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Dynamical Systems Methods for Electric Power Transmission: New Phenomenology and Future Smart Design

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Abstract

We overview recent results on dynamical systems approach to analysis and design of electricity grids. The first topic is the phenomenology of one type of short-term swing instabilities in multi-machine grids, which is termed the Coherent Swing Instability. The second topic is the design of electricity grid for management of multiple households. These apparently different topics are discussed from the common perspective of dynamical systems theory, that is, high-dimensional oscillatory systems with dominant inertia.

1. Introduction

An electricity grid is a large-scale man-made system for transmission of electric power, in which a huge number of dynamic components are interconnected such as generation plants and electrical loads. The stability of electricity grids is of basic importance for grid performance and is a well-established subject with a long history of research: see e.g. [1]. Understanding nonlinear dynamics of the grid is a key enabler to solving the stability problem, such as chaotic swings [2,3,4] and fractal-like basin boundaries [5,6]. In addition to this traditional problem, there is a growing trend of electricity grids in which ICT (Information and Communications Technology) can drastically change the current architecture and operation, such as the so-called *smart* grid vision [7] and informationization of energy [8]. In this new trend, dynamical systems methods are important for designing a future electricity grid that is robust against nonlinear responses including those listed above and cascading failures [9]. In this paper, we overview recent results on nonlinear problems in power grids engineering. There are two topics here: a

phenomenology of coupled swing dynamics [10,11] and a design of electricity grid [12]. The two topics are independent results and seem to be very different. However, we will see that they are closely related in common methodological ways: they can be studied with a similar mathematical model of high-dimensional nonlinear oscillatory systems with dominant inertial, and dynamical systems theory is crucial for solving both the nonlinear problems for electric power transmission.

2. New Phenomenology: Coherent Swing Instability in Multi-Machine Grids

The first topic is a new phenomenology of coupled swing dynamics in electricity grids with multiple synchronous machines and high-voltage transmission networks [10,11]. Coupled swing dynamics have attracted a lot of interests in the dynamical system community as an important example of multi degree-of-freedom, nearly integrable systems [13,14]. In [6], Ueda and his coworkers showed chaotic transients and fractal-like basin boundaries resulting from the nonlinearity of a simple multi-machine grid. Recently, we discovered a new route to the loss of transient stability in multi-machine grids that we termed the *Coherent Swing Instability* (CSI). This is an undesirable and emergent phenomenon of synchronous machines in an electricity grid, in which most of the machines in a sub-grid coherently lose synchronism with the rest of the grid after being subjected to a finite disturbance. We develop a minimal mathematical model of CSI for synchronous machines that are strongly coupled in a loop transmission network and weakly connected to the infinite bus (the outside of the grid): see Fig. 1. The model is similar to that used for analysis of mechanical pendulums hanged on a common wall. It is typical of high-dimensional oscillatory systems with dominant inertia and is called the nonlinear swing equations. An example of the CSI phenomenon obtained by numerical simulation of the model is shown in Fig. 2 from [10]. It provides a dynamical origin of CSI: it is related to escape from a potential well, or more precisely, to exit across a separatrix in the dynamical system for the amplitude of the weak nonlinear mode that governs the collective motion of the machines. The linear oscillations between strongly coupled machines then act as perturbations on the nonlinear mode. In this way, we reveal how the three different mode oscillations---local plant, inter-machine, and inter-area modes---interact to destabilize an electricity grid. The dynamics of CSI explain a part of the dynamical mechanism of cascading failure in interconnected power grids such as 2003 US-Canada blackout [9]: see [11] in details.

3. Future Smart Design for Management of Multiple Households

The second topic is a design of electricity grids consisting of multiple households and low-voltage distribution networks [12]. The penetration of small Distributed Energy Resources (DERs) such as solar panel, fuel cell, and Electric Vehicle (EV, as a storage/load device) has been increasing because it can reduce carbon emissions and increase the energy efficiency. Controlling a potentially huge number of DERs is a challenging problem for the future grid. To solve the problem, it needs to develop information architectures and control design that keep safe operations of individual DERs as well as the whole grid. For a low-voltage level (e.g. 200V in Japan), we are currently working on an electricity grid that consists of multiple households with DERs and distribution networks. An example of the electricity grid is shown in Fig. 3. Each household is equipped with facilities of energy generation (solar panel) and energy storage (battery), a load, and an interface circuit. The interface circuit includes a power converter system based on [15] and enables interconnection and synchronization between different households with own DERs. Because in Fig. 3 the cluster of three households is connected to the existing grid at one point, it is weakly coupled to the outside of the grid. This structure of electricity grid is the same as in Fig. 1 for high-voltage transmission networks. We have developed a mathematical model for the electricity grid and analyzed dynamical features of it that help us developing information architectures and control design for

it. From the common structural features, the developed model is close to the nonlinear swing equations used for CSI. A key point for dynamical analysis of the model is that it has both regular and stochastic excitations terms due to uncertain features of renewable generation and load consumption: see [12] in details.

Acknowledgements

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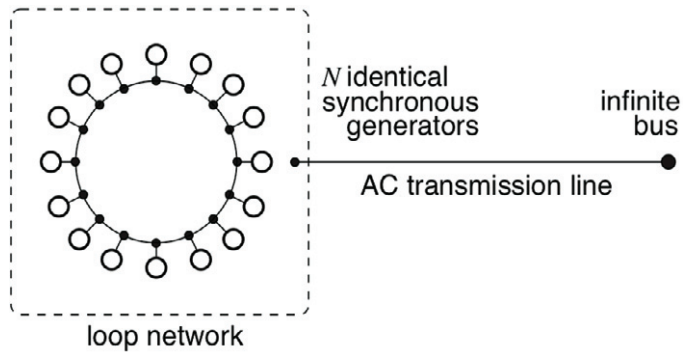
Fig. 1. Electricity grid with the loop topology [10]. Each small circle represents a synchronous machine supplying electric power. The grid consists of N small, identical generators, encompassed by the dotted box, which operate in the AC loop network and are connected to the infinite bus.
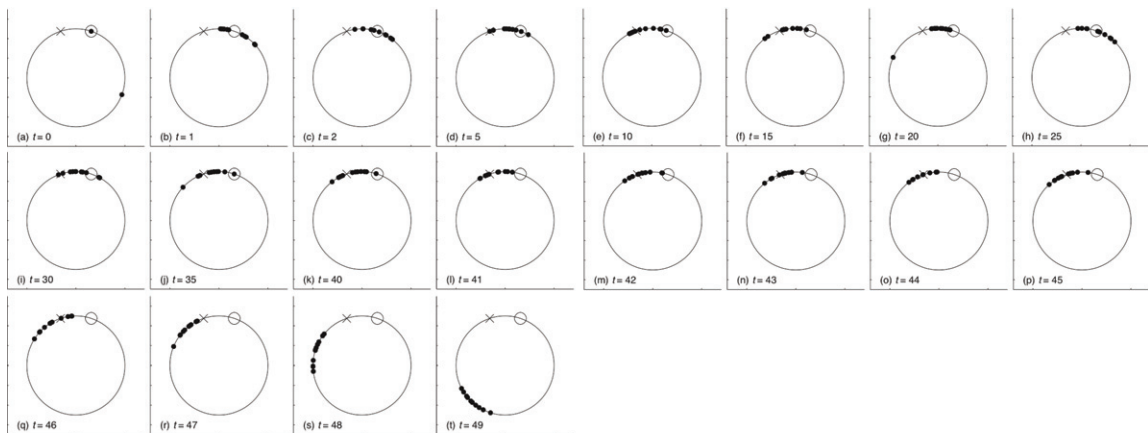

Fig. 2. Sequential snapshots of rotor angles $\delta_i(t)$ for coupled swing dynamics [10]. Solid points on the circle denote the rotor angles of $20(= N)$ generators in the loop grid. At the initial time (a), the 10th generator is locally disturbed, and its initial position $\delta_{10}(0)$ is denoted by the lower solid point, while all the other generators are at the elliptic equilibrium $\delta_i = \delta_c$ denoted by the circle. The rotor angles start to oscillate, keep the bounded oscillations in a while, and finally diverge in a unbounded manner (rotate spontaneously on the circle). This implies that all of the 20 generators lose synchronism with the outside of the grid coherently. This result is an example of global destabilization of a multi-machine grid caused by a single local disturbance.
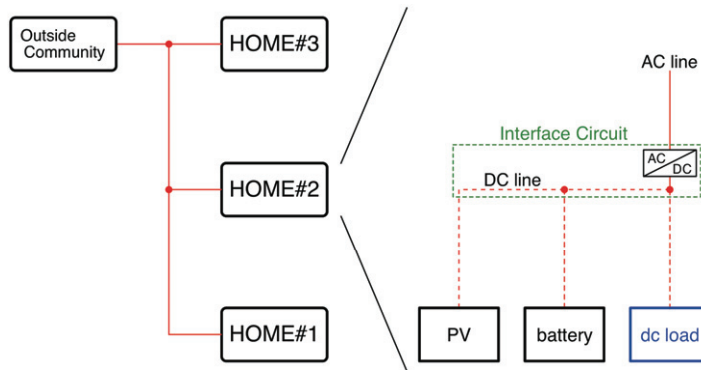
Fig. 3. Example of electricity grids consisting of multiple households with distributed energy resources (PV and battery) [12]. Each household with an interface circuit [15] is regarded as a synchronous machine with variable output. In this way, we study a common mathematical model for the two electricity grids with different scales and explore a universal mechanism for grid dynamics and control.