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ON A CERTAIN SIMPLE RELATIVE TRACE FORMULA FOR GSp(4)

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(JOINT WORK WITH KIMBALL MARTIN)

NOTATION

Let F be a number field and $\mathbb{A}$ be its ring of adeles. Let ψ be a non-trivial character of $\mathbb{A}/F$. Let E be a quadratic extension of F. Let $\kappa=\kappa_{E/F}$ denote the quadratic character of $\mathbb{A}^{\times}/F^{\times}$ corresponding to the quadratic extension E/F in the sense of class field theory. Let σ denote the unique non-trivial element in $\mathrm{Gal}\,(E/F)$ and take $\eta\in E^{\times}$ such that $\eta^{\sigma}=-\eta$.

For a non-archimedean place v of F, we denote by $\mathcal{O}_v$ the ring of integers in F_v, and by Ξ_v the characteristic function of $\mathrm{GSp}_4(\mathcal{O}_v)$.

For any algebraic group G, we will denote its center by Z.

1. SETUP

1.1. GSp(4) **and the Novodvorsky subgroups.** Let G be the group $\mathrm{GSp}\,(4)$, i.e. an algebraic group over F defined by

$$G=\{g\in \mathrm{GL}\,(4)\mid {}^tgJg=\lambda\,(g)\,J,\ \ \lambda\,(g)\in \mathrm{GL}(1)\},\quad \text{where } J=\begin{pmatrix}0 & 1_2\\ -1_2 & 0\end{pmatrix}.$$

Here ${}^tg$ denotes the transpose of g and $\lambda\,(g)$ is called the *similitude norm* of g.

Let us define the *upper and lower Novodvorsky (or split Bessel) subgroups*, resp. H and $\bar{H}$, of G by

$$H=\left\{\begin{pmatrix}a&0&0&0\\0&b&0&0\\0&0&b&0\\0&0&0&a\end{pmatrix}\begin{pmatrix}1_2 & X\\ 0 & 1_2\end{pmatrix}\mid a,b\in \mathrm{GL}(1),\ X\in \mathrm{Sym}^2\right\}$$

and

$$\bar{H}=\left\{\begin{pmatrix}a&0&0&0\\0&b&0&0\\0&0&b&0\\0&0&0&a\end{pmatrix}\begin{pmatrix}1_2 & 0\\ Y & 1_2\end{pmatrix}\mid a,b\in \mathrm{GL}(1),\ Y\in \mathrm{Sym}^2\right\}.$$

Here Sym^2 denotes the group of 2×2 symmetric matrices.

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1.2. **Quaternion similitude unitary groups and the Bessel subgroups.** For each $\epsilon \in F^\times$, let D_ϵ denote the quaternion algebra over F defined by

$$D_\epsilon = \left\{ \begin{pmatrix} a & b\epsilon \\ b^\sigma & a^\sigma \end{pmatrix} \mid a, b \in E \right\}.$$

We shall identify $a \in E$ with $\begin{pmatrix} a & 0 \\ 0 & a^\sigma \end{pmatrix} \in D_\epsilon$. We recall that $\{D_\epsilon\}_\epsilon$ gives a set of representatives for the isomorphism classes of quaternion algebras over F containing E, when ϵ runs over a set of representatives for $F^\times / \mathrm{N}_{E/F}(E^\times)$. Let $D_\epsilon \ni \alpha \mapsto \bar{\alpha} \in D_\epsilon$ denote the canonical involution of D_ϵ, i.e.,

$$\overline{\begin{pmatrix} a & b\epsilon \\ b^\sigma & a^\sigma \end{pmatrix}} = \begin{pmatrix} a^\sigma & -b\epsilon \\ -b^\sigma & a \end{pmatrix}.$$

We define the *quaternion similitude unitary group* G_ϵ of degree two over D_ϵ to be

$$G_\epsilon = \left\{ g \in \mathrm{GL}(2, D_\epsilon) \mid g^* \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g = \mu(g) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mu(g) \in \mathrm{GL}(1) \right\}$$

where $g^* = \begin{pmatrix} \bar{\alpha} & \bar{\gamma} \\ \bar{\beta} & \bar{\delta} \end{pmatrix}$ for $g = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$. We recall that the G_ϵ's are inner forms of $G = \mathrm{GSp}(4)$. When $\epsilon = 1$, we have $D_1 \simeq \mathrm{Mat}_{2\times 2}(F)$ and $G = \alpha G_1 \alpha^{-1}$ in $\mathrm{GL}_4(E)$ where

$$\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ \eta & -\eta & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & \eta & -\eta \end{pmatrix}.$$

We define the *upper* (resp. *lower*) *(anisotropic) Bessel subgroup* R_ϵ (resp. $\bar{R}_\epsilon$) of G_ϵ by

$$R_\epsilon = \left\{ \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} 1 & X \\ 0 & 1 \end{pmatrix} \mid a \in E^\times, X \in D_\epsilon^- \right\},$$

$$\bar{R}_\epsilon = \left\{ \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} 1 & 0 \\ Y & 1 \end{pmatrix} \mid a \in E^\times, Y \in D_\epsilon^- \right\},$$

where $D_\epsilon^- = \{X \in D_\epsilon \mid X + \bar{X} = 0\}$.

2. Relative Trace Formula

2.1. **Relative trace formula for G.** We define characters θ and ψ of $H(\mathbb{A})$ and $\bar{H}(\mathbb{A})$ by

$$\theta\left[\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & a \end{pmatrix} \begin{pmatrix} 1_2 & X \\ 0 & 1_2 \end{pmatrix}\right] = \kappa(ab)\psi\left[\mathrm{tr}\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} X\right)\right]$$

and

$$\psi\left[\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & a \end{pmatrix} \begin{pmatrix} 1_2 & 0 \\ Y & 1_2 \end{pmatrix}\right] = \psi\left[\mathrm{tr}\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} Y\right)\right].$$

For a cuspidal representation π of $G(\mathbb{A})/Z(\mathbb{A})$, we define the *upper and lower Novodvorsky periods* (with respect to θ^{-1} and ψ^{-1})

$$\mathcal{P}:\pi \longrightarrow \mathbb{C}, \qquad \mathcal{P}':\pi \longrightarrow \mathbb{C}$$

by

$$\mathcal{P}(\phi)=\mathcal{P}_{\theta^{-1}}(\phi)=\int_{Z(\mathbb{A})H(F)\backslash H(\mathbb{A})}\phi(h)\,\theta^{-1}(h)\,dh, \tag{2.1}$$

$$\mathcal{P}'(\phi)=\mathcal{P}'_{\psi^{-1}}(\phi)=\int_{Z(\mathbb{A})\bar{H}(F)\backslash \bar{H}(\mathbb{A})}\phi(\bar{h})\psi^{-1}(\bar{h})\,d\bar{h}. \tag{2.2}$$

Here we remark that the Novodvorsky periods necessarily vanish if π is not generic. If π is generic, then these are essentially the integrals that arise in Novodvorsky's integral representation [14] for GSp(4) × GL(1), i.e. the spinor L-functions $L(s,\pi)$ and $L(s,\pi,\otimes\kappa)$, evaluated at $s=\frac{1}{2}$. In particular we have

$$\mathcal{P}\not\equiv 0 \iff L(1/2,\pi\otimes\kappa)\neq 0,$$
$$\mathcal{P}'\not\equiv 0 \iff L(1/2,\pi)\neq 0.$$

For $f\in C_c^\infty(G(\mathbb{A}))$, we consider the associated kernel function

$$K(x,y)=K_f(x,y)=\sum_{\gamma\in Z(F)\backslash G(F)}\int_{Z(\mathbb{A})} f(x^{-1}\gamma y z)\,dz.$$

Then one side of the relative trace formula of our concern will be derived from

$$J(f)=\int_{Z(\mathbb{A})\bar{H}(F)\backslash\bar{H}(\mathbb{A})}\int_{Z(\mathbb{A})H(F)\backslash H(\mathbb{A})} K_f(\bar{h},h)\psi(\bar{h})^{-1}\theta(h)\,d\bar{h}\,dh. \tag{2.3}$$

At least formally, the relative trace formula is an identity derived from the geometric and spectral expansions of $K(x,y)$, of the form

$$J(f)=\sum_{\gamma\in\bar{H}(F)\backslash G(F)/H(F)} J_\gamma(f)=\sum_{\pi\ \mathrm{cusp}} J_\pi(f)+J_{\mathrm{nc}}(f).$$

Here each $J_\gamma(f)$ is a certain relative orbital integral, $J_{\mathrm{nc}}(f)$ denotes the non-cuspidal contribution, and

$$J_\pi(f)=\sum_\phi \mathcal{P}'(\pi(f)\phi)\overline{\mathcal{P}(\phi)}$$

where π is a cuspidal automorphic representation of $G(\mathbb{A})/Z(\mathbb{A})$ and ϕ runs over an orthonormal basis for π. In particular

$$J_\pi\not\equiv 0 \iff L(1/2,\pi)L(1/2,\pi\otimes\kappa)\neq 0.$$

2.2. **Relative trace formula for G_ϵ.** Let τ and ξ be the characters of R_ϵ and $\bar{R}_\epsilon$ defined by

$$\tau\left[\begin{pmatrix} a & 0\\ 0 & a\end{pmatrix}\begin{pmatrix} 1 & X\\ 0 & 1\end{pmatrix}\right]=\psi\left[\mathrm{tr}(-\eta X)\right]$$

and

$$\xi\left[\begin{pmatrix} a & 0\\ 0 & a\end{pmatrix}\begin{pmatrix} 1 & 0\\ Y & 1\end{pmatrix}\right]=\psi\left[\mathrm{tr}(-\eta^{-1}Y)\right].$$

For a cuspidal representation π of $G_\epsilon(\mathbb{A})/Z(\mathbb{A})$, we define the *upper and lower Bessel periods* (with respect to τ^{-1} and ξ^{-1})

$$\mathcal{P}_\epsilon:\pi\longrightarrow\mathbb{C}, \qquad \mathcal{P}'_\epsilon:\pi\longrightarrow\mathbb{C}$$

by

$$\text{(2.4)} \qquad \mathcal{P}_\epsilon(\phi) = \mathcal{P}_{\epsilon,\tau^{-1}}(\phi) = \int_{Z(\mathbb{A})R_\epsilon(F)\backslash R_\epsilon(\mathbb{A})} \phi(r)\tau^{-1}(r)\,dr,$$

$$\text{(2.5)} \qquad \mathcal{P}'_\epsilon(\phi) = \mathcal{P}'_{\epsilon,\xi^{-1}}(\phi) = \int_{Z(\mathbb{A})\bar{R}_\epsilon(F)\backslash \bar{R}_\epsilon(\mathbb{A})} \phi(\bar{r})\xi^{-1}(\bar{r})\,d\bar{r}.$$

Here we remark that on π, $\mathcal{P}_\epsilon \not\equiv 0$ if and only if $\mathcal{P}'_\epsilon \not\equiv 0$ since

$$\mathcal{P}'_\epsilon(\phi) = \overline{\mathcal{P}_\epsilon(\pi(w_\eta)\phi)} \quad \text{where} \quad w_\eta = \begin{pmatrix} 0 & -\eta^2 \\ 1 & 0 \end{pmatrix}.$$

Thus if $\mathcal{P}_\epsilon \not\equiv 0$, we simply say π *has a Bessel period (with respect to E).*

Let $f_\epsilon \in C_c^\infty(G_\epsilon(\mathbb{A}))$ and we consider the associated kernel function

$$K_\epsilon(x,y) = K_{f_\epsilon}(x,y) = \sum_{\gamma\in Z(F)\backslash G_\epsilon(F)} \int_{Z(\mathbb{A})} f_\epsilon(x^{-1}\gamma y z)\,dz.$$

Then the other side of the relative trace formula of our concern will be derived from

$$\text{(2.6)} \qquad J_\epsilon(f_\epsilon) = \int_{Z(\mathbb{A})\bar{R}_\epsilon(F)\backslash \bar{R}_\epsilon(\mathbb{A})} \int_{Z(\mathbb{A})R_\epsilon(F)\backslash R_\epsilon(\mathbb{A})} K_{f_\epsilon}(\bar{r},r)\xi(\bar{r})^{-1}\tau(r)\,d\bar{r}\,dr.$$

Ignoring convergence issues, (2.6) should have a geometric expansion of the form

$$J_\epsilon(f_\epsilon) = \sum_{\gamma_\epsilon\in \bar{R}_\epsilon(F)\backslash G_\epsilon(F)/R_\epsilon(F)} J_{\gamma_\epsilon}(f_\epsilon),$$

where the distributions $J_{\gamma_\epsilon}(f_\epsilon)$ are given by certain (relative) orbital integrals.

On the other hand, (2.6) should also have a spectral expansion of the form

$$J_\epsilon(f_\epsilon) = \sum_{\pi_\epsilon \text{ cusp}} J_{\pi_\epsilon}(f_\epsilon) + J_{\epsilon,\mathrm{nc}}(f_\epsilon)$$

where π_ϵ runs over the cuspidal automorphic representations of $G_\epsilon(\mathbb{A})/Z(\mathbb{A})$ and $J_{\epsilon,\mathrm{nc}}$ comprises the contribution from the non-cuspidal part of the spectrum. Then we have

$$J_{\pi_\epsilon}(f_\epsilon) = \sum_\phi \mathcal{P}'_\epsilon(\pi_\epsilon(f_\epsilon)\phi)\overline{\mathcal{P}_\epsilon(\phi)},$$

where ϕ runs over a suitable orthonormal basis for the space of π_ϵ. This implies that π_ϵ has a Bessel period if and only if $J_{\pi_\epsilon} \not\equiv 0$.

3. Results

Motivated by Böcherer's conjecture [3] and inspired by Jacquet's work [8], Shalika and the author made the following conjectures.

Conjecture 1. ([6, Conjecture 1.10]) *Given a generic cuspidal representation π of $G(\mathbb{A})/Z(\mathbb{A})$ such that $L(1/2,\pi)L(1/2,\pi\otimes\kappa) \neq 0$, there exists a Jacquet–Langlands transfer π_ϵ of π to some $G_\epsilon(\mathbb{A})/Z(\mathbb{A})$ which has a Bessel period with respect to E.*

Conversely, given a cuspidal representation π_ϵ of $G_\epsilon(\mathbb{A})/Z(\mathbb{A})$ which has a Bessel period with respect to E, there exists a generic Jacquet–Langlands transfer π of π_ϵ to $G(\mathbb{A})/Z(\mathbb{A})$ such that $L(1/2,\pi)L(1/2,\pi\otimes\kappa) \neq 0$.

Here, by Jacquet–Langlands transfer, we mean that $\pi_v \simeq \pi_{\epsilon,v}$ for almost all v. While the existence of the global Jacquet–Langlands transfer for G/Z and G_ϵ/Z is not yet known, this should follow from the completion of Arthur's Book Project [1] (for split SO(5) and inner forms) or, at least in the cases of Conjecture 1, the relative trace formula below.

This non-vanishing conjecture should be viewed as the global Gross–Prasad conjecture for (SO(5), SO(2)) (with the trivial character on SO(2)). While Conjecture 1 does not give a special value formula such as the ones conjectured by Böcherer [3], the following more general (if somewhat imprecise) conjecture should.

Conjecture 2. ([6, Conjecture 1.8], first relative trace formula identity) *For "matching" functions f and $(f_\epsilon)_\epsilon$, one has an identity of distributions*

$$J(f) = \sum_{\epsilon} J_\epsilon(f_\epsilon), \tag{3.1}$$

where these distributions are suitably regularized.

Here, for f to match with a family of functions $(f_\epsilon)_\epsilon$ ($\epsilon \in F^\times/N_{E/F}(E^\times)$) means the following. One defines a one-to-one correspondence between the set of "regular" double cosets $\bar{H}(F)\gamma H(F)$ for $G(F)$ and union over ϵ of the "regular" double cosets $\bar{R}_\epsilon(F)\gamma_\epsilon R_\epsilon(F)$ for G_ϵ. Then one says the functions f and $(f_\epsilon)_\epsilon$ match if the orbital integrals $J_\gamma(f) = J_{\gamma_\epsilon}(f_\epsilon)$ are equal whenever γ corresponds to γ_ϵ. Roughly, the regular double cosets are the ones for which the orbital integrals as defined above are convergent. Then in general, one wants to regularize the *singular* (non-convergent) orbital integrals and show an equality of these regularized orbital integrals to deduce (3.1).

Leaving the singular orbital integrals aside, to show the existence of sufficiently many matching functions becomes the main issue. It can be easily reduced to showing the existence of *local matching functions.* In particular, one would like to choose $f_v = \Xi_v$ and $f_{\epsilon,v} = \Xi_v$ (when $G_\epsilon(F_v) \simeq G(F_v)$) for almost all v and hence one needs to show the local Novodvorsky orbital integrals for Ξ_v equal the local Bessel orbital integrals for Ξ_v. This is known as the fundamental lemma for the unit element, and was established in [6].

Supposing now one has (3.1), one would like to deduce that

$$J_\pi(f) = J_{\pi_\epsilon}(f_\epsilon) \tag{3.2}$$

for suitable Jacquet-Langlands pairs π and π_ϵ. The fundamental lemma for the Hecke algebra established in [5] says that at almost all places we can vary our matching functions f and $(f_\epsilon)_\epsilon$ in the Hecke algebra. Thus the principle of infinite linear independence of characters (or, in our case, Bessel distributions) gives an equality of the form

$$\sum_{\pi\in\Pi} J_\pi(f) = \sum_{\epsilon} \sum_{\pi_\epsilon\in\Pi_\epsilon} J_{\pi_\epsilon}(f_\epsilon), \tag{3.3}$$

where Π and Π_ϵ denote certain near equivalence classes for π and π_ϵ. These near equivalence classes should be contained in the global L-packets of π and its transfers π_ϵ. This would follow, for instance, from the completion of Arthur's Book Project [1]. Strong multiplicity one for generic representations of GSp(4) (proven by Jiang and Soudry [10] for F totally real) says the left hand side of (3.3) has at most one term. On the other hand, the weak form of the local Gross–Prasad conjectures say the right hand side of (3.3) has at most one term (the strong form of local Gross–Prasad says which ϵ and π_ϵ should appear). Hence one obtains (3.2), from which one should be able to obtain the desired L-value formula as in the GL(2) cases in [9], [12] and [2]. We refer to [15] and [13] for local Gross-Prasad conjectures. See also Lapid–Offen [11] and the recent work of W. Zhang [17] for instances of deducing L-value formulas from Bessel identities in higher-dimensional unitary cases.

Let us state our main results. To make our statements as simple as possible, *we will assume strong multiplicity one for generic representations for arbitrary F, the near equivalence classes above are contained in global L-packets, and (the weak form of) the local Gross–Prasad conjectures for* (SO(5), SO(2)). We expect these assumptions will be validated in the near future with the completion of Arthur's Book Project [1].

Suppose π is generic, locally tempered everywhere, and supercuspidal at some place split in E/F. Let ϵ, π_ϵ be such that π_ϵ is the unique Jacquet–Langlands transfer, assumed to exist and be automorphic, determined at all local places by the local Gross–Prasad conjectures so as to have non-vanishing local Bessel periods.

Theorem 1. *There exists a class of matching functions f and f_ϵ such that the Bessel identity* (3.2) *holds.*

We note that our choice of matching functions guarantees the geometric and spectral expansions of our trace formulas are convergent without any regularization of integrals. To get from (3.2) to a special value formula, a detailed study of local Bessel distributions as in [9], [12], [2], [11] or [17] is needed. At present, we merely conclude

Corollary 1. *Suppose now that E/F is split at each archimedean place. Then*

$$L(1/2,\pi)L(1/2,\pi\otimes\kappa)\neq 0$$

if and only if π_ϵ has a Bessel period with respect to E.

Thus we establish Conjectures 1 and 2 under certain assumptions.

We remark that, by completely different methods, Ginzburg–Jiang–Rallis [7] made substantial progress towards the global Gross–Prasad conjecture for (SO($2n+1$), SO(2)). However, they assume that their representations of SO($2n+1$) and SO(2) transfer to *cuspidal* representations of GL($2n$) and GL(2). Under these hypotheses, they obtain one direction of the global Gross–Prasad conjecture, and partial results for the converse direction. Our Corollary 1 establishes both directions of the global Gross–Prasad conjecture for the case (SO(5), SO(2)) (under our local hypotheses) in the case that the SO(2) representation is trivial, whence the SO(2) representation does not transfer to a cuspidal representation of GL(2). Thus, there is no overlap of this result with the results of [7].

We hope to remove our local assumptions and eventually obtain an L-value formula with future work on these trace formulas. We also remark that W. Zhang [16] also recently established a global Gross–Prasad conjecture for certain unitary groups under some local assumptions by using a simple relative trace formula.

This note is an excerpt from [4], to which we refer the reader for details.

References

[1] J. Arthur, The endoscopic classification of representations. *Amer. Math. Soc. Colloquium Publications*, 61. Amer. Math. Soc., Providence, R. I. (2013).

[2] E. Baruch and Z. Mao, Central value of automorphic L-functions. *Geom. Funct. Anal.* 17 (2007), no. 2, 333–384.

[3] S. Böcherer, Bemerkungen über die Dirichletreihen von Koecher und Maaß, *Math. Gottingensis Schrift.* SFB. Geom. Anal. Heft **68**, 1986.

[4] M. Furusawa and K. Martin, On central critical values of the degree four L-functions for GSp(4): a simple trace formula. *Preprint.*

[5] M. Furusawa, K. Martin, and J. A. Shalika, On central critical values of the degree four L-functions for GSp(4): the fundamental lemma, III. *Mem. Amer. Math. Soc.* **225** (2013), no. 1057, x+134 pp.

[6] M. Furusawa and J. A. Shalika, On central critical values of the degree four L-functions for GSp(4): the fundamental lemma. *Mem. Amer. Math. Soc.* **164** (2003), no. 782, x+139 pp.

[7] D. Ginzburg, D. Jiang and S. Rallis, On the nonvanishing of the central value of the Rankin-Selberg L-functions. II. *Automorphic representations, L-functions and applications: progress and prospects*, 157191, Ohio State Univ. Math. Res. Inst. Publ., 11, de Gruyter, Berlin, 2005.

[8] H. Jacquet, *Sur un résultat de Waldspurger*, Ann. Sci. École Norm. Sup. (4) **19** (1986), 185–229.

[9] H. Jacquet and N. Chen. Positivity of quadratic base change L-functions. *Bull. Soc. Math. France*, 129(1):33–90, 2001.

[10] D. Jiang and D. Soudry, The multiplicity-one theorem for generic automorphic forms of GSp(4). *Pacific J. Math.*. **229** (2007), no. 2, 381–388.

[11] E. Lapid and O. Offen, Compact unitary periods. *Compos. Math.* 143 (2007), no. 2, 323–338.

[12] K. Martin and D. Whitehouse, Central L-values and toric periods for GL(2). *Int. Math. Res. Not.* IMRN **2009**, no. 1, 141–191.

[13] C. Moeglin and J.-L. Waldspurger, La conjecture locale de Gross–Prasad pour les groupes spéciaux orthogonaux : le cas général. *Astérisque* **347**, Sur les conjectures de Gross et Prasad. II. 167–216 (2012).

[14] M. E. Novodvorsky, Automorphic L-functions for symplectic group GSp(4), *Automorphic forms, representations and L-functions* (Proc. Sympos. Pure Math., Oregon State Univ., Corvallis, Ore., 1977), Part 2, pp. 87–95, Proc. Sympos. Pure Math., XXXIII, Amer. Math. Soc., Providence, R.I., 1979.

[15] D. Prasad and R. Takloo-Bighash, Bessel models for GSp (4). *J. Reine. Agnew. Math.* **655** (2011), 189–243.

[16] W. Zhang, Fourier transform and the global Gross–Prasad conjecture for unitary groups. *Preprint.*

[17] W. Zhang, Automorphic period and the central value of Rankin-Selberg L-function. *Preprint.*

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