$ for raw correlation and partial correlation with threshold $\tau = 1/\sqrt{N}$ are presented in Fig. C.14.



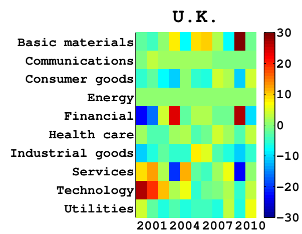
(a) $\boldsymbol{v}_2$, $\tau = 0$, (R.C.)



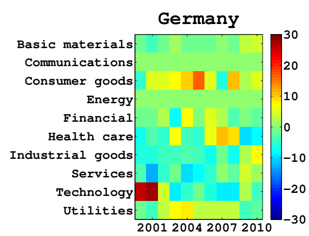
(b) $\boldsymbol{v}_2$, $\tau = 0$, (P.C.)



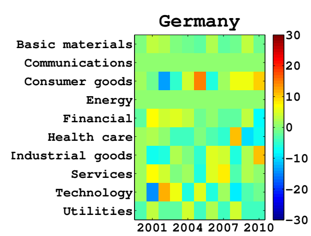
(c) $\boldsymbol{v}_2$, $\tau = 0$, (R.C.)



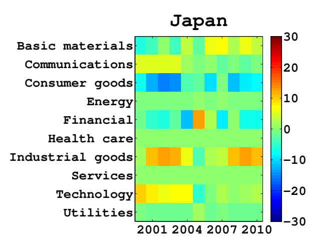
(d) $\boldsymbol{v}_2$, $\tau = 0$, (P.C.)



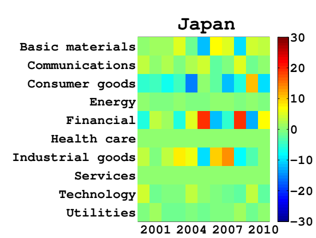
(e) $\boldsymbol{v}_2$, $\tau = 0$, (R.C.)



(f) $\boldsymbol{v}_2$, $\tau = 0$, (P.C.)

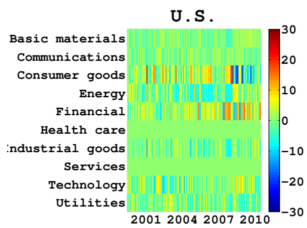


(g) $\boldsymbol{v}_2$, $\tau = 0$,, (R.C.)

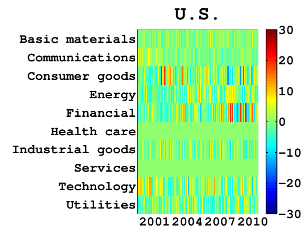


(h) $\boldsymbol{v}_2$, $\tau = 0$, (P.C.)

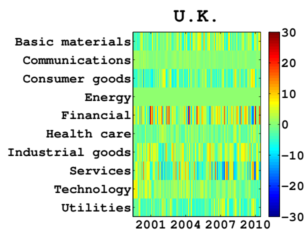
Fig. C.11. The yearly based SDR calculated from $\boldsymbol{v}_2$ with $\tau = 0$ for both raw and partial correlation for the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets.



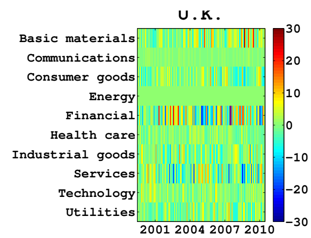
(a) $\boldsymbol{v}_2, \tau = 0,$ (R.C.)



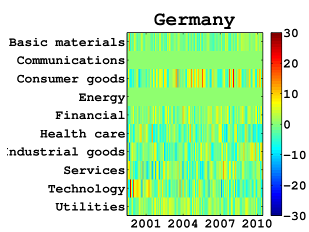
(b) $\boldsymbol{v}_2, \tau = 0,$ (P.C.)



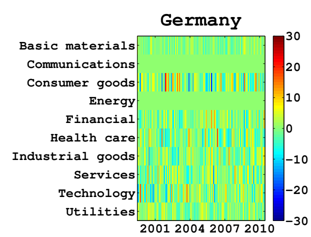
(c) $\boldsymbol{v}_2, \tau = 0,$ (R.C.)



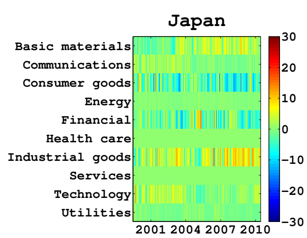
(d) $\boldsymbol{v}_2, \tau = 0,$ (P.C.)



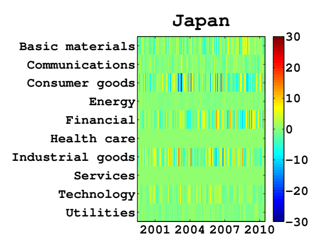
(e) $\boldsymbol{v}_2, \tau = 0,$ (R.C.)



(f) $\boldsymbol{v}_2, \tau = 0,$ (P.C.)

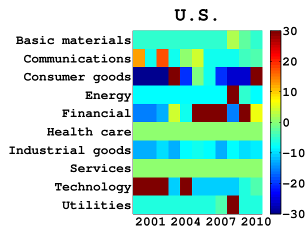


(g) $\boldsymbol{v}_2, \tau = 0,,$ (R.C.)

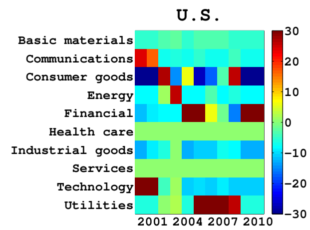


(h) $\boldsymbol{v}_2, \tau = 0,$ (P.C.)

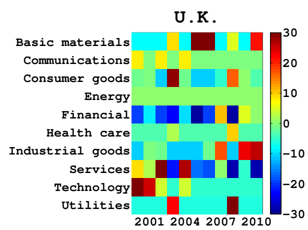
Fig. C.12. The monthly based SDR calculated from $\boldsymbol{v}_2$ with $\tau = 0$ for both raw and partial correlation for the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets.



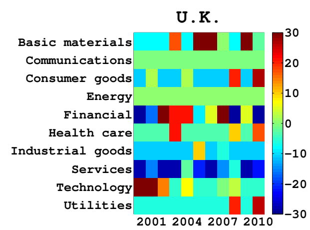
(a) $\boldsymbol{v}_2$, $\tau = 1/\sqrt{N}$, (R.C.)



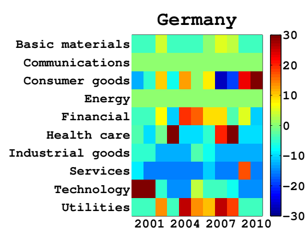
(b) $\boldsymbol{v}_2$, $\tau = 1/\sqrt{N}$, (P.C.)



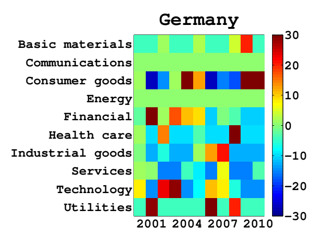
(c) $\boldsymbol{v}_2$, $\tau = 1/\sqrt{N}$, (R.C.)



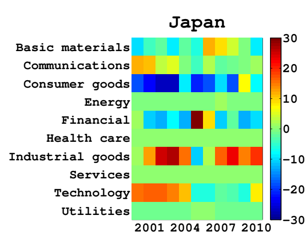
(d) $\boldsymbol{v}_2$, $\tau = 1/\sqrt{N}$, (P.C.)



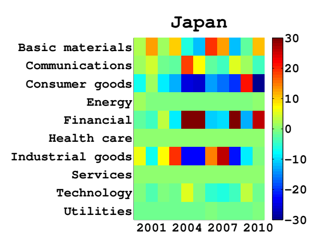
(e) $\boldsymbol{v}_2$, $\tau = 1/\sqrt{N}$, (R.C.)



(f) $\boldsymbol{v}_2$, $\tau = 1/\sqrt{N}$, (P.C.)

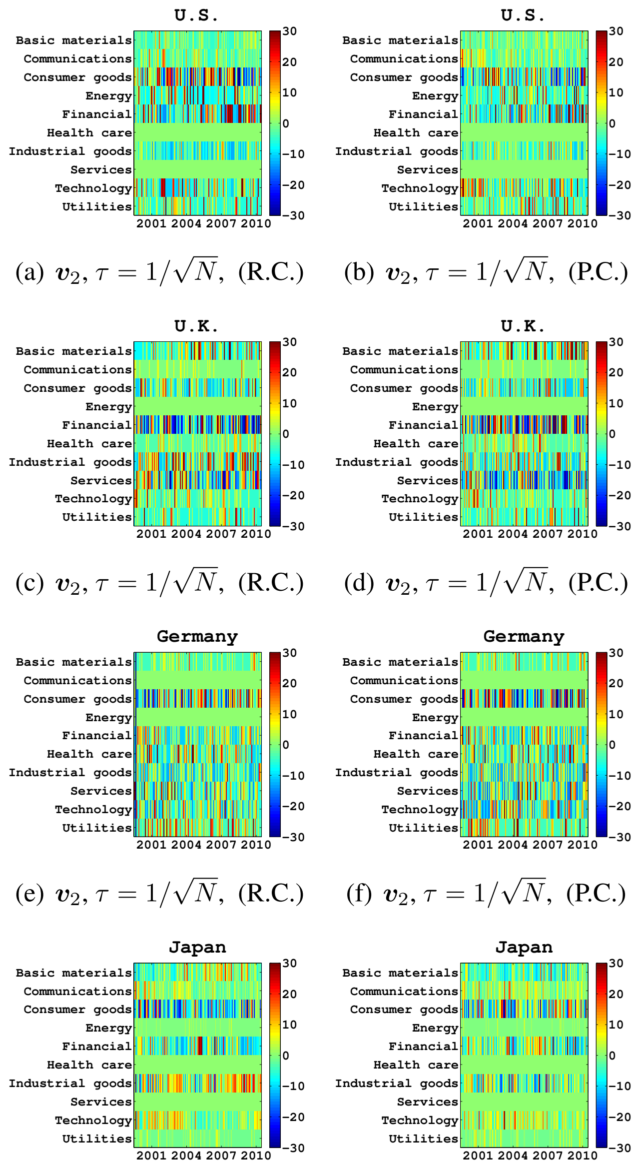


(g) $\boldsymbol{v}_2$, $\tau = 1/\sqrt{N}$, (R.C.)



(h) $\boldsymbol{v}_2$, $\tau = 1/\sqrt{N}$, (P.C.)

Fig. C.13. The yearly based SDR calculated from $\boldsymbol{v}_2$ with $\tau = 1/\sqrt{N}$ for both raw and partial correlation for the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets.



(a) $\boldsymbol{v}_2, \tau = 1/\sqrt{N},$ (R.C.) (b) $\boldsymbol{v}_2, \tau = 1/\sqrt{N},$ (P.C.)

(c) $\boldsymbol{v}_2, \tau = 1/\sqrt{N},$ (R.C.) (d) $\boldsymbol{v}_2, \tau = 1/\sqrt{N},$ (P.C.)

(e) $\boldsymbol{v}_2, \tau = 1/\sqrt{N},$ (R.C.) (f) $\boldsymbol{v}_2, \tau = 1/\sqrt{N},$ (P.C.)

(g) $\boldsymbol{v}_2, \tau = 1/\sqrt{N},$ (R.C.) (h) $\boldsymbol{v}_2, \tau = 1/\sqrt{N},$ (P.C.)

Fig. C.14. The monthly based SDR calculated from $\boldsymbol{v}_2$ with $\tau = 1/\sqrt{N}$ for both raw and partial correlation for the U.S. (a,b), U.K. (c,d), Germany (e,f), and Japanese (g,h) stock markets.

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