数理解析研究所講究録
第 1886 巻 2014 年 43-59

The Effect of Reversible Investment on Credit Risk[1]

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1 Introduction

A firm's decision to invest is one of the most important issues in finance. Recently, a real option based approach is so widely accepted in corporate finance to illustrate investment decision under uncertainty that it is by no means a new concept anymore. Earlier works usually consider a simple strategy such as entry and exit option (e.g. Brennan and Schwartz (1985) and Dixit (1989)). Subsequent research adopted optimal switching theory to illustrate a firm's sequential decision of investment under uncertainty (e.g. Brekke and Øksendal (1994), Duckworth and Zervos (2001), and Zervos (2003)). Yet, none of these studies is concerned with the default of a levered firm.

Modeling default time and credit risk is another crucial theme in finance, and has been studied extensively for decades. They are usually classified into two categories: structural models and reduced-form models. While the latter postulates credit events exogenously, which allows tractability for practitioners, the former seems more attractive on theoretical grounds as it estabilishes a link between economic fundamentals and the endogenous valuation of financial claims. Structural models originated from the seminal works of Black and Scholes (1973) and Merton (1974), and achieved great progress thereafter. The celebrated works of Leland (1994) and Leland and Toft (1996) especially made a breakthrough in modeling credit risk as they incorporate endogenous default boundary and optimal capital structure, and their work has been extended in numerous ways.

In this paper, we incorporate the optimal switching model of Vath and Pham (2007) into the credit risk model of Leland (1994) to capture a firm's investment opportunity and their impacts on the valuation of contingent claims. Namely, equity holders opt for when to switch between two different diffusion regimes in which both drift and diffusion coefficients differ, and this decision involves switching costs. For tractability, we only deal with the case in which both coefficients of one regime dominate those of another one. A default boundary, apparently different from that of Leland (1994), and switching thresholds are determined endogenously. There exist a few papers that adopted optimal switching of diffusion regime, but they are not rich enough to examine various problems. For instance, only diffusion coefficient is controlled in Leland (1998) and Ericsson (2000), even without switching costs, and the switching is irreversible in Ericsson (2000). In He (2011), only drift coefficient is controlled, which requires the effort of the manager, and the main issue of the paper is the optimal contracting between manager and shareholders.

[1]This paper is abbreviated version of Jeon and Nishihara (2013), and was supported by KAKENHI 23310103.

In Childs and Mauer (2008), both coefficients are controlled by the manager, but there is no switching costs, and the asset value is given as an arithmetic Brownian motion, which does not fit into the framework of Leland (1994). Guo et al. (2005) presumed regime shifts of the demand shock, but the shifts are exogenously given in their work.

Conflicts of interest between shareholders and creditors occur since shareholders switch regime to maximize their own interests. In other words, the condition in which equity value is maximized does not coincide with that in which debt value is maximized. Although we do not measure the agency costs of debt explicitly,[2] an extreme case of an agency problem is presented. When an asset substitution problem becomes severe, equity holders in the regime of higher coefficients switch to the regime of lower ones with negative switching costs, i.e. sell a portion of production facilities, right before the default, and the liquidation value of the firm decreases because of this agency problem. This result implies that equity holders expropriate from debt holders, which is consistent with Jensen and Meckling (1976).

We also investigate overinvestment and underinvestment problems by comparing the switching triggers of an unlevered firm with those of a levered firm. When the volatilities in two regimes differ considerably, the investment trigger of a levered firm is lower than that of an unlevered firm, which implies that investment timing of a levered firm is earlier than that of an unlevered firm. The risk shifting problem arises from the feature of equity holders as residual claimants, and is in line with Jensen and Meckling (1976). Meanwhile, the investment trigger of a levered firm is higher than that of an unlevered firm when the project is not risky enough, i.e. when there is little gap of volatility between the two regimes. This result implies that the investment timing of a levered firm is later than that of an unlevered firm. This is because only shareholders bear the cost of investment while the benefit from the project is shared with creditors. This underinvestment problem is consistent with the well-known claim of Myers (1977).

Our model can resolve the problem of structural models pointed out by Huang and Huang (2002) and Eom et al. (2004), namely the wide variations in the expected yield spreads depending on the credit grade of the bonds, and this is one of the most important contribution of the present paper. In our model, default occurs only in one regime, the one with lower coefficients of diffusion, provided the investment is reversible. Hence, we can regard the bond in a regime in which default might occur as speculative grade bonds, and that in another regime as investment grade bonds. Our analysis shows that the spreads of the speculative grade bond with optimal switching are lower than the spreads of the speculative grade bonds without switching, because the default boundary of the firm with an option to invest is lower than that without the option. Meanwhile, if the asset substitution problem is severe, a firm of investment grade switches regimes and defaults instantaneously, as explained before, and this leads to higher spreads of the investment grade bonds than that without optimal switching. The fact that agency problems increase the yield spreads is in line with Leland (1998), and furthermore, the impact of agency problems on yield spreads is considerable in our model, while it was insignificant in Leland (1998). To

[2]To analyze this, we have to compare the firm value under first-best policy which maximizes firm value with that under second-best policy which maximizes equity value. In the framework of Leland (1994), however, first-best policy is never to default, and this is the reason why we do not measure agency costs of debt explicitly here.

sum up, a firm's option to invest lowers the yield spreads of speculative grade bonds, and asset substitution problems with an option to disinvest raises the yield spreads of investment grade bonds.

The remainder of this paper is organized as follows: A formulation for optimal switching is provided as a preliminary in Section 2.1, and this is applied to the benchmark model of an unlevered firm in Section 2.2. The extension to the case of a levered firm is investigated in Section 3.1. The issue of conflict of interests is examined in Section 3.2, and both overinvestment and underinvestment problems are demonstrated in Section 3.3. The empirical implication of our model is summarized in Section 4, and the conclusion is given in Section 5.

2 Benchmark model: An unlevered firm

Before analyzing credit risk with optimal switching, we present the case of an unlevered firm as a benchmark model in this section. First, we introduce a formulation of optimal switching as a preliminary, and proceed with the valuation of a firm based on the optimal switching.[3]

2.1 Preliminaries: A formulation of optimal switching

We formulate an optimal switching problem on an infinite horizon with filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ satisfying the usual conditions and a set of regimes given as $\mathbb{I}_m = \{1, \cdots, m\}$. A switching control is a double sequence $\alpha = (\tau_n, \iota_n)_{n\geq 1}$ where $(\tau_n)_{n\geq 1} \in \mathcal{T}$ is an increasing sequence of stopping times with $\tau_n \to \infty$ representing the decision on when to switch, and $(\iota_n)_{n\geq 1} \in \mathbb{I}_m$ are $\mathcal{F}_{\tau_n}$-measurable representing the decision on where to switch. We denote the set of switching controls by $\mathcal{A}$. Given an initial state-regime $(x, i) \in \mathbb{R}^d \times \mathbb{I}_m$ and a switching control $\alpha \in \mathcal{A}$, the controlled process $X^{x,i}$ is the solution to

$$
\begin{aligned}
& dX_t = \mu(X_t, I_t^i)dt + \sigma(X_t, I_t^i)dW_t, \qquad t \geq 0, \quad X_0 = x, \\
\text{where} \quad & I_t^i = \sum_{n\geq 0} \iota_n \mathbf{1}_{[\tau_n, \tau_{n+1})}(t), \qquad t \geq 0, \quad I_{o^-}^i = i,
\end{aligned}
$$

and W is a standard Brownian motion on $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. We assume that $\mu_i(\cdot) := \mu(\cdot, i)$ and $\sigma_i(\cdot) := \sigma(\cdot, i)$ for $i \in \mathbb{I}_m$ satisfy the Lipschitz condition.

We suppose that the regime affects not only diffusion but also the reward function f : $\mathbb{R}^d \times \mathbb{I}_m \to \mathbb{R}$, and $f_i(\cdot) := f(\cdot, i)$ is assumed to be Lipschitz continuous and satisfy a linear growth condition. Switching from regime i to regime j incurs a constant cost denoted by g_{ij}, with the convention $g_{ii} = 0$. The triangular condition,

$$
g_{ik} < g_{ij} + g_{jk} \qquad \text{for } j \neq i, k, \tag{2.1}
$$

must be satisfied to prevent any redundant switching. A switching cost can be negative, and one can easily show that $g_{ij} + g_{ji} > 0$ for $i \neq j$.

[3]The formulation of optimal switching presented here is based on Pham (2009). For the details, refer to Vath and Pham (2007), Pham et al. (2009), Djehiche et al. (2009), and Bayraktar and Egami (2010).

The expected total profit of running the system given the initial state (x, i) and using the impulse control $\alpha \in \mathcal{A}$ is

$$J(x,i,\alpha) = \mathbb{E}\Big[\int_0^\infty e^{-rt} f(X_t^{x,i}, I_t^i)dt - \sum_{n=1}^{\infty} e^{-r\tau_n} g_{\iota_{n-1}\iota_n}\Big],$$

and the objective is to maximize this expected total profit over $\mathcal{A}$, which can be described by the value function

$$v_i(x) := v(x,i) = \sup_{\alpha\in\mathcal{A}} J(x,i,\alpha), \qquad x \in \mathbb{R}^d, \quad i \in \mathbb{I}_m.$$

Applying the dynamic programming principle, this problem can be rewritten as follows:

$$v_i(x) = \sup_{\alpha\in\mathcal{A}} \mathbb{E}\Big[\int_0^\theta e^{-rt} f(X_t^{x,i}, I_t^i)dt - \sum_{\tau_n\leq\theta} e^{-r\tau_n} g_{\iota_{\tau_{n-1}}\iota_{\tau_n}} + e^{-r\theta} v(X_\theta^{x,i}, I_\theta^i)\Big], \tag{2.2}$$

where θ is any stopping time. Furthermore, it can be shown that for each $i \in \mathbb{I}_m$ the value function $v_i(x)$ is a viscosity solution to the system of variational inequalities

$$\min\Big[rv_i - \mathcal{L}_i v_i - f_i, \quad v_i - \max_{j\neq i}(v_j - g_{ij})\Big] = 0, \qquad x \in \mathbb{R}^d, \quad i \in \mathbb{I}_m, \tag{2.3}$$

where $\mathcal{L}_i$ is the generator of the diffusion X in the regime i.[4] The switching region and the continuation region can be described, respectively, as follows:

$$\begin{aligned} \mathcal{S}_i &:= \{x \in \mathbb{R}^d : v_i(x) = \max_{j\neq i}(v_j - g_{ij})(x)\}, \\ \mathcal{C}_i &:= \{x \in \mathbb{R}^d : v_i(x) > \max_{j\neq i}(v_j - g_{ij})(x)\}. \end{aligned}$$

It can also be proved that the value function v_i is a viscosity solution to $rv_i - \mathcal{L}_i v_i - f_i = 0$ on $\mathcal{C}_i$, and that if the function σ_i is uniformly elliptic, v_i is C^2 on $\mathcal{C}_i$.

Although a formulation of optimal switching is presented under a general condition here, we only deal with a simple case in the remainder of this paper. Namely, a one-dimensional geometric Brownian motion with two different diffusion regimes and an identical operational regime are considered.

2.2 An unlevered firm

Suppose that a stochastic process $(X_t)_{t\geq 0}$ describing the asset value of a firm is given as a one-dimensional geometric Brownian motion, and that there exist two different diffusion regimes in which the drift and diffusion coefficients differ from each other. The coefficients are assumed to be positive constants. Then, the dynamics of the asset value in each regime $i \in \{1, 2\}$ can be described as follows:

$$dX_t = \mu_i X_t dt + \sigma_i X_t dW_t, \qquad X_0 = x.$$

We assume that the firm generates cash flow at the rate of δX_t at time t for some constant $\delta \in (0, \infty)$[5] in both regimes, i.e. $f_i(x) = \delta x$ for $i \in \{1, 2\}$, which implies identical operational

[4] Refer to Pham (2009) for the proof of this theorem.

[5] This assumption is also adopted in Leland and Toft (1996), Leland (1998), and Duffie and Lando (2001).

regimes. All agents in our model are assumed risk neutral, and a risk-free rate is given as a constant $r > \mu_i$ for $i \in \{1,2\}$ to ensure that value functions are finite and satisfy a linear growth condition.

First, let us consider the case without optimal switching, i.e. the one in which a firm does not have an option to invest in production facilities. A straightforward calculation shows that the expected present value of cash flow generated from the asset in each regime $i \in \{1,2\}$ is as follows:

$$\bar{V}_i^U(x) := \mathbb{E}\Big[\int_0^\infty e^{-rt}\delta X_t^x dt\Big] = \frac{\delta x}{r-\mu_i}. \tag{2.4}$$

We add superscript U to distinguish the value related to an unlevered firm from those related to the equity value and debt value of a levered firm, which will be presented in the next section. Note that $\bar{V}_i^U$ in (2.4) is a particular solution of the second-order ordinary differential equation

$$rw - \mathcal{L}_i w - f_i = 0, \tag{2.5}$$

whose general solution (without second member f_i) is of the form

$$w(x) = Ax^{\alpha_i} + Bx^{\beta_i} \tag{2.6}$$

for some constants A, B, and where

$$\alpha_i = \frac{1}{2} - \frac{\mu_i}{\sigma_i^2} + \sqrt{\Big(\frac{1}{2} - \frac{\mu_i}{\sigma_i^2}\Big)^2 + \frac{2r}{\sigma_i^2}} > 1, \qquad \beta_i = \frac{1}{2} - \frac{\mu_i}{\sigma_i^2} - \sqrt{\Big(\frac{1}{2} - \frac{\mu_i}{\sigma_i^2}\Big)^2 + \frac{2r}{\sigma_i^2}} < 0.$$

Now, we shall examine the case with optimal switching, i.e. the one in which a firm has an option to invest in production facilities. Equity holders would switch diffusion regime paying switching costs to maximize expected profits, and thus, their objective function in this case can be described as follows:

$$v_i(x) = V_i^U(x) := \sup_{\alpha\in\mathcal{A}} \mathbb{E}\Big[\int_0^\infty e^{-rt}\delta X_t^{x,i} dt - \sum_{n=1}^\infty e^{-r\tau_n} g_{\iota_{n-1}\iota_n}\Big]. \tag{2.7}$$

For tractability, we postulate that the coefficients of diffusion in one regime dominate those in the other, and without loss of generality, we suppose that regime 2 dominates regime 1, i.e. $\mu_2 > \mu_1$ and $\sigma_2 > \sigma_1$. Apparently, equity holders prefer regime 2 to regime 1.

There are a few cases depending on the signs of switching costs. If both switching costs are positive, i.e. $g_{12} > 0$ and $g_{21} > 0$, we can conjecture that there exist a switching threshold $x_1 \in (0,\infty)$ at which the firm switches from regime 1 to regime 2, i.e. $\mathcal{S}_1 = [x_1,\infty)$. In regime 2, however, the firm never switches to regime 1 as in the previous case, i.e. $\mathcal{S}_2 = \emptyset$. Value function in regime 1 is also a particular solution of (2.5), and thus, the value function in each regime $i \in \{1,2\}$ can be represented as follows:

$$V_1^U(x) = \begin{cases} \bar{V}_1^U(x) + A_1^U x^{\alpha_1}, & x \in (0,x_1), \\ V_2^U(x) - g_{12}, & x \in [x_1,\infty), \end{cases} \tag{2.8}$$

$$V_2^U(x) = \bar{V}_2^U(x), \qquad x \in (0,\infty), \tag{2.9}$$

where

$$x_1 = \frac{\alpha_1 g_{12}}{(\alpha_1 - 1)\delta\left(\frac{1}{r-\mu_2} - \frac{1}{r-\mu_1}\right)}, \qquad A_1^U = \left(\frac{1}{r-\mu_2} - \frac{1}{r-\mu_1}\right)\delta x_1^{1-\alpha_1} - g_{12} x_1^{-\alpha_1}. \tag{2.10}$$

The coefficient A_1^U and the switching threshold x_1 in (2.10) are determined by value matching and smooth pasting condition of V_1^U in (2.8) at x_1. Note that V_1^U takes the form of (2.6), and B_1^U, the coefficient of x^{β_1}, equals 0 from $\lim_{x\to 0} V_1^U(x) = 0$.

The former case corresponds to irreversible investment in the sense that a firm never returns to regime 1 after switching to regime 2. Now we investigate the case in which an investment is reversible, that is, switching from both regimes occurs. If switching from regime 2 to regime 1 involves negative costs, while switching from regime 1 to regime 2 incurs positive costs, i.e. $g_{21} < 0$ and $g_{12} > 0$, we can conjecture that there are two triggers. That is, x_1 at which the firm switches from regime 1 to regime 2, and $x_2 \in (0, x_1)$ at which the firm switches from regime 2 to regime 1. This implies $\mathcal{S}_1 = [x_1, \infty)$ and $\mathcal{S}_2 = (0, x_2]$, and the value functions that are particular solutions of (2.5) can be represented as follows:

$$V_1^U(x) = \begin{cases} \bar{V}_1^U(x) + A_1^U x^{\alpha_1}, & x \in (0, x_1), \\ V_2^U(x) - g_{12}, & x \in [x_1, \infty), \end{cases} \tag{2.11}$$

$$V_2^U(x) = \begin{cases} V_1^U(x) - g_{21}, & x \in (0, x_2], \\ \bar{V}_2^U(x) + B_2^U x^{\beta_2}, & x \in (x_2, \infty) \end{cases} \tag{2.12}$$

where

$$x_2 = \frac{\beta_2 (g_{21} + g_{12} y^{\alpha_1})}{(\beta_2 - 1)\delta\left(\frac{1}{r-\mu_2} - \frac{1}{r-\mu_1}\right)(y^{\alpha_1 - 1} - 1)}, \qquad x_1 = \frac{x_2}{y}, \tag{2.13}$$

$$B_2^U = \frac{\alpha_1 g_{12} x_1^{-\beta_2} - (\alpha_1 - 1)\delta\left(\frac{1}{r-\mu_2} - \frac{1}{r-\mu_1}\right) x_1^{1-\beta_2}}{\alpha_1 - \beta_2}, \tag{2.14}$$

$$A_1^U = \left(\frac{1}{r-\mu_2} - \frac{1}{r-\mu_1}\right)\delta x_1^{1-\alpha_1} + B_2 x_1^{\beta_2 - \alpha_1} - g_{12} x_1^{-\alpha_1}, \tag{2.15}$$

The coefficients A_1^U, B_2^U and switching thresholds x_1, x_2 in (2.13) to (2.15) are determined by value matching and smooth pasting conditions of V_1^U in (2.11) and V_2^U in (2.12) at x_1 and x_2, respectively. Note that V_2^U takes the form of (2.6), and A_2^U, the coefficient of x^{α_2}, equals 0 from $\lim_{x\to\infty} V_2^U(x)/\bar{V}_2^U(x) = 1$. An auxiliary variable y used in (2.13) is determined by the nonlinear equation presented in Appendix A.1 of the original paper. The fact that the firm has an option to switch from regime 2 to regime 1 with negative costs implies that the firm that has invested in production facilities has an option to sell a portion of them, that is, investment is reversible. We can say that investment reversibility improves as the sum of two switching costs decreases, and our model integrates a wide range of investment reversibility by virtue of this feature. A detailed explanation and implication of the optimal switching will be presented in the following section along with the case of a levered firm.

3 Credit risk model: A levered firm

In this section, we apply the optimal switching illustrated in the previous section to the case of a levered firm in the framework of Leland (1994), and analyze how the parameters affect the triggers, equity value, and credit spreads. Furthermore, we examine the well-known issues in finance such as conflicts of interest, and overinvestment and underinvestment problems.

3.1 A levered firm

As the previous section, we first demonstrate the case without optimal switching, i.e. the model of Leland (1994). The firm issues debt to exploit tax shields, but it incurs bankruptcy costs, and optimal capital structure is determined by the trade-off. For tractability, we assume that the debt is issued as a consol bond.[6] Denoting a constant tax rate and a coupon by $\theta \in (0,1)$ and c, respectively, it is well known that the initial equity value of the firm in each regime $i \in \{1,2\}$ can be represented as follows:

$$
\begin{aligned}
v_i(x) = \bar{V}_i^E(x) &:= \sup_{\tau \in \mathcal{T}} \mathbb{E}\Big[\int_0^\tau e^{-rt}\{\delta X_t^x + (\theta-1)c\}dt\Big] \\
&= \frac{\delta x}{r-\mu_i} + \frac{(\theta-1)c}{r} + \bar{B}_i^E x^{\beta_i}, \qquad (x \geq \bar{d}_i)
\end{aligned}
\tag{3.1}
$$

where

$$
\bar{B}_i^E = -\frac{\delta}{(r-\mu_i)\bar{d}_i^{\beta_i-1}} - \frac{(\theta-1)c}{r\bar{d}_i^{\beta_i}}, \qquad \bar{d}_i = \frac{(\theta-1)c\beta_i(r-\mu_i)}{r(1+\beta_i)\delta}.
\tag{3.2}
$$

$\bar{d}_i$ in (3.2) denotes the default boundary in each regime $i \in \{1,2\}$, and obviously, $\bar{V}_i^E(x) = 0$ for $x < \bar{d}_i$.

Let us denote the default time by $\tau_i^{\bar{d}} := \inf\{t > 0 | X_t \leq \bar{d}_i\}$. Provided a fraction $\gamma \in [0,1]$ of the assets are lost when default occurs, debt value of the firm in each regime $i \in \{1,2\}$ can be described as follows:

$$
\begin{aligned}
\bar{V}_i^D(x) &:= \mathbb{E}\Big[\int_0^{\tau_i^{\bar{d}}} e^{-rt}c\,dt + (1-\gamma)V_i^U(\bar{d}_i)e^{-r\tau_i^{\bar{d}}}\Big] \\
&= \frac{c}{r} + \bar{B}_i^D x^{\beta_i}, \qquad (x \geq \bar{d}_i)
\end{aligned}
\tag{3.3}
$$

where

$$
\bar{B}_i^D = \frac{(1-\gamma)\delta}{(r-\mu_i)\bar{d}_i^{\beta_i-1}} - \frac{c}{r\bar{d}_i^{\beta_i}}.
\tag{3.4}
$$

It is straightforward that $\bar{V}_i^D(x) = (1-\gamma)V_i^U(\bar{d}_i)$ for $x < \bar{d}_i$. We add superscripts E and D to distinguish the value related to equity from that related to debt. Note that $\bar{V}_i^E(x)$ and $\bar{V}_i^D(x)$ in (3.1) and (3.3) are also particular solutions to (2.5).

Now, we shall illustrate how the value of equity and debt change when optimal switching is included. For tractability, we do not consider debt restructuring when the diffusion regime is

[6] Leland and Toft (1996) showed that the fact that debt is issued as a consol bond does not harm any virtue of Leland (1994).

switched, which is beyond the scope of the present paper. Ericsson (2000) and Childs and Mauer (2008) also did not consider the restructuring of debt when the diffusion regime is switched.

As before, we postulate that regime 2 dominates regime 1 in the sense that the coefficients of diffusion in regime 2 dominates those in regime 1. It is obvious that equity holders prefer regime 2 to regime 1 because of their feature as residual claimants.

If both switching costs are positive, i.e. $g_{12} > 0$ and $g_{21} > 0$, we can conjecture that $\mathcal{S}_1 = [x_1, \infty)$ and $\mathcal{S}_2 = \emptyset$ by the same argument in (2.8) and (2.9). The value function of equity holders in regime 1 is also a particular solution of (2.5), and the equity value in each regime $i \in \{1, 2\}$ can be represented as follows:

$$V_1^E(x) = \begin{cases} 0, & x \in (0, d_1), \\ \frac{\delta x}{r-\mu_1} + \frac{(\theta-1)c}{r} + A_1^E x^{\alpha_1} + B_1^E x^{\beta_1}, & x \in [d_1, x_1), \\ V_2^E(x) - g_{12}, & x \in [x_1, \infty), \end{cases} \tag{3.5}$$

$$V_2^E(x) = \bar{V}_2^E(x), \qquad x \in (0, \infty), \tag{3.6}$$

where

$$d_1 = \frac{\bar{d}_2}{z}, \qquad x_1 = \frac{d_1}{y}, \tag{3.7}$$

$$A_1^E = \frac{\left(\frac{1}{r-\mu_1} - \frac{1}{r-\mu_2}\right)\delta d_1^{\beta_1} x_1 - \left(\frac{\delta d_1}{r-\mu_1} + \frac{(\theta-1)c}{r}\right)x_1^{\beta_1} - \bar{B}_2^E d_1^{\beta_1} x_1^{\beta_2} + g_{12} d_1^{\beta_1}}{d_1^{\alpha_1} x_1^{\beta_1} - d_1^{\beta_1} x_1^{\alpha_1}}, \tag{3.8}$$

$$B_1^E = -\frac{\delta d_1^{1-\beta_1}}{r-\mu_1} - \frac{(\theta-1)c d_1^{-\beta_1}}{r} - A_1^E d_1^{\alpha_1 - \beta_1}. \tag{3.9}$$

Apparently, the default boundary in regime 1, d_1 in (3.7), differs from that without optimal switching, $\bar{d}_1$ in (3.2), since the firm now has an option to invest in production facilities. Note that the difference between $V_1^E(x)$ in (3.5) and $\bar{V}_1^E(x)$ in (3.1) arises not only from the fact that the firm has an option to invest in facilities but also from the change in the default boundary. The default boundary d_1, switching threshold x_1, and coefficients A_1^E, B_1^E in (3.7) to (3.9) are determined simultaneously by the value matching and smooth pasting conditions of V_1^E in (3.5) at d_1 and x_1. Auxiliary variables y and z used in (3.7) can be calculated by nonlinear simultaneous equations provided in Appendix A.2 of the original paper.

Debt value is also affected by the possible change in diffusion regimes, i.e. the shareholders' decision to invest, and can be represented as follows:

$$V_1^D(x) = \begin{cases} (1-\gamma)V_1^U(x), & x \in (0, d_1), \\ \frac{c}{r} + A_1^D x^{\alpha_1} + B_1^D x^{\beta_1}, & x \in [d_1, x_1), \\ V_2^D(x), & x \in [x_1, \infty), \end{cases} \tag{3.10}$$

$$V_2^D(x) = \bar{V}_2^D(x), \qquad x \in (0, \infty), \tag{3.11}$$

where

$$B_1^D = \frac{x_1^{\alpha_1}(1-\gamma)\left(\frac{\delta d_1}{r-\mu_1} + A_1^U d_1^{\alpha_1}\right) - \frac{c}{r} x_1^{\alpha_1} - \bar{B}_2^D x_1^{\beta_2} d_1^{\alpha_1}}{x_1^{\alpha_1} d_1^{\beta_1} - x_1^{\beta_1} d_1^{\alpha_1}}, \tag{3.12}$$

$$A_1^D = \bar{B}_2^D x_1^{\beta_2 - \alpha_1} - B_1^D x_1^{\beta_1 - \alpha_1}. \tag{3.13}$$

The coefficients A_1^D in (3.13) and B_1^D in (3.12) are determined by the value matching conditions of V_1^D in (3.10) at d_1 and x_1. The fact that the smooth pasting condition is not involved implies that the optimization is carried out in the shareholders' interest.

The former case corresponds to irreversible investment in the sense that the firm never switches to regime 1 after switching to regime 2, and now we shall illustrate a case with reversible investment. If switching from regime 2 to regime 1 involves negative costs, while switching from regime 1 to regime 2 incurs positive costs, i.e. $g_{21} < 0$ and $g_{12} > 0$, we can conjecture $\mathcal{S}_1 = [x_1, \infty)$ and $\mathcal{S}_2 = (0, x_2]$ for $x_2 < x_1$ by the same argument as the case of an unlevered firm in (2.11) and (2.12). The value functions of equity holders which are particular solutions of (2.5) can be represented as follows:

$$V_1^E(x) = \begin{cases} 0, & x \in (0, d_1), \\ \frac{\delta x}{r-\mu_1} + \frac{(\theta-1)c}{r} + A_1^E x^{\alpha_1} + B_1^E x^{\beta_1}, & x \in [d_1, x_1), \\ V_2^E(x) - g_{12}, & x \in [x_1, \infty), \end{cases} \tag{3.14}$$

$$V_2^E(x) = \begin{cases} V_1^E(x) - g_{21}, & x \in (0, x_2], \\ \frac{\delta x}{r-\mu_2} + \frac{(\theta-1)c}{r} + B_2^E x^{\beta_2}, & x \in (x_2, \infty), \end{cases} \tag{3.15}$$

where

$$d_1 = \frac{\frac{\alpha_1(\theta-1)c}{r}(y^{\beta_2-\beta_1} - 1) - \alpha_1(g_{12}y^{\beta_2} + g_{21})z^{\beta_1}}{(\alpha_1 - 1)\delta\left\{\left(\frac{1}{r-\mu_1} - \frac{1}{r-\mu_2}\right)z^{\beta_1-1}(y^{\beta_2-1} - 1) - \frac{y^{\beta_2-\beta_1}-1}{r-\mu_1}\right\}}, \quad x_2 = \frac{d_1}{z}, \quad x_1 = \frac{x_2}{y}, \tag{3.16}$$

$$A_1^E = \frac{(x_1^{\beta_1}x_2^{\beta_2} - x_1^{\beta_2}x_2^{\beta_1})\left(\frac{\delta d_1}{r-\mu_1} + \frac{(\theta-1)c}{r}\right) - d_1^{\beta_1}\left\{\left(\frac{1}{r-\mu_1} - \frac{1}{r-\mu_2}\right)\delta(x_1x_2^{\beta_2} - x_1^{\beta_2}x_2) + (g_{12}x_2^{\beta_2} + g_{21}x_1^{\beta_2})\right\}}{d_1^{\beta_1}(x_1^{\alpha_1}x_2^{\beta_2} - x_1^{\beta_2}x_2^{\alpha_1}) - d_1^{\alpha_1}(x_1^{\beta_1}x_2^{\beta_2} - x_1^{\beta_2}x_2^{\beta_1})}, \tag{3.17}$$

$$B_1^E = -\frac{\delta d_1^{1-\beta_1}}{r-\mu_1} - \frac{(\theta-1)cd_1^{-\beta_1}}{r} - A_1^E d_1^{\alpha_1-\beta_1}, \tag{3.18}$$

$$B_2^E = \left(\frac{1}{r-\mu_1} - \frac{1}{r-\mu_2}\right)\delta x_1^{1-\beta_2} + A_1^E x_1^{\alpha_1-\beta_2} + B_1^E x_1^{\beta_1-\beta_2} + g_{12}x_1^{-\beta_2}. \tag{3.19}$$

The default boundary considering optimal switching, d_1 in (3.16), is also different from the default boundary which does not reflect optimal switching, $\bar{d}_1$ in (3.2). Note that default occurs only in regime 1 in this case. Intuitively, the firm would switch to regime 1 which involves negative costs right before default rather than default in regime 2. The default boundary d_1, switching thresholds x_1, x_2, and the coefficients A_1^E, B_1^E, B_2^E in (3.16) to (3.19) are determined simultaneously by value matching and smooth pasting condition of V_1^E in (3.14) and V_2^E in (3.15) at d_1, x_1, and x_2. Auxiliary variables y and z used in (3.16) can be calculated by nonlinear simultaneous equations presented in Appendix A.3 of the original paper, which involves numerical calculation.

In the former analysis, it was assumed implicitly that $d_1 < x_2$. Yet, it is also possible that $d_1 > x_2$ depending on the parameters. If this is the case, the firm in regime 2 switches to regime 1 when x hits x_2 and defaults immediately since $x_2 < d_1$. Thus, there are two default boundaries in this case while there is only one in the former case. The default boundaries are d_1 for the firm which has never switched to regime 2, and x_2 for the one which has switched to regime 2. Strictly speaking, x_2 is a switching threshold, but it can also be interpreted as a default boundary here since the default occurs right after the switching to regime 1. Value functions of equity holders in this case can be represented as follows:

$$V_1^E(x) = \begin{cases} 0, & x \in (0, d_1), \\ \frac{\delta x}{r-\mu_1} + \frac{(\theta-1)c}{r} + A_1^E x^{\alpha_1} + B_1^E x^{\beta_1}, & x \in [d_1, x_1), \\ V_2^E(x) - g_{12}, & x \in [x_1, \infty), \end{cases} \tag{3.20}$$

$$V_2^E(x) = \begin{cases} -g_{21}, & x \in (0, x_2], \\ \frac{\delta x}{r-\mu_2} + \frac{(\theta-1)c}{r} + B_2^E x^{\beta_2}, & x \in (x_2, \infty), \end{cases} \tag{3.21}$$

where

$$x_2 = \frac{\beta_2\{\frac{(\theta-1)c}{r} + g_{21}\}}{(1-\beta_2)\frac{\delta}{r-\mu_2}}, \qquad d_1 = \frac{x_2}{z}, \qquad x_1 = \frac{d_1}{y}, \tag{3.22}$$

$$B_2^E = -\frac{\delta x_2^{1-\beta_2}}{r-\mu_2} - \left\{\frac{(\theta-1)c}{r} + g_{21}\right\} x_2^{-\beta_2}, \tag{3.23}$$

$$B_1^E = \frac{\{B_2^E x_1^{\beta_2} - (\frac{1}{r-\mu_1} - \frac{1}{r-\mu_2})\delta x_1 - g_{12}\} d_1^{\alpha_1} + \{\frac{\delta d_1}{r-\mu_1} + \frac{(\theta-1)c}{r}\} x_1^{\alpha_1}}{x_1^{\beta_1} d_1^{\alpha_1} - x_1^{\alpha_1} d_1^{\beta_1}}, \tag{3.24}$$

$$A_1^E = -B_1^E d_1^{\beta_1-\alpha_1} - \frac{\delta d_1^{1-\alpha_1}}{r-\mu_1} - \frac{(\theta-1)c d_1^{-\alpha_1}}{r}. \tag{3.25}$$

The thresholds and the coefficients in (3.22) to (3.25) are determined in a similar way as before, and the auxiliary variables y and z used in (3.22) can be calculated by nonlinear simultaneous equations presented in Appendix A.4 of the original paper.

Debt value with reversible investment has nothing to do with $d_1 < x_2$ or $d_1 > x_2$, and can be represented as follows:

$$V_1^D(x) = \begin{cases} (1-\gamma)V_1^U(x), & x \in (0, d_1), \\ \frac{c}{r} + A_1^D x^{\alpha_1} + B_1^D x^{\beta_1}, & x \in [d_1, x_1), \\ V_2^D(x), & x \in [x_1, \infty). \end{cases} \tag{3.26}$$

$$V_2^D(x) = \begin{cases} V_1^D(x), & x \in (0, x_2], \\ \frac{c}{r} + B_2^D x^{\beta_2}, & x \in (x_2, \infty), \end{cases} \tag{3.27}$$

where

$$B_1^D = \frac{(x_1^{\alpha_1} x_2^{\beta_2} - x_1^{\beta_2} x_2^{\alpha_1})\left\{(1-\gamma)(\frac{\delta d_1}{r-\mu_1} + A_1^U d_1^{\alpha_1}) - \frac{c}{r}\right\}}{(x_1^{\alpha_1} x_2^{\beta_2} - x_1^{\beta_2} x_2^{\alpha_1}) d_1^{\beta_1} - (x_1^{\beta_1} x_2^{\beta_2} - x_1^{\beta_2} x_2^{\beta_1}) d_1^{\alpha_1}}, \tag{3.28}$$

$$A_1^D = (1-\gamma)\left(\frac{\delta d_1^{1-\alpha_1}}{r-\mu_1} + A_1^U\right) - \frac{c}{r} d_1^{-\alpha_1} - B_1^D d_1^{\beta_1-\alpha_1}, \tag{3.29}$$

$$B_2^D = A_1^D x_1^{\alpha_1-\beta_2} + B_1^D x_1^{\beta_1-\beta_2}. \tag{3.30}$$

The coefficients A_1^D, B_1^D, and B_2^D in (3.28) to (3.30) are determined by the value matching conditions of V_1^D in (3.26) and V_2^D in (3.27) at d_1, x_1, and x_2.

3.2 Conflicts of Interests

We have illustrated n the previous subsection that conflicts of interest occur in the sense that maximization of equity value does not coincide with that of debt value, because optimal switching is carried out in the equity holders' interest. Numerous studies have dealt with agency problems of debt, and to measure agency costs of debt explicitly, we have to compare the firm value under the first-best policy, which maximizes firm value, with that under the second-best policy, which maximizes equity value. In the framework of Leland (1994), however, the first-best policy that maximizes firm value is not to default, and this is why we do not measure the agency costs of debt explicitly in terms of the differences between the first-best and second-best policies. We rather illustrate the extreme case of agency problems in our model here.

We have demonstrated that default never occurs in regime 2 if investment is reversible. This is because a firm would rather switch to regime 1, which involves negative costs, right before default than default in regime 2. This corresponds to the case of $d_1 > x_2$ that we examined earlier. If this is the case, shareholders make a profit from the sales of production facilities right before default, and the liquidation value of the firm that creditors receive will be based on regime 1, which is apparently lower than that based on regime 2.[7] The fact that shareholders expropriate from creditors accords with Jensen and Meckling (1976). This problem is more likely to occur when reversibility of investment is low, since the switching threshold x_2 gets lower as investment reversibility worsens.

To verify this problem, we use the benchmark parameters used in the comparative statics, which is omitted in this abbreviated version. Among the parameters, we change g_{21} from -20 to -10 and observe the impact on the disinvestment trigger, which converges to the default boundary, equity value, and credit spreads of the firm in regime 2.

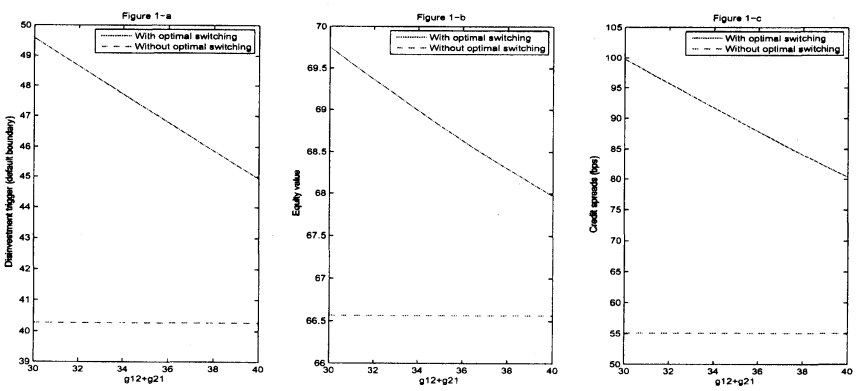

Figure 1: The Impact of conflicts of interests on disinvestment trigger, equity value, and credit spreads.

[7]Depending on the bond covenants, the disposition of assets might be restricted, especially before declaring bankruptcy, as noted by Smith and Warner (1979). If this is the case and $x_2 < d_1$, the optimal policy of equity holders will be same with the case of irreversible investment, even though the switching cost g_{21} is negative. Since securing debt is not the main issue of this paper, we assume that there is no restriction regarding the investment policy of the firm.

We can see in Figure 1-a that $x_2(< d_1)$ decreases as investment reversibility worsens, because the firm in regime 2 has less incentive to switch to regime 1 as investment reversibility worsens. In Figure 1-b, we can see that the equity value of the firm in regime 2 decreases as investment reversibility worsens, which is a straightforward result, but is still higher than that without optimal switching since the switching is implemented to maximize equity holders' interest. We can clearly see in Figure 1-c that the credit spreads of the firm with optimal switching is higher than those without optimal switching, and they decrease as investment reversibility worsens because of the decrease in the default boundary.

3.3 Overinvestment, Underinvestment

Jensen and Meckling (1976) pointed out the overinvestment problem by showing that a firm might be willing to accept projects with negative net present values if the expected payoff of shareholders increases at the expense of creditors. Meanwhile, Myers (1977) demonstrated the problem of underinvestment by showing that a firm financed with risky debt would pass up valuable investment opportunities that could make a positive net contribution to the market value of the firm. In this subsection, we capture both overinvestment and underinvestment problems by comparing the investment trigger of an unlevered firm and that of a levered firm.

It is well known that equity holders are more likely to exploit projects at the expense of creditors when the new projects are risky. To examine this problem, we use the benchmark parameters except for σ_{21} that we change from 0.2 to 0.25.

We can clearly see in Figure 2 that the investment trigger of a levered firm is much lower than that of an unlevered firm, which implies that the investment timing of a levered firm is earlier than that of an unlevered firm, and the problem exacerbates as the investment reversibility lowers. Furthermore, the gap between two triggers widens as the difference of volatility in the two regimes increases. This result reveals that equity holders are more likely to invest in risky projects when the firm is a levered one, shifting their risks to debt holders, and the problem becomes severe as the gap of volatility increases.

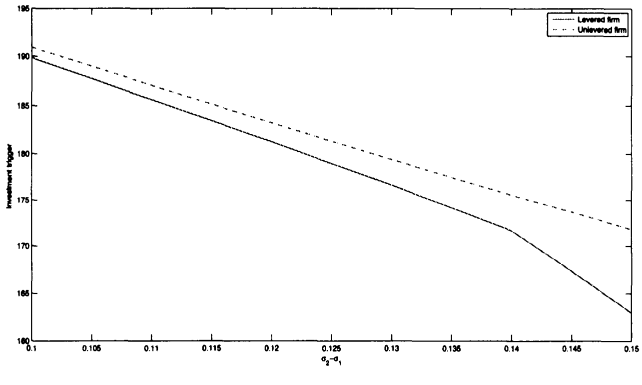

Figure 2: Investment triggers representing overinvestment problem.

It is also known that equity holders may forgo profitable investment opportunities when projects are not risky enough. To verify this problem, we suppose that $\mu_1 = 0.03$, $\sigma_1 = \sigma_2 = 0.1$, $g_{12} = 10$, $g_{21} = -9$, and let μ_2 vary from 0.034 to 0.038. This assumption implies that it is possible to raise the expected growth rate without raising volatility, which is ideal for creditors, while it might not be for equity holders that bear the investment costs.

We can see in Figure 3 that the investment trigger of a levered firm is higher than that of an unlevered firm, which implies that the firm defers timing of investment when debt is issued. Moreover, the gap between two triggers widens as the differences in expected growth rates in two regimes increases. This result arises from the fact that only equity holders bear the costs of investment while profits from the investment are shared with debt holders.

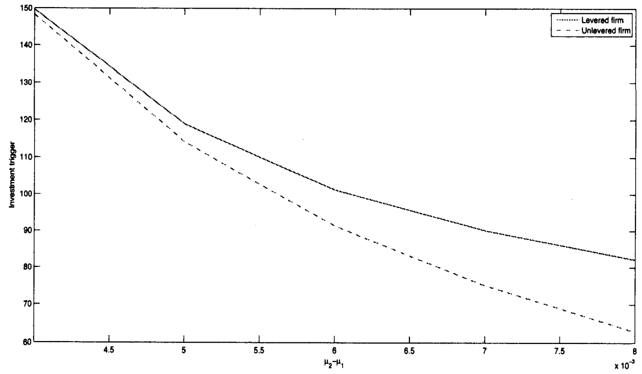

Figure 3: Investment triggers representing underinvestment problem.

4 Empirical Implications

In spite of the virtue of the theoretical backgrounds, structural models have been criticized for the lack of empirical validity. This is because they usually do not provide sufficient spreads, and reduced-form models can be considered to avert this problem as they postulate credit events exogenously so that the default time becomes totally inaccessible stopping time. In addition, the problem can be resolved in the framework of structural models by adopting jump diffusion or introducing imperfect information. Other research attempted to illustrate the spreads by non-default components, such as liquidity, taxes, call and conversion features, or even macroeconomic conditions and the business cycle. In this section, we address how credit risk modeling with optimal switching can enhance the empirical validity of structural models, especially in terms of the relationship between yield spreads and credit grade of the bonds, based on various features of the model that we have examined in the previous section.

Jones et al. (1984) argue that the Merton (1974) model fits better for junk bonds since it has greater incremental explanatory power for riskier bonds. Huang and Huang (2002), who empirically tested various structural models, concluded that credit risks account for only a small fraction of the observed spreads for investment grade bonds, while they account for a much larger fraction of the observed spreads for speculative grade bonds. Eom et al. (2004) also empirically analyzed various structural models, and verified that the spreads predicted by Leland and Toft's (1996) model are often either ludicrously small or incredibly large. It is natural to deduce that the ludicrously small and incredibly large spreads are generated from the bonds of speculative grade and investment grade, respectively. They conclude that the crucial problem of structural models that has to be overcome is to raise the average predicted spread relative to the model of Merton (1974), which cannot generate sufficiently high spreads, without overstating the risks associated with volatility, leverage, or coupon.

As explained before, default occurs only in regime 1 in our model. Hence, we can regard the bonds in regime 1 and regime 2 as speculative grade and investment grade bonds, respectively. To clarify the impact of optimal switching on yield spreads of speculative bonds, let us compare yield spreads of the firm in regime 1 with optimal switching and those without optimal switching. We use the benchmark parameters except for $g_{21} = -10$ to reflect the low investment reversibility in the real world.

We can see in Figure 4 that the yields of the firm in regime 1, i.e. those of speculative grade bonds, with optimal switching are lower than the yields without optimal switching. This is because the default boundary of a firm with an option to invest is lower than that without the option. Note that the default boundary decreases as regime 2 becomes more profitable for equity holders, as we examined in the comparative statics.

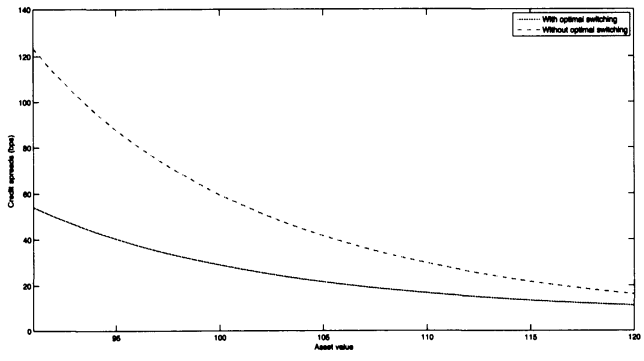

Figure 4: Credit spreads of speculative grade bonds.

Next, we investigate yield spreads of investment grade bonds, i.e. those of the firm in regime 2. We use the same parameters as in the former case.

Figure 5 shows that the yield spreads of the firm in regime 2 with optimal switching is higher than the yield spreads without optimal switching. This result arises from the agency problem of debt combined with the firm's option to disinvest, that is, default of the firm right after switching to regime 1. Note that this problem is more likely to occur when investment reversibility is low. Considering that almost none of the investment projects are perfectly reversible in the real world, it is reasonable to consider the agency problem of debt as one of the factors that raises the yield spreads of investment grade bonds. Leland (1998) also addressed the agency cost of debt to explain the observed yields that are much higher than yields generated from Leland (1994), but the effects were insignificant. In contrast, the impact of agency problems on our model is considerable.

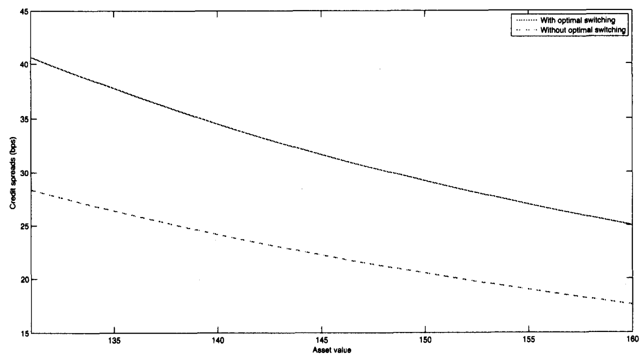

Figure 5: Credit spreads of investment grade bonds.

Combining these results, we can resolve the problem of structural models pointed out by Jones et al. (1984), Huang and Huang (2002), and Eom et al. (2004), i.e. wide variations in yield spreads depending on the credit grade of bonds. Yield spreads of speculative grade bonds decrease from the firm's option to invest, while those of investment grade bonds increase from agency problems combined with the firm's option to disinvest.

5 Conclusion

In this paper, we proposed the credit risk model with optimal switching between two different diffusion regimes. By allowing negative switching cost, we can integrate a wide range of investment reversibility in the framework. The default boundary and switching thresholds are endogenously determined, and we presented comparative statics regarding diffusion regimes, switching costs, and investment reversibility. Conflicts of interests between shareholders and creditors appear, and both overinvestment and underinvestment problems are examined by comparing investment triggers of an unlevered firm and a levered firm. Based on these features, our model resolves the problem of structural models pointed out by Jones et al. (1984), Huang and Huang (2002), and Eom et al. (2004), namely, the wide variations in yield spreads depending on the credit grade of the bonds. The yield spreads of speculative grade bonds decrease since the default boundary lowers because of an option to invest, and those of investment grade bonds increase because of agency problems combined with an option to disinvest.

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