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Coexistence in a diffusive Lotka-Volterra prey-predator system with a protection zone

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1 Introduction

In this article, we discuss the following Lotka-Volterra prey-predator system:

$$
\text{(P)}\begin{cases}
u_t = \Delta[(1+k\rho(x)v)u] + u(\lambda\chi_{\overline{\Omega}_0} + \nu\chi_{\overline{\Omega}\setminus\overline{\Omega}_0} - u - b(x)v) & \text{in } \Omega\times(0,\infty),\\
v_t = \Delta v + v(\mu + cu - v) & \text{in } \Omega\setminus\overline{\Omega}_0\times(0,\infty),\\
\partial_n u = 0 & \text{on } \partial\Omega\times(0,\infty),\\
\partial_n v = 0 & \text{on } \partial(\Omega\setminus\overline{\Omega}_0)\times(0,\infty),\\
u(x,0) = u_0(x)\geq 0 & \text{in } \Omega,\\
v(x,0) = v_0(x)\geq 0 & \text{in } \Omega\setminus\overline{\Omega}_0,
\end{cases}
$$

where Ω and Ω_0 are bounded domains in $\mathbb{R}^N (N\geq 1)$ with smooth boundaries $\partial\Omega$ and $\partial\Omega_0$ respectively, and satisfy the following condition: if $N\geq 2$, then $\overline{\Omega}_0\subset\Omega$; if $N=1$ and $\Omega=(a_1,a_2)$ for $a_1<a_2$, then $\Omega_0=(a,a_2)$ for some $a\in(a_1,a_2)$. Moreover, n is the outward unit normal vector on the boundary and $\partial_n=\partial/\partial n$; $k\geq 0$, $\lambda>0$, $\mu>0$, $\nu>0$ and $c>0$ are all constants; $\chi_{\overline{\Omega}_0}$ and $\chi_{\overline{\Omega}\setminus\overline{\Omega}_0}$ are characteristic functions; $\rho(x)$ is a smooth function in $\overline{\Omega}$ with $\partial_n\rho=0$ on $\partial\Omega$ and $b(x)$ is a Hölder continuous function in $\overline{\Omega}$. We assume that $\rho(x)=b(x)=0$ in $\overline{\Omega}_0$ since v is not defined in Ω_0. On the other hand, we assume that $\rho(x)>0$ and $b(x)>0$ in $\overline{\Omega}\setminus\overline{\Omega}_0$ and that both $\rho(x)/b(x)$ and $b(x)/\rho(x)$ are bounded in $\overline{\Omega}\setminus\overline{\Omega}_0$.

In (P), $u(x,t)$ and $v(x,t)$ are unknown functions representing population densities of prey and predator respectively; λ and ν denote the intrinsic growth rates of the prey species in $\overline{\Omega}_0$ and $\overline{\Omega}\setminus\overline{\Omega}_0$ respectively and μ denotes the intrinsic growth rate of the predator species; $b(x)$ and c represent the coefficients of prey-predator interaction; the homogeneous Neumann boundary condition means that no individual can cross the boundary.

In the first equation of (P), the nonlinear diffusion term $k\Delta[\rho(x)vu]$ means that the movements of individuals of the prey species in $\overline{\Omega}\setminus\overline{\Omega}_0$ are affected by population pressure from the predator species. Such a nonlinear diffusion term is usually called a cross-diffusion term, which was first introduced in Shigesada et al. [10].

In (P), only the prey species can go into the subregion Ω_0 of the habitat Ω. This means that the prey species is protected from predation in Ω_0. We call the subregion Ω_0 a protection zone. The effect of a protection zone on population models was first studied mathematically by Du and Shi [4]. Since their work was published, several mathematicians have studied population models with a protection zone for an endangered species under the assumption that the intrinsic growth rate of the endangered species is the same in the two regions Ω_0 and $\Omega\setminus\overline{\Omega}_0$ (see [2], [3], [7], [11]). On the other hand, the author [8] has studied the stationary problem of (P), where the intrinsic growth rates of the prey species are given separately in the two regions Ω_0 and $\Omega\setminus\overline{\Omega}_0$.

The stationary problem of (P) is

$$
\text{(SP)}\begin{cases}
\Delta[(1+k\rho(x)v)u]+u(\lambda\chi_{\overline{\Omega}_0}+\nu\chi_{\overline{\Omega}\setminus\overline{\Omega}_0}-u-b(x)v)=0 & \text{in } \Omega,\\
\Delta v+v(\mu+cu-v)=0 & \text{in } \Omega\setminus\overline{\Omega}_0,\\
\partial_n u=0 & \text{on } \partial\Omega,\\
\partial_n v=0 & \text{on } \partial(\Omega\setminus\overline{\Omega}_0).
\end{cases}
$$

The main purpose of this article is to make a resume of recent results obtained by the author [8]. Define

$$
U=(1+k\rho(x)v)u. \tag{1.1}
$$

Then (SP) is rewritten in the following form:

$$
\text{(EP)}\begin{cases}
\Delta U+\dfrac{U}{1+k\rho(x)v}\left(\lambda\chi_{\overline{\Omega}_0}+\nu\chi_{\overline{\Omega}\setminus\overline{\Omega}_0}-\dfrac{U}{1+k\rho(x)v}-b(x)v\right)=0 \text{ in } \Omega,\\
\Delta v+v\left(\mu+\dfrac{cU}{1+k\rho(x)v}-v\right)=0 & \text{in } \Omega\setminus\overline{\Omega}_0,\\
\partial_n U=0 & \text{on } \partial\Omega,\\
\partial_n v=0 & \text{on } \partial(\Omega\setminus\overline{\Omega}_0).
\end{cases}
$$

For $p>N$, we define

$$
X_1=W_n^{2,p}(\Omega)\times W_n^{2,p}(\Omega\setminus\overline{\Omega}_0), \tag{1.2}
$$

where $W_n^{2,p}(O)=\{w\in W^{2,p}(O):\partial_n w=0 \text{ on } \partial O\}$. It is said that (U,v) is a positive solution of (EP) if $(U,v)\in X_1$, $U>0$ in $\overline{\Omega}$, $v>0$ in $\overline{\Omega}\setminus\Omega_0$ and (U,v) satisfies (EP). We call (u,v) a positive solution of (SP) if (U,v) is a positive solution of (EP) and u is defined by (1.1). From a biological point of view, a positive solution of (SP) means a coexistence state of prey and predator.

For $q \in L^\infty(\Omega)$, we denote by $\lambda_1^N(q,\Omega)$ the first eigenvalue of $-\Delta+q$ over Ω with the homogeneous Neumann boundary condition. Moreover, we define

$$\lambda_\infty^*(k) = \begin{cases} \inf\limits_{\phi\in S} \dfrac{\displaystyle\int_\Omega |\nabla\phi|^2 dx + \frac{1}{k}\int_{\Omega\setminus\overline{\Omega}_0} \frac{b(x)}{\rho(x)}\phi^2 dx}{\displaystyle\int_{\Omega_0} \phi^2 dx} & \text{if } k>0, \\ \lambda_1^D(\Omega_0) & \text{if } k=0, \end{cases}$$

where $S = \{\phi \in H^1(\Omega) : \int_{\Omega_0}\phi^2 dx > 0\}$ and $\lambda_1^D(\Omega_0)$ is the first eigenvalue of $-\Delta$ over Ω_0 with the homogeneous Dirichlet boundary condition (when $N=1$, the homogeneous Dirichlet boundary condition $\phi(a) = \phi(a_2) = 0$ is replaced by $\phi(a) = \phi'(a_2) = 0$, but we use the same symbol $\lambda_1^D(\Omega_0)$). We are now in a position to state the main results of this article.

Theorem 1.1. *The following results hold true:*

(i) *Suppose that* $\lambda \geq \lambda_\infty^*(k)$. *For any* $\mu>0$ *and* $\nu>0$, (SP) *has at least one positive solution.*

(ii) *Suppose that* $0<\lambda<\lambda_\infty^*(k)$. *For any* $\nu>0$, *there exists a positive number* μ^* *such that* (SP) *has at least one positive solution if* $0<\mu<\mu^*$ *and has no positive solution if* $\mu \geq \mu^*$. *Furthermore,* μ^* *satisfies*

$$\lambda_1^N\left(\frac{b(x)\mu^* - \lambda\chi_{\overline{\Omega}_0} - \nu\chi_{\overline{\Omega}\setminus\overline{\Omega}_0}}{1+k\rho(x)\mu^*}, \Omega\right) = 0.$$

Theorem 1.2. $\lambda_\infty^*(k)$ *is continuous and strictly decreasing with respect to* $k \geq 0$ *and satisfies*

$$\lim_{k\to\infty} k\lambda_\infty^*(k) = \frac{1}{|\Omega_0|}\int_{\Omega\setminus\overline{\Omega}_0} \frac{b(x)}{\rho(x)} dx.$$

Theorem 1.3. *Suppose that* $\lambda > \lambda_\infty^*(k)$. *Let* (u_μ, v_μ) *be any positive solution of* (SP) *for each* $\mu>0$.

(i) *If* $k>0$, *then*

$$\lim_{\mu\to\infty} u_\mu = U_{\lambda,k} \ \text{ in } C^1(\overline{\Omega}_0),$$
$$\lim_{\mu\to\infty} u_\mu = 0 \ \text{ uniformly in any compact subset of } \overline{\Omega}\setminus\overline{\Omega}_0,$$
$$\lim_{\mu\to\infty} v_\mu = \infty \ \text{ uniformly in } \overline{\Omega}\setminus\Omega_0,$$

where $U_{\lambda,k}$ *is the unique positive solution of*

$$\Delta U + U\left\{\chi_{\overline{\Omega}_0}(\lambda - U) - \frac{b(x)}{k\rho(x)}\chi_{\overline{\Omega}\setminus\overline{\Omega}_0}\right\} = 0 \ \text{ in } \Omega, \quad \partial_n U = 0 \ \text{ on } \partial\Omega.$$

(ii) *If* $k = 0$, *then*

$$\lim_{\mu\to\infty} u_\mu = W_\lambda \ \textit{uniformly in } \overline{\Omega} \quad \textit{and} \quad \lim_{\mu\to\infty}(v_\mu - \mu) = 0 \ \textit{uniformly in } \overline{\Omega}\setminus\Omega_0.$$

Here, W_λ *is defined by* $W_\lambda = 0$ *in* $\overline{\Omega}\setminus\Omega_0$ *and* $W_\lambda = w_\lambda$ *in* Ω_0, *where* w_λ *is the unique positive solution of*

$$\begin{cases} \Delta w + w(\lambda - w) = 0 \ \textit{ in } \Omega_0, \\ w = 0 \ \textit{ on } \partial\Omega_0 \ \textit{ if } N \geq 2, \quad w(a) = w'(a_2) = 0 \ \textit{ if } N = 1. \end{cases}$$

We remark that Theorems 1.1–1.3 were obtained in [8]. We refer to [8] for the proof of Theorems 1.2 and 1.3.

Theorem 1.1 implies that the environments inside the protection zone are more important for the prey species than those outside it. Moreover, Theorems 1.1 and 1.2 assert that even if λ is very small, the existence of a positive solution of (SP) is guaranteed for any $\mu > 0$ and $\nu > 0$ if the cross-diffusion effect is sufficiently large. Theorem 1.3 implies that as $\mu \to \infty$, the prey species concentrates inside the protection zone and the two species become spatially segregated under the assumption $\lambda > \lambda_\infty^*(k)$.

When $N = 1$, we can obtain the following theorem on the profiles of $U_{\lambda,k}$ and w_λ in Ω_0.

Theorem 1.4. *Suppose that* $\lambda > \lambda_\infty^*(k)$, $N = 1$, $\Omega = (a_1, a_2)$ *and* $\Omega_0 = (a, a_2)$ *with* $a_1 < a < a_2$. *Then both* $U_{\lambda,k}$ *and* w_λ *are convex upwards and strictly increasing in* (a, a_2).

This article is organized as follows. In Section 2, we will give an outline of the proof of Theorem 1.1. In Section 3, we will prove Theorem 1.4.

2 Proof of Theorem 1.1

2.1 Local bifurcation

The following change of variables in (EP) is convenient for obtaining positive solutions by using the bifurcation theory:

$$\tau = \frac{\nu}{\lambda}. \tag{2.1}$$

Then (EP) is rewritten in the following form:

$$(\mathrm{EP})_\tau \begin{cases} \Delta U + f_\tau(\lambda, U, v) = 0 & \text{in } \Omega, \\ \Delta v + f(U, v) = 0 & \text{in } \Omega\setminus\overline{\Omega}_0, \\ \partial_n U = 0 & \text{on } \partial\Omega, \\ \partial_n v = 0 & \text{on } \partial(\Omega\setminus\overline{\Omega}_0), \end{cases}$$

where

$$\begin{cases} f_\tau(\lambda, U, v) = \dfrac{U}{1+k\rho(x)v}\left(\lambda(\chi_{\overline{\Omega}_0} + \tau\chi_{\overline{\Omega}\setminus\overline{\Omega}_0}) - \dfrac{U}{1+k\rho(x)v} - b(x)v\right), \\ f(U,v) = v\left(\mu + \dfrac{cU}{1+k\rho(x)v} - v\right). \end{cases}$$

We will use λ as a bifurcation parameter to obtain positive solutions which bifurcate from the constant solution curve

$$\Gamma_v = \{(\lambda, U, v) = (\lambda, 0, \mu) : \lambda > 0\}.$$

For any $\tau > 0$, we define

$$\Sigma_\tau = \left\{(\lambda, \mu) \in [0,\infty)^2 : \lambda_1^N\left(\frac{b(x)\mu - \lambda(\chi_{\overline{\Omega}_0} + \tau\chi_{\overline{\Omega}\setminus\overline{\Omega}_0})}{1+k\rho(x)\mu}, \Omega\right) = 0\right\}. \tag{2.2}$$

Then the following lemma holds true.

Lemma 2.1. *For any $\tau > 0$, the set Σ_τ forms an unbounded curve and can be expressed as $\Sigma_\tau = \{(\lambda^*(\mu,\tau), \mu) : \mu \geq 0\}$, where $\lambda^*(\mu,\tau)$ is continuous and strictly increasing with respect to $\mu \geq 0$ and satisfies $\lambda^*(0,\tau) = 0$ and $\lim_{\mu\to\infty}\lambda^*(\mu,\tau) = \lambda^*_\infty(k)$.*

Let $\lambda^* = \lambda^*(\mu,\tau)$ be the positive number defined in Lemma 2.1 and let ϕ^* be a positive solution of

$$-\Delta\phi^* + \frac{b(x)\mu - \lambda^*(\chi_{\overline{\Omega}_0} + \tau\chi_{\overline{\Omega}\setminus\overline{\Omega}_0})}{1+k\rho(x)\mu}\phi^* = 0 \text{ in } \Omega, \quad \partial_n\phi^* = 0 \text{ on } \partial\Omega.$$

Moreover, we define

$$\psi^* = (-\Delta + \mu I)^{-1}_{\Omega\setminus\overline{\Omega}_0}\left[\frac{c\mu}{1+k\rho(x)\mu}\phi^*\right],$$

where I is the identity mapping and $(-\Delta+\mu I)^{-1}_{\Omega\setminus\overline{\Omega}_0}$ is the inverse operator of $-\Delta+\mu I$ over $\Omega \setminus \overline{\Omega}_0$ subject to the homogeneous Neumann boundary condition. Then we can prove the following local bifurcation property by applying the local bifurcation theorem of Crandall and Rabinowitz [1] to $(\mathrm{EP})_\tau$.

Lemma 2.2. *Positive solutions of $(\mathrm{EP})_\tau$ bifurcate from Γ_v if and only if $\lambda = \lambda^*$. Precisely, all positive solutions of $(\mathrm{EP})_\tau$ near $(\lambda^*, 0, \mu) \in \mathbb{R} \times X_1$ can be expressed as*

$$\Gamma_\delta = \{(\lambda, U, v) = (\lambda(s), s(\phi^* + U(s)), \mu + s(\psi^* + v(s))) : s \in (0,\delta)\}$$

for some $\delta > 0$, where X_1 is the Sobolev space defined by (1.2). Here $(\lambda(s), U(s), v(s))$ is a smooth function with respect to s and satisfies $(\lambda(0), U(0), v(0)) = (\lambda^*, 0, 0)$ and $\int_\Omega U(s)\phi^* dx = 0$.

2.2 A priori estimates and non-existence result of positive solutions

By virtue of the maximum principle due to Lou and Ni [6], we can derive the following a priori estimates of positive solutions of $(\mathrm{EP})_\tau$.

Lemma 2.3. *Let $\theta \in (0,1)$. Then there exist two positive constants C_1 and C_2 such that any positive solution (U, v) of $(\mathrm{EP})_\tau$ satisfies*

$$v > \mu \ \text{ in } \overline{\Omega} \setminus \Omega_0, \quad \|U\|_{C^{1,\theta}(\overline{\Omega})} \leq C_1 \quad \text{and} \quad \|v\|_{C^{1,\theta}(\overline{\Omega}\setminus\Omega_0)} \leq C_2.$$

Here, if $k > 0$, then C_1 can be chosen independently of $\mu > 0$.

Furthermore, we can prove the following non-existence result of positive solutions.

Lemma 2.4. *If $\lambda \leq \lambda^*(\mu, \tau)$, then $(\mathrm{EP})_\tau$ has no positive solution.*

2.3 Completion of the proof of Theorem 1.1

Firstly, we will prove the following proposition.

Proposition 2.5. *$(\mathrm{EP})_\tau$ has at least one positive solution if and only if $\lambda > \lambda^*(\mu, \tau)$.*

Proof of Proposition 2.5. Define

$$E = C_n^1(\overline{\Omega}) \times C_n^1(\overline{\Omega} \setminus \Omega_0),$$

where $C_n^1(\overline{O}) = \{w \in C^1(\overline{O}) : \partial_n w = 0 \text{ on } \partial O\}$. For the local bifurcation branch Γ_δ which was obtained in Lemma 2.2, let $\Gamma \subset \mathbb{R} \times E$ denote the maximal connected set satisfying

$$\Gamma_\delta \subset \Gamma \subset \{(\lambda, U, v) \in (\mathbb{R} \times E) \setminus \{(\lambda^*, 0, \mu)\} : (U, v) \text{ is a solution of } (\mathrm{EP})_\tau\}.$$

By virtue of the strong maximum principle, we can verify that

$$\Gamma \subset \mathbb{R} \times P_\Omega \times P_{\Omega\setminus\overline{\Omega}_0} \tag{2.3}$$

holds, where $P_O = \{w \in C_n^1(\overline{O}) : w > 0 \ \text{ in } \overline{O}\}$. Define

$$Y = \left\{(\phi, \psi) \in E : \int_\Omega \phi\phi^* dx = 0\right\}, \tag{2.4}$$

that is, Y is the supplement of span $\{(\phi^*, \psi^*)\}$ in E. It follows from the global bifurcation theory of Rabinowitz [9] that one of the following non-excluding properties holds (see Rabinowitz [9] and Theorem 6.4.3 in López-Gómez [5]):

(1) Γ is unbounded in $\mathbb{R} \times E$.

(2) There exists a constant $\bar{\lambda} \neq \lambda^*$ such that $(\bar{\lambda}, 0, \mu) \in \Gamma$.

(3) There exists $(\tilde{\lambda}, \tilde{\phi}, \tilde{\psi}) \in \mathbb{R} \times (Y \setminus \{(0,\mu)\})$ such that $(\tilde{\lambda}, \tilde{\phi}, \tilde{\psi}) \in \Gamma$.

Case (2) is impossible because of (2.3). Due to (2.3), (2.4) and $\phi^* > 0$, case (3) is also impossible. Thus, case (1) must hold. Therefore, we see from (2.3) and Lemmas 2.3 and 2.4 that $(\mathrm{EP})_\tau$ has at least one positive solution if and only if $\lambda > \lambda^*$. Hence the proof of Proposition 2.5 is complete. □

We are now ready to prove Theorem 1.1.

Proof of Theorem 1.1. On account of (2.1) and Proposition 2.5, we notice that (SP) has at least one positive solution if and only if $\lambda > \lambda^*(\mu, \nu/\lambda)$. We first prove part (i) for any fixed $\lambda \geq \lambda^*_\infty(k)$. Lemma 2.1 yields

$$\lambda^* \left(\mu, \frac{\nu}{\lambda}\right) < \lim_{\mu \to \infty} \lambda^* \left(\mu, \frac{\nu}{\lambda}\right) = \lambda^*_\infty(k) \leq \lambda$$

for any $\mu > 0$ and $\nu > 0$. Therefore, (SP) has at least one positive solution for any $\mu > 0$ and $\nu > 0$.

We next prove part (ii) for any fixed $\lambda \in (0, \lambda^*_\infty(k))$ and $\nu > 0$. By Lemma 2.1, we see that $\lambda^*(\mu, \nu/\lambda)$ is continuous and strictly increasing with respect to $\mu \geq 0$ and satisfies

$$\lambda^* \left(0, \frac{\nu}{\lambda}\right) = 0 < \lambda \quad \text{and} \quad \lim_{\mu \to \infty} \lambda^* \left(\mu, \frac{\nu}{\lambda}\right) = \lambda^*_\infty(k) > \lambda.$$

Thus, we can find a positive number μ^* such that $\lambda > \lambda^*(\mu, \nu/\lambda)$ holds if $0 < \mu < \mu^*$ and $\lambda \leq \lambda^*(\mu, \nu/\lambda)$ holds if $\mu \geq \mu^*$, and moreover, μ^* satisfies

$$\lambda_1^N \left(\frac{b(x)\mu^* - \lambda\chi_{\overline{\Omega}_0} - \nu\chi_{\overline{\Omega}\setminus\overline{\Omega}_0}}{1 + k\rho(x)\mu^*}, \Omega \right) = 0$$

because of (2.2) and Lemma 2.1. This means that (SP) has at least one positive solution if $0 < \mu < \mu^*$ and has no positive solution if $\mu \geq \mu^*$. □

3 Proof of Theorem 1.4

Proof of Theorem 1.4. We recall that $U_{\lambda,k}$ is the unique positive solution of

$$\begin{cases} U''_{\lambda,k} + U_{\lambda,k} \left\{ \chi_{[a,a_2)}(\lambda - U_{\lambda,k}) - \dfrac{b(x)}{k\rho(x)}\chi_{(a_1,a)} \right\} = 0 \ \text{ in } (a_1, a_2), \\ U'_{\lambda,k}(a_1) = U'_{\lambda,k}(a_2) = 0 \end{cases}$$

and w_λ is the unique positive solution of

$$\begin{cases} w''_\lambda + w_\lambda(\lambda - w_\lambda) = 0 \ \text{ in } (a, a_2), \\ w_\lambda(a) = w'_\lambda(a_2) = 0. \end{cases}$$

Then we can easily see that $0 < U_{\lambda,k} < \lambda$ and $0 < w_\lambda < \lambda$ in (a, a_2). Thus

$$U''_{\lambda,k} = -U_{\lambda,k}(\lambda - U_{\lambda,k}) < 0 \quad \text{and} \quad w''_\lambda = -w_\lambda(\lambda - w_\lambda) < 0 \quad \text{in } (a, a_2).$$

Hence we obtain $U'_{\lambda,k} > 0$ and $w'_\lambda > 0$ in (a, a_2) because of $U'_{\lambda,k}(a_2) = w'_\lambda(a_2) = 0$. Therefore, we get the conclusion. $\square$

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