Finance and the Optimal Investment Decisions of a Firm Under Imperfect Competition

Toshio Watanabe

Hirao School of Management, Konan University
E-mail: toshio@center.konan-u.ac.jp

ABSTRACT

Both John Maynard Keynes and Hyman Minsky emphasized the effects of long-run expectations and financial structures on investment decisions. Minsky alleged that financial booms and collapses become inevitable in market economies and developed a new theory called the "financial instability hypothesis." In this paper, we construct an optimal-investment model under conditions of imperfect competition and explicitly represent the financing of investment. Our model demonstrates the micro-foundation of the investment theory of Keynes and Minsky. We show that both a firm's expected rate of growth and bank behavior play crucial roles in determining investment. For example, when the risk premium of a bank reacts elastically to the expected growth rate of demand, fluctuations in investment increase significantly and the range of fluctuation exceeds that in a perfect capital market. This means that the bank's behavior amplifies the fluctuations of the economy.

Keywords: expected growth rate of demand, bank behavior, investment decisions
JEL Classification Numbers: D92, E12

1 Introduction

There is no doubt that financial factors influence the real economy. The woes of the Japanese economy in the 1990's and the subprime loan problems that emerged in the latter half of the 2000's both demonstrate and symbolize the influence of the instability of financial markets on real economies.

Previously, Keynes (1936) emphasized the effects of long-run expectations and financial structures on investment decisions; however, Minsky (1975) pointed out that Keynes mainly focused on the following aspects: (1) decision-making by each person under conditions of uncertainty; (2) the financing of a firm's investments; and (3) the cyclical behavior of the real economy. Minsky criticized the standard IS/LM model for neglecting to consider these matters. He alleged that financial booms and collapses become inevitable in market economies and developed a new theory called the financial instability hypothesis.

Minsky's ideas led to the development of various mathematical models of this issue.[1] For example, Taylor and O'Connell (1985) focused on long-run expectations and household portfolios. They showed that the more elastic the reaction of the substitution between equity and money to the profit rate, the higher the possibility of financial instability. Many of the models mentioned above incorporate loan or equity markets and extend the IS/LM model. They basically describe financial instability as an unstable steady state in the dynamic model.

The purpose of this paper is to analyze the relationship between investment decisions and finance. We focus on two aspects. First, our aim is to build an optimal-investment model of an imperfectly competitive firm. This assumption clarifies the importance of demand and gives a micro-foundation to the investment theories of Keynes and Minsky. Our analysis is in contrast to the argument that focuses on the supply side. Second, we suppose an imperfect capital market and explicitly deal with the financing of the investment. We analyze the effects of bank behavior and long-run expectations on investment decisions and compare them with the situation in a perfect capital market.

Since Jorgenson (1963, 1967), there have been many papers concerned with optimal investment decisions. For example, Hayashi (1985) and Osterberg (1989) developed models under an imperfect capital market. However, most of these papers assume a competitive firm. There is scarcely any work that analyzes the investment decisions of imperfectly competitive firms.

Adachi (2003), one of the few such papers, provided an outline of the investment model for an imperfectly competitive firm. He analyzed the investment decisions of the firm simultaneously with its choice of technique, concluding that the expected growth rate of demand might be interpreted as corresponding to Keynes' "animal spirits" (Keynes, 1936, p. 161), which reflect the state of the firm's long-run expectations. Unfortunately, this model cannot investigate the effects of financial factors due to its assumption of a perfect capital market.

On the other hand, Asada and Semmler (1995) built an investment model of a monopolistic firm under an imperfect capital market. They indicated that chaotic features emerge as optimal paths of investment and debt-capital ratio. In this model, the expected demand is represented by the linear function of capital stock, so that the expected growth rate of demand is always equal to the growth rate of capital stock.

However, as Keynes pointed out, long-run expectations can easily change. In general, there is a difference between the expected growth rate of demand and the growth rate of capital stock. In this paper, the expected growth rate of demand may be interpreted as corresponding to long-run expectations. We assume that expected growth rate of demand is an independent variable.

[1] Other than Taylor and O'Connell (1985), discussed below, these include Delli and Gallegati (1990), Downe (1987), and Lavoie (1995).

We construct a simple optimal-investment model and investigate the effect of the expected growth rate of demand. Assuming an imperfect capital market, the objective function of the firm includes the debt-capital ratio, and the investment depends on financial as well as real factors. The expected growth rate of demand affects the investment decisions of the firm not only directly but also indirectly through bank behavior. The shift of the optimal path in response to change in the expected growth rate of demand is different from that occurring under a perfect capital market.

This paper is organized in the following manner. Section 2 presents an investment-decision model. We derive the objective function of the firm from the arbitrage condition between riskless assets and equity in the household. Next, we model the profit maximization of an imperfectly competitive firm and the financing of the investment. Section 3 sets the Hamiltonian function and derives the optimal investment path. Section 4 analyzes a steady state and compares it with one under a perfect capital market. Finally, Section 5 summarizes the results.

2 The Value of a Firm and the "Maximum Problem"

2.1 The Value of a Firm and the Objective Function

The firm maximizes the market value of debt B_t plus equity E_t at time $t = 0$.[2] We assume that equity is issued only when the firm is established. The firm borrows funds from a bank.

The value of the firm V_t at time t is represented as

$$V_t = B_t + z_t E, \tag{1}$$

where z_t is the equity price and E is constant.

The firm stands to profit by production. The gross profits Π_t go to the bank as interest r_t and to the adjustment costs of investment C_t. The remainder is distributed to stockholders as dividends D_t or held back as retained earnings RE_t. All investment I_t must be financed through either retained earnings or debt issue. These relations are represented as

$$\Pi_t = C_t + r_t B_t + RE_t + D_t, \tag{2}$$

[2]There are other assumptions. For example, the firm maximizes equity. The firm has debts. The interests of stockholders may conflict with those of creditors under an imperfect capital market. Taking into account these situations, we adopt the assumption of the maximization of the market value of the firm, as in Osterberg (1989).

$$I_t = \dot{B}_t + RE_t. \tag{3}$$

On the other hand, both riskless asset and equity are held by the household.[3] The allocation of these assets is decided in the situation wherein the rates of return become equal. The arbitrage condition is written as

$$\frac{\dot{z}E + D}{zE} = i, \tag{4}$$

where i is the return rate of the riskless asset.

We will now derive the objective function of the firm. Taking the time derivative of equation (1) and taking into account equations (3) and (4), we have

$$\dot{V}_t = I_t - RE_t + iz_t E - D_t. \tag{5}$$

Substituting equations (1) and (2) into (5), we have

$$\dot{V}_t = iV_t - (\Pi_t - (r_t - i)B_t - I_t - C_t). \tag{6}$$

Integrating equation (6), we can express the objective function as

$$V_0 = \int_0^\infty \{\Pi_t - (r_t - i)B_t - I_t - C_t\} \exp(-it)dt. \tag{7}$$

The firm decides on an investment that maximizes V_0.

2.2 The Maximum Problem of the Firm under Imperfect Competition

The representative firm under imperfect competition will face expected demand curves over future periods when it makes investment decisions.[4] In this section, we examine how the imperfectly competitive firm determines its output and price, given the existing capital stock.

[3] The riskless assets include deposits and national bonds.

[4] This formulation follows Adachi (2003).

Let us denote the expected demand of the firm as Y_t^e and the price of product as p_t at a given point of time. Then, the expected demand function of the firm will be written as

$$Y_t^e = A_t p_t^{-\eta}, \tag{8}$$

where A_t denotes the expected demand at a given price and η is the price elasticity of demand. A change in A_t indicates a shift in the expected demand curve. In the following equation, we assume η to be constant.

Suppose that the firm determines output Y_t to be equal to expected demand Y_t^e. We can then rewrite (8) in the form of an inverse demand function, as

$$p_t = \left(\frac{Y_t}{A_t}\right)^{-\frac{1}{\eta}}. \tag{9}$$

The firm uses capital stock K_t and labor N_t to produce the output. We assume the Cobb–Douglas production function,

$$Y_t = K_t^{\alpha} N_t^{(1-\alpha)}, 0 < \alpha < 1. \tag{10}$$

The short-run profit Π_t of the firm is given by

$$\Pi_t = p_t Y_t - \omega N_t, \tag{11}$$

where ω is the nominal wage rate. The short-run decision of the firm under imperfect competition is to determine output and price in order to maximize profit, given the stock of capital. Maximizing Π_t with respect to Y_t subject to the constraint in (9) yields[5]

$$Y_t = \left\{ A_t^{-\frac{1}{\eta}} \left(\frac{\eta}{\eta-1}\right) \cdot \omega \cdot \left(\frac{1}{1-\alpha}\right) \cdot K_t^{-\frac{\alpha}{1-\alpha}} \right\}^{-\frac{(1-\alpha)\eta}{m}}, \eta > 1, m = 1-\alpha+\alpha\eta > 1. \tag{12}$$

[5] $\eta > 1$ is a necessary condition for satisfying profit maximization.

For analytical convenience, we define the ratio of profit to capital as $\pi_t = (p_t Y_t - \omega N_t) / p_t^k K_t$, where p_t^k is the price of capital. Since we set $p_t^k = 1$, without loss of generality, the rate of return on capital can be expressed as

$$\pi(k_t) = \frac{m}{(\eta-1)(1-\alpha)} \cdot \left\{ \left(\frac{\eta}{\eta-1} \right) \cdot \left(\frac{1}{1-\alpha} \right) \right\}^{\frac{-\eta}{m}} \left(\frac{\omega^{(1-\eta)(1-\alpha)}}{k_t} \right)^{\frac{1}{m}}, k_t = \frac{K_t}{A_t}; \quad (13)$$
$$\Pi_t = \pi(k_t) K_t, \pi_k < 0;$$

where k_t denotes the ratio of capital to the level of expected demand. The rate of return π_t is a decreasing function of k_t, and $\frac{1}{m}$ represents the elasticity of π_t with respect to k_t.[6]

2.3 The Adjustment Costs of Investment and the Imperfect Capital Market

The investment decisions of the firm are made on the basis of expected demand and costs over the periods during which the newly installed equipment will be used.

First, we consider the adjustment costs of investment. These are the costs involved in sourcing and setting up new equipment and training labor to operate it. We assume that the adjustment costs depend on the level of investment and capital stock and are expressed as a homogenous function of degree of one. Then, the total adjustment cost C_t is written as

$$C(I_t, K_t) = C\left(\frac{I_t}{K_t}, 1 \right) K_t = c(g_t) K_t, g_t = \frac{I_t}{K_t}, \quad (14)$$

where g_t is the investment per unit of capital. We call this variable g_t, the capital accumulation rate. The adjustment cost per unit of capital $c(g_t)$ is assumed to have the following properties:

$$c' > 0, c'' > 0. \quad (15)$$

If we ignore the depreciation of capital, we have

[6] The effect of the nominal wage rate ω on profit rate is represented by $\pi_\omega < 0$. However, for the sake of simplicity, we assume that the nominal wage rate ω is constant. We focus on the effect of the capital-expected demand ratio k_t on the profit rate.

$$\dot{K}_t = g_t K_t. \tag{16}$$

Second, we consider the financing of the investment. Under an imperfect capital market, a bank cannot have perfect knowledge of a firm. If the firm fails in its investments and goes bankrupt, the bank cannot collect the interest or the principal. Kalecki and Keynes called these risks "lender's risks" (Kalecki, 1937, p. 442; Keynes, 1936, p. 134). On this basis, the bank's loan rate is set higher than the return rate of riskless assets.

The bank's risk premium μ for lending is high when the possibility of the firm's bankruptcy is high. We assume that the risk premium depends on the debt-capital ratio and the long-run expectations of the bank. When the debt-capital ratio is high and the long-run expectations of the bank worsen, the risk premium is high. The risk premium is represented by

$$\begin{aligned} &\mu = \mu(e, b_t), \frac{B_t}{K_t} = b_t, \\ &\mu_e < 0, \mu_{ee} < 0, \mu_{b_t} > 0, \mu_{b_t b_t} > 0. \end{aligned} \tag{17}$$

where b_t is the debt-capital ratio and e is the long-run expectations of the bank.

The decreasing degree of risk premium for long-run expectations μ_e expands more than proportionally as long-run expectations e increase. On the other hand, the increasing degree of risk premium to the debt-capital ratio μ_{b_t} increases more than proportionally as b_t increases.

The long-run expectations of the bank are made on the basis of how much profit the firm makes in future and how well the firm might do with its investments. It may be said that these elements depend on the expected growth rate of demand, of which the long-run expectations of the bank e are a function. This is represented by

$$e = e(a), e_a > 0, \tag{18}$$

$$\frac{\dot{A}_t}{A_t} = a. \tag{19}$$

We suppose that the loan rate is the sum of the return rate of riskless assets and the risk premium. Taking into account equations (17) and (18), we can express the bank's loan rate as

$$r_t = i + \mu(e(a), b_t). \tag{20}$$

2.4 A Model of Investment Decisions

In this section, we will formulate an optimal-investment model. First, we consider the objective function. Substituting equation (20) for (6), we obtain

$$\begin{aligned} &\dot{V} = iV_t - \{\pi(k_t) - \mu(e(a), b_t)b_t - \phi(g_t)\}K_t, \\ &\phi(g_t) = g_t + c(g_t). \end{aligned} \tag{21}$$

Considering the relation of $K_t = k_t A_t$ and putting $A_0 = 1$, without any loss of generality, we derive the following equation for the value of the firm:[7]

$$V_0 = \int_0^\infty \{\pi(k_t) - \mu(e(a), b_t)b_t - \phi(g_t)\}k_t \exp\{-(i-a)t\}dt. \tag{22}$$

We assume the following condition:

$$i - a > 0. \tag{23}$$

Next, let us consider constraint conditions. First, taking the time derivative of the capital-expected demand ratio k_t, we have

$$\dot{k}_t = (g_t - a)k_t. \tag{24}$$

Similarly, taking the time derivative of the debt-capital ratio b_t, we have

$$\frac{\dot{b}_t}{b_t} = \left(\frac{\dot{B}_t}{B_t} - \frac{\dot{K}_t}{K_t}\right). \tag{25}$$

The firm is financed partly by retained earnings. We assume that the firm obtains a certain rate of investment funding from these earnings. Denoting the ratio of retained earnings to investment by the variable γ, we have

$$RE_t = \gamma I_t. \tag{26}$$

[7] A_0 denotes the level of the expected demand at time $t = 0$.

Then, taking into account equations (3), (25) and (26), we have

$$\dot{b}_t = (1-\gamma-b_t)g_t. \tag{27}$$

To sum up, the problem of investment decisions is to maximize

$$\max_{g_t} V_0 = \int_0^\infty \{\pi(k_t) - \mu(e(a), b_t)b_t - \phi(g_t)\}k_t \exp\{-(i-a)t\}dt \tag{28}$$

subject to constraints

$$\dot{k}_t = (g_t - a)k_t, \quad \text{and} \tag{29}$$

$$\dot{b}_t = (1-\gamma-b_t)g_t. \tag{30}$$

In this problem, g_t is controlled by the firm, while k_t and b_t are state variables.[8] The objective function of firm (28) includes b_t as the variable of function μ. This means the financial factor affects the investment decisions.

On the other hand, under a perfect capital market, there is no asymmetric information. The risk premium μ is equal to 0. The optimal investment decision under a perfect capital market is to maximize

$$\max_{g_t} V_0 = \int_0^\infty \{\pi(k_t) - \phi(g_t)\}k_t \exp\{-(i-a)t\}dt, \tag{31}$$

subject to the constraint:

$$\dot{k}_t = (g_t - a)k_t. \tag{32}$$

In a perfect capital market, the debt-capital ratio does not affect investment decisions. In section 4, below, we make a comparison of the effect of the expected

[8] For the sake of simplicity, we consider the case where the dividends are positive, $D_t = \Pi_t - C_t - r_t B_t - \gamma I_t > 0$. When I_t is constant, a rise in γ will decrease D_t.

growth rate of demand on the optimal investment path under an imperfect and under a perfect capital market.

3 Optimal Investment Decisions

3.1 The Steady State and Optimal Path

In order to solve the problem described above under an imperfect capital market, we set up the present value Hamiltonian:

$$\begin{aligned} H_t = [\{\pi(k_t) - \mu(e(a), b_t)b_t - \phi(g_t)\}k_t + \lambda_t(g_t - a)k_t \\ + q_t(1 - \gamma - b_t)g_t]\exp\{-(i-a)t\}, \end{aligned} \tag{33}$$

where λ_t and q_t are shadow prices of k_t and b_t, respectively. The first-order conditions for a maximum of V_0 are

$$H_{g_t} = -\phi'(g_t)k_t + \lambda_t k_t + q_t(1 - \gamma - b_t) = 0, \tag{34a}$$

$$\dot{\lambda}_t = \lambda_t(i - g_t) - \{\pi'(k_t)k_t + \pi(k_t) - \mu(e(a), b_t)b_t - \phi(g_t)\}, \tag{34b}$$

$$\dot{q}_t = q_t(i - a + g_t) + \mu(e(a), b_t)k_t + \mu_{b_t}(e(a), b_t)b_t k_t. \tag{34c}$$

The transversality conditions are

$$\lim_{t\to\infty} \lambda_t k_t \exp\{-(i-a)t\} = 0, \quad \lim_{t\to\infty} q_t b_t \exp\{-(i-a)t\} = 0. \tag{34d}$$

Taking the time derivative of equation (34a) and taking into account equations (29), (30), (34a), (34b) and (34c), we have

$$\dot{g}_t = \frac{\phi'(g_t)(i - g_t) - (\pi' k_t + \pi(k_t) - \phi(g_t)) + \mu(e(a), b_t)(1 - \gamma) + \mu_{b_t}(e(a), b_t)b_t(1 - \gamma - b_t)}{\phi''}. \tag{35}$$

Equation (35) describes the dynamics of the capital accumulation rate, but it does not include q_t, which is the costate variable of b_t. This means that the dynamic system comprises variables g_t, k_t and b_t and is completed by equations (29), (30), and (35).

Next, we analyze the character of a steady state. We have the following steady-state relationships:

$$\phi'(g^*) = \frac{\pi' k^* + \pi^* - \phi(g^*) - \mu(e(a), b^*) b^*}{i - g^*}, \tag{36a}$$

$$g^* = a, \quad \text{and} \tag{36b}$$

$$1 - \gamma = b^*. \tag{36c}$$

From equations (36b) and (36c), in the steady state, it is evident that the capital accumulation rate is equal to the expected growth rate of demand and that the debt-capital ratio is equal to one minus the ratio of retained earnings to investment.

In equation (36a), the left-hand side indicates the marginal cost of investment and the right-hand side indicates the marginal value of the capital, that is, Tobin's marginal q. It should be noted that in this model, Tobin's marginal q depends on the financial factor.

Let us examine whether the optimal path converges into the steady state. Making linear approximations near the steady state, we obtain the following coefficient matrix M,

$$M = \begin{bmatrix} \phi''(i - g^*) & -(\pi'' k^* + 2\pi') & 0 \\ k^* & 0 & 0 \\ 0 & 0 & -g^* \end{bmatrix}. \tag{37}$$

Further, we have the determinant of matrix M,

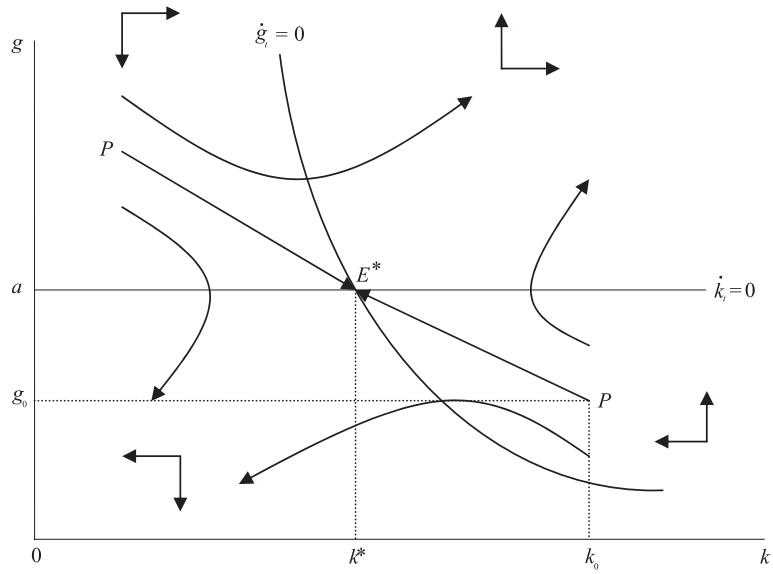
$$\det M = -g^* k^* (\pi'' k^* + 2\pi') > 0.$$

From equations (23) and (36b), the following condition is satisfied at the steady state:

$$i - g^* > 0.^{9} \tag{38}$$

[9]Supposing that $i - a < 0$, the inequality — $i - g^* < 0$ — is satisfied at the steady state. From equation (22), we know that the value of the firm V_0 does not converge to a finite value at the steady state.

Figure 1. Steady State.



Then, there are two cases for the eigenvalues of the characteristic equation.[10] The first is the case where one is a positive real number and the others are negative real numbers. The second is the case where one is a positive real number and the others are conjugate complex numbers with a negative real number. In these cases, the firm can set the initial value of g_t to converge to a steady state.

Let us analyze this system by using a phase diagram. The slope of $\dot{g}_t = 0$ near the steady state on the *Okg* plane in Figure 1 is calculated from equation (35) in the following manner:

$$\left.\frac{dg}{dk}\right|_{\dot{g}_t=0} = \frac{\pi'' k^* + 2\pi'}{\phi''(i - g^*)} < 0 \tag{39}$$

The locus of points where $\dot{k}_t = 0$ is a horizontal line on the *Okg* plane. The intersection of these loci denoted by $E^*(k^*, g^*)$ represents the steady state.

However, currently, the normal interest rate in Japan is extremely low. It may be said that condition (38) is unlikely to hold true in actuality.

[10]See the Appendix for the proof.

Figure 2. Phase Diagram in the Case of $1-\gamma<\bar{b}$.

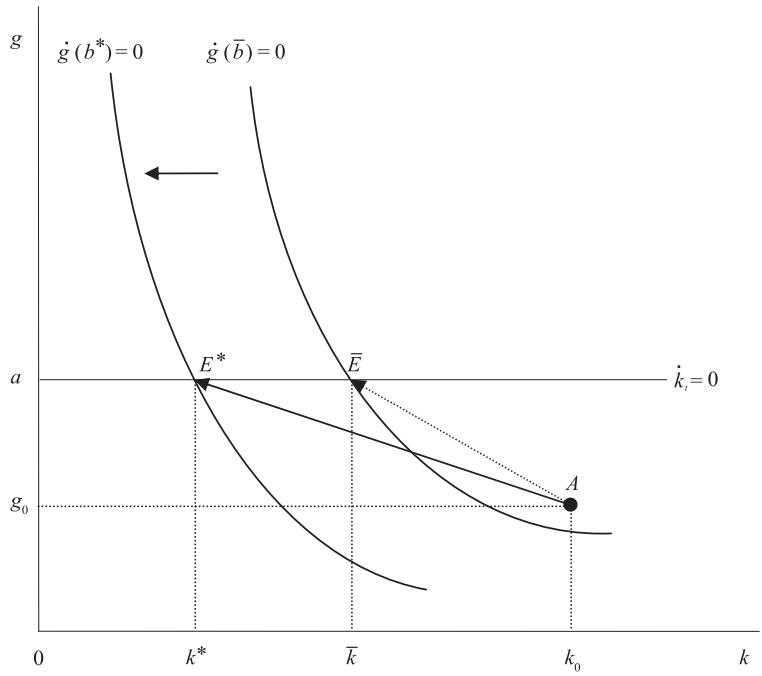


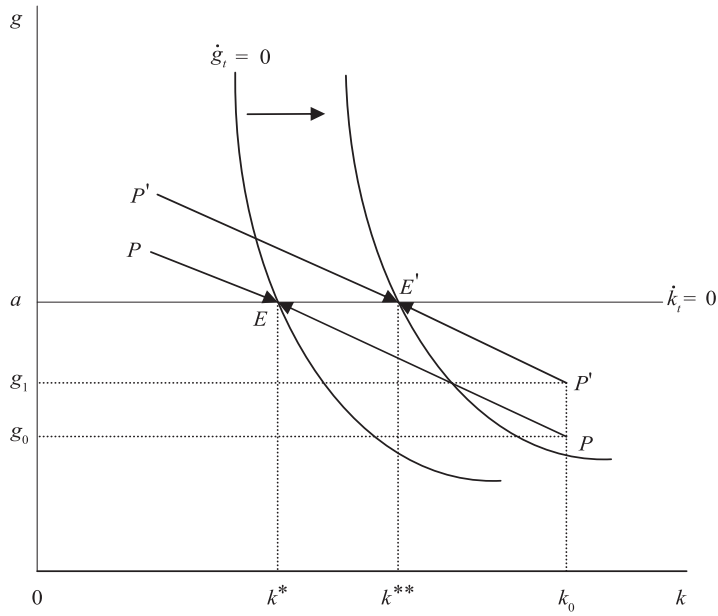
Figure 1 depicts $\dot{g}_t=0$ in the case of a steady state when the debt-capital ratio satisfies $1-\gamma=b^*$.[11] In this case, there are two optimal paths for satisfying the optimal conditions and the transversality conditions. One path converges to a steady state and the other to $k_t=0$. We eliminate the latter case because it is trivial.

To summarize these results, the optimal investment path is represented by the saddle path *PP* in Figure 1. This implies that there are unique initial levels of capital accumulation rates for each initial value of k_t. For example, if the initial capital-expected demand ratio is k_0, the firm sets an optimal capital accumulation rate g_0.

Incidentally, the debt-capital ratio does not always satisfy $1-\gamma=b^*$. Suppose $\bar{b}$ represents a debt-capital ratio that is higher than $b^*(\bar{b}>b^*)$. Figure 2 shows both the $\dot{g}_t=0$ curve in the case of $1-\gamma=b^*$ and that in the case of $1-\gamma<\bar{b}$. The latter curve shifts to the right compared with the former. The point at the

[11] Figure 1 represents that case where the solutions of the charcteristic equation are one positive and two negative real numbers.

Figure 3. The Effect of the Ratio of Retained Earnings to Investment γ on Investment.



intersection of the $\dot{g}_t = 0$ curve in the case of $1-\gamma < \bar{b}$ and the $\dot{k}_t = 0$ line is represented by point $\bar{E}$. However, this point is not the steady state.

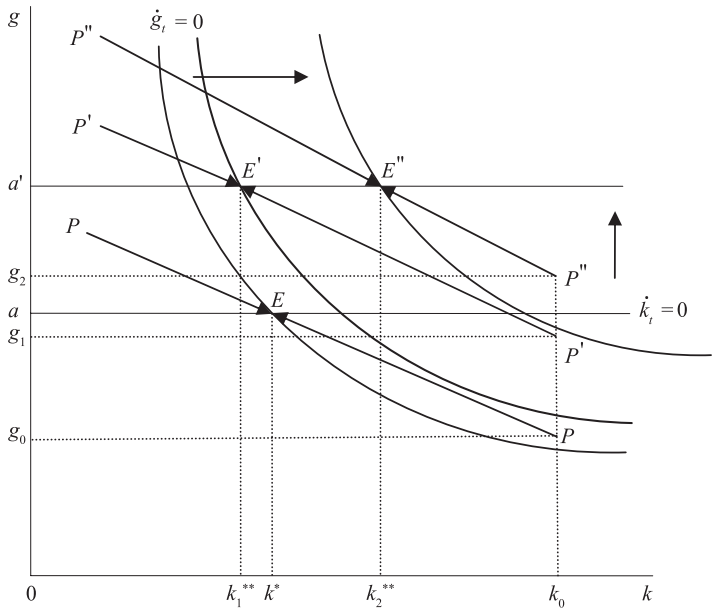
Next, we investigate the transition process to the steady state when the debt-capital ratio satisfies $1-\gamma < \bar{b}$. Suppose the initial point is represented by point A and the debt-capital ratio satisfies $1-\gamma < \bar{b}$. When these conditions are satisfied, the debt-capital ratio decreases as time passes. Then, the $\dot{g}_t = 0$ curve shifts to the left. The shift stops when the debt-capital ratio becomes $1-\gamma = b^*$. This time, the steady state is achieved. In this process, the investment operates along the AE^* line. This indicates that the decrease in the debt-capital ratio and the increase in investment occur simultaneously in this economy.

3.2 Comparative Analysis of the Steady State

In this section, we examine the effect of exogenous variables on the steady state. We assume the changes in parameters are unexpected and permanent, and, for the sake of simplicity, that the debt-capital ratio satisfies $1-\gamma = b^*$ in the following account.[12]

[12]As shown in Figure 2, the $\dot{g}_t = 0$ curve shifts in the case of $1-\gamma \neq b^*$. Although there are differences between the optimal path in the case of $1-\gamma = b^*$ and in the case of $1-\gamma \neq b^*$, the optimal

Figure 4. The Effect of the Expected Growth Rate of Demand a on Investment.



First, let us examine the effect of the ratio of retained earnings to investment γ on investment. When γ rises, the $\dot{g}_t = 0$ curve will shift rightward, as shown in Figure 3. Then, the saddle path PP shifts up to $P'P'$. Although the capital accumulation rate in the steady state remains unchanged, g_t will increase for any given initial value of k_t. For example, if the initial value of k_t is k_0, then g_t will increase from g_0 to g_1. This result is caused by the assumption of an imperfect capital market and contrasts with the results of the Modigliani–Miller Theorem (Modigliani and Miller, 1958). The same change of the saddle path occurs in the case of the decrease of riskless asset interest i.

Next, we examine how the expected growth rate of demand a affects the investment. When a increases, the $\dot{g}_t = 0$ curve shifts rightward and the $\dot{k}_t = 0$ line shifts upward, as shown in Figure 4. The capital accumulation rate in the steady state increases and the saddle path shifts upward. The capital accumulation rate g_t will increase for any given initial value of k_t. The influence on the capital-expected demand ratio is ambiguous and is calculated in the following manner:

paths of investment are similar and the economy finally converges to the steady state. For the sake of simplicity, we assume that the debt-capital ratio satisfies $1-\gamma = b^*$ and the focus is on the effect of exogenous variables on the movement of g_t and k_t.

$$\frac{\partial k^*}{\partial a} = \frac{-g^* k^* \{\phi''(i-g^*) + e'(a)\mu_e(e(a), b^*)b^*\}}{\det M} \left(b^* = 1-\gamma\right). \tag{40}$$

The sign of the above equation depends on the reaction of the risk premium to the expected growth rate. The sign becomes positive when the long-run expectations of the bank are elastic to the expected growth rate of demand (i.e., when $e'(a) > 0$ is large) and the risk premium of the bank is elastic to the bank's long-run expectations (the absolute figure of $\mu_e(e(a),b^*)$ is large). When these conditions are satisfied, the $\dot{g}_t = 0$ curve shifts rightward dramatically. The capital-expected demand ratio increases and the optimal path shifts upward significantly. Then, high capital accumulation can be realized in the economy.

Proposition 1. *The investment per unit of capital g_t is related positively to γ and a and negatively to i. Thus, the investment function of the firm under imperfect competition may be represented as*

$$g_t = G(i, \gamma, a),$$

where

$$\frac{\partial G}{\partial i} < 0, \frac{\partial G}{\partial \gamma} > 0, \frac{\partial G}{\partial a} > 0.$$

4 Comparisons with a Perfect Capital Market

As Keynes pointed out, long-run expectations often change. The expected growth rate of demand may be interpreted as corresponding to Keynes' "animal spirits," which reflect the state of the long-run expectations of the firm.

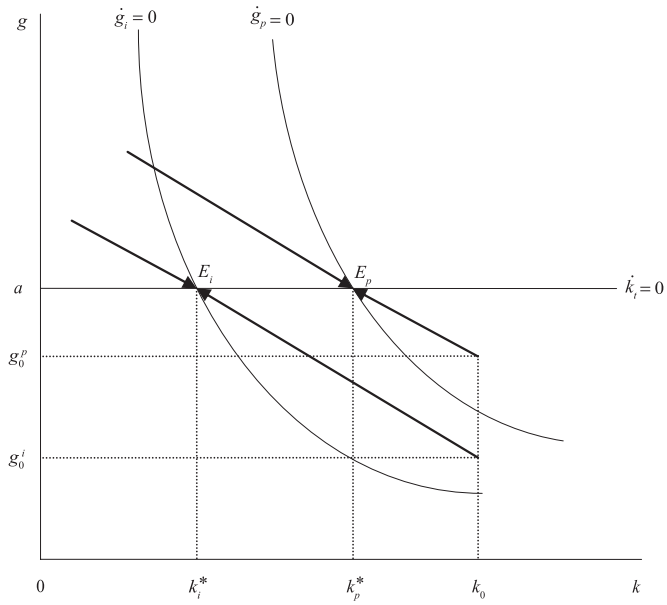
In this paper, we suppose that the change in investment is mainly caused by change in the expected growth rate of demand and is expressed as the shifting of the optimal investment path. In this situation, the fluctuation of the investment will be represented by volatility in the optimal path.

Next, we make a comparison of the effect of the character of the capital market on the investment. Under a perfect capital market, the $\dot{g}_t = 0$ curve is represented as $\dot{g}_p = 0$; under an imperfect capital market, this curve is represented as $\dot{g}_i = 0$.

The investment decisions of the firm under a perfect capital market are formulated as the problem of maximizing equation (31) subject to (32). The objective function (31) does not include financial factors, and the dynamic system comprises two differential equations (for $\dot{g}_t$ and $\dot{k}_t$).

Here, $\dot{k}_t$ is the same as under an imperfect capital market; however, $\dot{g}_t$ changes and is written in the following manner:

Figure 5. The Steady State Under Perfect and Imperfect Capital Markets.



$$\dot{g}_p = \phi'(g_t)(i - g_t) - (\pi' k_t + \pi(k_t) - \phi(g_t)). \tag{41}$$

We have the following steady-state relationships:

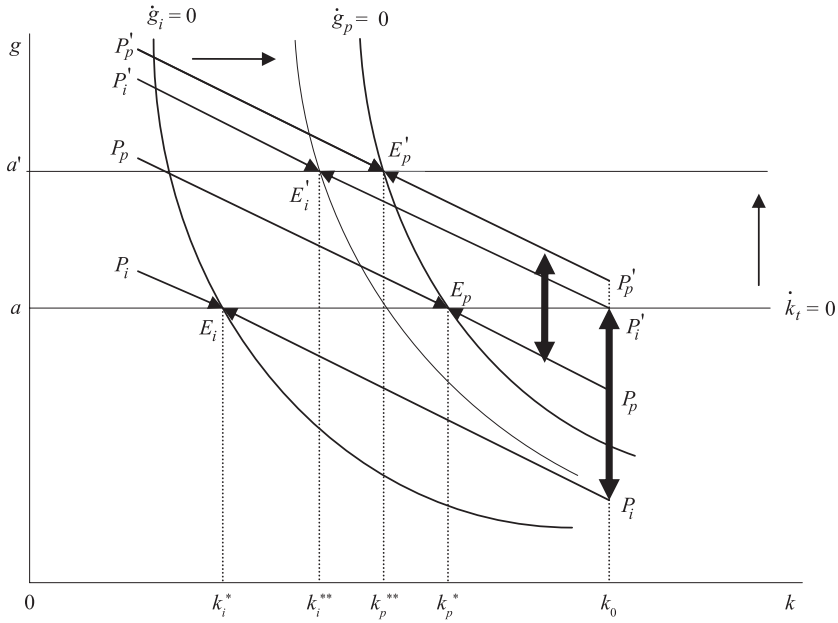
$$\phi'(g^*) = \frac{\pi' k^* + \pi^* - \phi(g^*)}{i - g^*}, \tag{42}$$

$$g^* = a. \tag{43}$$

In a steady state, the capital accumulation rate is equal to the expected growth rate of demand. This result is the same as that under an imperfect capital market.

Compared with (36a), there is no risk premium μ on the right-hand side of (42). Figure 5 shows the steady state under each market on the same plane. The $\dot{g}_i = 0$ curve is located on the left side of $\dot{g}_p = 0$. The optimal investment path under an imperfect capital market is presented below. This is where the level of investment is lower than that under a perfect capital market for any given capital-expected demand ratio. The reason is that there is a risk premium

Figure 6. The Effect of the Expected Growth Rate of Demand on the Optimal Investment Path.



under an imperfect capital market. The rise of a debt-capital ratio increases the risk premium and decreases the present value of investment returns and Tobin's marginal q including financial factors. On the other hand, under a perfect capital market, Tobin's marginal q does not change due to the debt-capital ratio.

Next, we examine the effect of the expected growth rate of demand on the optimal investment path. Under a perfect capital market, the increase in the expected growth rate of demand only causes $\dot{k}_t = 0$ to shift upward. As seen in Figure 6, the steady state changes from E_p to E_p' and the capital-expected demand ratio decreases. The optimal investment path P_pP_p shifts up to $P_p'P_p'$.

Under an imperfect capital market, the increase in the expected growth rate of demand shifts both the $\dot{k}_t = 0$ line and the $\dot{g}_i = 0$ curve simultaneously. The $\dot{g}_i = 0$ curve cannot go over the $\dot{g}_p = 0$ curve because of the existence of the risk premium μ.

As we discussed in the previous section, the $\dot{g}_i = 0$ curve shifts dramatically when the risk premium of the bank is elastic to the expected growth rate of demand (when $e'(a) > 0$ and the absolute figure of $\mu_e(e(a),b_t)$ are large). Meeting these conditions, the $\dot{g}_i = 0$ curve closes to $\dot{g}_p = 0$ and the capital accumulation

rate closes to the level of perfect capital market. In contrast, the decrease in the expected growth rate of demand causes the $\dot{g}_i = 0$ curve to shift leftward and move away from the $\dot{g}_p = 0$ curve. The optimal path shifts downward and the level of investment decreases dramatically.

Figure 6 depicts an example of the shift of the $\dot{g}_i = 0$ curve and the optimal investment paths. Depending on the bank's reaction to the expected growth rate of demand, the investment path moves between $P_i P_i$ and $P_i' P_i'$. The more elastically the risk premium reacts to the expected growth rate of demand, the larger the range of the optimal investment path. Figure 6 shows that the range of the investment under an imperfect capital market is larger than under a perfect capital market. This means that bank behavior amplifies the fluctuation of investment and has an important influence on the real economy.

Proposition 2. *Investment changes dramatically under an imperfect capital market when the risk premium of the bank is elastic to the expected growth rate of demand (when $e'(a) > 0$ and the absolute figure of $\mu_e(e(a), b_t)$ are large). In this situation, the range will be larger than that under a perfect capital market.*

5 Conclusions

In this paper, we discussed investment decisions under an imperfect capital market and imperfectly competitive firms. Keynes and Minsky focused on the effects of long-run expectations and financial factors. We also considered debt, bank behavior, and the expected growth rate of demand.

We pointed out that change in the expected growth rate of demand changes the capital accumulation rate significantly when the risk premium of a bank is elastic to long-run expectations with regard to demand. This means that bank behavior amplifies fluctuation of the economy. If the stability of the investment is desirable, the central bank should implement its monetary policy in response to change in the expected growth rate of demand.

There are a few possible extensions of our model; for example, incorporating the issue of new shares. In particular, its extension to a general equilibrium model should be a high priority for future research in the area.

Appendix

We derive the characteristic equation as eigenvalues h from coefficient matrix M:

$$L = -h^3 + (a_1 - g^*)h^2 + (a_1 g^* + a_2 k^*)h + a_2 k^* g^*, \tag{A1}$$

$$a_1 = \phi''(i - g^*), \tag{A2}$$
$$a_2 = -(\pi'' k^* + 2\pi') > 0.$$

Since the determinant of matrix M is positive, at least one root becomes positive. The characteristic equation is represented by

$$L = -(h - h_1)(h^2 + b_1 h + b_2), \tag{A3}$$

where h_1 is a positive root ($h_1 > 0$).

When the second element of the right-hand side of equation (A3) has two roots that are negative real numbers, the following conditions are satisfied:

$$b_1 > 0; \quad b_2 > 0.$$

Expanding equation (A3), we correspond coefficients to equation (A1). We have

$$a_1 - g^* = h_1 - b_1, \tag{A4}$$

$$a_1 g^* + a_2 k^* = b_1 h_1 - b_2, \tag{A5}$$

$$a_2 k^* g^* = b_2 h_1. \tag{A6}$$

We assume the following condition:

$$i - g^* > 0.$$

Then, equation (A2) satisfies $a_1 > 0$. Considering $a_2 > 0$ and $h_1 > 0$, we can derive $b_2 > 0$ from equation (A6); substituting these results into equation (A5), we can derive $b_1 > 0$.

References

Adachi, Hideyuki. 2003. "Investment Decisions and the Choice of Technique of the Firm Under Imperfect Competition." *Kobe University Economic Review* 49: 1–18.

Asada, Toichiro and Willi Semmler. 1995. "Growth and Finance: An Intertemporal Model." *Journal of Macroeconomics* 17: 623–49.

Delli Gatti, Domenico and Mauro Gallegati. 1990. "Financial Instability, Income Distribution and the Stock Market." *Journal of Post Keynesian Economics* 12: 356–74.

Downe, Edward. 1987. "Minsky's Model of Financial Fragility : A Suggested Addition." *Journal of Post Keynesian Economics* 9: 440–54.

Hayashi, Fumio. 1985. "Corporate Finance Side of the q Theory of Investment." *Journal of Public Economics* 27: 261–80.

Jorgenson, Dale. 1963. "Capital Theory and Investment Behavior." *American Economic Review* 53: 247–59.

Jorgenson, Dale. 1967. "The Theory of Investment Behavior." In *Determinants of Investment Behavior*, edited by Robert Ferber, 129–55. New York: Columbia University Press.

Kalecki, Michael. 1937. "The Principle of Increasing Risk." *Economica* 14: 440–47.

Keynes, John M. 1936. *The General Theory of Employment, Interest, and Money.* London: Macmillan.

Lavoie, Marc. 1995. "Interest Rates in Post-Keynesian Models of Growth and Distribution." *Metroeconomica* 46: 146–77.

Minsky, Hyman P. 1975. *John Maynard Keynes*. New York: Columbia University Press.

Modigliani, Franco and Merton Miller. 1958. "The Cost of Capital, Corporation Finance and the Theory of Investment." *American Economic Review* 48: 261–97.

Osterberg, William P. 1989. "Tobin's q, Investment, and the Endogenous Adjustment of Financial Structure." *Journal of Public Economics* 40: 293–318.

Taylor, Lance and Stephen A. O'Connell. 1985. "A Minsky Crisis." *Quarterly Journal of Economics* 100: 871–85.