The catuṣkoṭi in a bilattice

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1. Introduction

The *catuṣkoṭi* (1) is a fundamental principle in Buddhist logic, often called the *tetralemma* or "four-corners" in the West ("四句分別" in Japanese). In the catuskoti, four alternatives, possibilities, or options are considered. Namely, that some proposition holds, that it fails, that it both holds and fails, that it neither holds nor fails. Or, *true*, *false*, *both*, *neither*.

According to the traditional conception in the Western logic, inherited by *classical logic* in the present age, a proposition is *bivalent*. It is either true or false, and not both. From this viewpoint, the third and fourth alternatives in the catuskoti are not possible options. They contradicts the basic principles of classical logic, the law of non-contradiction and the law of excluded middle. But such contradictory options do not appear to be taken as a plain absurdity in Buddhist texts. For example, Nāgārjuna employs the catuskoti many times in the course of argumentation in his highly theoretical work, *Mūlamadhyamakakārikā* (hereafter, MMK). So it should be possible to make *logical* sense of the catuskoti.

Non-classical logics may be useful here. In recent years, Graham Priest, with Jay Garfield, has been developing a framework of four-valued semantics to understand the catuskoti (Priest, 2010; Garfield & Priest, 2009; Priest, 2002). Using four *status predicates* defined in terms of truth predicate, he succeeds in formulating the four corners as mutually exclusive and jointly exhaustive options. But the four-valued semantics is very weak. Cotnoir (forthcoming) points out that it cannot justify intuitively valid principles of inference actually used in MMK, and proposes to regard the four corners as a four-valued Boolean algebra. The resultant logic is of course classical logic, hence those principles become valid. However, to appeal to classical logic is to be back where we started. As discussed later, it is hard to see how to express the four options in the vocabulary of classical logic.

The present paper is an attempt at finding a suitable logical vocabulary for the catuskoti. First, it must be able to articulate the catuskoti as the enumeration of four mutually exclusive and jointly exhaustive possibilities. Second, the underlying logic should be sufficiently strong for validating intuitively legitimate inferences. I will try out an algebraic structure of *bilattice* and a logical vocabulary developed on the structure.

I do not claim that the analysis of the catuskoti in terms of bilattices is faithful to Buddhist texts such as MMK. At this stage, it is only a sort of exercise in formal logic. Examination of whether or how it is useful to interpret and understand actual usage of the catuskoti in the texts is reserved for another occasion to consider.

2. Priest's logic of the catuskoti

In this section, Priest's logic of the catuskoti is introduced and a criticism by Cotnoir is examined.

2.1 Usage of the catuskoti

Let us first look at a few passages from MMK in which the catuskoti is used (2) .

> Everything is real and is not real / Both real and not real / Neither real nor not real. / This is Lord Buddha's teaching. (XVIII, 8)
>
> "Empty" should not be asserted / "Nonempty" should not be asserted / Neither both nor neither should be asserted / They are only used nominally. (XXII, 11)
>
> Having passed into nirvana, the Victorious Conquerer / Is neither said to be existent / Nor said to be nonexistent / Neither both nor neither are said. (XXV, 17)

In each passage four options are considered. In the first quote, all options are affirmed. All are denied in the second and third. A distinction should be made clear here. It is one thing to hold that there are four possibilities and quite another to assert or deny one or some, or even more, all of them.

In the literature, the positive use of the catuskoti in which all the four alternatives are affirmed is interpreted as aiming to undermine the conventional viewpoint and to lead the reader to the insight that there is a conventional truth (reality) and an ultimate truth (reality). The negative catuskoti, often called the fourfold negation, in which all are denied can be understood as a *reductio* argument to reveal that a whole doctrine of, say, emptiness or existence is illusory (Deguchi, Garfield & Priest, 2008; Garfield & Priest, 2009; Priest, 2010; Cotnoir, forthcoming).

It is now important to note that, for these arguments to have intended effects, it should be taken by the reader to be *unusual* to affirm or reject all the kotis. In other words, it should have been by and large accepted, as an underlying assumption of theoretical discourse, that for any proposition, there are four options that are mutually exclusive and jointly exhaustive (see Priest, 2010; Ruegg, 1977, Appendix II). In what follows, I use the word "catuskoti" only to refer to this assumption (not its positive and negative use). And here I discuss only how to understand the catuskoti in this sense. The more profound problem of seeing how the arguments through the catuskoti work is beyond the reach of the paper.

2.2 Status predicates and four-valued semantics

Needless to say, to make logical sense of the catuskoti is already a problem. The simple-minded formulation of four alternatives is:

$$A \text{ (true)} \quad \neg A \text{ (false)} \quad A \wedge \neg A \text{ (both)} \quad \neg(A \vee \neg A) \text{ (neither)}$$

The third and fourth alternatives are apparently contradictory. Worse, they are equivalent under the assumption of De Morgan law ($\neg(A \vee B) \equiv \neg A \wedge \neg B$) and double negation law ($\neg\neg A \equiv A$). They also entail the first and second ($A \wedge B$ entails both A and B). Thus, these formulas hardly express mutually exclusive options.

Several attempts have been made by modern commentators (3). For example, Robinson (1957) uses quantified propositions instead of structureless formulas. Staal (1975) suggests that intuitionistic logic may help. Westerhoff (2009) claims that two kinds of negation are used in the catuskoti. However as Priest (2010) shows, these interpretations fail to fulfill the exclusiveness requirement. There are entailment or collapsing among those *kotis*.

The task is to find a new vocabulary and logic suitable for the catuskoti. Priest (2010) introduces the set of *status predicates* and gives their interpretations using four-valued semantics. Let $\mathbf{T}$ be a truth predicate, "is true". Then $\mathbf{T}\langle A\rangle$ is the sentence "A is true", where $\langle A\rangle$ is the name of A. The falsity predicate $\mathbf{F}$ is defined by $\mathbf{F}\langle A\rangle = \mathbf{T}\langle\neg A\rangle$. That A is false is that the negation of A is true. Then he defines four predicates in terms of the truth and falsity predicates:

$$\begin{aligned}
T\langle A\rangle &= \mathbf{T}\langle A\rangle \wedge \neg\mathbf{F}\langle A\rangle && : A \text{ is true (and not false)}\\
F\langle A\rangle &= \neg\mathbf{T}\langle A\rangle \wedge \mathbf{F}\langle A\rangle && : A \text{ is false (and not true)}\\
B\langle A\rangle &= \mathbf{T}\langle A\rangle \wedge \mathbf{F}\langle A\rangle && : A \text{ is both true and false}\\
N\langle A\rangle &= \neg\mathbf{T}\langle A\rangle \wedge \neg\mathbf{F}\langle A\rangle && : A \text{ is neither true nor false}
\end{aligned}$$

Now the catuskoti, the assumption that the four possibilities are mutually exclusive and jointly exhaustive, can be expressed as the following schemata:

$$\text{(C1)} \quad T\langle A\rangle \vee F\langle A\rangle \vee B\langle A\rangle \vee N\langle A\rangle \qquad\qquad \text{(C2)} \quad \neg(S_1\langle A\rangle \wedge S_2\langle A\rangle),$$

where S_1 and S_2 are distinct status predicates.

This is the logical vocabulary Priest presents for the catuskoti. Let us now look at its semantics. Consider a four-element lattice depicted by the Hasse diagram:

$$\begin{tikzcd} & t & \\ b \arrow[ur] & & n \arrow[ul] \\ & f \arrow[ul] \arrow[ur] & \end{tikzcd}$$

consisting of t (true), f (false), b (both), and n (neither). Conjunction $\wedge$ is interpreted as the meet operation (greatest lower bound) on this lattice and disjunction $\vee$ is join (least upper bound). Negation $\neg$ maps t to f and vice versa, but n to itself and b to itself. The set D of designated values is $\{t, b\}$. An inference is *valid* if it preserves the designated values from its premisses to conclusion.

This is, indeed, semantics of the logic *First Degree Entailment* (FDE). Priest calls his logic of the catuskoti FDES (FDE plus status predicates). As to interpretation of the truth predicate $\mathbf{T}$ in terms of which status predicates are defined, he requires the following:

- If the value of A is in D (t or b), so is that of $\mathbf{T}\langle A\rangle$;
- Otherwise, the value of $\mathbf{T}\langle A\rangle$ is f.

These conditions generate the following table that indicates the values of T, B, N and F:

A	$T\langle A\rangle$	$B\langle A\rangle$	$F\langle A\rangle$	$N\langle A\rangle$
t	t or b	f	f	f or b
b	b or f	t or b	b or f	f or b
f	f	f	t or b	f or b
n	f	f	f	t

You will see that if A takes some value then the statement to the effect that it takes that value is designated. For example, if A is f then the value of $F\langle A\rangle$ is t or b, hence designated (Look at the diagonal from top left to bottom right). And more importantly, the formal conditions of the catuskoti are validated. That is, (C1) and (C2) always take one of the designated values whichever value A takes.

Thus, Priest succeeds in formulating the catuskoti formally, employing the vocabulary of status predicates and four-valued semantics. He also presents a natural-deduction style proof theory that is complete with respected the above semantics of FDES. Furthermore he proceeds to examine logical nature of the positive and negative catuskoti argument introducing the fifth predicate E and the fifth value e (emptiness). But as said before I do not step into that.

2.3 FDE is too weak

Cotnoir (forthcoming) argues against Priest that the four-element lattice "cannot ground the logic at play in MMK" since FDE (or FDES) does not justify many of inference principles used by Nāgārjuna. We shall look at some examples [(4)] .

Modus ponens $A \supset B, A \models B$.

> When there is change, there is motion. / Since there is change in the moving, ... / Motion is in that which is moving. (II, 2)

Modus tollens $A \supset B, \neg B \models \neg A$.

> If apart from the cause of form, there were form/ Form would be without cause. / But nowhere is there an effect / Without a cause.
> If apart from form / There were a cause of form / It would be a cause without an effect. / But there are no causes without effects.
> ... Form itself without cause / Is not possible or tenable. (IV, 2–5)

The conclusion is obtained by two consecutive applications of modus tollens.

Hypothetical syllogism $A \supset B, B \supset C \models A \supset C$.

Reductio ad absurdum $A \supset (B \wedge \neg B) \models \neg A$.

> If motion is in the mover. / There would have to be a twofold motion ...
> If there were a twofold motion. / The subject of that motion would be twofold. (XXI, 5–6)

This argument is intended to undermine the initial antecedent ("motion is in the mover") on the ground that it leads to an absurd consequence that the motion of a single individual requires two movers. It can be seen as an instance of hypothetical syllogism as well. (Reductio is problematic in itself because, as the catuskoti shows, some contradictions have to be tolerated in some way. Thus reduction to a contradiction is not necessarily reduction to absurdity. We shall return to this point later.)

These instances of intuitively valid inference rules involve the conditional. Since the vocabulary of FDES contains no conditional, Cotnoir tries out the material conditional, i.e. $A \supset B := \neg A \vee B$. The inference principles above all turn to be invalid. For example, let the values of A and B be b and n respectively. Then $\neg A$ is b, hence $A \supset B (= \neg A \vee B)$ is t. Under this valuation, the premises of modus ponens are designated, but the conclusion is not. So modus ponens is invalid.

Thus FDE (or FDES, the status predicates are irrelevant here) as it is, is too weak to be faithful to the whole argumentation of MMK. Cotnoir's solution is bold. He retains the four-element lattice as the basic semantic structure and the set of designated values $\{t, b\}$. But the lattice is now taken as a Boolean algebra. While conjunction and disjunction are invariably meet and join, negation becomes to toggle b and n as well as t and f. The logic endorsed by this semantics is classical logic. All arguments above are justified.

Unfortunately his proposal has an obvious drawback: It faces a difficulty in expressing the catuskoti (exclusive and exhaustive four possibilities) in the vocabulary. As we have seen, the operators of classical logic ($\wedge$, $\vee$ and $\neg$) could hardly achieve the purpose. How about adding the status predicate? The truth predicate $\mathbf{T}$ in FDES is *transparent* in the sense that it satisfies the unrestricted T-schema. And as is well known, the unrestricted T-schema leads to triviality in any truth theory based on classical logic. Moreover even if one could somehow introduce a truth predicate $\mathbf{T}$ by restricting T-schema so that pathological sentences like the Liar or Curry's sentence are excluded, the catuskoti would collapse in classical logic.

To see this, notice that, in the present setting, $\mathbf{F}\langle A\rangle$ ($= \mathbf{T}\langle\neg A\rangle$) is equivalent to $\neg\mathbf{T}\langle A\rangle$ (they entail each other) (5) . Now each koti is rewritten like this:

$$\begin{aligned}
T\langle A\rangle &= \mathbf{T}\langle A\rangle \wedge \neg\mathbf{F}\langle A\rangle && \equiv \mathbf{T}\langle A\rangle \wedge \neg\neg\mathbf{T}\langle A\rangle \equiv \mathbf{T}\langle A\rangle \wedge \mathbf{T}\langle A\rangle \equiv \mathbf{T}\langle A\rangle \\
F\langle A\rangle &= \neg\mathbf{T}\langle A\rangle \wedge \mathbf{F}\langle A\rangle && \equiv \neg\mathbf{T}\langle A\rangle \wedge \neg\mathbf{T}\langle A\rangle \equiv \neg\mathbf{T}\langle A\rangle \\
B\langle A\rangle &= \mathbf{T}\langle A\rangle \wedge \mathbf{F}\langle A\rangle && \equiv \mathbf{T}\langle A\rangle \wedge \neg\mathbf{T}\langle A\rangle \\
N\langle A\rangle &= \neg\mathbf{T}\langle A\rangle \wedge \neg\mathbf{F}\langle A\rangle && \equiv \neg\mathbf{T}\langle A\rangle \wedge \neg\neg\mathbf{T}\langle A\rangle \equiv \mathbf{T}\langle A\rangle \wedge \neg\mathbf{T}\langle A\rangle,
\end{aligned}$$

where $\equiv$ denotes logical equivalence (mutual entailment). The third and fourth alternatives are equivalent, and they entail the first and second. The catuskoti collapses. The devise of status predicates would not work on classical logic.

It might be said that, although the catuskoti cannot be formulated in the vocabulary of classical logic, it is embodied at the level of semantics. Every sentence takes just one of four truth values. They represent the four, mutually exclusive and jointly exhaustive possibilities. In this line of thought, the catuskoti is taken to be a semantic category at meta level, which would perhaps be ineffable at object level (6) . This might be a viable option. But I do not see whether or how it is successful. Instead, I pursue the original idea of expressing the catuskoti in the vocabulary of formal language at object level (7) .

3. The catuskoti in a bilattice

Here is my proposal. Following Priest and Cotnoir, I use four-valued semantics. But the four-element set is now regarded as a *bilattice*, a structure equipped with two lattice ordering. On the bilattice we can define new negation and conditional that allow us to express the catuskoti and justify the requisite inference principles. Since the new vocabulary are so strong that classical logic can be represented in the logic via a translation, we cannot live with a transparent truth predicate, much less the status predicates. Note, however, that use of status predicates is not required. It suffices if the catuskoti is expressible in the vocabulary.

3.1 Bilattices

Definition 1. A *bilattice* is a structure $\mathcal{B} = \langle B, \leq, \leq', \neg \rangle$, where $|B| \geq 2$, $\langle B, \leq \rangle$ and $\langle B, \leq' \rangle$ are complete lattices and the unary operator $\neg$ on B satisfies:

$$x \leq y \Rightarrow \neg y \leq \neg x; \quad x \leq' y \Rightarrow \neg x \leq' \neg y; \quad \neg\neg x = x.$$

We write $\wedge, \vee$ and $\wedge', \vee'$ for the meet and join operators with respect to $\leq$ and $\leq'$ respectively. And when $\mathcal{B}$ has the maximum and minimum elements with respect to $\leq$ and $\leq'$, we denote them by t, f, t', f' respectively.

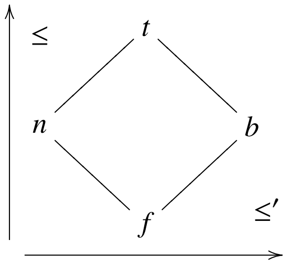
The notion of bilattices was introduced by Ginsberg (1988) as a general framework for many applications. The thought was to represent difference in the amount of knowledge or information, and truth degree in one and the same structure (hence two orderings). Fitting further investigated and applied the notion for logic programming and other purposes (Fitting, 1990, 1991, etc.) . I follow Arieli & Avron (1996)'s approach based on the notion of *logical* bilattices.

Definition 2. A *prime bifilter* of a bilattice $\mathcal{B}$ is a non-empty subset $\mathcal{F} \subsetneq B$ such that:

$$x \wedge y \in \mathcal{F} \iff x \in \mathcal{F} \text{ and } y \in \mathcal{F} \iff x \wedge' y \in \mathcal{F}$$
$$x \vee y \in \mathcal{F} \iff x \in \mathcal{F} \text{ or } y \in \mathcal{F} \iff x \vee' y \in \mathcal{F}$$

A pair $\langle \mathcal{B}, \mathcal{F} \rangle$ of a bilattice and its prime filter is called a *logical bilattice*.

Now look at our four-element structure in this way:



Four

Let us denote this structure by **Four**. That is, **Four** $= \langle\{t, b, n, f\}, \leq, \leq', \neg\rangle$, where $\langle\{t, b, n, f\}, \leq, \neg\rangle$ is the four-valued semantic structure for FDE, and $\leq'$ is yet another ordering on $\{t, b, n, f\}$ as indicated by the diagram. The meet operator $\wedge'$ associated to $\leq'$ behaves like this (dually for the join $\vee'$):

$$t \wedge' b = t \quad b \wedge' f = f \quad t \wedge' f = b \wedge' n = t \wedge' n = f \wedge' n = n.$$

It is easily verified that **Four** is a logical bilattice with the prime bifilter $\{t, b\}$. Moreover, **Four** is the most basic logical bilattice in the following sense:

Theorem 3 (Arieli and Avron)**.** Let $\langle\mathcal{B}, \mathcal{F}\rangle$ be a logical bilattice. Then there exists a unique homomorphism $h : \mathcal{B} \to$ **Four** such that $h(x) \in \{t, b\} \iff x \in \mathcal{F}$.

This theorem tells that **Four** plays an analogous role to that of the two-element $\{t, f\}$ among Boolean algebras. So it is well-motivated to turn her eyes on bilattices when one considers what kind of structure the set of four truth values should take.

Given a propositional language and a logical bilattice $\langle\mathcal{B}, \mathcal{F}\rangle$, a valuation v is a function that maps each propositional variable to an element of $\mathcal{B}$ and extends to the set of all formulas in an obvious way. Validity of (multi-conclusioned) inference is defined as follows:

Definition 4. Let $\langle\mathcal{B}, \mathcal{F}\rangle$ be a logical bilattice, and Γ, Δ finite sets of formulas. $\Gamma \models_{\mathcal{B}} \Delta$ if for every valuation over $\mathcal{B}$ such that $v(A) \in \mathcal{F}$ for every $A \in \Gamma$, there exists some $B \in \Delta$ such that $v(B) \in \mathcal{F}$. $\Gamma \models \Delta$ if $\Gamma \models_{\mathcal{B}} \Delta$ for every logical bilattice $\langle\mathcal{B}, \mathcal{F}\rangle$.

By the previous Theorem, the validity $\models$ reduces to the validity in **Four**:

Theorem 5 (Arieli and Avron)**.** $\Gamma \models \Delta \iff \Gamma \models_{\mathbf{Four}} \Delta$.

3.2 The catuskoti in the bilattice Four

To represent the catukoti in the vocabulary of bilattices, we introduce one more operator – called *conflation* in the literature. A conflation inverts the second ordering but preserves the first. It is, as it were, orthogonal to $\neg$.

Definition 6. A *conflation* is a unary operation $-$ on a bilattice $\mathcal{B}$ such that:

$$x \leq' y \Rightarrow -y \leq' -x; \quad x \leq y \Rightarrow -x \leq -y \quad --x = x; \quad -\neg x = \neg - x.$$

A bilattice with a conflation is called *classical* if for every $x \in \mathcal{B}$, $x \vee -\neg x = t$.

If we define $-$ on **Four** by $-t = t$, $-b = n$, $-n = b$, and $-f = f$, then $-$ is a conflation and **Four** is classical. Notice that in **Four**, $-\neg$ is just classical negation in the four-valued Boolean algebra, that is, it toggles b and n as well as t and f. Let us put $\sim A = -\neg A$.

Now we define, instead of status predicates, a set of (composite) operators that represents the four alternatives. For any formula A, let

$$T(A) = A \wedge -A$$

$$B(A) = A \wedge \neg A$$

$$N(A) = \sim A \wedge -A$$

$$F(A) = \sim A \wedge \neg A.$$

We call these the *status operators*. Then the following truth value table is obtained. One can see that the operators behave like Priest's status predicates:

A	$T(A)$	$B(A)$	$N(A)$	$F(A)$
t	t	f	f	f
b	f	b	n	f
n	f	n	b	f
f	f	f	f	t

Look at the diagonal. For any formula A, A takes a value x if and only if the instance of the corresponding status formula $X(A)$ takes a "designated" value (an element of the bifilter $\{t, b\}$). And it is immediate that the following schemata is valid in **Four**:

$$\text{(C'1)} \quad T(A) \vee B(A) \vee N(A) \vee F(A) \qquad\qquad \text{(C'2)} \quad \neg(S_1(A) \wedge S_2(A)),$$

where S_1 and S_2 are distinct status operators. This is the catuskoti in the bilattice **Four**. The operators embody the four possibilities that are mutually exclusive and jointly exhaustive.

3.3 Validity of inferences

Now turn to the inferences. We present two options. The first is to use material conditional defined by $\sim = -\neg$. The second is to define conditional as a primitive operator.

Using the classical negation $\sim$ Recall that the composite operator $\sim\, = -\neg$ is just classical negation. So if we define $x \supset y := \,\sim x \vee y$, it behaves just like the material conditional in classical logic. Then the inferences listed above are all validated, with the occurrences of negation $\neg$ replaced by $\sim$. For example, reductio ad absurdum should be written as $A \supset (B \wedge \sim B) \models \,\sim A$.

Indeed, the whole of classical logic can be represented within the logic of **Four** via a translation. Let $\mathcal{L}$ be the language of classical logic with operators $\{\wedge, \vee, \sim\}$. For any $A \in \mathcal{L}$, we define its translation $\varphi(A)$ in the language of **Four** inductively as:

$$\varphi(p) := p \quad \varphi(B \wedge C) := \varphi(B) \wedge \varphi(C) \quad \varphi(B \vee C) := \varphi(B) \vee \varphi(C) \quad \varphi(\sim B) := -\neg\varphi(B).$$

Then we have it that:

Proposition 7. Let $A_1, \ldots, A_n, B_1, \ldots, B_m \in \mathcal{L}$. Then

$$A_1, \ldots, A_n \models_{\mathbf{CL}} B_1, \ldots, B_m \iff \varphi(A_1), \ldots, \varphi(A_n) \models_{\mathbf{Four}} \varphi(B_1), \ldots, \varphi(B_m),$$

where $\models_{\mathbf{CL}}$ denotes the consequence of classical logic.

Proof. ($\Leftarrow$) Obvious. ($\Rightarrow$) Given a valuation v on **Four**, define a classical valuation v_φ by $v_\varphi(p) = t$ if $v(p) \in \{t, b\}$, and $v_\varphi(p) = f$ otherwise. Then we can prove by induction that

$$v_\varphi(A) = t \iff v(\varphi(A)) \in \{t, b\},$$

for any $A \in \mathcal{L}$. From this the desired implication follows immediately. □

Conditional as primitive We introduce a conditional in two steps following Arieli and Avron. Given a logical bilattice $\langle \mathcal{B}, \mathcal{F} \rangle$, define the binary operator $\supset$ by:

$$x \supset y := \begin{cases} y & \text{if } x \in \mathcal{F} \\ t & \text{otherwise.} \end{cases}$$

This is almost satisfactory. It validates modus ponens and hypothetical syllogism. But modus tollens fails. So we introduce a stronger conditional. Let

$$x \to y := (x \supset y) \wedge (\neg y \supset \neg x).$$

Modus tollens is valid with this conditional as well as the two rules above (8) .

Things are a little complicated with reductio ad absurdum. $A \to (B \wedge \neg B) \models \neg A$ is not valid (Consider the case where $v(A) = v(B) = n$). But when we denote by $\bot$ a proposition that always takes the value f, the reductio of the form $A \to \bot \models \neg A$ is valid. Thus in the logic of bilattices, it is $\bot$, not $B \wedge \neg B$, that plays the role of "absurdity". Recall that a contradiction of the form $A \wedge \neg A$ is the third alternative $B(A)$ of the catuskoti. In our framework such a contradictory possibilities is not excluded as "absurd". Then what are there "absurd" propositions? Rejecting all the kotis is "absurd". (C'1) always takes the value t, hence $\neg(T(A) \vee B(A) \vee N(A) \vee F(A))$ is always f. Accepting more than one koti is "absurd" too. $S_1(A) \wedge S_2(A)$ is always f for any two distinct operators S_1 and S_2.

4. Conclusion

We have searched for a logical vocabulary that (1) allows an object-level formulation of the catuskoti and (2) accommodates various inferences used in MMK as justified. Priest's logic of the catuskoti failed on (2) as Cotnoir criticized. But his proposal did not fulfill the requirement (1). The framework based on the notion of bilattices satisfies the both requirements in virtue of the status operators and newly defined negation and conditional.

It remains to make logical sense of the positive and negative use of the catuskoti in our bilattice machinery. It will be a challenge for to reject all the kotis and to accept more than one koti are "absurd" in the bilattice **Four**. The negative use does not appear to cause any problem as it is presumably a reductio argument. But, indeed, what is it that is rejected through the argument? Is $\neg A$ in the conclusion an appropriate formalization of what is rejected? And the positive use immediately faces a difficulty since it is not a reductio. What does it mean to assert an "absurd" thing? I expect that introducing the fifth value in the bilattice will work as in Priest's catuskoti. But this is for the future occasion.

$*$ I thank Professor Tetu Makino for this paper is conceived in the course of discussion with him.

(1) In the following, I omit the diacriticals in the word "catuskoti" except in the bibliography.

(2) All quotes from MMK are taken from the translation by Garfield (1995).

(3) See Ruegg (1977, Appendix II) for a survey.

(4) What follows is not an exhaustive list of inference principles Nāgārjuna endorsed. For a more comprehensive list, see Robinson (1957).

(5) According to its truth condition, $\mathbf{T}\langle\neg A\rangle$ is designated only if $\neg A$ is designated. Since $\neg$ is classical negation, it follows that A is not designated. Then $\mathbf{T}\langle A\rangle$ is f, and hence $\neg\mathbf{T}\langle A\rangle$ is t. Thus $\mathbf{F}\langle A\rangle$ entails $\neg\mathbf{T}\langle A\rangle$. The converse is similar. In both cases it is crucial that the negation is classical.

(6) For Cotnoir, each of the four truth values is, formally, a pair of the classical value 0, 1, which

reflects both of the conventional and ultimate perspective. It explains why there are four (two times two) possibilities. Makino (2015) also uses pairs of truth values (or valuations) as classification of formulas to formalize his modified version of the catuskoti.
(7) Some more comments on Cotnoir's paper. Besides the above inferences, he also mention principles involving quantification, which is not dealt with here. And he suggests another approach employing some relevant conditional that can coexist with a transparent truth predicate. See Beall (2009) for an attempt at finding such a conditional.
(8) See Arieli & Avron (1996) for a sequent-style proof theory and Hilbert-style axiomatization of the whole logic with $\supset$ and $\rightarrow$. The system is almost the relevant logic **R** except for it lacks the contraction axiom $(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$ of **R**.

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