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On local newforms for unramified $\mathrm{U}(2,1)$

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1 Introduction

Local newforms play an important role in the theory of automorphic representations. Roughly speaking, a newform for an irreducible generic representation π of a p-adic group is a vector which attains the L-factor of π via Rankin-Selberg integral. The existence of newforms was known only for $\mathrm{GL}(n)$ until the work of Roberts and Schmidt [11] for $\mathrm{GSp}(4)$. In this note, we study newform theory for unramified $\mathrm{U}(2,1)$.

This note is a survey of the author's work [6], [7], [9], [8] on newrofms for unramified $\mathrm{U}(2,1)$. Let G denote the unramified unitary group in three variables defined over a p-adic field of odd residual characteristic. Newforms for an irreducible generic representation (π, V) of G is defined by using a family of open compact subgroups $\{K_n\}_{n\geq 0}$ of G, which is an analog of paramodular subgroups of $\mathrm{GSp}(4)$. For each non-negative integer n, we denote by $V(n)$ the space of K_n-fixed vectors. The smallest integer such that $V(n)$ is not trivial is called the conductor of π. We write N_π for the conductor of π, and call $V(N_\pi)$ the space of newforms for π. An algebraic structure of $V(n)$ was studied in [6] and [9], for example, the multiplicity one theorem for newforms and the dimension formula for $V(n)$, $n \geq N_\pi$.

Our main concern is the relation of newforms and Rankin-Selberg factors. Gelbart and Piatetski-Shapiro in [4] attached a family of Rankin-Selberg integrals to an irreducible generic representation π of G, and defined L and ε-factors for π. In *loc. cit.* they showed that the spherical vector attains the L-factor of π when π is an unramified principal series representation. But there were no results for ramified representations. In this note, we establish a theory of newforms for Gelbart and Piatetski-Shapiro's integral. We see that (i) the newform for an irreducible generic representation π of G attains the L-factor of π (Theorem 4.1) (ii) the conductor of π coincides with the exponent of q^{-s} of the ε-factor, where q denotes the cardinality of the residue field (Theorem 4.3).

We summarize the contents of this paper. In section 2, we recall from [1] the theory of Rankin-Selberg integral introduced by Gelbart, Piatetski-Shapiro and Baruch. In section 3, we define newforms for G and recall their basic properties. In section 4, we show Theorems 4.1 and 4.3 assuming Lemma 4.2, which is proved in section 5.

2 Rankin-Selberg integral

In this section, we recall from [1] the theory of Rankin-Selberg integral for $\mathrm{U}(2,1)$ introduced by Gelbart, Piatetski-Shapiro and Baruch.

2.1 Notation

We use the following notation. Let F be a non-archimedean local field of characteristic zero, $\mathfrak{o}_F$ its ring of integers, and $\mathfrak{p}_F$ the maximal ideal in $\mathfrak{o}_F$. We fix a uniformizer ϖ_F in F, and denote by $|\cdot|_F$ the absolute value of F normalized so that $|\varpi_F| = q_F^{-1}$, where q_F is the cardinality of the residue field $\mathfrak{o}_F/\mathfrak{p}_F$. Throughout this paper, we assume that the characteristic of $\mathfrak{o}_F/\mathfrak{p}_F$ is different from two.

Let $E = F[\sqrt{\epsilon}]$ be the quadratic unramified extension over F, where $\sqrt{\epsilon}$ is a non-square unit in $\mathfrak{o}_F$. We denote by $\mathfrak{o}_E$, $\mathfrak{p}_E$ the analogous objects for E. Then ϖ_F is a uniformizer of E, and the cardinality of $\mathfrak{o}_E/\mathfrak{p}_E$ is equal to q_F^2. So we abbreviate $\varpi = \varpi_F$ and $q = q_F$. We realize (the group of F-points of) the unramified unitary group in three variables defined over F as

$$G = \mathrm{U}(2,1) = \{g \in \mathrm{GL}_3(E) \mid {}^t\overline{g}Jg = J\}.$$

Here we denotes by $^-$ the non-trivial element in $\mathrm{Gal}(E/F)$ and

$$J = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Let B be the upper triangular Borel subgroup of G, U its unipotent radical, and T the group of the diagonal elements in G. For a non-trivial additive character ψ_E of E, we also denote by ψ_E the character of U defined by $\psi_E(u) = \psi_E(u_{12})$, for $u = (u_{ij}) \in U$. For an irreducible generic representation (π, V) of G, we write $\mathcal{W}(\pi, \psi_E)$ for the Whittaker model of π associated to (U, ψ_E).

2.2 Zeta integrals

Let $\mathcal{C}_c^\infty(F^2)$ be the space of locally constant, compactly supported functions on F^2. For an irreducible generic representation (π, V) of G, Gelbart and Piatetski-Shapiro introduced a family of zeta integrals which has the form $Z(s, W, \Phi)$ ($W \in \mathcal{W}(\pi, \psi_E)$, $\Phi \in \mathcal{C}_c^\infty(F^2)$) as follows:

We identify the subgroup

$$H = \left\{ \begin{pmatrix} a & 0 & b \\ 0 & 1 & 0 \\ c & 0 & d \end{pmatrix} \in G \right\}$$

of G with $\mathrm{U}(1,1)$. Since $\mathrm{SU}(1,1)$ is isomorphic to $\mathrm{SL}_2(F)$, we can write any element h in H as

$$h = \begin{pmatrix} b & 0 \\ 0 & \bar{b}^{-1} \end{pmatrix} \begin{pmatrix} \sqrt{\epsilon} & 0 \\ 0 & 1 \end{pmatrix} h_1 \begin{pmatrix} \sqrt{\epsilon}^{-1} & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.1}$$

where $b \in E^\times$ and $h_1 \in \mathrm{SL}_2(F)$. For $\Phi \in \mathcal{C}_c^\infty(F^2)$ and $h \in H$, we define a function $f(s, h, \Phi)$ on $\mathbf{C}$ by

$$f(s, h, \Phi) = |b|_E^s \int_{F^\times} \Phi((0, r)h_1)|r|_E^s d^\times r$$

by using the decomposition of h in (2.1). We note that the definition of $f(s, h, \Phi)$ is independent of the choices of $b \in E^\times$ and $h_1 \in \mathrm{SL}_2(F)$.

Set $B_H = B \cap H$ and $U_H = U \cap H$. Then U_H is the unipotent radical of the Borel subgroup B_H of $H = \mathrm{U}(1,1)$. For $W \in \mathcal{W}(\pi, \psi_E)$ and $\Phi \in \mathcal{C}_c^\infty(F^2)$, we define zeta integral $Z(s, W, \Phi)$ by

$$Z(s, W, \Phi) = \int_{U_H \backslash H} W(h) f(s, h, \Phi) dh.$$

Then $Z(s, W, \Phi)$ absolutely converges to a function in $\mathbf{C}(q^{-2s})$ if $\mathrm{Re}(s)$ is sufficiently large.

2.3 L and ε-factors

We recall the definition of L and ε-factors attached to an irreducible generic representation (π, V) of G. Set

$$I_\pi = \langle Z(s, W, \Phi) \mid W \in \mathcal{W}(\pi, \psi_E), \Phi \in \mathcal{C}_c^\infty(F^2), \psi_E : \text{non-trivial} \rangle.$$

Then I_π is a fractional ideal of $\mathbf{C}[q^{-2s}, q^{2s}]$ which contains 1. Thus, there exists a polynomial $P(X)$ in $\mathbf{C}[X]$ such that $P(0) = 1$ and $I_\pi = (1/P(q^{-2s}))$. We define *the L-factor $L(s, \pi)$ of π* by

$$L(s, \pi) = \frac{1}{P(q^{-2s})}.$$

To define ε-factor of π, we recall the functional equation. Let ψ_F be a non-trivial additive character of F. For $\Phi \in \mathcal{C}_c^\infty(F^2)$, we denote by $\hat{\Phi}$ its Fourier transform with respect to ψ_F. Then there exists $\gamma(s, \pi, \psi_F, \psi_E) \in \mathbf{C}(q^{-2s})$ which satisfies

$$\gamma(s, \pi, \psi_F, \psi_E) Z(s, W, \Phi) = Z(1 - s, W, \hat{\Phi}),$$

for all $W \in \mathcal{W}(\pi, \psi_E)$ and $\Phi \in \mathcal{C}_c^\infty(F^2)$.

By using the above functional equation, we define *the* ε*-factor* $\varepsilon(s, \pi, \psi_F, \psi_E)$ *of* π by

$$\varepsilon(s, \pi, \psi_F, \psi_E) = \gamma(s, \pi, \psi_F, \psi_E) \frac{L(s, \pi)}{L(1 - s, \widetilde{\pi})},$$

where $\widetilde{\pi}$ is the contragradient representation of π. By [7], we obtain $L(s, \widetilde{\pi}) = L(s, \pi)$, and hence

$$\varepsilon(s, \pi, \psi_F, \psi_E) = \gamma(s, \pi, \psi_F, \psi_E) \frac{L(s, \pi)}{L(1 - s, \pi)}. \tag{2.2}$$

Thus, we can show the following proposition by the standard argument:

Proposition 2.3. *The* ε*-factor* $\varepsilon(s, \pi, \psi_F, \psi_E)$ *is a monomial in* q^{-2s} *of the form*

$$\varepsilon(s, \pi, \psi_F, \psi_E) = \pm q^{-2n(s-1/2)},$$

with some $n \in \mathbf{Z}$.

3 Newforms

In this section, we introduce a family of open compact subgroups of G, and define the notion of newforms for irreducible generic representations of G. We summarize the basic properties of newforms for G, which are an analog of those for $\mathrm{GL}(n)$ and $\mathrm{GSp}(4)$.

3.1 Newforms

Newforms for G are defined by the following open compact subgroups $\{K_n\}_{n \geq 0}$ of G. For each non-negative integer n, we define an open compact subgroup K_n of G by

$$K_n = \begin{pmatrix} \mathfrak{o}_E & \mathfrak{o}_E & \mathfrak{p}_E^{-n} \\ \mathfrak{p}_E^n & 1 + \mathfrak{p}_E^n & \mathfrak{o}_E \\ \mathfrak{p}_E^n & \mathfrak{p}_E^n & \mathfrak{o}_E \end{pmatrix} \cap G.$$

Remark 3.1. The definition of K_n is inspired by the paramodular subgroups of $\mathrm{GSp}(4)$, which is used in [11]. We also note that the group $\begin{pmatrix} \mathfrak{o}_E & \mathfrak{o}_E & \mathfrak{p}_E^{-n} \\ \mathfrak{p}_E^n & 1 + \mathfrak{p}_E^n & \mathfrak{o}_E \\ \mathfrak{p}_E^n & \mathfrak{p}_E^n & \mathfrak{o}_E \end{pmatrix}^\times$ is a conjugate of the subgroup of $\mathrm{GL}_3(E)$ which is used to define newforms for $\mathrm{GL}_3(E)$ in [5].

For an irreducible generic representation (π, V) of G, we set

$$V(n) = \{v \in V \mid \pi(k)v = v,\ k \in K_n\},\ n \geq 0.$$

Then it follows from [9] that there exists a non-negative integer n such that $V(n)$ is not zero.

Definition 3.2. We define *the conductor of* π by

$$N_\pi = \min\{n \geq 0 \mid V(n) \neq \{0\}\}.$$

We call $V(N_\pi)$ the space of *newforms* for π and $V(n)$ that of *oldforms*, for $n > N_\pi$.

3.2 Basic properties of newforms

We recall some basic properties of newforms from [6] and [9]. Firstly, the growth of dimensions of oldforms for generic representations π is independent of π, as in the cases of GL(n) and GSp(4) (see [2], [10], [11]). The following dimension formula for oldforms holds:

Proposition 3.3 ([6], [9]). *Let (π, V) be an irreducible generic representation of G. For $n \geq N_\pi$, we have*

$$\dim V(n) = \left\lfloor \frac{n - N_\pi}{2} \right\rfloor + 1.$$

In particular, $V(N_\pi)$ and $V(N_\pi + 1)$ are one-dimensional.

Secondly, newforms for G are test vectors for the Whittaker functional. We say that a function W in $\mathcal{W}(\pi, \psi_E)$ is a newform if W is fixed by K_{N_π}. The following proposition is important to the application to the theory of zeta integral:

Proposition 3.4 ([6]). *Suppose that the conductor of ψ_E is $\mathfrak{o}_E$. Then for all nonzero newforms W in $\mathcal{W}(\pi, \psi_E)$, we have*

$$W(1) \neq 0.$$

3.3 Zeta integral of newforms

We apply newforms for G to the theory of zeta integral. We suppose that the conductor of ψ_E is $\mathfrak{o}_E$. One of the nice properties of the subgroups $\{K_n\}_{n\geq 0}$ is that $K_{n,H} = K_n \cap H$ is a maximal compact subgroup of H for all n. Set $T_H = T \cap H$. Then we have an Iwasawa decomposition $H = U_H T_H K_{n,H}$, for any n. There exists an isomorphism

$$t : E^\times \simeq T_H; a \mapsto \begin{pmatrix} a & & \\ & 1 & \\ & & \bar{a}^{-1} \end{pmatrix}$$

For $W \in \mathcal{W}(\pi, \psi_E)$ and $\Phi \in \mathcal{C}_c^\infty(F^2)$, we obtain

$$Z(s, W, \Phi) = \int_{E^\times} \int_{K_{n,H}} W(t(a)k) f(s, k, \Phi) |a|_E^{s-1} dk d^\times a.$$

For $n \geq 0$, we denote by Φ_n the characteristic function of $\mathfrak{p}_F^n \oplus \mathfrak{o}_F$. If W a newform in $\mathcal{W}(\pi, \psi_E)$, then we have

$$Z(s, W, \Phi_{N_\pi}) = \mathrm{vol}(K_{n,H}) Z(s, W) L_E(s, 1). \tag{3.5}$$

Here $L_E(s, 1) = 1/(1 - q^{-2s})$ is the L-factor of the trivial representation of $E^\times$ and

$$Z(s, W) = \int_{E^\times} W(t(a)) |a|_E^{s-1} d^\times a. \tag{3.6}$$

We note that Proposition 3.4 implies that the integral $Z(s, W)$ does not vanish for any non-zero newforms in $\mathcal{W}(\pi, \psi_E)$.

If ψ_F has conductor $\mathfrak{o}_F$, then we have $\hat{\Phi}_{N_\pi} = q^{-N_\pi} \mathrm{ch}_{\mathfrak{o}_F \oplus \mathfrak{p}_F^{-N_\pi}}$, and hence

$$Z(1 - s, W, \hat{\Phi}_{N_\pi}) = q^{-2N_\pi(s-1/2)} Z(1 - s, W, \Phi_{N_\pi}) \tag{3.7}$$

by (3.5).

4 Main results

In this section, we show our two main theorems, which describe L and ε-factors of irreducible generic representations of G in terms of newforms and conductors.

4.1 L-factors and newforms

We show that zeta integrals of newforms attain L-factors. We normalize Haar measures on $E^\times$ and $K_{n,H}$ so that the volumes of $\mathfrak{o}_E^\times$ and of $K_{n,H}$ are one respectively. Then the following holds:

Theorem 4.1 ([8]). *Suppose that ψ_E has conductor $\mathfrak{o}_E$. Let π be an irreducible generic representation of G and W the newform in $\mathcal{W}(\pi, \psi_E)$ such that $W(1) = 1$. Then we have*

$$Z(s, W, \Phi_{N_\pi}) = L(s, \pi).$$

Theorem 4.1 is reduced to the following lemma:

Lemma 4.2. *With the notation as above, we have*

$$Z(s, W, \Phi_{N_\pi})/L(s, \pi) = 1 \text{ or } 1/L_E(s, 1).$$

We postpone the proof of Lemma 4.2 to the next section.

Proof of Theorem 4.1. We further assume that ψ_F has conductor $\mathfrak{o}_F$. Suppose that $Z(s, W, \Phi_{N_\pi})/L(s, \pi) = 1/L_E(s, 1)$. Then by (2.2), we obtain

$$\begin{aligned} \varepsilon(s, \pi, \psi_F, \psi_E) &= \gamma(s, \pi, \psi_F, \psi_E)\frac{L(s, \pi)}{L(1-s, \pi)} \\ &= \frac{Z(1-s, W, \hat{\Phi}_{N_\pi})}{Z(s, W, \Phi_{N_\pi})}\frac{L(s, \pi)}{L(1-s, \pi)} \\ &= q^{-2N_\pi(s-1/2)}\frac{L_E(s, 1)}{L_E(1-s, 1)}. \end{aligned}$$

The last equality follows from (3.7). This contradicts Proposition 2.3 which implies that $\varepsilon(s, \pi, \psi_F, \psi_E)$ is monomial. Thus we get $Z(s, W, \Phi_{N_\pi}) = L(s, \pi)$, as required. □

4.2 ε-factors and conductors

We show that the exponent of q^{-2s} of the ε-factor of an irreducible generic representation π of G coincides with the conductor of π. Applying the argument in the proof of Theorem 4.1, we obtain the following:

Theorem 4.3 ([8]). *Suppose that ψ_E and ψ_F have conductors $\mathfrak{o}_E$ and $\mathfrak{o}_F$ respectively. For any irreducible generic representation π of G, we have*

$$\varepsilon(s, \pi, \psi_F, \psi_E) = q^{-2N_\pi(s-1/2)}.$$

5 Proof of Lemma 4.2

In this section, we explain how to prove Lemma 4.2.

5.1 Evaluation of L-factors

We shall evaluate $L(s,\pi)$, for each irreducible generic representation (π, V) of G. The L-factor $L(s,\pi)$ is defined as the greatest common divisor of the zeta integrals $Z(s,W,\Phi)$. For $W \in \mathcal{W}(\pi,\psi_E)$ and $\Phi \in \mathcal{C}_c^\infty(F^2)$, there exist $W_i \in \mathcal{W}(\pi,\psi_E)$ and $\Phi_i \in \mathcal{C}_c^\infty(F^2)$ $(1 \le i \le m)$ such that

$$Z(s,W,\Phi) = \sum_{i=1}^{m} Z(s,W_i) f(s,1,\Phi_i).$$

By the theory of zeta integral for GL(1), we have

$$f(s,1,\Phi_i) \in L_E(s,1)\mathbf{C}[q^{-2s}, q^{2s}].$$

Recall that we defined

$$Z(s,W) = \int_{E^\times} W(t(a))|a|_E^{s-1} d^\times a,$$

for $W \in \mathcal{W}(\pi,\psi_E)$. To estimate $Z(s,W)$, we can apply the theory of Kirillov model for GL(2).

An irreducible generic representation of G is supercuspidal, or else a subrepresentation of a parabolically induced representation from B. The Levi component T of B is isomorphic to $E^\times \times \mathrm{U}(1)$. For a quasi-character μ_1 of $E^\times$ and a character μ_2 of U(1), we denote by $\mathrm{Ind}_B^G \mu_1 \otimes \mu_2$ the corresponding parabolically induced representation. According to the classification of representations of G, we have the following evaluation of the shape of L-factors:

Proposition 5.1. *Let π be an irreducible generic representation of G.*
(i) If π is supercuspidal, then $L(s,\pi)$ divides $L_E(s,1)$.
(ii) If π is a proper submodule of $\mathrm{Ind}_B^G \mu_1 \otimes \mu_2$, then $L(s,\pi)$ divides $L_E(s,\mu_1)L_E(s,1)$.
(iii) If $\pi = \mathrm{Ind}_B^G \mu_1 \otimes \mu_2$, then $L(s,\pi)$ divides $L_E(s,\mu_1)L_E(s,\overline{\mu_1}^{-1})L_E(s,1)$.

5.2 Calculation of zeta integral of newforms

Let W be the newform in $\mathcal{W}(\pi,\psi_E)$ such that $W(1) = 1$. We shall compute $Z(s,W,\Phi_{N_\pi})$. Suppose that π has conductor zero. Then $\pi = \mathrm{Ind}_B^G(\mu_1 \otimes 1)$, for some unramified quasi-character μ_1 of $E^\times$. In this case, newforms in $\mathcal{W}(\pi,\psi_E)$ are just spherical Whittaker functions. In [4], Gelbart and Piatetski-Shapiro showed that

$$Z(s,W,\Phi_0) = L_E(s,\mu_1)L_E(s,\overline{\mu_1}^{-1})L_E(s,1)$$

by using Casselman-Shalika's formula for spherical Whittaker functions in [3]. We therefore obtain $Z(s, W, \Phi_0) = L(s, \pi)$ because of Proposition 5.1.

From now on, we assume that N_π is positive. By (3.5), we have

$$Z(s, W, \Phi_{N_\pi}) = Z(s, W) L_E(s, 1),$$

and hence it is enough to compute $Z(s, W)$. One can easily observe that

$$Z(s, W) = \int_{E^\times} W(t(a)) |a|_E^{s-1} d^\times a = \sum_{i=0}^{\infty} W(t(\varpi^i)) q^{2i(1-s)}. \tag{5.2}$$

So we shall give a recursion formula for $W(t(\varpi^i))$, $i \geq 0$, in terms of two "Hecke eigenvalues" λ and ν.

We abbreviate $N = N_\pi$. Let us define the eigenvalue λ. We define a level raising operator $\theta' : V(N) \to V(N+1)$ by

$$\theta' v = \int_{K_{N+1}} \pi(k) v dk, \ v \in V(N),$$

and a level lowering operator $\delta : V(N+1) \to V(N)$ by

$$\delta w = \int_{K_N} \pi(k) w dk, \ w \in V(N+1).$$

Since $\dim V(N) = 1$, there exists $\lambda \in \mathbf{C}$ such that

$$\lambda v = \delta \theta' v,$$

for all $v \in V(N)$.

Next, we define the eigenvalue ν. Put

$$\zeta = \begin{pmatrix} \varpi & & \\ & 1 & \\ & & \varpi^{-1} \end{pmatrix} \in G.$$

We define the Hecke operator T on $V(N+1)$ by

$$T v = \int_{K_{N+1} \zeta K_{N+1}} \pi(k) v dk, \ v \in V(N+1).$$

Because $\dim V(N+1) = 1$, there exists ν in $\mathbf{C}$ such that

$$T v = \nu v,$$

for all $v \in V(N+1)$.

With the notation as above, we obtain the following recursion formula for $W(t(\varpi^i))$, $i \geq 0$.

Proposition 5.3. *Let (π, V) be an irreducible generic representation of G whose conductor N_π is positive. For any newform W in $\mathcal{W}(\pi, \psi_E)$, we have*

$$(\nu + q^2 - \lambda)c_i + q(\nu + q^2 - q^3)c_{i+1} = q^5 c_{i+2}, \ i \geq 0,$$
$$(\nu - q^3)c_0 = q^4 c_1,$$

where $c_i = W(t(\varpi^i))$, $i \geq 0$.

By (5.2) and Proposition 5.3, we can describe the zeta integral of newforms in terms of λ and ν:

Proposition 5.4. *Let (π, V) be an irreducible generic representation of G whose conductor N_π is positive and W its newform in $\mathcal{W}(\pi, \psi_E)$ such that $W(1) = 1$. Then we have*

$$Z(s, W) = \frac{1 - q^{-2s}}{1 - \dfrac{\nu + q^2 - q^3}{q^2} q^{-2s} - \dfrac{\nu + q^2 - \lambda}{q} q^{-4s}}.$$

In particular,

$$Z(s, W, \Phi_{N_\pi}) = \frac{1}{1 - \dfrac{\nu + q^2 - q^3}{q^2} q^{-2s} - \dfrac{\nu + q^2 - \lambda}{q} q^{-4s}}.$$

5.3 Proof of Lemma 4.2

We have seen that Lemma 4.2 holds for the unramified principal series representations.

Let π be an irreducible generic representation of G. We assume that N_π is positive. Proof of Lemma 4.2 is done by comparing Propositions 5.1 and 5.4. Suppose that π is supercuspidal or a subrepresentation of $\mathrm{Ind}_B^G \mu_1 \otimes \mu_2$, for some ramified quasi-character μ_1 of $E^\times$. Then it follows from Proposition 5.1 that $L(s, \pi) = 1$ or $L_E(s, 1)$. By definition, we have $Z(s, W, \Phi_{N_\pi})/L(s, \pi) \in \mathbf{C}[q^{-2s}, q^{2s}]$. So we get

$$Z(s, W, \Phi_{N_\pi})/L(s, \pi) = 1 \text{ or } 1/L_E(s, 1),$$

by Proposition 5.4.

Suppose that π is a subrepresentation of $\mathrm{Ind}_B^G \mu_1 \otimes \mu_2$, for unramified μ_1. Then we can regard newforms for π as functions in $\mathrm{Ind}_B^G \mu_1 \otimes \mu_2$. Due to [6], non-zero newforms f in π satisfy $f(1) \neq 0$. By using this property of newforms, we can compute the eigenvalues ν and λ explicitly, and Lemma 4.2 follows.

References

[1] E. M. Baruch. On the gamma factors attached to representations of $\mathrm{U}(2,1)$ over a p-adic field. *Israel J. Math.*, 102:317–345, 1997.

[2] W. Casselman. On some results of Atkin and Lehner. *Math. Ann.*, 201:301–314, 1973.

[3] W. Casselman and J. Shalika. The unramified principal series of p-adic groups. II. The Whittaker function. *Compositio Math.*, 41(2):207–231, 1980.

[4] S. Gelbart and I. Piatetski-Shapiro. Automorphic forms and L-functions for the unitary group. In *Lie group representations, II (College Park, Md., 1982/1983)*, volume 1041 of *Lecture Notes in Math.*, pages 141–184. Springer, Berlin, 1984.

[5] H. Jacquet, I. Piatetski-Shapiro, and J. Shalika. Conducteur des représentations du groupe linéaire. *Math. Ann.*, 256(2):199–214, 1981.

[6] M. Miyauchi. Conductors and newforms for non-supercuspidal representations of unramified $\mathrm{U}(2,1)$, preprint, arXiv:1112.4899v1, 2011.

[7] M. Miyauchi. On epsilon factors of supercuspidal representations of unramified $\mathrm{U}(2,1)$, arXiv:1111.2212v1. *accepted for publication in Trans. Amer. Math. Soc.*, 2012.

[8] M. Miyauchi. On L-factors attached to generic representations of unramified $\mathrm{U}(2,1)$, preprint, 2012.

[9] M. Miyauchi. On local newforms for unramified $\mathrm{U}(2,1)$, arXiv:1105.6004v1. *accepted for publication in Manuscripta Math.*, 2012.

[10] M. Reeder. Old forms on GL_n. *Amer. J. Math.*, 113(5):911–930, 1991.

[11] B. Roberts and R. Schmidt. *Local newforms for GSp(4)*, volume 1918 of *Lecture Notes in Mathematics*. Springer, Berlin, 2007.