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On a function space with the hypograph topology

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1 Introduction

The study of topologies on function spaces plays a significant role in geometric functional analysis. Since function spaces are frequently infinite-dimensional, the theory of infinite-dimensional topology has made meaningful contributions to it. Indeed, several function spaces have been shown to be homeomorphic to typical infinite-dimensional spaces. From the end of 1980s to the beginning of 1990s, many researchers investigated topological types of function spaces of real-valued continuous functions on countable spaces endowed with the pointwise convergence topology, see [8]. In this article, we define a hypograph of a map from a compact metrizable space to a dendrite and discuss the topology of the hypograph space. We can consider that hypograph spaces give certain geometric aspect to function spaces with the pointwise convergence topology. This article is a résumé of the joint work with K. Sakai and H. Yang [6].

Throughout the article, all maps are continuous, but functions are not necessarily continuous. Let X be a compact metrizable space and Y be a dendrite with an end point $\mathbf{0}$. Recall that a *dendrite* is a Peano continuum, namely a connected, locally connected, compact metrizable space, containing no simple closed curves. An *end point* of a space has an arbitrarily small open neighborhood whose boundary is a singleton. It is well-known that each pair of distinct points of a dendrite is connected by the unique arc [12, Chapter V, (1.2)]. We denote the unique arc of two points x, y in the dendrite Y by $[x, y]$, where it is the constant path if $x = y$.

For each function $f : X \to Y$, we define the *hypograph* $\downarrow f$ of f as follows:

$$\downarrow f = \bigcup_{x \in X} \{x\} \times [\mathbf{0}, f(x)] \subset X \times Y.$$

When f is continuous, the hypograph $\downarrow f$ is closed in $X \times Y$. We denote the set of maps from X to Y by $\mathrm{C}(X, Y)$ and the hyperspace of non-empty closed sets in $X \times Y$ endowed with the Vietoris topology by $\mathrm{Cld}(X \times Y)$. Then we have

$$\downarrow \mathrm{C}(X, Y) = \{\downarrow f \mid f \in \mathrm{C}(X, Y)\} \subset \mathrm{Cld}(X \times Y).$$

Let $\overline{\downarrow \mathrm{C}(X, Y)}$ be the closure of $\downarrow \mathrm{C}(X, Y)$ in $\mathrm{Cld}(X \times Y)$. In the case that Y is the closed unit interval $\mathbf{I} = [0, 1]$ and $\mathbf{0} = 0$, we can regard

$$\downarrow \mathrm{USC}(X, \mathbf{I}) = \{\downarrow f \mid f : X \to \mathbf{I} \text{ is upper semi-continuous}\}$$

as the subspace in $\mathrm{Cld}(X \times \mathbf{I})$. Let $\mathbf{Q} = \mathbf{I}^{\mathbb{N}}$ be the Hilbert cube and $\mathbf{c}_0 = \{(x_i)_{i \in \mathbb{N}} \in \mathbf{Q} \mid \lim_{i \to \infty} x_i = 0\}$. Z. Yang and X. Zhou [10, 11] showed the following theorem:

THEOREM 1.1. *Suppose that the set of isolated points of X is not dense. Then* $\overline{\downarrow\mathrm{USC}(X,\mathbf{I})} = \overline{\downarrow\mathrm{C}(X,\mathbf{I})}$ *and the pair* $(\overline{\downarrow\mathrm{USC}(X,\mathbf{I})}, \downarrow\mathrm{C}(X,\mathbf{I}))$ *is homeomorphic to* $(\mathbf{Q}, \mathbf{c}_0)$.

For spaces W_1 and W_2, the symbol (W_1, W_2) means that $W_2 \subset W_1$. A pair (W_1, W_2) of spaces is homeomorphic to (Z_1, Z_2) if there exists a homeomorphism $f : W_1 \to Z_1$ such that $f(W_2) = Z_2$. We generalize their result as follows:

MAIN THEOREM. *If X is infinite and locally connected, then the pair* $(\overline{\downarrow\mathrm{C}(X,Y)}, \downarrow\mathrm{C}(X,Y))$ *is homeomorphic to* $(\mathbf{Q}, \mathbf{c}_0)$.

2 Preliminaries

The topological characterizations for pairs of infinite-dimensional spaces goes back to the uniqueness of cap sets and f-d cap sets due to R.D. Anderson [1], and now, has reached the one of absorbing pairs for each Borel class, refer to [2, 3]. In this section, we shall introduce the notion of strong universality and absorbing pair for the proof of the main theorem. For each open cover $\mathcal{U}$ of a space Z, a map $f : W \to Z$ is *$\mathcal{U}$-close* to $g : W \to Z$ provided that for any $w \in W$, both of $f(w)$ and $g(w)$ are contained in some $U \in \mathcal{U}$. When $Z = (Z, d)$ is a metric space, for each $\epsilon > 0$, a map $f : W \to Z$ is said to be *ϵ-close* to $g : W \to Z$ if $d(f(w), g(w)) < \epsilon$ for all $w \in W$. Let (W_1, W_2) be a pair of spaces, and $\mathcal{C}_1$ and $\mathcal{C}_2$ be classes of spaces. We say that (W_1, W_2) is *strongly $(\mathcal{C}_1, \mathcal{C}_2)$-universal* if the following condition holds:

(su) Let $Z_1 \in \mathcal{C}_1$, $Z_2 \in \mathcal{C}_2$, K a closed subset of Z_1, and $f : Z_1 \to W_1$ a map such that the restriction $f|_K$ of K is a Z-embedding. Then for every open cover $\mathcal{U}$ of W_1, there exists a Z-embedding $g : Z_1 \to W_1$ such that g is $\mathcal{U}$-close to f, $g|_K = f|_K$ and $g^{-1}(W_2) \setminus K = Z_2 \setminus K$.

It is said that a closed subset A of W is a *Z-set* in W if for each open cover $\mathcal{U}$ of W, there exists a map $f : W \to W$ such that f is $\mathcal{U}$-close to the identity map id_W and $f(W) \cap A = \emptyset$. A countable union of Z-sets is called a *Z_σ-set*. In addition, a *Z-embedding* is an embedding whose image is a Z-set. A pair (W_1, W_2) is *$(\mathcal{C}_1, \mathcal{C}_2)$-absorbing* provided that the following conditions are satisfied:

(1) $W_1 \in \mathcal{C}_1$ and $W_2 \in \mathcal{C}_2$;

(2) W_2 is contained in a Z_σ-set in W_1;

(3) (W_1, W_2) is strongly $(\mathcal{C}_1, \mathcal{C}_2)$-universal.

Denote the class of compact metrizable spaces by $\mathcal{M}_0$, and the one of separable metrizable absolute $F_{\sigma\delta}$-spaces by $\mathcal{F}_{\sigma\delta}$. According to Theorem 1.7.6 of [3], the following can be established.

THEOREM 2.1. *Let W_1 and Z_1 be topological copies of the Hilbert cube $\mathbf{Q}$. If pairs (W_1, W_2) and (Z_1, Z_2) are $(\mathcal{M}_0, \mathcal{F}_{\sigma\delta})$-absorbing, then they are homeomorphic.*

The following fact is well known.

FACT 1. The pair $(\mathbf{Q}, \mathbf{c}_0)$ is $(\mathcal{M}_0, \mathcal{F}_{\sigma\delta})$-absorbing.

Combining Theorem 2.1 with Fact 1, we need to show the following conditions:

(1) $\overline{\downarrow\mathrm{C}(X,Y)}$ is homeomorphic to $\mathbf{Q}$ and $\downarrow\mathrm{C}(X,Y)$ is an $F_{\sigma\delta}$-set in $\overline{\downarrow\mathrm{C}(X,Y)}$;

(2) $\downarrow\mathrm{C}(X,Y)$ is contained in a Z_σ-set in $\overline{\downarrow\mathrm{C}(X,Y)}$;

(3) $(\overline{\downarrow\mathrm{C}(X,Y)}, \downarrow\mathrm{C}(X,Y))$ is strongly $(\mathcal{M}_0, \mathcal{F}_{\sigma\delta})$-universal.

3 The space $\overline{\downarrow C(X, Y)}$ is homeomorphic to the Hilbert cube

This section is devoted to proving the following theorem:

THEOREM 3.1. *If X has no isolated points, then $\overline{\downarrow C(X, Y)}$ is homeomorphic to $\mathbf{Q}$.*

First, we have the following proposition:

PROPOSITION 3.2. *If X has no isolated points, then $\overline{\downarrow C(X, Y)}$ is an AR.*

Sketch of proof. Observe that $\overline{\downarrow C(X, Y)}$ is a Peano continuum. According to the the Wojdysławski Theorem [13], see Theorem 5.3.14 of [7], the hyperspace $\mathrm{Cld}(\overline{\downarrow C(X, Y)})$ is an AR. Then we have the retraction

$$\bigcup : \mathrm{Cld}(\mathrm{Cld}(X \times Y)) \ni \mathcal{A} \mapsto \bigcup \mathcal{A} \in \mathrm{Cld}(X \times Y)$$

and $\bigcup(\mathrm{Cld}(\overline{\downarrow C(X, Y)})) = \overline{\downarrow C(X, Y)}$. It follows that $\overline{\downarrow C(X, Y)}$ is a retract of $\mathrm{Cld}(\overline{\downarrow C(X, Y)})$, which implies that $\overline{\downarrow C(X, Y)}$ is an AR. □

We say that a subset Z is *homotopy dense* in a space W if there exists a homotopy $h : W \times \mathbf{I} \to W$ such that $h(w, 0) = w$ and $h(w, t) \in Z$ for every $w \in W$ and $t > 0$. Using the same technique as [5, Theorem 4.1], we have the following:

PROPOSITION 3.3. *If X has no isolated points, then $\downarrow C(X, Y)$ is homotopy dense in $\overline{\downarrow C(X, Y)}$.*

Let d_X and d_Y be admissible metrics on X and Y, respectively. We use an admissible metric ρ on $X \times Y$ as follows:

$$\rho((x, y), (x', y')) = \max\{d_X(x, x'), d_Y(y, y')\} \text{ for each } x, x' \in X \text{ and } y, y' \in Y.$$

Since X and Y are compact, the hyperspace $\mathrm{Cld}(X \times Y)$ admits the Hausdorff metric ρ_H induced by ρ. For each $A \in \mathrm{Cld}(X \times Y)$, we define a set-valued function $A : X \to \mathrm{Cld}(Y) \cup \{\emptyset\}$ as follows:

$$A(x) = \{y \in Y \mid (x, y) \in A\} \in \mathrm{Cld}(Y) \cup \{\emptyset\}.$$

The following is the key lemma of this article.

LEMMA 3.4 (The Digging Lemma). *Let $\phi : Z \to \downarrow C(X, Y)$ be a map of a paracompact Hausdorff space Z. If X has a non-isolated point x_∞, then for each map $\epsilon : Z \to (0, 1)$, there exist maps $\psi : Z \to \downarrow C(X, Y)$ and $\delta : Z \to (0, 1)$ such that for each $z \in Z$,*

(a) $\rho_H(\phi(z), \psi(z)) < \epsilon(z)$,

(b) $\psi(z)(x) = \{\mathbf{0}\}$ *for all $x \in X$ with $d_X(x, x_\infty) < \delta(z)$.*

A space Z has the *disjoint cells property* provided that for any maps $f, g : \mathbf{Q} \to Z$ of the Hilbert cube and any open cover $\mathcal{U}$ of Z, there exist maps $f', g' : \mathbf{Q} \to Z$ such that f' and g' are $\mathcal{U}$-close to f and g, respectively, and $f'(\mathbf{Q}) \cap g'(\mathbf{Q}) = \emptyset$.

PROPOSITION 3.5. *If X has no isolated points, then $\overline{\downarrow C(X, Y)}$ has the disjoint cells property.*

Sketch of proof. Let $f, g : \mathbf{Q} \to \overline{\downarrow\mathrm{C}(X,Y)}$ be maps and $\epsilon > 0$. Since $\downarrow\mathrm{C}(X,Y)$ is homotopy dense in $\overline{\downarrow\mathrm{C}(X,Y)}$ by Proposition 3.3, we can obtain maps $f' : \mathbf{Q} \to \downarrow\mathrm{C}(X,Y)$ that is ϵ-close to f, and $g' : \mathbf{Q} \to \downarrow\mathrm{C}(X,Y)$ that is $\epsilon/3$-close to g. Taking a non-isolated point $x_\infty \in X$ and applying the Digging Lemma 3.4, we can find a map $g'' : \mathbf{Q} \to \downarrow\mathrm{C}(X,Y)$ such that g'' is $\epsilon/3$-close to g' and $g''(z)(x_\infty) = \{\mathbf{0}\}$ for all $z \in \mathbf{Q}$. Define a map $g''' : \mathbf{Q} \to \overline{\downarrow\mathrm{C}(X,Y)} \setminus \downarrow\mathrm{C}(X,Y)$ as follows:

$$g'''(z) = g''(z) \cup \{x_0\} \times \{y \in Y \mid d_Y(y, \mathbf{0}) \leq \epsilon/3\}.$$

Then f' and g''' are ϵ-close to f and g, respectively, and $f'(\mathbf{Q}) \cap g'''(\mathbf{Q}) = \emptyset$. □

Combining Propositions 3.2 and 3.5 with Toruńczyk's characterization of the Hilbert cube [9], we can obtain Theorem 3.1.

4 The space $\downarrow\mathrm{C}(X,Y)$ is an $F_{\sigma\delta}$-set in $\overline{\downarrow\mathrm{C}(X,Y)}$

In this section, we show the following proposition:

PROPOSITION 4.1. *The space $\downarrow\mathrm{C}(X,Y)$ is an $F_{\sigma\delta}$-set in $\overline{\downarrow\mathrm{C}(X,Y)}$.*

Sketch of proof. For each $\delta, \epsilon > 0$, define $\mathcal{A}(\delta, \epsilon) \subset \overline{\downarrow\mathrm{C}(X,Y)}$ as follows:

- $A \in \mathcal{A}(\delta, \epsilon)$ provided that for each $x_1, x_2 \in X$ with $d_X(x_1, x_2) < \delta$, if $y_i \in A(x_i)$ and $y_i \notin [\mathbf{0}, z_i]$ for any $z_i \in A(x_i) \setminus \{y_i\}$, $i = 1, 2$, then $d_Y(y_1, y_2) \leq \epsilon$.

Then it is closed in $\overline{\downarrow\mathrm{C}(X,Y)}$ and we have

$$\downarrow\mathrm{C}(X,Y) = \bigcap_{n\in\mathbb{N}} \bigcup_{m\in\mathbb{N}} \mathcal{A}(1/m, 1/n).$$

Hence $\downarrow\mathrm{C}(X,Y)$ is an $F_{\sigma\delta}$-set in $\overline{\downarrow\mathrm{C}(X,Y)}$. □

5 The space $\downarrow\mathrm{C}(X,Y)$ is contained in a Z_σ-set in $\overline{\downarrow\mathrm{C}(X,Y)}$

We use the following lemma for detecting Z-sets in $\overline{\downarrow\mathrm{C}(X,Y)}$.

LEMMA 5.1. *Suppose that $F = E \cup Z$ is a closed set in $\overline{\downarrow\mathrm{C}(X,Y)}$ such that Z is a Z-set in $\overline{\downarrow\mathrm{C}(X,Y)}$, and for each $A \in E$, there exists a point $a \in X$ with $A(a) = \{\mathbf{0}\}$. Then F is a Z-set in $\overline{\downarrow\mathrm{C}(X,Y)}$.*

PROPOSITION 5.2. *If X has no isolated points, then $\downarrow\mathrm{C}(X,Y)$ is contained in some Z_σ-set in $\overline{\downarrow\mathrm{C}(X,Y)}$.*

Sketch of proof. Take a countable dense set $D = \{d_n \mid n \in \mathbb{N}\}$ in X. For each $n, m \in \mathbb{N}$,

$$F_{n,m} = \{\downarrow f \in \downarrow\mathrm{C}(X,Y) \mid d_Y(f(d_n), \mathbf{0}) \geq 1/m\}$$

is a Z-set in $\downarrow\mathrm{C}(X,Y)$ due to the Digging Lemma 3.4. Then the closure $\overline{F_{n,m}}$ is a Z-set in $\overline{\downarrow\mathrm{C}(X,Y)}$ because $\downarrow\mathrm{C}(X,Y)$ is homotopy dense in $\overline{\downarrow\mathrm{C}(X,Y)}$ by Proposition 3.3. Moreover, we have

$$F = \bigcap_{n\in\mathbb{N}} \bigcap_{m\in\mathbb{N}} (\downarrow\mathrm{C}(X,Y) \setminus F_{n,m}) = \{X \times \{\mathbf{0}\}\}.$$

It follows from Lemma 5.1 that the closure $\overline{F}$ is a Z-set in $\overline{\downarrow\mathrm{C}(X,Y)}$. □

6 The pair $(\overline{\downarrow C(X,Y)}, \downarrow C(X,Y))$ is strongly $(\mathcal{M}_0, \mathcal{F}_{\sigma\delta})$-universal

We needs the following lemma to verify the strong $(\mathcal{M}_0, \mathcal{F}_{\sigma\delta})$-universality of $(\overline{\downarrow C(X,Y)}, \downarrow C(X,Y))$.

LEMMA 6.1. *Let $x_m, x_\infty \in X$, $m \in \mathbb{N}$, such that $\{r_m = d_X(x_m, x_\infty)\}_{m\in\mathbb{N}}$ is a strictly decreasing sequence conversing to 0, and let $y_0 \in Y \setminus \{\mathbf{0}\}$ such that $d_Y(\mathbf{0}, y_0) \le 1$. Suppose that $g : Z \to \mathbf{Q}$ is an injection from a space Z into the Hilbert cube $\mathbf{Q}$ and $\delta : Z \to (0,1)$ is a map. Then there exists a map $\Phi : Z \to \overline{\downarrow C(X, [\mathbf{0}, y_0])}$ satisfying the following conditions:*

(1) *Φ is injective;*

(2) *$\rho_H(\Phi(z), X \times \{\mathbf{0}\}) \le \delta(z)$ for all $z \in Z$;*

(3) *$\Phi(z)(x) = \{\mathbf{0}\}$ for all $x \in X$ with $d_X(x, x_\infty) \ge r_{2k}$ and $z \in Z$ with $2^{-k} \le \delta(z) \le 2^{-k+1}$, $k \in \mathbb{N}$;*

(4) *$z \in g^{-1}(\mathbf{c}_0)$ if and only if $\Phi(z) \in \downarrow C(X, [\mathbf{0}, y_0])$;*

(5) *$\Phi(z)(x_\infty) = \{y \in [\mathbf{0}, y_0] \mid d_Y(y, \mathbf{0}) \le \delta(z)\}$ for all $z \in Z$.*

PROPOSITION 6.2. *If X has no isolated points, then $(\overline{\downarrow C(X,Y)}, \downarrow C(X,Y))$ is strongly $(\mathcal{M}_0, \mathcal{F}_{\sigma\delta})$-universal.*

Sketch of proof. Let $Z \in \mathcal{M}_0$, $C \in \mathcal{F}_{\sigma\delta}$, K a closed subset of Z, $0 < \epsilon$ and $\Phi : Z \to \overline{\downarrow C(X,Y)}$ a map such that $\Phi|_K$ is a Z-embedding. We shall construct a Z-embedding $\Psi : Z \to \overline{\downarrow C(X,Y)}$ so that Ψ is ϵ-close to Φ, $\Psi|_K = \Phi|_K$ and $\Psi^{-1}(\downarrow C(X,Y)) \setminus K = C \setminus K$.

Since $\Phi(K)$ is a Z-set in $\overline{\downarrow C(X,Y)}$, we may assume that $\Phi(K) \cap \overline{\Phi(Z \setminus K)} = \emptyset$. Define $\delta(z) = \min\{\epsilon, \rho_H(\Phi(z), \Phi(K))\}/4$. Since $\downarrow C(X,Y)$ is homotopy dense in $\overline{\downarrow C(X,Y)}$ by Proposition 3.3, there exists $h : Z \to \overline{\downarrow C(X,Y)}$ such that $\rho_H(h(z), \Phi(z)) \le \delta(z)$ and $h(Z \setminus K) \subset \downarrow C(X,Y)$.

Take a non-isolated point $x_\infty \in X$. By the Digging Lemma 3.4, we can obtain $\psi : Z \setminus K \to \downarrow C(X,Y)$ and $r : Z \setminus K \to (0,1)$ so that

(a) $\rho_H(h(z), \psi(z)) \le \delta(z)$,

(b) $\psi(z)(x) = \{\mathbf{0}\}$ for all $x \in X$ with $d_X(x, x_\infty) < r(z)$.

Let $Z_k = \{z \in Z \mid 2^{-k} \le \delta(z) \le 2^{-k+1}\} \subset Z \setminus K$. Since x_∞ is a non-isolated point, we can choose $x_m \in X \setminus \{x_\infty\}$ so that $r_m = d_X(x_m, x_\infty) < \min\{1/m, d_X(x_{m-1}, x_\infty), r(z) \mid z \in Z_m\}$. Since $(\mathbf{Q}, \mathbf{c}_0)$ is strongly $(\mathcal{M}_0, \mathcal{F}_{\sigma\delta})$-universal by Fact 1, we can take am embedding $g : Z \to \mathbf{Q}$ so that $g^{-1}(\mathbf{c}_0) = C$. Choose $y_0 \in Y \setminus \{\mathbf{0}\}$ with $d_Y(\mathbf{0}, y_0) \le 1$. Using Lemma 6.1, we can obtain $\psi' : Z \setminus K \to \overline{\downarrow C(X, [\mathbf{0}, y_0])}$ satisfying the following conditions:

(1) ψ' is injective;

(2) $\rho_H(\psi'(z), X \times \{\mathbf{0}\}) \le \delta(z)$ for all $z \in Z \setminus K$;

(3) $\psi'(z)(x) = \{\mathbf{0}\}$ for all $x \in X$ with $d_X(x, x_\infty) \ge r_{2k}$ and $z \in Z_k$, $k \in \mathbb{N}$;

(4) $z \in C \setminus K$ if and only if $\psi'(z) \in \downarrow C(X, [\mathbf{0}, y_0])$;

(5) $\psi'(z)(x_\infty) = \{y \in [\mathbf{0}, y_0] \mid d_Y(y, \mathbf{0}) \le \delta(z)\}$ for all $z \in Z \setminus K$.

Define $\Psi|_{Z\setminus K}$ by $\Psi(z) = \psi(z) \cup \psi'(z)$. □

7 Remarks

In this section, we will give some remarks on the main theorem. For more details, refer to [4]. Z. Yang and X. Zhou [11] proved the stronger result as follows:

THEOREM 7.1. *The pair* $(\downarrow\mathrm{USC}(X,\mathbf{I}),\downarrow\mathrm{C}(X,\mathbf{I}))$ *is homeomorphic to* $(\mathbf{Q},\mathbf{c}_0)$ *if and only if the set of isolated points of* X *is not dense.*

It is unknown whether the same result holds or not in the general case. However, the author [4] shows the following theorem (Z. Yang [10] proved the case that $Y = \mathbf{I}$).

THEOREM 7.2. *The space* $\downarrow\mathrm{C}(X,Y)$ *is a Bare space if and only if the set of isolated points of* X *is dense.*

Sketch of proof. The "only if" part follows from the same argument as Section 5. In fact, if the set of isolated points of X is not dense, then $\downarrow\mathrm{C}(X,Y)$ is a Z_σ-set in itself, and hence it is not a Bare space.

Next, we show the "if" part. Let X_0 be the set of isolated points in X and $\mathcal{F}$ be the finite subsets of X_0. For each $F \in \mathcal{F}$ and $n \in \mathbb{N}$, we define

$$U_{F,n} = \{A \in \overline{\downarrow\mathrm{C}(X,Y)} \mid d_Y(y,\mathbf{0}) < 1/n \text{ for all } x \in X \setminus F \text{ and } y \in A(x)\}.$$

Then $U_{F,n}$ is open in $\overline{\downarrow\mathrm{C}(X,Y)}$ and $U_n = \bigcup_{F\in\mathcal{F}} U_{F,n}$ is dense in $\overline{\downarrow\mathrm{C}(X,Y)}$. Observe that the G_δ-set $G = \bigcap_{n\in\mathbb{N}} U_n \subset\downarrow\mathrm{C}(X,Y)$ is a Baire space and dense in $\downarrow\mathrm{C}(X,Y)$. Consequently, $\downarrow\mathrm{C}(X,Y)$ is a Baire space. □

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