Synthetic Aperture Radar Interferometry Time-series for Surface Displacement Monitoring: Data interpretation and improvement in accuracy

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Synthetic Aperture Radar Interferometry Time-series for Surface Displacement Monitoring:

Data interpretation and improvement in accuracy

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Kazuya Ishitsuka
Abstract

A synthetic aperture radar (SAR) system emits microwaves to the Earth’s surface and observes backscattering signals. As a result of synthetic aperture processing with matched filtering, SAR provides a reflectivity map of the Earth’s surface with high spatial resolution. SAR interferometry (InSAR) is a method of estimating surface displacements from the phase difference between two SAR observations. A displacement image of an earthquake was mapped by spaceborne SAR interferometry for the first time approximately 20 years ago. Since then, InSAR has become an essential technique in the geodetic study of the Earth. However, there are still limitations such as interferometric decorrelation and atmospheric path delay, which prevent the estimation of surface displacement with millimeter accuracy.

InSAR time-series analysis has been developed to improve the estimation accuracy of surface displacement. Although many algorithms have been presented, the main idea is to exploit high-quality pixels with less decorrelation, which are sometimes referred to as persistent scatterers (PSs). Taking this approach, other nuisance terms including atmospheric effects can be approximately estimated and removed.

This thesis contributes to the extension of the effectiveness of InSAR time-series analysis. One topic of the thesis is to map a surface displacement field accurately and propose a post-processing interpretation using spatio-temporal displacement patterns, which are related to the local geology. Two displacement fields are studied for this purpose. The first is a natural surface uplift on the Bangkok plain after severe ground subsidence due to groundwater extraction. The other is ground displacement of the Hatchobaru geothermal field, the largest geothermal field in Japan.

The analysis of the Bangkok plain reveals a slight uplift at a rate of approximately 10 mm/year from November 2007 to December 2010. The area of uplift was approximately 250 km² in size and is one of the largest natural uplift phenomena in the world. In this area, the temporal displacement pattern can be divided according to two mechanisms: a long-term surface rebound and a seasonal displacement having a yearly cycle. The thesis further analyzes lateral aquifer connectivity from the temporal pattern of surface displacement.

The ground surface around the Hatchobaru geothermal field is covered with vegetation, making it generally difficult to obtain a large number of PSs. Spatial adaptive filtering that depends on the noise magnitude of pixels is thus applied. Analysis suggests that the ground around the geothermal field subsided at a rate of approximately 15 mm/year from July 2007 to December 2010. Moreover, sharp boundaries of the displacement area are recognized that may correspond to fault traces. The result shows the effectiveness of InSAR time-series analysis for displacement monitoring of a geothermal field and investigating the subsurface structure on a field scale.

In addition to the time-series analysis of coherent pixels, change detection analysis is performed using time-series statistics of incoherent pixels. A large natural disaster such as an earthquake or
volcanic eruption can severely damage buildings and infrastructure. To map damaged areas soon after an earthquake is vital in terms of taking proper measures and understanding the disaster mechanism. The thesis develops a method of detecting a disaster area using coherence and its statistics derived from a pre-seismic period. The method is applied to areas of liquefaction in the Kanto region, thus demonstrating its effectiveness.

Another topic of the thesis, in addition to the interpretation of a surface displacement map, is the improvement in estimation accuracy afforded by the use of polarimetry. The stability of the scattering properties of a target depends on polarimetry. It can thus be assumed that a greater number of coherent pixels can be exploited in estimating surface displacement using polarimetry. In addition, an increasing number of satellites, including ALOS, TerraSAR-X and Radarsat-2, are operating in multi-polarimetric mode. Analysis with polarimetry is thus becoming a more realistic option.

The first algorithm developed in this thesis uses co-polarized components of polarimetry. Ideal PSs such as objects having single- or double-bounce scattering reflect co-polarized components equally, and the phase stability is thus almost uniform. The new algorithm estimates pixels with ideal scattering and is applied to ALOS/PALSAR data acquired in full polarimetric mode. As a result, the variance of estimated displacement velocity improves at 86% of coherent pixels.

The second algorithm uses both co- and cross-polarized components. This algorithm based on maximum likelihood theory objectively determines the phase noise of each polarimetry, and estimates the surface displacement velocity considering scattering characteristics pixel by pixel. The algorithm is applied to an area of groundwater extraction in Japan, and it is seen that the number of coherent pixels increases. Consequently, the area subsidence is well identified using co- and cross-polarized interferograms.

These studies demonstrate the effectiveness of InSAR time-series analysis for the monitoring of surface displacement, and improve the monitoring capability of the analysis.
Acknowledgement

The thesis is the summary of the outcomes during the five years I have studied SAR interferometry in my master and Ph.D. course in Kyoto university. I would like to express my sincere thankfulness to those who support the research.

I greatly acknowledge Prof. Toshifumi Matsuoka for fruitful advices and helps on the research. He is the person who introduces me SAR interferometry, when I enter his laboratory. Through the researches with him, I found research is a worthwhile and challenging job. All opportunities to study SAR interferometry owe his support.

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<td>Weighting coefficients between HH-HH and HV-HV interferograms</td>
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<tr>
<td>$\beta$</td>
<td>Weighting coefficients between HH-HH and VV-VV interferograms</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The absolute value of complex coherence (coherence)</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Instant coherence affected by short-term scatterers’ motion</td>
</tr>
<tr>
<td>$\gamma_{cp}$</td>
<td>Complex coherence</td>
</tr>
<tr>
<td>$\gamma_{diff}$</td>
<td>Difference of coherence</td>
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<td>$\gamma_{geom}$</td>
<td>Coherence due to geometrical effects</td>
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<td>$\gamma_{pc}$</td>
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<td>$\gamma^p$</td>
<td>Coherence of pre-seismic datasets</td>
</tr>
<tr>
<td>$\gamma^c$</td>
<td>Coherence of co-seismic datasets</td>
</tr>
<tr>
<td>$\gamma_{avg}$</td>
<td>Average of coherence difference</td>
</tr>
<tr>
<td>$\gamma_{thre}$</td>
<td>Threshold of coherence difference</td>
</tr>
<tr>
<td>$\gamma_{std}$</td>
<td>The standard deviation of coherence difference</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The angle between two SAR locations</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>The spectral exponent of scale-invariant signal</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>The eigenvalues of the covariance matrix</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Wavenumber</td>
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<tr>
<td>$\theta$</td>
<td>The incidence angle</td>
</tr>
<tr>
<td>$\theta_{sq}$</td>
<td>The squint angle</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>The damping coefficient</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
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<tr>
<td>$\xi$</td>
<td>The axis perpendicular to slant-range and azimuth direction</td>
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<td>$\xi_k$</td>
<td>Sub-pixel location in perpendicular to slant-range and azimuth direction</td>
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<td>$\rho$</td>
<td>the coefficient of eigenvalues</td>
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<tr>
<td>$\sigma_{dcr}$</td>
<td>The standard deviation of phase noise due to decorrelation</td>
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<td>$\sigma_{dcr,pol}$</td>
<td>The standard deviation of decorrelated phase in a certain polarimetric state</td>
</tr>
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\( \sigma_{\text{los}} \) The standard deviation of random scatterers’ motion in LOS direction
\( \sigma_m \) The standard deviation of model parameters
\( \sigma_{r,\text{short}} \) The standard deviation of short-term scatterers’ motion
\( \sigma_{r,\text{long}} \) The standard deviation of long-term scatterers’ motion
\( \sigma_\phi \) The standard deviation of interferometric phase
\( \sigma_{ts} \) The standard deviation of estimated time-series displacement
\( \sigma_v \) The standard deviation of surface displacement velocity
\( \sigma_A \) The standard deviation of amplitude time-series
\( \sigma_{\Delta h} \) The standard deviation of scatterer height
\( \tau \) Pulse duration of signal in range direction
\( \nu \) Phase of a SAR image
\( \nu_{\text{scat}} \) Random phase shift due to scattering
\( \phi \) Interferometric phase
\( \phi_0 \) Expectation value of interferometric phase
\( \phi_{\text{atm}} \) Interferometric phase due to changes in atmospheric condition
\( \phi_{\text{dcr}} \) Interferometric phase due to decorrelation
\( \phi_{\text{disp}} \) Interferometric phase due to surface displacement
\( \phi_{\text{height}} \) Interferometric phase due to scatterer height with respect to a DEM
\( \phi_{\text{inc}} \) Increment phase vector
\( \phi_{\text{orb}} \) Interferometric phase due to orbital trajectories
\( \phi_{\text{obcr}} \) Phase vector of differential interferograms after correcting scatterer height
\( \phi_{\text{topo}} \) Interferometric phase due to topography
\( \phi_{\text{ts}} \) Time-series phase vector of differential interferograms
\( \tilde{\phi} \) Phase discontinuities after the congruence operation
\( \psi \) Unwrapped phase
\( \ell_i \) A integer coefficient
\( \Delta \theta \) The difference in incidence angle
\( \Delta \theta_{\text{sq}} \) Difference in squint angle
\( \Delta f_c \) Difference in central frequency
\( \Delta f_{\text{dop}} \) Difference in Doppler frequency
\( \Delta h \) Scattering height with respect to a reference surface (digital elevation model)
\( \Delta r \) Geometrical resolution in (slant) range direction
\( \Delta r_g \) Geometrical resolution in ground range direction
\( \Delta x \) Geometrical resolution in azimuth direction
\( \Delta R \) Distance change in range direction between two SAR acquisitions
\( \Omega \) The decorrelation rate
\( a_{\text{atm}} \) The coefficient of a function describing atmospheric effects
The coefficient of a function describing long-term displacement
The coefficient of the exponential function describing long-term displacement
The coefficient of the linear function describing long-term displacement
The coefficient of a function describing seasonal displacement
The coefficient of a function describing atmospheric effects
The coefficient of a function describing long-term displacement
The coefficient of the exponential function describing long-term displacement
The coefficient of a function describing seasonal displacement
The coefficient of a function describing atmospheric effects
The coefficient of a function describing atmospheric effects
Expectation of residual phases of displacement time-series
The central frequency of the carrier signal
Power contribution of covariance matrix for double-bounce scattering
Power contribution of covariance matrix for helix scattering
Power contribution of covariance matrix for surface scattering
Power contribution of covariance matrix for volumetric scattering
Complex reflectivity
Imaginary unit
The coefficient for the threshold of coherence difference
The coefficient for scatterer height
model parameters
A real coefficient
The coefficient for surface displacement velocity
The mean of amplitude time-series
A real value described in phase unwrapping operation
A constant value for the congruence operation
Range (slant-range) direction
Sub-pixel location of a persistent scatterer in range direction
Time
Surface displacement velocity
Speed of the light
Satellite platform velocity
Complex vector composed by two complex SAR data
Azimuth direction
Azimuth location where a scatterer is illuminated by the beam center
Sub-pixel location of a persistent scatterer in azimuth direction
The complex scattering vector
$A$ Amplitude of a SAR image
$B$ Baseline
$B_g$ The parallel component of baseline
$B_\perp$ The perpendicular component of baseline (geometrical or normal baseline)
$B_{lc}$ The critical baseline
$B_\perp c$ The mean of geometrical baseline
$C$ Complex signal in a SAR image
$D$ The distance between pixels
$D_p$ A constant describing increase in long-term scatterers’ motion
$D_A$ The amplitude dispersion index
$G$ Data kernel to estimate time-series displacements
$H$ Data kernel to estimate a displacement velocity and a scatterer height
$H_d$ Designed matrix of $\frac{d}{dt}$
$J$ Objective function for phase unwrapping
$K_c$ The chirp rate
$L$ The antenna length in azimuth direction
$N_{damp}$ The number of the damping coefficients
$N_{int}$ The number of (differential) interferograms
$N_{mdl}$ The number of model parameters
$N_{pix}$ The number of pixels to calculate coherence
$N_{sar}$ The number of SAR images
$P$ The rank of the matrix $H_d$
$N_{is}$ The number of measurements for time-series displacements
$R$ The range distance between SAR sensor and a scatterer
$R_0$ Closest distance between SAR sensor and Earth’s surface
$S_{i,j}$ Complex scattering coefficient for transmitted ($j$) and observed ($i$) polarization
$T$ Temporal baseline
$\bar{T}$ The mean of temporal baseline
$U_{lt}$ Long-term components of displacement time-series
$U_{se}$ Seasonal components of displacement time-series
$W_e$ The electromagnetic width
$W_{i,j}$ Weights of the minimum Lp-norm algorithm between $i$th and $j$th pixels
$Z$ A constant for the posterior probability function of interferometric phase
$r_c(\cdot)$ Compressed signal in range direction
$r_r(\cdot)$ Reference signal in range direction
$r_t(\cdot)$ Transmitted signal in range direction
$r_s(\cdot)$ Received signal in range direction
F( ) Misfit vector for model parameters
J( ) The Jacobian
M( ) The marginal likelihood
L( ) The likelihood
Const A constant of integration
Cov Covariance matrix of two complex SAR data
Li₂ Euler’s dilogarithm
pdf Probability density function
SNR Signal to noise ratio
Chapter 1

Introduction

Surface displacements of the Earth greatly affect human society. Land subsidence and uplift due to groundwater use, for instance, can damage infrastructure and residential buildings, while landslides are a threat to human life. Surface displacements also provide clues with which to understand the dynamics of nature. Plate motion causes strain accumulation, which can appear as surface displacement. In particular, earthquakes and volcanic eruptions often cause destructive displacements. Meanwhile, the dynamics of natural phenomena provides information on how the land has been shaped. Surface displacements are also closely related with the development of natural resources. Displacements in an aquifer region or area of development of geothermal or oil/gas are caused by the migration of subsurface fluid and can thus be used to monitor the distribution of subsurface fluid. Moreover, pressure and volume changes in a reservoir can be inferred from the magnitude of surface displacements and physical parameters of the surrounding media.

The importance of the measurement of surface displacements has been well recognized, and surface displacement has long been measured with techniques that have been state of the art at the time. Standard techniques including leveling, triangulation, trilateration, and the use of ground positioning systems offer accurate and reliable measurements of surface displacements. Despite the successful application of the above standard geodetic measurements, it has been difficult to obtain a synoptic view of displacements owing to the limitation of the spatial density of measurements. The operation of synthetic aperture radar (SAR) onboard satellites has allowed the development of two novel techniques, namely SAR interferometry (InSAR) and InSAR time-series analysis, which satisfy the demand of monitoring with high spatial density.

InSAR analysis uses the phase difference between two SAR data acquired in a satellite repeat-pass configuration [Bamler and Hartl, 1998; Massonnet and Feigl, 1998]. Displacements along the line of sight are estimated from the phase differences between two overlapping SAR images taken with similar viewing geometries after subtracting terrain and orbital contributions. The effectiveness of the InSAR technique has been demonstrated for displacements generated by earthquakes, volcanic eruptions, and large ground subsidences. However, because of the instability of the estimation accuracy, there are a variety of displacements that cannot be detected employing such analysis. Phenomena
that are difficult to detect by InSAR analysis include moderate ground subsidence, regular strain accumulations due to plate motion, slow landslides, and the tilt of a single building. These phenomena involve small motion (on the order of a few centimeters or less than a centimeter) and displacement over a small spatial area. As shown later, the difficulty arises because InSAR analysis attempts to solve an ill-posed problem.

InSAR time-series analysis has been developed for the more accurate estimation of surface displacement using time-series phase information of coherent targets [Kampes, 2005; Ferretti, 2014]. Compared with standard InSAR analysis, the advantages of InSAR time-series analysis are not only the accuracy of the measurements but also the high spatial resolution, no limitation on the spatial gradient of displacement, and the extraction of the temporal variation of displacement. It has been recognized that SAR observations are suitable for periodic monitoring because they are made in repeat passes of the satellite. The improved estimation accuracy increases the monitoring capability. The first InSAR time-series analysis, referred to as permanent scatterer (persistent scatterer) SAR interferometry analysis, was proposed by Ferretti et al. (2000) and Ferretti et al. (2001). Since then, several types of InSAR time-series analysis have been proposed [e.g., Berardino et al., 2002; Schmidt and Bürgmann, 2003; Werner et al., 2003; Lanari et al., 2004; Hooper et al., 2004; Hooper et al., 2007]. These methods have been successfully applied to a variety of geophysical phenomena. However, one of the limitations is that there are fewer coherent pixels in a non-urban area, and pixels that can be used to measure accurate surface displacement are sometimes sparsely distributed. Advanced InSAR time-series analysis has recently been proposed to increase the number of pixels used to measure surface displacement [Hooper, 2008; Ferretti et al., 2011]. InSAR time-series analysis is now a powerful technique used to monitor a variety of surface displacement phenomena.

The aim of this thesis is to further improve the monitoring capability of InSAR time-series analysis. The thesis first reviews InSAR time-series analysis from the viewpoint of data analysis. The basic concept is easy to understand; however, it is necessary to understand InSAR imaging properties and the meaning of each step in InSAR time-series processing to attain high accuracy. The capability of the displacement monitoring of InSAR time-series analysis is then improved in terms of the interpretation of displacement and accuracy. For interpretation of InSAR time-series displacement, spatio-temporal interpretation using the temporal variation of spatially dense surface displacement is proposed. To improve estimation accuracy, processing algorithms that use polarimetry are proposed.

The strong advantage of InSAR time-series analysis in the study of geophysical phenomena is the accurate measurements of surface displacement with high spatial density and large spatial coverage. The thesis proposes a post-processing interpretation using the spatial and temporal patterns of displacements. Employing the method, two displacement phenomena are proposed, one being natural surface uplift that covers the whole metropolitan area of Bangkok. Ground subsidence due to excessive groundwater extraction has been severe problem especially in developing countries [Waltham, 2002; Galloway and Burbey, 2011]. Recently, several countries have taken measures to mitigate groundwater extraction, and the amount of ground subsidence has been reduced [Taniguchi, 2011].
However, surface rebound has been reported where ground subsidence has occurred [Chen et al., 2007]. The phenomenon has not been fully investigated. InSAR time-series analysis is thus employed to map and understand the displacement trend. The other phenomenon is subsidence around the Hatchobaru geothermal field, located in the south of Japan. There is growing interest in geothermal development because of increasing concern about the emission of greenhouse gases and sustainable energy. Geothermal energy is a promising energy source that produces sustainable energy by circulating extracted thermal fluid. The observation of surface displacement that is likely associated with subsurface fluid migration would thus help produce sustainable energy.

Two new methods of InSAR time-series analysis are then described using multi-polarimetric interferograms to increase the estimation accuracy and spatial density. Even though InSAR time-series analysis is a powerful technique for displacement monitoring, one limitation of the technique is the spatial density of estimated displacement in sub-urban and non-urban areas, because the analysis uses coherent phase information. In recent years, several satellites equipped with SAR, which can acquire multi-polarimetric data, have been launched. These include ALOS, ENVISAT, Radarsat-2, and TerraSAR-X. Additionally, recently launched SAR satellite missions such as ALOS-2 and Sentinel-1 will operate in multi-polarimetric acquisition mode. It is expected that InSAR time-series analysis, which uses single polarimetry, will evolve to analysis employing multi-polarimetry. The first new method relates to the analysis of interferograms using co-polarized components (HH-HH and VV-VV interferograms). This method is based on the observation that the decorrelation magnitudes of co-polarized interferograms are almost identical. The second method relates to the analysis of interferograms using arbitrary combinations of multi-polarimetric interferograms. The interferometric phases derived from different polarimetry differ in the magnitude of decorrelation depending on target scatterer characteristics. The developed method considers the difference in decorrelation magnitude for each polarimetry.

1.1 Contributions

The thesis makes contributions to the literature in three main areas. The first is the mapping and interpretation of surface displacements inferred from InSAR time-series analysis (No. 1–5 in the following). The second is an improvement of the algorithm for damage detection using temporal changes of decorrelated (incoherent) scatterers (No. 6 in the following). The third is the development of a new algorithms for InSAR time-series analysis using polarimetry (No. 7 and 8 in the following). Specifically, the studies reported in this thesis make the following contributions.

1. It is found that the ground surface in Bangkok uplifted at a rate of approximately 1 cm/year from October 2007 to December 2010.

2. It is found that the long-term uplift is likely caused by natural groundwater recovery, because of the correlation between the area of uplift and the area of groundwater recovery.
3. It is found that seasonal displacements are superimposed on the uplift in part of Bangkok and the spatial pattern is likely controlled by aquifer connectivity.

4. It is found that the ground surface around the Hatchobaru geothermal field subsided at a rate of approximately 1.5 cm/year from July 2007 to December 2010.

5. It is found that the spatial pattern of surface displacement around the Hatchobaru geothermal field is likely controlled by a fault.

6. Soil liquefaction associated with the 2011 Tohoku earthquake in the Kanto region is mapped from statistical changes in coherence.

7. A new algorithm for InSAR time-series analysis that uses both HH-HH and VV-VV interferograms is designed and found to provide better estimation accuracy of surface displacements.

8. An improved algorithm of InSAR time-series analysis based on maximum likelihood theory that uses all HH-HH, HV-HV and VV-VV interferograms is designed.

1.2 Thesis roadmap

Chapter 2 briefly overviews SAR, InSAR and InSAR time-series analyses. The basic concept of SAR imaging is described to clarify the properties of the data. The basic concept of InSAR analysis is then described. The limitations of InSAR analysis are especially described and provide the motivation to develop InSAR time-series analysis. The concept and processing flow of InSAR time-series analysis are then described. The review of InSAR time-series analysis describes a variety of approaches for the limitation of InSAR analysis and gives the basis of each approach. Additionally, the review describes the characteristics of the processing employed in this study. Among several InSAR time-series analysis, the thesis uses a method based on persistent scatterer interferometry. The basic processing flow used in the thesis is described in this chapter. Finally, each processing step is simulated to describe its meaning.

Chapter 3 investigates surface displacement and its interpretation on the Bangkok plain. Natural surface rebound on the Bangkok plain is detected employing persistent scatterer interferometry. Both spatial and temporal patterns of surface displacement and characterized reservoir connectivity are analyzed. This study has been published in Geochemistry, Geophysics, Geosystems [Ishitsuka et al., 2014a].

Chapter 4 describes recent surface displacement in the Hatchobaru geothermal field. Since a coherent scatterer (a persistent scatterer) physically corresponds to a point-like target such as an artificial structure, areas covered with natural targets including a geothermal area are difficult targets for InSAR time-series analysis in terms of the number of pixels used to measure surface displacement. The thesis applies spatial adaptive filtering to increase the number of coherent pixels in measuring surface displacement.

Chapter 5 presents the detection of an area of soil liquefaction associated with the 2011 Tohoku
earthquake from changes in the interferometric correlation. A new algorithm that considers ordinal changes in the factor is developed and the accuracy of the detection method is examined by applying the method to the area of soil liquefaction resulting from the 2011 Tohoku earthquake. This study has been published in Earth Planets Space [Ishitsuka et al., 2012].

Chapter 6 presents a new algorithm of persistent scatterer interferometry using both HH-HH and VV-VV interferograms. The use of both HH-HH and VV-VV interferograms increases the total number of interferograms used and thus the accuracy of the estimation of surface displacement. The method is applied to ALOS/PALSAR images acquired in full polarimetric mode and covering Aomori prefecture in northern Japan. This study has been published in IEEE Geoscience and Remote Sensing Letters [Ishitsuka et al., 2014b].

Chapter 7 describes another new algorithm for persistent scatterer interferometry using three different polarimetric SAR interferograms: HH-HH, HV-HV and VV-VV interferograms. To incorporate both co- and cross-polarized components, the algorithm considers the difference in the decorrelation magnitude depending on polarimetry. The method is examined by applying it to a site in Aomori where ground subsidence have occurred due to groundwater extraction. The results are validated with leveling data.

Chapter 8 summarizes the thesis and makes suggestions for future work.
Chapter 2

Background

2.1 Synthetic aperture radar imaging

Routine repeat-pass SAR measurements made from satellites began with the satellite ERS-1. ERS-1 was launched in July 1991 and acquired a large number of SAR data suitable for InSAR analysis. The Japan Earth Resources Satellite (JERS-1) was launched from Japan in February 1992 and continued to acquire data until October 1998. Since then, several satellites equipped with SAR (e.g., ALOS, ENVISAT, RADARSAT, and TerraSAR-X) have continuously acquired SAR images, thus making InSAR and InSAR time-series analysis widely applicable. Recently, successive SAR missions have been launched: Sentinel-1 has operated since its launch in April 2014 and ALOS-2 since its launch in May 2014. It has been said that the golden age of operational SAR observations has arrived. SAR data have thus provided a basis for earth science and engineering.

A satellite with SAR orbits about 600–700 km above the Earth at a speed of about 7–8 km/s. SAR emits microwave pulses onto the antenna’s illumination footprint in a side-looking manner with respect to the flight track. The pulses are emitted repeatedly at a certain frequency (about 1–10 kHz), which is called the pulse repetition frequency (PRF). SAR observes the signal backscattered from the Earth’s surface in the meantime between pulse emissions. The direction along the flight track generally is referred to as the azimuth direction, while the direction perpendicular to the track is referred to as the range direction (Figure 2.1). The ground range is the range in the direction parallel to the illuminated surface, and the slant range is in the same direction as the pulses are emitted (Figure 2.1). The frequencies of widely used microwaves correspond to the L-band (wavelength of approximately 24 cm), C-band (wavelength of approximately 5 cm), and X-band (wavelength of approximately 3 cm). A variety of SAR observation modes have been developed and operated. This thesis describes the strip map mode that is a current standard observation mode for InSAR and InSAR time-series analysis. The strip map mode illuminates the radar footprint along the flight path with a fixed off-nadir and squint angle, and the radar footprint covers a strip on the surface as the satellite moves. The strip map mode is sometimes divided into two modes depending on the imaging geometry.
One geometry is referred to as the boresight geometry while the other is the squinted geometry. The boresight mode emits pulses in a direction perpendicular to the flight track, while the antenna in the squinted geometry is oriented at a certain angle (i.e., the squint angle $\theta_{sq}$ is not zero). The thesis describes the basic concept of the strip map mode with boresight geometry because this geometry is simpler and allows a better understanding of the concept of the synthetic aperture.

At first, let me briefly describe the spatial resolution without applying any compression processing. Assuming that a rectangular pulse with the duration of $\tau$ is emitted, the geometrical resolution in the slant range direction is

$$\Delta r = \frac{v_l \tau}{2}$$  \hspace{1cm} (2.1)

where $v_l$ is the speed of light. When we assume the pulse duration of $3 \times 10^{-5} \text{sec}$, the range resolution is about 4500 m. If we want to obtain a fine range resolution data, the pulse resolution must be small. On the other hand, signal-to-noise ratio (SNR) of signal have to be enough large to distinguish a target even in noisy condition. High SNR can be attained by raising the average transmitted power. This can be done either by raising peak power, or by increasing the pulse duration. Peak power is often constrained by physical limitation, so increase in the pulse duration is a practical solution to increase SNR. The SNR and resolution in the system is conflict. The geometrical resolution of azimuth direction ($\Delta \alpha$) is also described from the antenna beam width ($\frac{\lambda}{L}$) and the slant range distance between satellite and a target ($R$).

$$\Delta \alpha = \frac{\lambda R}{L}$$  \hspace{1cm} (2.2)

where $\lambda$ is the wave length and $L$ is the antenna dimension along the azimuth direction. When we assume the satellite flying 700 km above the Earth and emitting 10 cm wave length from the antenna with azimuth width of 5 m leads to 35000 m.

The technique to reconstruct spatially high resolution image is referred as SAR focusing or synthetic aperture radar processing. The basic assumption of the processing is that observed signal is the coherent superposition from individual backscattered signals. Below, I simply describe the idea of SAR focusing using matched filtering. In range direction, the chirp signal whose frequency is modulated linearly is emitted.

$$r_c(t) = \exp \left( j \pi \left(2f_c t + K_c t^2 \right) \right) \text{rect} \left( \frac{t}{\tau} \right)$$  \hspace{1cm} (2.3)

where $j$ is an imaginary unit, $f_c$ is the carrier frequency, and $K$ is the chirp rate. The instantaneous frequency of the signal is

$$f = \frac{1}{2\pi} \frac{d \left(2\pi f_c t + \pi K_c t^2 \right)}{dt}$$  \hspace{1cm} (2.4)

$$= f_c + K_c t$$  \hspace{1cm} (2.5)

Then, the bandwidth of the chirp signal is $K_c \tau$ hertz. The backscattered signal from a point target...
Figure 2.1: The configuration of SAR data acquisition.

located at the distance from the satellite of $R$ is

$$r_s(t) = \exp \left( j \pi \left( 2f_c \left( t - \frac{2R}{v_l} \right) + K_c \left( t - \frac{2R}{v_l} \right)^2 \right) \right) \text{rect} \left( \frac{t - \frac{2R}{v_l}}{\tau} \right)$$

(2.6)

which can be simplified by shifting frequency domain (heterodyne operation) as

$$r_s(t) = \exp \left( -j \frac{2f_c}{v_l} R \right) \exp \left( K_c \left( t - \frac{2R}{v_l} \right)^2 \right)$$

(2.7)

To compress the observed signal in time domain, matched filtering is used. The matched filtering is the cross-correlation of a received signal and a reference signal. This is the same way that an arbitrary signal buried in noisy background can detect by cross-correlating with a replica (reference) signal. A reference signal is the complex conjugate of observed signal as follows:

$$r_h(t) = r_s(t)^*$$

(2.8)
The range compression of the observed signal can be performed with the convolution of the observed and the reference signal.

\[ r_c(t) = \int_{-\infty}^{\infty} r_s(u) r_h(t-u) du \]  

(2.9)

\[ = \exp \left( -j \frac{4\pi}{\lambda} R \right) \text{sinc} \left( \pi K_c \left( t - \frac{2R}{v_l} \right) \right) \]  

(2.10)

The compressed signal is a sinc function, which is a compact shape with a sharp peak. The time when the sinc function crosses zero is \( \frac{1}{K_c \tau} \), thus the value of \( \frac{1}{K_c \tau} \) is referred as range resolution in time domain. In distance domain, range resolution \( \Delta r \) is defined based on the zero crossing of the sinc function.

\[ \Delta r = \frac{v_l}{K_c \tau} \]  

(2.11)

Range resolution become fine, when the chirp rate or pulse duration is large. Phase term is preserved in exponentially term (the first term), which is the range distance measured by carrier frequency. Time band product \( K_c \tau^2 \), the ratio of uncompressed and compressed signal, is useful index which describes how signal can be focused.

In stead, in azimuth direction, the variation in frequency of multiple locations due to the Doppler effect associated with the movement of the sensor relative to the Earth. As a satellite flies, the distance between a satellite and a target change according to the hyperbolic equation.

\[ R(x) = \sqrt{R_0^2 + (x - x_0)^2} \]  

(2.12)

\[ \approx R_0 + \frac{(x - x_0)^2}{2R_0} \]  

(2.13)

Thus, phase of azimuth signal varies with the distance \( -\frac{4\pi}{\lambda} R(x) \). According to the hyperbolic equation, the azimuth signal become also chirp signal as is the range signal. The Doppler centroid frequency \( f_{dop} \), which corresponds to the carrier frequency \( f_c \) in range, is derived as follows:

\[ f_{dop} = -\frac{4\pi}{\lambda} \frac{1}{2\pi} \frac{dR(x)}{dx} \]  

(2.14)

\[ \approx -\frac{2v_{sat}^2 x_{dop}}{\lambda R_0} \]  

(2.15)

\[ \approx -\frac{2v_{sat} \sin \theta_{sq}}{\lambda} \]  

(2.16)

where \( x_{dop} \) is azimuth location where a target is illuminated by the center of beam pattern, and \( \theta_{sq} \) is squint angle. \( v_{sat} \) is the sensor velocity on a satellite. Here, since we assume \( \theta_{sq} \) is zero, the Doppler central frequency \( f_{dop} \) is also zero. Similarly, Doppler bandwidth \( B_{dop} \) can be derived as:

\[ B_{dop} = \frac{2v_{sat} \cos \theta_{sq}}{L} \]  

(2.17)
When \( \theta_q \) is zero, Doppler bandwidth is \( \frac{2v_{sat}}{L} \). Once Doppler bandwidth is known, the resolution in azimuth direction is derived as the reciprocal of the bandwidth. Therefore, azimuth resolution is \( \frac{L}{2v_{sat}} \) in time, and

\[
\Delta x = \frac{L}{2}
\]  

in distance. Interestingly, azimuth resolution depends only on sensor width \( L \), and is independent on other system parameters. As the consequence of the azimuth focusing, azimuth signal is focused in a location where Doppler centroid frequency is zero, which is referred as zero-Doppler plain (Figure 2.1). The details about SAR processing is written in Franceschetti and Lanari (1999) and Cumming and Wong (2005).

Compressed signal is a two dimensional sinc function in both range and azimuth direction. Since the Fourier transform of sinc function is a boxcar shape, the spectrum of compressed signal is rectangular in range and azimuth frequency domain. Above discussions are about a observed signal from a single target with a unit reflectivity. True observed SAR data consists of a lot of targets with its reflectivity. Considering acquisition geometry, SAR image is the projection of three dimensional target reflectivity onto range and azimuth direction [Bamler and Hartl, 1998]. And, the image is band-pass filtered by the impulse response in range and azimuth direction, because frequency domain of two dimensional sinc function is two dimensional rectangular function.

The resulting SAR image, which is referred as single look complex (SLC), consists of amplitude and phase components. Below, I note properties of amplitude and phase. The amplitude is related with the magnitude of reflectivity in a pixel. Generally, the amplitude of man-made structures are stronger, while that of vegetation is weaker. Since a pixel contains a lot of scatterers, the amplitude as the sum of each scatterer contributions is considered as a random variable, and the fading is referred as speckle. I also note the side lobe of neighboring pixels. As shown in Equation 2.10 and the discussions of an azimuth signal, the impulse response of a pixel is two dimensional sinc function. The target showing strong reflectivity would affect the information of neighboring pixels whose reflectivity is weak. On the other hand, the phase is related with the distance between the satellite and a resolution cell, and scattering characteristics of a resolution cell. The observed phase is wrapped and ranged from 0 to \( 2\pi \). Even though the phase is a function of distance between the satellite and a scatterer on the surface, the absolute distance is difficult to measure using phase value, because it is difficult to estimate the cycle of phase and signal delay due to atmospheric effects. The phases become meaningful when we compare two observations (i.e., interferometry).

For describing the amplitude and phase statistics, I consider the assumption of distributed scatterers (or Gaussian scatterers). Statistics of distributed scatterers are derived from three assumptions: (a) A lot of scatterers exist within a pixel, (b) there is no dominant scatterer in a pixel, and (c) all scatterer is not correlated [Bamler and Hartl, 1998]. The assumption (a) is valid for the SAR image, because the size of a resolution pixel is greatly larger than the wavelength of the radar signal. Whether the assumptions (b) and (c) are valid depends on the target. Especially, the vegetation area suits for these...
assumptions (b) and (c). Assuming distributed scatterers, real and imaginary observations independently obey Gaussian distribution, which is referred as circular Gaussian distribution. That results in the probability distribution of amplitude is Rayleigh distribution, while the intensity (the square of amplitude) obeys the exponential distribution whose mean and standard deviation are identical. On the other hand, the phase values have the uniform distribution between 0 and $2\pi$.

The limitation of SAR observation should be noted. In principal, SAR cannot distinguish targets at the same slant range distance. Thus, the edge of a tall building appears in the near range compared with the location where it is situated, and a small building at nearer ground range appears at far range compared with the tall building (layover). The top of the mountain tend to lean toward the satellite, and the slope at the near ground range looks steeper and the that of far ground range (foreshortening).

Below, the spatial resolution about a SAR image on the ground is described. Since SAR flight trajectory can be assumed to be parallel to the ground, the azimuth resolution is same as the ground resolution in azimuth direction. On the other hand, the ground range resolution is different from the slant-range resolution depending on the acquisition geometry.

\[ \Delta r_g = \frac{\Delta r}{\sin \theta} \]  

(2.19)

where $\Delta r_g$ is the ground range resolution, and $\theta$ is the incidence angle. Thus, the ground range resolution of far range pixel is coarser than that of near range pixel in a flat place. Topography affects the incidence angle and ground range resolution. Considering topography, the ground range resolution at the near range slope is finer than that at the far range slope.

### 2.2 Synthetic aperture radar interferometry

InSAR analysis uses the phase informations of two observations to estimate physical parameters. The difference of the two data acquisitions (e.g., acquisition geometry, time, frequency) determine the physical parameter to be estimated. As shown later, the topography is sensitive to the difference of acquisition geometry of the satellite, thus the generation of the Earth’s topography map has been the primary purpose of the method historically. Currently, there is no doubt that displacement measurement is one of main topics of InSAR analysis. The method to estimate surface displacement is sometimes referred as differential SAR interferometry (DInSAR). Although the the definition of the terms, InSAR and DInSAR are not exactly same, in this thesis, I use the terms, InSAR and DInSAR analysis for indicating the method to estimate surface displacements using two data acquired at the different time.

Let me write the observed complex signal $C$ in a pixel as follows:

\[ C = A \exp(j\nu) \]  

(2.20)

where $A$ and $\nu$ is the amplitude and the phase. The phase is proportional to the geometrical distance
Figure 2.2: The configuration of two SAR acquisitions for SAR interferometry, and the definition of baseline. The figure is described in zero-doppler plane. Parallel ray approximation (far field approximation) is used for the definition of baseline. $B_\perp$ and $B_\parallel$ are the perpendicular and parallel component of baseline.

$R$ and the random phase shift due to scattering $\nu_{\text{scat}}$.

$$\nu = -\frac{4\pi}{\lambda} R + \nu_{\text{scat}}$$  \hspace{1cm} (2.21)

Interferometric processing is defined as the product of the complex signal at an acquisition and the complex conjugate of the complex signal at an another acquisition.

$$C_1 C_2^* = A_1 A_2 \exp(j\phi)$$  \hspace{1cm} (2.22)

where $\phi = \nu_1 - \nu_2$ is referred as interferometric phase and the resulting image is called as interferogram. Assuming that the phase measurement is repeated under the same condition, it would yield the same result, and phase due to scattering characteristics is deterministic. The condition is referred that the imaging is coherent. If we assume that two coherent SAR images are acquired at the same location and the different timing, the resulting phase difference represents the range change between the first and the second observation ($\Delta R$).

$$\phi = \exp \left( -\frac{4\pi}{\lambda} \Delta R \right)$$  \hspace{1cm} (2.23)

where $\Delta R$ equals $R_1 - R_2$. The minus sign comes in because $\phi$ is defined as the phase delay, which decreases as $\Delta R$ increases.

Surface displacement, the major interest of this study, leads the range difference. In addition, the range difference is induced by orbital trajectories, topography and changes in atmospheric condition.
The resulting interferometric phase is the sum of the contributions.

\[ \phi = \phi_{\text{disp}} + \phi_{\text{orb}} + \phi_{\text{topo}} + \phi_{\text{atm}} \]  

(2.24)

where \( \phi_{\text{disp}}, \phi_{\text{orb}}, \phi_{\text{topo}}, \) and \( \phi_{\text{atm}} \) are phases due to displacement, orbital trajectories, topography, and atmospheric effects. Below, I describe the main components of the interferometric phase in more detail.

**Interferometric phase**

**Phase due to orbital trajectories:**
The orbital phase is associated with the difference in distance from flat earth due to different orbital trajectories. Generally, the effect is the largest contribution to the range difference between two observations. Assuming that the range directions of two data are parallel, which is referred as far-field approximation, the range difference can be approximated as follows:

\[ \phi_{\text{orb}} = -\frac{4\pi}{\lambda}B_{\parallel}\sin(\delta) \]  

(2.25)

\[ = -\frac{4\pi}{\lambda}B_{\parallel} \]  

(2.26)

where \( B_{\parallel} \) is the parallel component of baseline (Figure 2.2). Since \( B_{\parallel} \) often has the order of \( 10^2 \) m, the orbital effect largely appears in the interferometric phase. The orbital phase pattern is subtracted using the information about the location of orbital path.

**Phase due to topography:**
The single SAR data cannot resolve scatterers at the same range. On the other hand, interferometry provides the height information of scatterers and it can be considered that the method constructs three dimensional image of scatterers [Bamler and Hartl, 1998]. This is the idea of the generation of digital elevation model. The basic concept is the parallax, and the the measurement of InSAR analysis is accurate by using phase information. Let me consider the approximation about the relationship between interferometric phase and the axis perpendicular to radar illumination (\( \xi \)).

\[ \xi \approx \frac{R}{B_{\perp}} \Delta R \]  

(2.27)

\[ \approx \frac{\lambda R}{4\pi B_{\perp}} \phi \]  

(2.28)

where \( B_{\perp} \) is the perpendicular component of baseline (Figure 2.2). I substitute \( h = \xi \sin \theta \) into Equation 2.28, where \( h \) is height direction, and obtain the relationship called phase-to-height sensitivity:

\[ \frac{\partial \phi}{\partial h} = \frac{4\pi B_{\perp}}{\lambda R \sin \theta} \]  

(2.29)

Then, the topographic phase associated with the scatterer height with respect to a reference surface can be described as follows:

\[ \phi_{\text{topo}} = -\frac{4\pi B_{\perp} \Delta h}{\lambda R \sin \theta} \]  

(2.30)
where $\Delta h$ is scatterer height or the error in digital elevation (Figure 2.2). As shown above, we can resolve a single scatterer center with two SAR images. During InSAR processing, the effect is subtracted using external digital elevation model (DEM).

**Phase due to atmospheric effects:**
From the macroscopic view point, the radar path delay occurs due to the atmospheric refractive index variation. As is widely known, the refractive index is a macroscopic parameter that describe the ratio of radar velocities between two propagation medium. It is known that the change in refractive index of SAR observation is caused by (1) the hydrostatic effect, which is influenced by the total pressure and temperature of the atmosphere, (2) the wet effect, which is induced by the partial pressure and temperature of the water vapor, (3) the liquid water content, and (4) the ionospheric effect, which has dispersion nature (i.e., frequency dependent) [Hanssen, 2001]. Sometimes, the effects (1)-(3) are summarized as the tropospheric effects. Among them, the wet effect is the major contribution that changes the refractive index of the troposphere [Goldstein, 1995; Tarayre and Massonnet, 1996]. This is due to the fact that the molecular dipole moment of $H_2O$ varies the refractive index of microwave. The physical mechanism of changes in the atmospheric refractive index can be divided into two components: one is the change is the propagation velocity, the other is the geometrical delay due to the path bending. For the tropospheric effects, it has been confirmed that the change in the propagation delay is generally dominant in observed phase delay. On the other hand, the bending effects of the ionospheric effects are also dominant mainly in the low frequency radar (e.g., L-band systems). The bending effect sometimes leads the location error in addition to the interferometric phase shift [Meyer et al., 2006; Raucoules and Michele, 2010].

**Decorrelation**
The coherent condition is the essential assumption of InSAR analysis. However, the observed image is not necessarily coherent, because the reflectivity of a pixel would change due to temporal changes of scatterers or changes in observation geometry. The loss of coherence has been known as decorrelation, and it makes interferometric phases random values. Then, because of the randomness, the interpretation of interferograms becomes difficult. On the other hand, decorrelation provides information about changes in scatterers’ reflectivity. For instance, the decorrelation would provide useful information to detect disaster area, which is discussed in Chapter 5 in the thesis. Below, the qualitative index to measure the degree of coherent condition: the coherence is described. Then, important relationships between the coherence and interferometric phase and the source of decorrelation are described.

**The definition of coherence and its properties:**
The complex coherence $\gamma_{cp}$ between two complex signals $C_1$ and $C_2$ is defined as follows:

$$\gamma_{cp} = \frac{E[C_1C_2^*]}{E[C_1C_1^*]E[C_2C_2^*]}$$  \hspace{1cm} (2.31)
where $E[\cdot]$ is the ensemble average. The ensemble average can be measured as the expectation value under stationary condition. Thus, in the practical InSAR data processing, the coherence is calculated with neighboring pixels within a certain rectangular window assuming a homogeneous area (statistics of SAR data is equivalent within the area). Generally, the absolute value of complex coherence $\gamma$ is estimated as follows:

$$\gamma = \frac{|\sum_{n=1}^{N_{\text{pix}}} C_{1n} C_{2n}^*|}{\sqrt{\sum_{n=1}^{N_{\text{pix}}} |C_{1n}|^2 \sum_{n=1}^{N_{\text{pix}}} |C_{2n}|^2}}$$

(2.32)

where $N_{\text{pix}}$ is the number of pixels to calculate the coherence, and $n$ is the pixel number within the window. The absolute value of complex coherence (coherence) is ranged from 0 to 1. The coherence is the normalized sum of complex interferogram. If phases of pixels used for the calculation is identical, the coherence becomes larger, while coherence become smaller if phases are random.

Although InSAR analysis generally estimates the coherence with the rectangular window in the spatial domain, the coherence in InSAR time-series analysis is calculated in the temporal domain, which is sometimes referred as temporal coherence. The definition of the temporal coherence is described in the next section.

The above description shows that the coherence in the practical data processing is approximated. Thus, we have to care that the estimated coherence has biased. The previous study showed that the degree of the bias of coherence depends on the number of pixels and the 'true' coherence [Touzi and Lopez, 1996]. The lower 'true coherence' and the fewer number of pixels leads the overestimation of coherence. In addition to the bias of the estimated coherence, it has been shown that the standard deviation of coherence becomes larger with decreasing the 'true' coherence [Touzi and Lopez, 1996].

**The source of decorrelation:**
The coherence has been known as the product of several effects [Zebker and Villasenor, 1992]:

$$\gamma = \gamma_{\text{therm}} \gamma_{\text{geom}} \gamma_{\text{temp}}$$

(2.33)

where $\gamma_{\text{therm}}$, $\gamma_{\text{geom}}$, and $\gamma_{\text{temp}}$ are coherence due to thermal, geometrical and temporal effect.

The thermal effect is induced by noise of the sensor. Data acquisition induces additional noise that is known as random values.

$$\gamma_{\text{therm}} = \frac{S\text{NR}}{S\text{NR} + 1}$$

(2.34)

where SNR indicates signal-to-noise ratio. SNR of the SAR system is determined by the ratio of the received signal power and the power of thermal noise [Hanssen, 2001]. The power of the received signal relates the radar equation which is a function of the transmitted power, the directivity gain, the distance between antenna and a scatterer, the illumination area and the normalized radar cross section. The power of the thermal noise is a function of the receiver noise temperature and the range bandwidth. In case of ERS-1/2 system, SNR is estimated as 11.7 [Hanssen, 2001].

The geometrical effect is induced by baseline separation of two data acquisitions. As shown in the previous section, the impulse response of SAR data is band-pass filtered. In other words, each
Figure 2.3: The spectral shift in the ground range direction induced by difference in incidence angle. 
(a) Geometrical interpretation of ground range projection of observed signals (represented as sinusoidal function for simplicity). (b) The frequency spectrum in slant-range and ground-range axis. 

pixel in a SAR image has its own band in frequency domain. The baseline separation of two SAR acquisitions causes the change in incidence angle.

\[
\Delta \theta = \frac{B \tan \theta}{R}
\]

(2.35)

where \( \Delta \theta \) is the difference in the incidence angle. Because of the baseline separation and corresponding incidence angle separation, the three dimensional configuration of two range spectra are not identical. Assuming that distributed targets are in a resolution cell on the ground, we could compare the difference of two spectra by projecting them onto ground range direction [Figure 2.3 a]. Some parts of two spectra are not overlapped because of the difference in incidence angle [Figure 2.3 b]. Since only overlapped signal provides interferometric information, the difference of two spectra cause decorrelation due to geometrical SAR configuration. Quantitatively, the decorrelation effect is proportional to the perpendicular component of baseline [Li and Goldstein, 1990; Zebker and Villasenor, 1992].

\[
\gamma_{\text{geom}} = 1 - \frac{2 \Delta r B_{\perp}}{\lambda R \tan \theta}
\]

(2.36)

The critical baseline \( B_{\perp,c} \) is defined as the perpendicular component of baseline whose coherence is zero.

\[
B_{\perp,c} = \frac{\lambda R \tan \theta}{2 \Delta r}
\]

(2.37)

The Equations 2.36 and 2.37 are derived based on distributed scatterers. Coherence of an ideal persistent scatterer is not affected by geometrical configuration as shown in the following section.
Figure 2.4: Theoretical coherence as a function of system wavelength and the standard deviation of scatterers’ motion.

The temporal effect is caused by the temporal change in scatterer location and reflectivity. Growth of vegetation and surface change due to disaster damage are examples of the temporal change. Assuming that a number of pixel are redistributed with Gaussian-statistic motion, the temporal coherence can be a function of the motion variance and the wavelength [Zebker and Villasenor, 1992] (Figure 2.4).

\[
\gamma_{\text{temp}} = \exp \left( -\frac{1}{2} \left( \frac{4\pi}{\lambda} \right)^2 \sigma_{\text{los}}^2 \right)
\]  \quad (2.38)

where \( \sigma_{\text{los}}^2 \) is the variance of scatterers’ motion in line of sight direction. One of the implications of the equation is the sensitivity of wavelength to the scatterer motions. If the wavelength is longer, the loss of coherence is smaller.

In addition to the theoretical decorrelation of an interferogram, the temporal variation of the temporal loss of coherence have been shown [e.g., Rocca, 2007; Lavalle et al., 2012]. For instance, if a random scatterer motion (or Brownian motion) can be divided into short-term (shorter than data acquisition span) and long-term (motions constantly occurred in the whole acquisition span) effects, and the variance of long-term motion increases with time, the variance of a scatterer can be described as

\[
\sigma_{\text{los}}^2 = \sigma_{\text{r,short}}^2 + D_p \sigma_{\text{r,long}}^2 t
\]  \quad (2.39)

where \( t \) is time, and \( D_p \) is a real value that describe how increase in the variance of long-term scatterers’ motion. Assuming that a pixel has a lot of scatterer so that the normal approximation holds, the coherence as a function of time become exponential function [Rocca, 2007; Morishita and Hanssen,
Figure 2.5: Theoretical loss of coherence as a function of wavelength and time. The standard deviation of scatterers’ motion are assumed to be 0.2 cm for long-term motion, and 0.1 cm for short term motion.

\[ \gamma(t) = \gamma_0 \exp\left(-\frac{t}{\Omega}\right) \]  

(2.40)

where \( \gamma_0 \) is the instant coherence affected by the short-term motion variance and the wavelength, while \( \Omega \) is the decorrelation rate, which is a function of \( \sigma^2_{r,\text{long}} \), \( v \) and the wavelength. Figure 2.5 is an example of theoretical variance in temporal effect of coherence, when the standard deviation of scatterers’ motion is 0.2 cm in long-term and 0.1 cm in short-term effects.

**Statistics for interferometric phase:**

Decorrelation makes interferometric phases random values. Here, I describe the statistics of interferometric phase assuming distributed scatterers as is used to describe SAR data statistics in the previous section. The probability distribution function of the complex signal of distributed scatterers is circular Gaussian distribution, which result in uniform distribution of a phase of SAR data. However, interferometric phase of distributed scatterers, which is the phase of the product of two circular Gaussian signals, is no longer uniformly distributed, as long as the two complex signals are correlated to some extent.
Figure 2.6: Probability density function of interferometric phase as a function of coherence.

Under the assumption that the complex signal $C_1$ and $C_2$ are jointly circular Gaussian, their joint probability distribution function (pdf) can be described as follows [Bamler and Hartl, 1998]:

$$pdf(w) = \frac{1}{\pi^{2|\text{Cov}|}} \exp \left( -w^T \text{Cov}^{-1} w \right)$$  \hspace{1cm} (2.41)

where $w$ is the complex vector, and the $\text{Cov}$ is the covariance matrix that are defined as:

$$w = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$  \hspace{1cm} (2.42)

$$\text{Cov} = E[ww^T]$$  \hspace{1cm} (2.43)

$$= \begin{pmatrix} E[|C_1|^2] & \gamma_c \sqrt{E[|C_1|^2]} E[|C_2|^2] \\ \gamma_c \sqrt{E[|C_1|^2]} E[|C_2|^2] & E[|C_2|^2] \end{pmatrix}$$  \hspace{1cm} (2.44)

With this Gaussian assumption and the consideration of the pdf of $C_1 C_2^*$, the pdf for the interferometric phase and its standard deviation can be described [Just and Bamler, 1994; Bamler and Hartl, 1998]. Below, I write the equations that describe the statistics for interferometric phase of a single-look interferogram without derivation. The pdf of the interferometric phase is

$$pdf(\phi) = \frac{1 - \gamma^2}{2\pi} \frac{1}{1 - |\gamma|^2 \cos^2 (\phi - \phi_0)} \left( 1 + \frac{\gamma \cos (\phi - \phi_0) \arccos (\gamma \cos (\phi - \phi_0))}{\sqrt{1 - \gamma^2 \cos^2 (\phi - \phi_0)}} \right)$$  \hspace{1cm} (2.45)

where $\phi_0$ is the expectation value of interferometric phase. When the interferometric phase is ranged from $-\pi$ to $\pi$, the variance of the interferometric phase ($\sigma^2_{\phi}$) is

$$\sigma^2_{\phi} = \frac{\pi^2}{3} - \pi \arcsin (\gamma) + \arcsin^2 (\gamma) - \frac{\text{Li}_2(\gamma^2)}{2}$$  \hspace{1cm} (2.46)
Figure 2.7: An example of one dimensional phase unwrapping. A black dashed line is the wrapped phase and a red dashed line is the unwrapped phase.

Figure 2.8: Examples of phase integration of phase gradients in a closed path, and a residue. Values indicate wrapped phase. (a) Phase without residues. The integration of phase difference in a close path (e.g., \( \sum_{i} q_i \)) is zero. (b) Phase with a positive residue. The integration of phase difference in a close path is one.

\[
\begin{array}{cccc}
\text{a} & q_4 & 0.1 \rightarrow 0.2 & 0.3 \\
q_1 & q_2 & 0.1 \rightarrow 0.2 & 0.4 \\
\text{b} & q_4 & 0.1 \rightarrow 0.2 & 0.3 \\
q_1 & q_3 & 0.2 \rightarrow 0.4 & 0.3
\end{array}
\]

Figure 2.6 is the example of the pdf of the interferometric phase with \( \phi_0 = 0 \). The pdf and the standard deviation show that the accuracy of interferometric phase is a function of coherence. That is why coherence has been used for the indicator for estimating the quality of interferograms. Moreover, random phases due to decorrelation tend to cause the phase unwrapping error, which leads to wrong estimation of surface displacement.

**Phase unwrapping**

Since observed phase is originally wrapped by modulo \( 2 \pi \), we need to unwrap to obtain absolute phase. Although wrapping absolute (unwrapped) phase is straightforward, unwrapping observed wrapped phase is often difficult due to its non-uniqueness and non-linearity. The following equa-
tion is the general description of the unwrapping procedure of discretized data.

\[ \psi_i = \phi_i + 2\pi \ell_i \]  \hspace{1cm} (2.47)

where \( \psi_i \) and \( \phi_i \) are unwrapped and wrapped phase and \( \ell_i \) is an integer coefficient. The most common notion is to assume that unwrapped phase has local phase gradients of less than \( \pi \) radian. In other words, the sampling rate is high enough to avoid phase aliasing over all of the data. And, considering unwrapping phase (phase difference) during InSAR processing, another assumption is applied that the absolute phase (phase difference) value at a certain reference pixel within a analyzed area is known (or can be assumed) before unwrapping procedure. We need to use such a priori knowledge about a reference point, because observed phase difference is generally affected by various effects such as sensor characteristics or atmospheric propagation delay over a scene. If these assumptions can be correct, we can uniquely unwrap phase in one dimension (Figure 2.7).

In two or three dimension, however, we have a problem for unwrapping procedure, that is the existence of residues. A residue is a point around which integrating phase gradient in a closed path does not return zero (Figure 2.8). If no residues are found in the entire wrapped phase, wrapped phase can be uniquely unwrapped by using any algorithms. That means resulting unwrapped phases is obtained from any integral paths. However, the existence of residue makes the unwrapped phase non-unique, (i.e., the result depends on calculation path). Thus, unwrapping procedure should be done carefully, because the precision of unwrapping could directly affect the accuracy of estimated parameters.

The phase unwrapping problem mathematically can be described as the optimization problem that minimizes the difference between unwrapped phase gradients and wrapped phase gradients. Thus, the objective function \( J \) of phase unwrapping is

\[ J = \sum |(\psi_i - \psi_j) - (\phi_i - \phi_j)|^p \]  \hspace{1cm} (2.48)

where \( \psi_i \) is unwrapped phase at a pixel \( i \). \(|^p\) means that the function is minimized from the viewpoint of Lp-norm. A lot of unwrapping algorithms have been proposed for two dimension (e.g., branch-cut algorithm [Goldstein et al., 1988], minimum cost flow algorithm [Costantini 1998; Chen and Zebker, 2000], minimum Lp-norm algorithm [Ghiglia and Romero, 1996], least square algorithm [Ghiglia and Romero, 1994]). The major difference of these algorithm is the number of \( p \). the most robust algorithm is the least square algorithm, which minimize the function by means of \( p = 2 \), while the minimum cost flow algorithm minimize the objective function. The successful branch-cut algorithm equal to minimizing the function using \( p = 0 \), which is the same concept of the minimum Lp-norm algorithm of \( p = 0 \). However, since it has been verified that there is no guarantee of the existence of the solution that minimizing the function using \( p = 0 \), the results of the branch-cut algorithm and the minimum Lp-norm algorithm may not be identical because of phase unwrapping error.
An example of InSAR processing flow

Here, I briefly describe an example of processing flow of InSAR analysis (Figure 2.9). A slave single look complex (SLC) of a SAR image are coregistered with respect to a master SLC. Then, interferograms are created according to Equation 2.22. The spectral filtering in range direction is applied in order to mitigate the geometrical effect of decorrelation, when interferograms are created. Phases due to orbital trajectories and topography are subtracted using simulated phases created by orbital data and DEM. If the orbital phases cannot be removed using the external data, the orbital phases are estimated and subtracted iteratively. Spatial filtering such as the adaptive filtering [Goldstein and Werner, 1998] is applied in order to smooth interferograms. Subsequently, phase unwrapping is performed. Generally, decorrelated area are masked prior to the unwrapping in order to prevent phase unwrapping error. Atmospheric effects are estimated and subtracted assuming some correlations with topography. The resulting interferogram after subtracting simulated orbital, topographic and atmospheric phase is called as the differential interferogram. Then, the differential interferograms in radar coordinate are converted to the longitude and latitude coordinate.
Major limitation of SAR interferometry for estimating surface displacement

InSAR analysis provides the spatially dense surface displacement map. However, there are some limitations that prevent operational monitoring with InSAR analysis. The limitations are closely related with the properties of interferogram as described below. Thus, this is also about the summary of this section and the reason why InSAR time-series analysis is necessary.

**Phase randomness due to decorrelation:**
As shown above, decorrelation increases phase variance. Therefore, the identification of deterministic phase components such as atmospheric effects or phase due to scatterer height become difficult. Moreover, phase unwrapping error induced by decorrelation causes severe wrong estimation of surface displacement. The unwrapping error happens not only at a decorrelated pixel, but also at an entire interferogram. Once phase unwrapping occurs, there is no versatile way to detect unwrapping error locations. InSAR analysis only works under coherent conditions, where the received echoes are correlated between two SAR images. The current strategy to mitigate decorrelation is to apply multi-look procedure or other spatial filtering to interferogram. The multi-look procedure is the method to decrease the variance of phase by \(\sqrt{N}\) times at the cost that the resolution becomes \(N\) times coarser. Similarly, any spatial filtering reduce the geometrical resolution for decreasing variance. And, there is no guarantee that the mean value of filtered interferometric phase is identical to the 'true' phase value. Therefore, the estimated phase may have biased by decreasing the variance of interferometric phase.

**The inaccuracy of external interferometric phase model:**
In InSAR processing, deterministic components of interferometric phase except displacement phases are simulated and subtracted. For instance, phase due to scatterer height are estimated using a digital elevation model, atmospheric effects are estimated using some spatial function (e.g., a model assuming the correlation with topography). However, there are the inaccuracies of these external information that limit the accuracy of estimated displacement. Especially, it has been known that atmospheric effects are difficult to simulate and cause severe variation in interferometric phase with about several centimeter scale.

### 2.3 Synthetic aperture radar interferometry time-series

Estimating surface displacements using InSAR analysis can be considered as the ill-posed inverse problem [Bamler and Hartl, 1998]. Because of the ill-posed problem, the estimation of surface displacements from interferometric phase is not straightforward without any a priori information. Therefore, InSAR analysis uses additional (external) information for estimating orbital and scatterer height effects, and subtracted them from interferometric phases. However, it is currently difficult to separate each phase contribution completely with the external data because of the inaccuracy of the external information. As a consequence, the problems about decorrelation and residual deterministic phases
remains to be major limitation of InSAR analysis. Multiple interferograms offer solutions for the limitations. One of the well-known technique is to average available interferograms, which is referred as interferogram stacking. The method reduces temporally random values which may correspond to atmospheric effects and decorrelation. However, severe atmospheric effects sometimes violate the assumption. And, isolated coherent pixels cannot be extracted, because interferograms are often filtered before the stacking.

Then, InSAR time-series analysis has been developed for estimating more accurate surface displacement and temporal variation of displacement. It has been reported that there are coherent scatterers that are not decorrelated in any temporal and geometrical baseline. The coherent scatterers mainly correspond to a man-made structure or a bared rock (e.g., Usai and Klees, 1999). InSAR time-series analysis detect the coherent scatterers by phase stability, and utilizes high quality phase information. Thus, coherent pixels used by InSAR time-series analysis is defined by phase behavior. Depending on the analysis, the coherent pixels have different names. Well-known names are persistent scatterers, permanent scatterers, point targets, or coherent targets. In this thesis, I mainly use persistent scatterers to refer the coherent pixels, because this is one of the names that has been widely used. By using high quality phase information, InSAR time-series analysis estimate and subtract deterministic phase contributions by utilizing time-series phase information.

**Properties of a persistent scatterer**

Here, I describe properties of an ideal persistent scatterer. First is the basic reason why coherence of a persistent scatterer is independent from geometrical baseline [Ferretti et al., 2001]. Assuming a pixel without affecting displacement, atmosphere, and temporal decorrelation, the coherence can be described as follows:

\[ \gamma = \frac{\int_{r_1}^{r_2} a_1 a_2 \exp(j\phi)dr}{|a_1||a_2|} \]  

with

\[ \phi = \frac{4\pi}{\lambda R \tan \theta} r \]
where \( r \) is range location of scatterer, and \( r_1 \) and \( r_2 \) are the nearest and the farthest range location of scatterers in a pixel. If amplitude is independent from range coordinate, the coherence can be simplified.

\[
\gamma = \int_{r_1}^{r_2} \exp\left(\frac{j 4\pi}{\Delta R \tan \theta} r\right) dr \tag{2.51}
\]

If \( r_1 \) is \(-\frac{\Delta r}{2}\) and \( r_2 \) is \(\frac{\Delta r}{2}\), coherence \( \gamma \) becomes

\[
\gamma = \left| \sin\left(\frac{k_r \Delta r}{2}\right) \right| \tag{2.52}
\]

where \( k_r \) is \(\frac{2\pi}{\Delta R \tan \theta}\). Equation 2.52 is coherence of distributed scatterers as a function geometrical baseline [Ferretti et al., 2001]. And, coherence become zero, when \( k_r \) is \(\pm \frac{\Delta r}{\Delta R}\). Coherence of persistent scatterers can be described as a model, when \( r_1 \approx r_2 \). In this case, coherence is not sensitive to geometrical baseline variation, and always high. Therefore, a persistent scatterer is sometimes referred as a point-wise scatterer (Figure 2.10).

As described in the previous section, InSAR analysis resolves scatterers in incidence angle direction. Let me apply this theory to the analysis of persistent scatterers. A persistent scatterer, which has point-wise shape, is located somewhere in a pixel. This location of a persistent scatterer affect interferometric phase [Perissin et al., 2006; Perissin and Rocca, 2006; Kampes, 2006; Perissin and Ferretti, 2007]. Now we assume that a point-wise scatterer that is located in a sub-pixel location of \( r_k \) in slant-range, \( x_k \) in azimuth and \( \xi_k \) in perpendicular to slant-range and azimuth direction, which are defined with respect to the center of a pixel (Figure 2.11). The effect of \( r_k \) can appear in phase, when there is difference in central frequency of the carrier signal. And, the effect be simply derived from
the distance change in range direction.

\[ \phi_{r_k} = -\frac{4\pi}{v_l} \Delta f_c r_k \]  (2.53)

where \( v_l \) is the speed of light, and \( \Delta f_c \) is the difference in central frequency of the carrier signal. For instance, both ERS and Envisat sensor are C-band, however there is the central frequency difference of about 31 MHz \([\text{Perissin et al.}, 2006]\). The effect of \( \xi_k \) is derived from the distance change in this direction associated with the change in incidence angle.

\[ \phi_{\xi_k} = -\frac{4\pi}{\lambda} \xi_k \Delta \theta \]  (2.54)

By substituting the relationship between baseline separation and change in incidence angle (Equation 2.35) into Equation 2.54,

\[ \phi_{\xi_k} = -\frac{4\pi B \xi_k}{R} \]  (2.55)

This equation also can be derived by substituting the geometrical relationship, \( h = \xi \sin \theta \) into Equation 2.30. The phase due to azimuth sub-pixel location \( x_i \) is also induced by the change in distance in the azimuth direction due to the difference in the squint angle \( \Delta \theta_{sq} \).

\[ \phi_{x_i} = -\frac{4\pi}{\lambda} x_i \Delta \theta_{sq} \]  (2.56)

The difference in the squint angle is caused by the difference in Doppler frequency \( \Delta f_{dop} \) \([\text{Perissin et al.}, 2006]\).

\[ \Delta f_{dop} \approx \frac{2v_{sat}}{\lambda} \Delta \theta_{sq} \]  (2.57)

where \( v_{sat} \) is the satellite platform velocity. By substituting Equation 2.57 into Equation 2.56, phase changes induced by sub-pixel position of azimuth direction \( x_k \) is derived as follows.

\[ \phi_{x_k} = -\frac{2\pi}{v_{sat}} \Delta f_{dop} x_k \]  (2.58)

In this thesis, I used data acquired with the same central frequency and the same Doppler frequency. Therefore, phases due to sub-pixel location of \( r_k \) and \( x_k \) were neglected, and phases due to the sub-pixel location of \( \xi_k \) were considered as the phase due to scattering height \( \Delta h \).

**Classification of InSAR time-series analysis**

Several InSAR time-series analysis have been proposed and have different names depending on the algorithm. Each algorithm has its own strategies to improve estimation accuracy, however it is also true that methods have been developed by being inspired by other methods. Therefore, it would be worth describing a brief review of InSAR time-series analysis. The analysis that have been commonly used are Permanent scatterer (persistent scatterer) interferometry (PSInSAR, PS-InSAR or PSI) \([\text{Ferretti et al.}, 2000; \text{Ferretti et al.}, 2001; \text{Werner et al.}, 2003]\), Small baseline subsets (SBAS or SB)
[Berardino et al., 2002; Schmidt and Bürgmann, 2003; Lanari et al., 2004], and Stanford method for persistent scatterer (StaMPS, PS-InSAR or PSI) [Hooper et al., 2004; Hooper et al., 2007]. Furthermore, advanced InSAR time-series analysis in terms of increase in the number of pixels to estimate surface displacement have been recently developed [Hooper, 2008; Ferretti et al., 2011]. A lot of InSAR time-series analysis (more than described above) have been developed, however there would be no doubt that the aboves are one of the most influential studies for InSAR time-series analysis [Ferretti et al., 2011]. Let me describe characteristics of each method.

(A) Permanent scatterer (persistent scatterer) interferometry:

Permanent scatterer (persistent scatterer) interferometry (PSInSAR, PS-InSAR or PSI) analysis proposed by Ferretti et al. (2000) and Ferretti et al. (2001) is the first paper which used highly coherent scatterers called permanent scatterers to extract time-series surface displacement phase. The major innovations of the method are as follows.

The use of coherent pixels with single-look interferograms

The method uses single-look interferograms. Single-look interferograms enable to estimate displacement with fine spatial resolution. Also, the high quality phase information of isolated coherent scatterers would not be filtered out. In order to process single-look interferograms and find isolated coherent scatterers, coherence should not be calculated in the spatial domain as is done in InSAR analysis. Therefore, they proposed to use coherence defined in the temporal domain. Coherence in the temporal domain allow to examine pixels that are coherent over time, which is referred as permanent (persistent) scatterers (PSs).

Solving the ill-posed problem using spatial, temporal and geometrical properties of interferometric phase

The analysis estimates and subtracts phases due to scatterer height and atmospheric phases (phases that cannot be removed by InSAR analysis). They estimated these phase contributions by introducing analytical and empirical phase models. As described in the previous section, phases due to scatterer heights are linearly correlated with geometrical baseline. Phases due to atmospheric inhomogeneity can often be described as the function in the spatial domain. Surface displacement is often linear with time, thus phases due to displacement are modeled with constant velocity. Ferretti et al. (2001) iteratively estimates unknown parameters of these models and extract each phase component. Compared with other time-series InSAR analysis, the method do not use any spatial and temporal filtering. Therefore, the high spatial resolution of SAR data is retained. Since the filtering is a technique to improve the stability of the solution at the sacrifice of the unbiased solution, their method can result in unbiased solution, which makes it easier to examine the accuracy of estimates.

Interferometric datasets selection with respect to a single master image

The method select interferometric datasets with respect to a single master image. Therefore, the coherent pixels that are (theoretically) independent to baseline conditions can be detected.
Avoiding phase unwrapping error as much as possible

Phase unwrapping error is one of the major error sources of InSAR analysis. It is known that phase unwrapping error likely occurs when a certain amount of residues exist [Ghiglia and Pritt, 1998]. Since decorrelation induces residues, unwrapping of decorrelated area must be avoided. InSAR analysis often masked low coherence area, however the boundaries between coherent and incoherent area are sometimes obscure and unwrapping errors often occur. On the other hand, the method applies phase unwrapping after the identification of PSs. Thus, unwrapping errors would be reduced.

Spatial and temporal filtering to remove atmospheric effects

Ferretti et al. (2000) introduced a method to remove atmospheric phase contributions using temporal and spatial filtering. Phases due to surface displacements and scatterer heights are estimated with interferometric phase after applying low-pass filtering in the temporal domain, while atmospheric phases are extracted from interferometric phase after applying low-pass filter in the spatial domain and high-pass filter in the temporal domain. These filters are based on the characteristics of the phase components (i.e., atmospheric effects appear spatially low frequency and temporally high frequency).

(B) Small baseline subsets analysis:

Small baseline criteria for selecting interferometric datasets

Small baseline subsets (SBAS) analysis [Berardino et al., 2002; Schmidt and Bürgmann, 2003; Lanari et al., 2004] is a method to estimate time-series surface displacement using interferometric datasets selected based on small baseline criteria. Small baseline criteria is a method to select interferometric datasets using maximum thresholds for geometrical and temporal baseline. The data selection is based on the assumption that pixels become decorrelated with increasing geometrical and temporal baselines. Therefore, the method do not necessarily use ideal PSs, because the method estimate surface displacement at coherent pixels only during a certain baseline criteria. In stead, the coherent pixels in SBAS are sometimes referred as slightly decorrelated filtered pixels [Hooper, 2008].

Identification of coherent pixels in the spatial domain

SBAS analysis proposed by Berardino et al. (2002) and Schmidt and Bürgmann (2003) uses multi-looked interferograms in the whole processing chain. Thus, SNR of decorrelated pixels would be improved, though the spatial resolution decreases. Moreover, coherent pixels in SBAS analysis [Berardino et al., 2002] are calculated in the spatial domain (using neighboring pixels within a certain window size), while coherent pixels in PSInSAR analysis [Ferretti et al., 2001] are determined in the temporal domain of a single pixel. In this sense, the concept of coherent pixels of this method is the similar as the standard InSAR analysis, rather than persistent scatterers.

Small baseline subsets analysis with single-look interferograms

Lanari et al. (2004) extended the SBAS analysis for single-look interferograms. They proposed to divide interferometric phases into spatially low-pass and high-pass components, and analyze these two components separately, then combined outputs finally. The spatially low-pass components are the
same as the multi-looked interferogram used in Berardino et al. (2002), and the same processing chain is used. On the other hand, the spatially high-pass components are the difference between single-look and multi-look differential interferograms. By analyzing high-pass components separately, the analysis enables to deals with the movement of single scatterer that was neglected in Berardino et al. (2002). Moreover, since atmospheric effects can be neglected in the high-pass component, the ill-posedness of the problem is improved in the analysis of high-pass component. For selecting coherent pixels in high-pass components, they defined coherence in the temporal domain. Although Ferretti et al. (2001) calculated coherence with wrapped phase, the method proposed to calculate temporal coherence using unwrapped phase. One of the factors to determine the accuracy of this analysis is whether low-pass components can be correctly created. In highly decorrelated area, it would be difficult to create low-pass components and there may be phase artifacts due to the filtering of decorrelated phases.

(C) Stanford method for persistent scatterer:

Redefinition of persistent scatterers
Hooper et al., (2004) and Hooper et al., (2007) developed InSAR time-series analysis that can be applied to natural terrains. It is known that the density of coherent pixels identified by the PSInSAR algorithm [Ferretti et al., 2001] is insufficient in natural terrain. This insufficient density becomes a problem when we analyze displacement around non-urban area such as volcanic region. Thus, they redefined PS as a pixel that behave as a coherent pixel if the amount of decorrelation can be reduced. The redefined PS includes pixels with a small amount of decorrelation, which are physically correspond to bared surface and trunks.

PS selection with probability density function
In addition, They also introduced PS selection based on the probability density function of coherence. Because decorrelation noise is a random variable, the coherence, which is a indicator to measure the amount of decorrelation is also a random and has uncertainty. In other words, even though a pixel has the small coherence value (e.g., 0.3), there is a finite possibility that the pixel is PS. For considering the uncertainty, they proposed a method to simulate the probability density function as a function of coherence, and determine a coherence threshold based on the probability density function.

Temporal and spatial filtering for extracting surface displacement
They also introduced a idea to extract surface displacement phase using characteristics of each phase component (e.g., atmospheric effects appear spatially low frequency and temporally high frequency) as is in PSInSAR and SBAS analysis [Ferretti et al., 2000; Berardino et al., 2002; Lanari et al., 2004]. However, compared with the analyses, they applied temporal and spatial filtering for isolating each phase component before and after PS identification and phase unwrapping, though the the above analyses apply the filter after phase unwrapping operation. To apply spatial and temporal filtering without using phase model, the method does not need a priori knowledge of displacement and displacements.
at a pixel with complex displacement behavior (e.g., periodic displacement) can be obtained, though PSInSAR analysis proposed by Ferretti et al. (2001) estimate coherent pixels whose displacement behavior is a priori known and can be modeled.

Other new concepts
Several new concepts have been proposed in Hooper et al., (2007). One of the ideas is three dimensional phase unwrapping [Hooper and Zebker, 2007], which aim at doing unwrap in three dimensional domain (both spatial and temporal domain). However, three dimensional phase unwrapping method that can apply a various situation without losing the reliability has not bee proposed yet. The other idea is to revive discarded pixels surrounding PS, which they called multiple pixel PS for increasing the number of pixels to estimate surface displacements. The idea of multi pixel PS is effective at areas where the spatial displacement pattern can be assumed to be continuous.

(D) Advanced InSAR time-series analysis:

Recently, InSAR time-series analysis has been extended in a way that allows the increase the density of coherent pixels to estimate displacement even in non-urban area. One of the common methods is the multi-temporal InSAR (MT-InSAR) analysis [Hooper, 2008], which incorporate PS and SBAS techniques in terms of data selection. The other is SqueeSAR [Ferretti et al., 2011]. In SqueeSAR analysis, they use all possible combination of SAR datasets and applies the space adaptive filtering, which imposes a filter only on noisy pixels sharing the similar amount of decorrelation preserving phase information of PSs. In both advanced time-series methods, the basic concept is to estimate surface displacements, not only using phases in highly coherent pixels, but also using phases in slightly decorrelated pixels as long as there is a chance to enhance signal-to-noise ratio of the signals.

InSAR time-series analysis used in this thesis

In this study, I developed and used InSAR time-series analysis that can be used in both urban area and natural terrain within the frame work of advanced InSAR time-series analysis. The philosophy of the time-series analysis used in this study is mainly influenced by persistent scatterer interferometry (e.g., Ferretti et al., 2000; Ferretti et al., 2001; Kampes, 2005; Ferretti et al., 2011) and inspired by other InSAR time-series analysis (e.g., Berardino et al., 2002; Wegmüller et al., 2010). Based on the philosophy and the influence from the other previous studies, I reconstructed the flow of analysis so that (i) we can examine the estimation accuracy, (ii) we can analyze displacement pattern in both urban and natural terrain, (iii) we can estimate non-linear surface displacement pattern. The basic flow of time-series InSAR analysis I used in this thesis is as follows: (1) SAR data combination (2) PS candidates selection (3) Differential interferograms creation (4) Space adaptive filtering (5) PS selection (6) Phase unwrapping (7) Removal of atmospheric phases (8) Estimation of displacement velocity and scatterer height (9) Estimation of time-series displacement. In the followings, I briefly explain each step of the basic flow of the analysis used in this study.
(1) SAR data combination

There are two major criteria to determine interferometric datasets: (i) small baseline approach [e.g., Berardino et al., 2002], and (ii) single master approach [e.g., Ferretti et al., 2000; Ferretti et al., 2001]. As described above, the small baseline approach selects datasets using a criteria about geometrical and temporal baseline. On the other hand, the single master approach selects datasets in a way that every SAR images creates interferograms with respect to an arbitrary SAR images. In the following analysis, I used the small baseline approach, because we can create larger number of interferograms than the single master approach, even if there are rather small number of SAR data at areas of interest.

(2) PS candidates selection

PS candidates are selected prior to PS selection in order to reduce computational cost. The candidates selection is performed based on amplitude statistics called as the amplitude dispersion index $D_A$.

$$D_A = \frac{\sigma_A}{m_A} \quad (2.59)$$

where $\sigma_A$ and $m_A$ are the standard deviation and the mean of the amplitude values ($A_k$). Suppose that a pixel has a reflectivity with the angular amount of zero (g and $\angle g = 0$) and gaussian phase noise with phase standard deviation of $\sigma_n$, a phase dispersion is nearly equal to a amplitude dispersion in the case of high SNR ($\sigma_n << |g|$). Although the coherent pixel is defined as its phase stability, coherence is underestimated if topographic or atmospheric phases are not properly compensated. And, coherent pixels should be evaluated pixel by pixel, not with surrounding pixels. The amplitude dispersion index is the practical indicator to evaluate phase stability, because the index is calculated pixel by pixel and phase stability is shown in the next section.

(3) Differential interferograms creation

Differential interferograms are created at selected PS candidate pixels with selected SAR datasets. This processing is almost same as InSAR analysis, however, the major difference is that interferograms are created using single-look SAR images. In some studies, no spectral filtering in range direction is imposed in order to identify originally coherent pixels that has high phase stability without filtering. However, I applied the spectral filtering when I create interferograms in order to extract larger number of coherent pixels. If the properties of PSs is the main interest, the spectral filtering is not be basically applied.

(4) Space adaptive filtering

I optionally used space adaptive filtering. Space adaptive filtering is the filtering that are imposed only on pixels with similar noise magnitude, which is sometimes referred as statistically homoge-
neous pixels (SHP). In other words, filtering are selectively applied to pixels depending on noise statistics. Therefore, the filtering avoid contaminating PSs by filtering with noisy pixels. On the other hand, noisy pixels can be filtered and improved SNR. Because of the above characteristics, the space adaptive filtering is especially useful for analyzing both PSs and distributed scatterers (i.e., natural terrains). More concretely, I first apply a statistical test to amplitude data stack in order to classify pixels sharing statistically similar features. Then, we imposed multi-looking filtering upon pixels classified into the same category. The classification with amplitude data stack is discussed in the next section. And, the application of the filtering to SAR data is shown in chapter 4.

(5) PS selection

As described above, PS is defined by its phase stability. To evaluate phase stability, we quantify the phase noise magnitude using a index called the coherence $\gamma$ defined in the temporal domain.

$$\gamma = \left| \frac{1}{N_{intf}} \sum_{i} \exp(j(\phi_{obs,i} - \phi_{model,i}(m))) \right|$$

(2.60)

where $\phi_{obs,i}$ and $\phi_{model,i}(m)$ are the observed wrapped phases in differential interferograms, and modeled wrapped phases, respectively. $m$ is the model parameter vector of modeled phase. Generally, we assume that modeled phases consist of phase due to surface displacements $\phi_{disp,i}$, scatterer heights with respect to a reference DEM $\phi_{topo,i}$, atmospheric effects $\phi_{atm,i}$, and decorrelation $\phi_{dcr,i}$.

$$\phi_{obs,i} = \phi_{disp,i} + \phi_{topo,i} + \phi_{atm,i} + \phi_{dcr,i}$$

(2.61)

One of the simplest models for the phase components are

$$\phi_{disp,i} = \frac{4\pi}{\lambda} v$$

(2.62)

$$\phi_{topo,i} = \frac{4\pi}{\lambda R \sin\theta} \Delta h$$

(2.63)

$$\phi_{atm,i} = a_{atm} \cdot r + b_{atm} \cdot x + c_{atm}$$

(2.64)

In this case, model parameters $m$ are displacement velocity $v$, scatterer height $\Delta h$ and coefficients for atmospheric effects $a_{atm}$, $b_{atm}$ and $c_{atm}$. The accuracy of the model parameter estimation and PS selection are discussed in the next chapter.

The temporal coherence measures the magnitude of phases that are not modeled. This quantification is based on the assumption that the rest of phase (phases that is not modeled) are ranged from $-\pi$ to $\pi$. The rest of phases consist of (A) phases due to decorrelation, (B) phases due to unmodeled nonlinear displacements, and (C) phases due to unmodeled atmospheric effects. Therefore, temporal coherence cannot measure the amount of deorrelation, if residual phases contains a large amount of (B) nonlinear phase and (C) atmospheric phase.
The definition of coherence in InSAR analysis is shown in Equation 2.31 and 2.32. The coherences defined by InSAR and the time-series analysis are identical, if we (a) calculate the ensemble average of in the temporal domain, and (b) neglect amplitude information $a_1$ and $a_2$, and (c) assume that phase differences $\epsilon_1 - \epsilon_2$ consist of (A)-(C) described in this section.

(6) Phase unwrapping

The phase unwrapping method used in InSAR analysis has been extended to sparse phases in InSAR time-series analysis. The density of PSs are important, because phase unwrapping generally assumes no aliasing (i.e., the difference between neighboring pixels are less than $\pi$). Phase aliasing induces phase discontinuity that avoid unique phase unwrapping. In this study, I mainly use minimum Lp-norm algorithm [Ghiglia and Romero, 1996] and minimum cost flow algorithm [Costantini and Rosen, 1999; Werner et al., 2002]. The minimum Lp-norm algorithm gives L0-norm solution that is expected to be the best solution. However, L0-norm minimization of two dimensional phase unwrapping has been known as nondeterministic polynomial-time hard (NP-hard) problem [Chen and Zebker, 2000]. Thus, it is sometimes impossible to obtain the exact solution in practical cases. The simulation of the algorithm is shown in the next section. On the other hand, minimum cost flow algorithm provides the approximate solution that is mostly same as L0-norm solution, and has less computational time.

The selection of a reference pixel is important for the accurate displacement estimation. If there is still residual atmospheric artifacts, estimated displacements become less accurate, as the distance from a reference pixel is longer [Kampes, 2005]. Thus, a reference pixel should be set near areas of interest where surface displacement occurs, or around the center of analyzed area.

The above two dimensional unwrapping show its effectiveness to InSAR time-series analysis. Strictly speaking, however, phase unwrapping of InSAR time-series displacement is three dimensional problem (two dimension in the spatial domain, and one dimension in the temporal domain). Three dimensional phase unwrapping methods have been proposed based on branch-cut algorithm [Huntley, 2001; Salfity et al., 2006a], integer least-square algorithm [Salfity et al., 2006b], quasi-L1-norm algorithm [Hooper and Zebker, 2007], and step-wise three-dimensional algorithm. The main difference between two dimensional and three dimensional phase unwrapping is distribution of pairs of residues. In two dimensional problem, path independent phase unwrapping is possible by restricting path integration between positive and negative residues (e.g., branch-cut). On the other hand, the processing is possible when residues are connected in the three dimension. Therefore, residue-cut in three dimension becomes surface, which is bounded by residues (referred as residue loop or phase singularity loop), while it is tree-like structure in two dimensional problem. The exact solution of three dimensional phase unwrapping would be obtained with L0-norm solution within the framework of optimization. That would be future work for more accurate InSAR time-series analysis [Hooper and Zebker, 2007].
(7) Removal of tropospheric effects

In this thesis, tropospheric effects are estimated and removed by fitting a spatial function describing the effects based on the statistical consideration. It is known that the behavior of atmospheric effects can be described using stochastic parameters such as the power spectrum, the autocorrelation and the fractal dimension [Hanssen, 2001]. The spatial correlation for tropospheric effects exhibits as follows.

\[ E[\phi_{atm}(P_1) - \phi_{atm}(P_2)]^2 = K D^\beta \]  \hspace{1cm} (2.65)

where \( E[] \) is the expectation operator, \( K \) is a positive constant and \( D \) is the distance between pixels \( P_1 \) and \( P_2 \). \( \beta \) is a coefficient. Equation 2.65 suggests that the shorter the distance between two pixels affected by atmospheric turbulence phenomena, the more similar their phase values. For a stationary process, Equation 2.65 can be measured as follows:

\[ E[\phi_{atm}(P_1) - \phi_{atm}(P_2)]^2 = 2R_{\phi\phi}(0) - 2R_{\phi\phi}(D) \]  \hspace{1cm} (2.66)

where \( R_{\phi\phi} \) is the autocorrelation function of atmospheric phases. Equation 2.66 means that the spatial correlation can be measured from the autocorrelation function.

Considering the above auto-correlation of tropospheric effects, it can be said that tropospheric phases would not have variation in short distance. Thus, one approach to remove the tropospheric effects is to fit and subtract an empirical function. Ferretti et al. (2001) proposed to apply a linear plane, when we analyze a small area (approximately 2 km-3 km square area).

\[ \phi_{atm}(r, x) = a_{atm} \cdot r + b_{atm} \cdot x + c_{atm} \]  \hspace{1cm} (2.67)

where \( a \) to \( c \) are coefficients of the function, and \( r \) and \( x \) is range and azimuth direction, respectively. This empirical model can be applied, only if we analyze a small area.

(8) Estimation of displacement velocity and scatterer height

Once interferometric phases are unwrapped, the relationship between phases \( (\phi_{obs}) \) and model parameters such as displacement velocity and scatterer height is linear. Thus, we could estimate these parameters by linear regression. If I described the modeled phases except model parameters are data kernel \( H (\phi_{obs} = Hm + \phi_{dcr}) \), model parameters can be estimated based on the least-square estimation as follows:

\[ m = (H^T H)^{-1} H^T \phi_{obs,i} \]  \hspace{1cm} (2.68)

The solution is unbiased and efficient estimator. The variance of the estimated model parameters \( \sigma_m^2 \) can be written as follows:

\[ \sigma_m^2 = E[(\bar{m} - m)(\bar{m} - m)^T] \]  \hspace{1cm} (2.69)
\[ = E[(H^T H)^{-1} H^T \phi_{dcr}((H^T H)^{-1} H^T \phi_{dcr})^T] \]  \hspace{1cm} (2.70)
\[ = \sigma_{dcr}^2 (H^T H)^{-1} \]  \hspace{1cm} (2.71)
where $\sigma^2_n$ is the variance of decorrelation phase vector $\phi_{dcr}$. It is known that $H^T H$ is a factor to determine the propagation of error in data to estimated model parameters.

(9) Estimation of time-series displacement

The method to retrieve time-series displacements depends on datasets selection method. However, without depending on the datasets selection, time-series surface displacement can be retrieved by inverting differential interferograms created in the different time span.

Here, we assume that there are $N_{intf}$ differential interferograms and $N_{sar}$ SAR images. Suppose that we estimate surface displacement at a pixel and phases associated with scatterer height are subtracted beforehand, incremental phase vector $\phi_{inc}$ and the vector of differential interferograms after the correction of scatterer height $\phi_{obcr}$ have $N_{intf} \times 1$ and $N_{sar} \times 1$ dimension. The data kernel matrix $G$ that describe the relationship between $\phi_{inc}$ and $\phi_{obcr}$ has $N_{intf} \times N_{sar}$ dimension.

$$
\phi_{obcr} = G\phi_{inc}
$$

(2.72)

$G$ is constructed by 1 and 0 using the time span of differential interferograms. $N_{intf}$ lines of $G$ matrix correspond to the time span of $N_{intf}$ differential interferograms, and an arbitrary column of a line become 1 if differential interferograms are created with the time span. Otherwise, columns are arranged by 0. The time-series differential interferometric phase $\phi_{ts}$ can be obtained by summing the incremental phase vector from preceding time steps.

$$
\phi_{ts,t} = \sum_{k=1}^{t} \phi_{inc,k}
$$

(2.73)

The stability of the inverse problem of Equation 2.72 is important to obtain reliable time-series displacement. Often, the number of interferograms $N_{intf}$ is greater than the number of SAR images $N_{sar}$. This use of redundant differential interferograms helps to improve model resolution of inversion and reduce the variance of model parameters. However, because of irregular acquisition of SAR data, the lack of datasets that covers a few SAR images causes rank deficiency (i.e., the inverse problem is a mixed-determined). Therefore, datasets selection and the selection of the threshold should be performed considering the stability of time-series displacements.

Inverse problems are often solved using a priori knowledge to stabilize the estimates (i.e., decrease the variance). For the inverse problem, damping of incremental phase is often use to mitigate atmospheric artifacts as well as stabilize estimates [Schmidt and Burgmann, 2003].

$$
\begin{bmatrix}
G \\
\kappa \frac{d}{dt}
\end{bmatrix}
\phi_{inc} =
\begin{bmatrix}
\phi_{obcr} \\
0
\end{bmatrix}
$$

(2.74)

where $\kappa$ is the damping coefficient. The damping of times-series displacement with a weighting coefficient is based on the empirical fact that the atmospheric effect appears in high frequency signal
in temporal domain. The weighting coefficient should be determined objectively, otherwise time-series displacements are over-damped or under-damped. One way to determine the objective function is Akaike’s Baysian Information Criterion (ABIC) defined as follows.

\[
ABIC(\kappa^2) = -2\log M(\kappa^2) + 2N_{damp}
\]

(2.75)

where \(M(\kappa^2)\) is the marginal likelihood and \(N_{damp}\) is the number of damping coefficients. Therefore, maximizing ABIC means maximizing the marginal likelihood as long as the number of the damping coefficients is constant.

### 2.4 Simulation of synthetic aperture radar interferometry time-series

It is important to understand properties of each processing in order to evaluate the accuracy of estimated displacement. Here, I simulated some of processing steps of InSAR time-series analysis used in this study. As described in the previous section, InSAR time-series analysis has been developed to overcome the limitations of InSAR analysis. The main limitation is decorrelation and the tropospheric effects. By using simulation data, I describe the accuracy of each processing depending on decorrelation, and how each processing mitigate tropospheric effects. At first, I show the relationship between phase stability and the amplitude dispersion index (Equation 2.59) for PS candidates selection. From the simulation, I describe the threshold of the index for selecting proper candidates of PSs. Second, I simulate the method to select statistically homogeneous pixels (SHP) for space adaptive filtering. Parametric and non-parametric methods are compared using the simulation. Third, I simulate PS selection under noise (decorrelation) condition. The accuracy of displacement velocity and scatterer height depending on noise magnitude is discussed. Also, properties of temporal coherence (Equation 2.60) is described. Fourth, the accuracy of minimum Lp-norm algorithm for two dimensional phase unwrapping and its unwrapping error is discussed. The importance of proper phase unwrapping is noticed. In fifth, the performance of methods to mitigate tropospheric phases are examined. In InSAR time-series analysis, tropospheric effects can be mitigated, however it is difficult to remove them completely. I show how we can mitigate tropospheric effects by InSAR time-series analysis and the relationship with processing parameters. Totally, this section shows the theoretical performance of InSAR time-series analysis, and estimated accuracy of displacements.

PS candidates selection (Processing step (2))

PS is defined by its phase stability. However, the measuring of phase stability in every pixel is not computationally efficient. Instead of phase stability, the stability of amplitude called the amplitude dispersion index is used.
To simulate the relationship between amplitude dispersion index and phase dispersion, I calculated the same condition as is in Ferretti et al. (2001). The complex signal is composed of a complex reflectivity and a circular gaussian noise characterized by the same noise magnitude for both real and imaginary components. For a complex reflectivity, phase is assumed to be zero and fixed to be one. In order to visualize the relationship under several SNR condition, noise standard deviation was gradually incremented from 0.05 to 0.8. Phase standard deviation and the amplitude dispersion index with its deviation were obtained from 5000 calculations of the simulation.

I calculated the simulation with the different number of SAR images. The amplitude dispersion and phase dispersion show almost same trend when the noise standard deviation of less than 0.3 (Figure 2.12). On the other hand, the standard deviation of noise increases with decreasing the number of SAR images (Figure 2.12). Thus, the larger the number of interferograms, the more accurate the index is.

**The summary of this simulation**
- The amplitude dispersion index is correlated with phase dispersion if noise magnitude is less than about 0.35.
- If the number of SAR data is smaller, the variance of amplitude dispersion increases.

**SHP selection in space adaptive filtering (Processing step (4))**

Amplitude time-series information becomes a good proxy for examining the noise magnitude of a pixel, Statistically homogeneous pixels (SHP) selection also use a statistical test of amplitude stack to classify neighboring pixels. The rigidity of classification would differ depending on the properties of statistical tests.

One way to classify the amplitude time-series statistics is to use a nonparametric test. Kolmogorov-Smirnov (KS) test, a nonparametric test of continuous and one-dimensional probability distribution,
is often used for the purpose. The null hypothesis of the statistical test is that the two samples are drawn from the same distribution.

The amplitude statistics is often described in Rice distribution (e.g., Ferretti et al., 2001). In low SNR, the distribution approaches Rayleigh distribution, while becomes Gauss-like distribution in high SNR. Thus, another way is the use of a parametric test. The method we used in the dissertation is the combination of the t-test and F-test, focusing on high SNR pixels whose amplitudes are drawn from Gauss distribution. The t-test has the null hypothesis that two samples drawn from Gauss distribution have the same mean value, while the F-test has the null hypothesis that two samples drawn from Gauss distribution have the same variance.

I examined the rigidity of these statistical tests with simulation data. Master amplitude time-series drawn from Gauss distribution are fixed to have the mean of 0.5 and the standard deviation of 0.15. Slave amplitude time-series data drawn from Gauss distribution have the mean value from -0.2 to +0.2, and the standard deviation from -0.1 to +0.3 with respect to the master data. The number of SAR image is assumed to be 15. In each mean and standard deviation value, we calculated the classification 50 times and counted the percentage that the null hypothesis was not rejected. The results of the simulation show that the combination of F-test and t-test provide more rigid classification compared with KS-test, if we can assume the amplitude values have Gauss distribution (Figure 2.13).

The summary of the simulation
- The rigidity of pixel noise classification depends on the properties of statistical tests.
- If we can assume Gaussian distribution of amplitude statistics, the parametric test has more rigid classification than the non-parametric test, as is expected.
Figure 2.14: The shape of objective function. x and y axis are displacement rate and DEM error (scatterer height). (a) without noise (b) with phase noise that has the standard deviation of 0.7 rad (c) with phase noise that has the standard deviation of 1 rad.

**PS selection (Processing step (5))**

PS selection is a processing to identify pixels with a small amount of decorrelation-induced noise. The processing step is also known as optimization problem to estimate deterministic phase components from observed phase. For understanding properties of the problem, I assumed an idealistic, but still realistic condition: interferometric phases consist of phases due to surface displacement with constant velocity, scatterer height and decorrelation noise drawn from Gauss distribution. Then, I estimated surface displacement velocity and scatterer height from simulated phase.

The problem to estimate surface displacement velocity and scatterer height is nonlinear problem, because phase is wrapped. To visualize the non-linearity of the problem, I mapped the shape of coherence as a function of estimated model parameters (Figure 2.14). Fixing displacement velocity and scatterer height, I mapped the figures by adding noise with standard deviation of 0, 0.7 and 1 radian. When we did not add the noise component, the maximum value of coherence is one, and the maximum coherence value decreases with increasing the amount noise. Moreover, Figure 2.14 c shows a local maximum in addition to the global maximum of coherence. Note that the noise component is a random value, thus the existence of the local maximum is also a probabilistic phenomenon.

Let me describe the accuracy of the estimated parameter. From the viewpoint of parameter estimation problem, the variance of estimated parameters increases with increasing noise magnitude [Aster et al., 2013]. If the estimation problem is linear, we can construct a linear relationship between the data noise variance and the variance of the estimated model parameters. On the other hand, when the estimation problem is nonlinear, it is generally difficult to describe the linear relationship between noise variance and the variance of estimated parameters analytically. However, as long as the noise amount is not too large and the nonlinearity of the problem is not too strong, we can approximately consider the linear relationship between noise and model parameters perturbation as described in
Figure 2.15: The variance of estimation of (a) displacement velocity (b) scatterer height in different noise level. Black line indicates theoretical value.

linear least square estimation problem [Aster et al., 2013].

The approximation allow us to evaluate estimation accuracy quantitatively. Assuming that $F(m)$ is model misfit vector as defined follows

$$F(m) = \left[ f_1(m) \quad \vdots \quad f_m(m) \right]$$

(2.76)

(2.77)

with

$$f_i(m) = s(G(m)_i - d_i) \quad i = 1, 2, \ldots, m$$

(2.78)

where $s(\cdot)$ is a function that describes the mis-fit between the data ($d$) and modeled data ($G(m)$).

If a small amount noise magnitude leads to small model perturbation in a nonlinear system, we can linearize the misfit vector ($F(m)$) as follows.

$$F(m^* + \Delta F) \approx F(m^*) + J(m^*)\Delta m$$

(2.79)

where $m^*$ is the optimal model parameters and $J(m)$ is the Jacobian

$$J(m) = \left[ \begin{array}{ccc} \frac{\partial f_1(m)}{\partial m_1} & \cdots & \frac{\partial f_m(m)}{\partial m_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m(m)}{\partial m_1} & \cdots & \frac{\partial f_m(m)}{\partial m_n} \end{array} \right]$$

(2.80)

Therefore, there is a linear relationship between changes in the misfit vector ($\Delta F$) and changes in model parameters ($\Delta m$).

$$\Delta F = F(m^* + \Delta m) - F(m^*) \approx J(m^*)\Delta m$$

(2.81)
In this case, \( J(m^*) \) in nonlinear estimation problem describe the linear relationship between data and model parameter. Therefore, we obtain the relationship between the covariance of model parameters and data as follows.

\[
\text{Cov}(m^*) \approx \text{Cov}(d) \left( J(m^*)^T J(m^*) \right)^{-1}
\]  

(2.82)

When we consider that modeled phases consist of linear displacement velocity and scatterer height, the relationships between noise variance and the variance of model parameters (\( v \) and \( \Delta h \)) are as follows.

\[
\sigma_v^2 \approx \frac{\sigma_{dcr}^2}{k^2 \sum_k (T_k - \bar{T})^2}
\]

(2.83)

\[
\sigma_{\Delta h}^2 \approx \frac{\sigma_{dcr}^2}{k_{\Delta h}^2 \sum_k (B_k - \bar{B}_\perp)^2}
\]

(2.84)

where \( \sigma_v^2 \) and \( \sigma_{\Delta h}^2 \) are the variance of the displacement velocity and the scatterer height. \( \sigma_{dcr}^2 \) is the phase noise variance due to decorrelation. \( k \) is the interferogram number, and \( \bar{T} \) and \( \bar{B}_\perp \) are the mean of temporal baseline and geometrical baseline. The coefficients \( k_v \) and \( k_{\Delta h} \) are

\[
k_v = \frac{4 \pi}{\lambda}
\]

(2.85)

\[
k_{\Delta h} = \frac{4 \pi}{\lambda R \sin \theta}
\]

(2.86)

For simulating this error propagation, I created 300 interferometric phases with constant displacement velocity and scatterer height. Simulated temporal baselines are uniformly distributed from -1500 days and 1500 days, and geometrical baselines are -1000 m and 1000 m. Then, I added Gaussian noise to the interferometric phases and estimated displacement velocity and scatterer height.
Variances of estimated parameters were obtained using 300 calculation of data having several noise
variance ranged from 0 to 1 radian with the increment of 0.1 radian. As a result of estimation, there is
good agreement between simulated variance and theoretical consideration of the variance according
to Equation 2.83 and 2.84 (Figure 2.15). Therefore, it can be said that the nonlinearity of the problem
is not too strong and the variance of estimated model parameters can be approximately described by
the linear relationship with noise variance.

Then, I simulated the relationship between noise variance and coherence. With increasing noise
magnitude, the coherence decreases and the variance of coherence increases (Figure 2.16 a). I also
calculated the relationship with different number of interferograms (10, 30 and 100 interferograms).
The result shows that coherence is larger when there are smaller number of interferograms even if
the amount of noise is same (Figure 2.16 b). This simulation means that coherence is biased and
overestimated, when the number of interferograms is not enough large.

The summary of this simulation
- PS selection is nonlinear parameter estimation problem.
- The nonlinearity of the problem increases with increasing phase noise magnitude.
- If phase noise magnitude is small, we can assume linear relationship between the variance of esti-
mated parameters and phase noise.
- Coherence is a probabilistic value, because coherence is obtained from random phase. The variance
increases with increasing phase noise magnitude.
- The quantitative relationship between phase noise standard deviation and coherence is a function
of the number of interferograms. In other word, the simulation describes the bias of coherence as is
observed in InSAR analysis when the number of interferograms is small.

Phase unwrapping - Minimum Lp-norm algorithm (Processing step (6))

The basic concept
The minimum Lp-norm algorithm considers phase unwrapping as a optimization problem. The solu-
tion of this algorithm minimizes the following objective function $J$:

$$ J = \sum |(\psi_i - \psi_j) - (\phi_i - \phi_j)|^p $$  \hspace{1cm} (2.87)

Thus, the condition that minimize the function can be denoted as:

$$ \delta J = p \sum ((\psi_i - \psi_j) - (\phi_i - \phi_j))((\psi_i - \psi_j) - (\phi_i - \phi_j))^{p-2} $$  \hspace{1cm} (2.88)

Thus, we can obtain $\delta J = 0$, when

$$ ((\psi_i - \psi_j) - (\phi_i - \psi_j))((\psi_i - \psi_j) - (\phi_i - \phi_j))^{p-2} = 0 $$  \hspace{1cm} (2.89)
This equation can also be written as

\[ W_{i,j}(\psi_i - \psi_j) = W_{i,j}(\phi_i - \phi_j) \]  

(2.90)

where

\[ W_{i,j} = |(\psi_i - \psi_j) - (\phi_i - \phi_j)|^{p-2} \]  

(2.91)

If \( W_{i,j} \) is regarded as weights, Equation 2.90 can be considered as weighted least square problem. And, weighting factor can be made from data itself, that is data-dependent. Although Equation 2.90 seems to be linear, Equation 2.90 is non-linear problem, because the weights are functions of the input data and the solution. The minimum Lp-norm algorithm solve this problem using iterative weighted least square algorithm. Preconditioned conjugate gradient method (PCG) was used to solve the weighted least square problem, because the designed matrix is symmetric, positive definite and often large.

**Congruence operation**

Estimated unwrapped phases should differ from observed wrapped phase only by an integer multiplying \( 2\pi \). This constraint is often referred as congruence. In other words, re-wrapped phase of estimated unwrapped phase should be identical to observed wrapped phase. Although L0- or L1-norm solution typically enforces congruence, it is better to enforce the congruence to unwrapped phase derived from minimum Lp-norm algorithm, because this algorithm iteratively performs weighted least-square optimization and the convergence to global minimum are not guaranteed, but local minimum are. If the solution does not agree with congruence, the post-processing congruence algorithm is effective. The simple way of the congruence operation is to subtract the unwrapped phase from wrapped phase, then the wrap the result, and add the unwrapped phase.

\[ \tilde{\phi}_i = \phi_i + W[\psi_i - \phi_i] \]  

(2.92)
where $\tilde{\phi}_i$ is unwrapped phase after the congruence operation and $W[ \cdot ]$ is wrapped operator. However, Equation 2.92 does not work well under the noisy condition. For instance, if the difference between wrapped and unwrapped phase are around $\pi$, the direction of congruence (i.e., the difference is enforced to be 0 or $2\pi$) depends on the magnitude of random noise. It may induce the unintended phase discontinuities. To avoid the problem as much as possible, the congruence operation is described as the inverse problem to search a constant value, $q$ such that minimizing phase discontinuities of $\tilde{\phi}_i$ after the following congruence operation.

$$\tilde{\phi}_i = \phi_i + q + W[\psi_i - \phi_i - q]$$  \hspace{1cm} (2.93)

Here, I searched the constant parameter, $q$ by using grid search method.

**Simulation of the minimum Lp-norm algorithm**

A simple model to understand the concept of minimizing Lp-norm is line fitting problem. Suppose
the situation that eight data are on the line of $y = 2.23x + 4.5$ and two data are outliers (Figure 2.17), I compared the results of line fitting problem by minimizing L2-norm and L0-norm. As expected, if we minimize L2-norm and estimate two coefficients, the solution cannot agree with the synthetic line. However, by minimizing L0-norm, we can estimate synthetic line, because L0-norm counts the number of crosspoint between the data and estimated line. I can also estimate the synthetic line by minimizing L1-norm in this case.

Then, phase unwrapping of two dimensional grid data was carried out as the validation of the program. Although InSAR time-series analysis deals with two dimensional sparse data, I examined two dimensional grid data, which appears in InSAR analysis. However, the implications obtained from this simulation can apply to both InSAR and InSAR time-series analysis. Here, wrapped phase with shear was simulated (Figure 2.18 a). Horizontal shear line exist at the midway between the top and the bottom of the image. Above the shear line, phases represent planer surface decreasing in value from the bottom-left toward the top-right. Below the shear line, the phases linearly increase from bottom-left to top-right. This synthetic wrapped phase cannot be uniquely unwrapped because they contain two negative residues located along the perceived shear line (Figure 2.18 b). By applying least-square unwrapping algorithm, it is impossible to retrieve synthetic data due to residues (Figure 2.19). The least-square solution underestimated the magnitude of unwrapped phase gradients, because it had spatially continuous phase even at the residues where the phase difference is over $\pi$. These unwrapping errors propagate all of the analyzed area. On the other hand, the solution derived from minimum Lp-norm algorithm successfully estimated synthetic unwrapped phase (Figure 2.20a, b). Note that estimated unwrapped phase contains a constant value with respect to simulated unwrapped phase.
Figure 2.20: Final solution of Unwrapping by using minimum Lp-norm algorithm. (a) Estimated unwrapped phase. (b) Rewrapped phase of the estimated unwrapped phase. (c) Data-dependent weighting matrix in x direction (horizontal direction). (d) Data-dependent weighting matrix in y direction (vertical direction).
Figure 2.21: An example of the effect of congruence operation. (a) Simulated wrapped phase (random gaussian noise was added to wrapped phase with shear line). (b) Unwrapped phase using minimum Lp-norm algorithm without congruence operation. (c) Unwrapped phase using minimum Lp-norm algorithm and congruence operation. Congruence operation was performed after all iteration.
Figure 2.22: The statistical results of phase unwrapping of phase with a constant gradient. The mean and standard deviation were calculated from 30 times unwrapping results. (a) The number of residues. (b) The rate of pixels in which the difference between estimated and simulated unwrapping phase are over $2\pi$ ($2\pi$ or $4\pi$ or ...). The results were derived from the congruence operation after each iteration (green) and final solution (blue).

Figure 2.23: The statistical results of phase unwrapping of phase with a constant gradient. The mean and standard deviation were calculated from 30 times unwrapping results. (a) The number of residues. (b) The rate of pixels in which the difference between estimated and simulated unwrapping phase are over $2\pi$ ($2\pi$ or $4\pi$ or ...). The results were derived from the congruence operation after each iteration (green) and final solution (blue).

Although the example of phase applied above can be retrieved the synthetic phase, minimum Lp-norm algorithm sometimes shows the convergence to a local minimum, and phase gradients is not congruence (i.e., the difference of phase gradients between converged unwrapped phase and initial wrapped phase is not an integer multiplying $2\pi$). The congruence operation should be applied to avoid such a physically inappropriate situation. To examine the advantage of congruence operation, we applied minimum Lp norm algorithm to simulated wrapped phase with shear line and noise (Figure...
Without congruence operation, we could retrieve simulated wrapped phase after the 100 iterations of minimum Lp-norm algorithm (Figure 2.21 b). On the other hand, if we applied the congruence operation after the convergence of the iteration, we could retrieve simulated wrapped phase (Figure 2.21 c).

We now have the two choices about the timing to apply the congruence operation. One is the application after each iteration of the Lp-norm algorithm, another is the application after final convergence of the algorithm. In order to compare the two choices, I tested the accuracy of phase unwrapping using two examples having data with the different magnitude of noises. This simulation also has a purpose to examine the effect of phase noise to phase unwrapping error.

One example is the phase with a constant gradient in 32 × 32 grids. This synthetic wrapped phase originally does not have residues. I added the noise with the standard deviation of 0.1 to 0.8 rad to the wrapped phase. Figure 2.22 a showed the number of residues in each phase noise level, while Figure 2.22 showed the rate of the unwrapping error that is the estimated and simulated unwrapping phase difference over 2π. The congruence operation was applied after each iteration (green in Figure 2.22) and the convergence (blue in Figure 2.22). Comparison of Figure 2.22 a and b showed that the unwrapping error start occurring when the residues are found. This comparison suggests us the basic and important need to care of the magnitude of phase noise for proper unwrapping. And, according to Figure 2.22 b, it is better to apply the congruence operation after the convergence of Lp-norm algorithm.

Second example is the phase with shear in addition to a constant phase gradient in 32 × 32 grids. Even without noise, four residues exist along phase shear line. Similarly, I added the noise with the standard deviation from 0.1 to 0.8 rad to the wrapped phase and examined the number of residues and unwrapping error. In Figure 2.23 a, the number of residues also start increasing at the noise with the standard deviation of 0.6 rad. However, the unwrapping error start increasing before increasing residues (Figure 2.23 a and b). The earlier occurrence of unwrapping error might be due to the pre-existence of residues in wrapped phase without noise. Also in this case, it is better to apply the congruence operation after the convergence of Lp-norm algorithm. Although the demerit of this choice is higher computational time (at most, 10 times bigger), I decide to apply the congruence operation after the final convergence of minimum Lp-norm algorithm.

These two examples showed that minimum Lp-norm algorithms works well especially under less magnitude of noise. However, higher magnitude of noise is likely to yield a larger number of residues, that sometimes produces significant unwrapping error. Note that the quantitative relationship between the magnitude of noise and unwrapping error depends on the data. Thus, it is important to check carefully whether the unwrapping error exist or not, when we apply the program to real data.

**Summary of this simulation**
- The minimum Lp-norm algorithm is a phase unwrapping method that minimize the difference between wrapped and unwrapped phase gradient in terms of L0-norm.
Figure 2.24: Examples of scale-invariant signals. (a) the spectral exponent is $\frac{8}{3}$. (b) the spectral exponent is $\frac{2}{3}$.

Figure 2.25: (a) An example of differential interferogram around Tokyo. (b) Power spectrum of the differential interferogram. The spectral exponent in vertical corresponds to azimuth direction, and horizontal corresponds to range direction.

- By using L0-norm minimization, outliers of data can be neglected. In case of phase unwrapping, outliers correspond to residues. - Residues are induced by shears of phase or phase noise (decorrelation).
- As the number of residues increase, the percentage of unwrapping error increases.
- It is better to apply the congruence operation after the iteration of the algorithm.

**Removal of the tropospheric effects (Processing step (7) and (9))**

**Plane approximation**

When we analyze small area of interest, the tropospheric effects are sometimes approximated as a flat plane. It seems to be correct, when we analyze small area about $2km \times 2km$ square. However, the
tropospheric effects are sometimes statistically treated as scale-invariant (fractal) process [Hanssen, 2001]. The scale-invariant signal consists of a variety size of similar characteristics, so the signal is no longer a flat plane. There is questions about how a flat plane approximate tropospheric effects, and how small should it be when we use a flat plane to approximate the effects.

A scale-invariant process is characterized by its power spectrum called power law behavior.

\[
P_{\phi}(\eta) \propto \eta^{-\epsilon}
\]

where \(\eta\) is wavenumber, and \(\epsilon\) is the spectral exponent. The scale-invariant signal is also called as fractal signal or scale signal. The spectral exponent \(\epsilon\) describe how fast the signal varies. In two dimension, the spectral exponent describes how rough the surface is. Figure 2.24 is examples of two dimensional scale-invariant signal. When the spectral exponent become small, the surface become rough, and it becomes purely Gaussian signal when \(\epsilon\) is zero.

In order to create statistical data, I first estimate properties of tropospheric effects from statistical view point. I created 20 differential interferograms with ALOS/PALSAR data acquired over Tokyo, Japan. Since there is no surface displacement during the SAR acquisitions, the main component of phase in differential interferograms is due to tropospheric effects. I assume that the tropospheric phase is the scale-invariant signal that obeys the power law (Equation 2.94). As described in Equation 2.94, the power law has two statistical parameters: the variance of phase amplitude and the spectral exponent. From the unwrapped differential interferograms, I estimate two parameters in a least square sense. Since the spectral exponent may have orientation dependence, I estimated the parameter in both range and azimuth direction.
Figure 2.27: Residual phase after subtracting a bilinear flat plane as a function of interferogram size.

Table 2.1: The statistical parameter of tropospheric phases as the scale invariant signal.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon ) (horizontal)</td>
<td>2.84</td>
<td>0.20</td>
</tr>
<tr>
<td>( \epsilon ) (vertical)</td>
<td>2.41</td>
<td>0.079</td>
</tr>
<tr>
<td>The variance of amplitude [rad]</td>
<td>0.31</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Figure 2.25 is an example of unwrapped phase and its power spectrum. Phase with the maximum amplitude of about 3 radian are visible (Figure 2.25 a). Power spectrum of the differential interferogram shows decrease in power with increasing wavenumber, which is typical characteristics of the power law (Figure 2.25 b). The two exponent spectrum estimated in vertical (azimuth) and horizontal (range) direction are estimated from power spectrum over wave number of 1 cycle/km, because high frequency signal is possibly associated with decorrelation. Then, I calculated the average and standard deviation of estimated parameters form 20 differential interferograms (Table 2.1).

As a result of estimation, the mean and the standard deviation of the spectral exponent in both range and azimuth direction was almost identical. The variance of amplitude had the average of 0.31 radian, which corresponds to 0.58 cm approximately. Without any mitigation of the tropospheric effects, the residual phase is directly the error of displacement estimation.

Then, I examined the flat plane approximation of tropospheric phase. A variety size of synthetic differential interferograms are created based on the statistical parameter obtained in Table 2.1. Figure
2.26 is examples of tropospheric phase in areas of 10 km square and 3 km square. From the figure, tropospheric phase is visually similar as a flat plane as the size of interferograms is small. A bilinear plane was fitted in a least square sense, and calculated the mean and standard deviation of the absolute value of residual phase. If the residual phase is small, the displacement estimates would be accurate. Figure 2.27 shows the residual phase as a function of the size of interferograms. As expected, the amount of residual phase increases with increasing the size of interferograms. For instance, if one would like to estimate the residual tropospheric phase less than 0.2 radian using the method, the size of interferograms should be less than about 10 km square area.

**The summary of this simulation**

- Phase due to tropospheric effects can be approximated as a flat plane, if the size of interferograms is smaller.
- The amount of the residual tropospheric phases becomes smaller as the size of interferograms is smaller. The threshold of the size can be determined depending on how accurate surface displacements are estimated.

**Temporal filtering using damping of time-series displacement**

Temporal filtering is imposed when time-series displacement is estimated from small baseline datasets selection (Equation 2.74). The temporal filtering can reduce tropospheric effects, because the effects appear as temporally high frequency signal. The magnitude of filtering is controlled by the damping coefficient, which is determined objectively by ABIC minimization (Equation 2.74 and 2.75). Here, I simulated and quantified how the processing can mitigate tropospheric effects.

To create simulation data for small baseline datasets selection, baseline distribution of synthetic data is needed. From ALOS/PALSAR data, I estimate that the standard deviation of geometrical baseline is about 1667 m. Then, geometrical baseline are assumed to obey Gaussian distribution with the standard deviation of 1667 m. The distribution of temporal baseline is arbitrary, because I did not impose small baseline criterion on the temporal direction.

Interferograms are created assuming that interferometric phase consists only of tropospheric effects, obeying the power law behavior that is described in Table 2.1. Area of interest is 30 km square. Tropospheric effects are simulated on SLCs, not interferograms. And interferograms are created with datasets selected with geometrical baseline is less than 1000 m. The damping coefficients are created as the average value of every pixel’s coefficient determined with ABIC minimization.

The performance was examined with the amount of (absolute value of) interferometric phase after the temporal filtering. Since simulated interferometric phases are composed of the tropospheric effects, the amount of phase would get close to zero when tropospheric effects are correctly mitigated. Table 2.2 shows the simulation result as a function of the number of SAR data. By imposing the temporal filtering, the amount of tropospheric phase is certainly decreased. More concretely, the
Table 2.2: Simulation result of the temporal damping. The first and second lines in each number of SAR data indicate the amount of residual phase after and before applying the temporal filtering.

<table>
<thead>
<tr>
<th>The number of SAR data</th>
<th>Mean [rad] After (Before)</th>
<th>STD [rad] After (Before)</th>
<th>Mean of the number of interferograms</th>
<th>STD of the number of interferograms</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.37 (0.63)</td>
<td>0.076 (0.053)</td>
<td>53.5</td>
<td>14.2</td>
</tr>
<tr>
<td>50</td>
<td>0.35 (0.62)</td>
<td>0.054 (0.037)</td>
<td>123.9</td>
<td>18.1</td>
</tr>
<tr>
<td>100</td>
<td>0.34 (0.63)</td>
<td>0.058 (0.032)</td>
<td>143.3</td>
<td>20.3</td>
</tr>
</tbody>
</table>

The mean of tropospheric phase is decreased by 58 %, 56 %, 55 % when the number of SAR data is 20, 50, and 100, respectively. The significant tendency of the improvement in performance depending on the number of data was not recognized, possibly because the temporal filtering is applied the gradient of the temporal change in phase, which is calculated with the previous and next SAR acquisitions.

**The summary of this simulation**

- The temporal filtering can decrease the amount of tropospheric effects by about 55-60 % (in the simulated data condition)
- The performance of the temporal filtering may be independent of the number of interferograms.

### 2.5 Summaries

In this chapter, I have summarized SAR imaging and InSAR analysis and discussed InSAR time-series analysis in terms of theoretical background and numerical simulation. Resulting data obtained from SAR imaging is the two-dimensional projection of three-dimensional reflectivity composing amplitude and phase information. InSAR analysis is the method to estimate range changes using phase difference of two SAR observations under coherent scatterers’ reflection. Range changes are caused not only by surface displacements but also by topography, orbital trajectories and atmospheric effects. These phase contributions are estimated and subtracted in InSAR analysis, however it is often difficult to remove these contributions with enough accuracy. Moreover, assuming distributed scatterers, geometrical and temporal decorrelation make interferometric phase random values.

InSAR time-series analysis provides effective solutions to overcome the limitations of InSAR analysis, and offers time-series displacement with higher accuracy. Instead of distributed scatterers, persistent scatterers are assumed in InSAR time-series analysis. Therefore, displacement trend can be estimated using time-series coherent phase information of persistent scatterers. InSAR time-series analysis used in this study is designed based on persistent scatterer interferometry, one of InSAR time-series analysis that can analyze both synoptic and detailed (high-spatial resolution) surface dis-
placement. Small baseline criteria is used for selecting interferometric datasets considering the use of ALOS/PALSAR images. PS candidates selection using amplitude statistics is one of the processing steps for determining the accuracy to select promising PS candidates. Through the simulation, I showed the number of SAR data is an important parameter that relates the accuracy of the candidates selection. Differential interferograms are created without multi-looking operation for retaining high spatial resolution of SAR data. Space adaptive filtering is optionally introduced for incorporating distributed scatterers in addition to persistent scatterers and increasing the number of coherent pixels to map surface displacements. The rigidity of statistically homogeneous pixels depending on statistical tests was shown using the simulation. PS selection is one of the important processing steps in the InSAR time-series analysis for determining the accuracy of estimated surface displacement. I simulated the quantitative relationship between the variance of the estimated parameters (displacement velocity and scatterer height) and the variance of noise due to decorrelation. The InSAR time-series analysis uses coherence defined in the temporal domain for measure decorrelation magnitude of interferometric phase of each pixel. The dependence of the coherence on the number of interferograms were shown in the simulation. Although simulation data is currently optimistic condition, I showed that InSAR time-series analysis have the ability to estimate surface displacement velocity with millimeter or sub-millimeter accuracy. Phase unwrapping is also important step to determine the accuracy of estimated displacements. Using minimum Lp-norm algorithm, phase unwrapping error induced by interferometric phase noise (decorrelation) was shown. The mitigation of tropospheric effects is also important processing step for determining estimated accuracy. With the simulation, I examined two mitigation methods: plane fitting and temporal filtering. Both methods can mitigate tropospheric effects, however, smaller size of analyzed area is important when the plane fitting method is used.

This chapter described how InSAR time-series analysis estimates surface displacements. The following chapters are studies for improving the monitoring capability of InSAR time-series analysis in terms of interpretation and estimation accuracy.
Chapter 3

Natural surface rebound in the Bangkok plain

3.1 Introduction

Land subsidence due to the compaction of overdrafted aquifer systems have occurred beneath many cities worldwide and have caused severe damage to buildings and infrastructures [Waltham, 2002; Galloway and Burbey, 2011]. In recent decades, several cities have taken measures for groundwater extraction to mitigate severe ground subsidence caused by groundwater extraction by, for instance, implementing artificial water injection schemes or regulating of groundwater pumping, and surface rebound in these regions has been reported [Amelung et al., 1999; Lu and Danskin, 2001; Mouèlic et al., 2002; Chen et al., 2007]. However, especially in the case of natural surface rebound, its spatial distribution and temporal evolution are still not clear.

The mechanism of surface rebound can be generally described as follows: As a result of decreased groundwater pumping or the water injection into a confined aquifer, the groundwater level recovers. This increase in the hydraulic head leads to an increase of pore pressure in the aquifer, and the expansion of its granular skeleton. However, because of compaction of the aquitard, ground subsidence continues for a while, although at a gradually decreasing rate. When the magnitude of the expansion exceeds that of the compaction, elastic rebound of the surface can be observed.

Synthetic aperture radar (SAR) interferometry (InSAR) and InSAR time-series analysis have been successfully used to monitor subsidence and uplift events, and to unravel the characteristics of the aquifers. Previous studies found the presence of unrecognized fault extension or other geological boundary from the spatial pattern of estimated surface displacement, or the elastic property of the aquifer [Lu and Danskin, 2001; Hoffmann et al., 2001; Bell et al., 2008]. In this study, using InSAR time-series analysis and applying a temporal model on the displacement time-series, I estimated the lateral connectivity of aquifer as well as the elastic properties of the surrounding media.

I focused on Bangkok, the capital city of Thailand, which is situated on flat low land (Figure
Figure 3.1: (a) Topography around Bangkok plane. Black rectangular stands for analyzed area of ALOS/PALSAR images. Yellow dots denote the location of the monitoring wells to measure the vertical movement and groundwater level. (b) Time-series groundwater level at the second shallowest aquifer (Phra Pradang aquifer) measured at six monitoring wells in Bangkok [Giao et al., 2012]. The locations of monitoring wells correspond to the number in a of this Figure.
Figure 3.2: Distribution of the groundwater level change in the third shallowest aquifer (Nakorn Luang aquifer) for the period from 1997 to 2009. The shaded area shows the groundwater level recovery zone (the positive contour value means the drawdown, while the negative value means the recovery comparing to the initial level of 1980) after Giao (2010). White dashed rectangular indicates covered area of SAR images, and yellow circle indicates a reference point used in this study.
3.1 a) and underlain by a multi-aquifer system consisting of several confined aquifers and aquitards [Sinsakul, 2000]. In this region, previous studies have reported ground subsidence due to excessive groundwater pumping [Phien-wej et al., 2006; Taniguchi et al., 2009] and subsequent groundwater level recovery after 1997 [Giao, 2010; Giao et al., 2012] (Figure 3.1 b, 3.2) that resulted from several measures to regulate and reduce groundwater use by the government [Taniguchi, 2011]. I first obtained the spatio-temporal pattern of the surface rebound in this area by using InSAR time-series analysis of ALOS/PALSAR images acquired from November 2007 to December 2010. Then, I interpreted this estimated pattern of natural surface rebound using a temporal model.

3.2 Methodology

In order to measure the recent surface displacement in Bangkok, I performed the interferometric point target analysis (IPTA) [Werner et al., 2003; Wegmüller et al., 2010], one of the InSAR time-series methods, on 11 PALSAR images acquired from November 2007 to December 2010 from an ascending orbit (Figure 3.3).

Depending on the electro-magnetic wave scattering characteristics of targets on the Earth’s surface, phases in SAR images are degraded by noise, which caused decorrelation in interferograms. IPTA can mitigate decorrelation-induced phase inaccuracy by finding pixels with coherent targets over time, called as persistent scatterers (PS). Physically, they correspond to man-made buildings or bared surface, therefore, displacement analysis at the target of urban area is one of the suitable targets.
for InSAR time-series analysis using IPTA.

First, I selected 45 interferometric pairs with a perpendicular baseline of less than 1000 m (Figure 3.3). Next, I selected the persistent scatterer (PS) candidates using the criterion of an amplitude dispersion index [Ferretti et al., 2001] of less than 0.5, and created differential interferograms at the PS candidate pixels. Then, I removed the residual orbital fringes. As an external digital elevation model (DEM), I used the SRTM3 DEM with an interval of 3 arc-seconds [Farr and Kobrick, 2000]. After the residual orbital fringes were removed, the DEM errors and the displacement rates were iteratively estimated in a least squares sense. The pixels having the standard deviation of more than 0.4 rad were discarded. Then, phase unwrapping was carefully performed with the minimum cost flow algorithm [Costantini and Rosen, 1999; Werner et al., 2002]. I set a reference point at the northwest of analyzed area (A yellow circle in Figure 3.2) where the change in groundwater level from 1997 to 2009 is minimal. Finally, displacement time-series were estimated by inverting each unwrapped differential interferogram [Schmidt and Bürgmann, 2003]. To mitigate atmospheric effects, I smoothed the temporal changes in phase under the assumption that the atmospheric effect would have a high temporal frequency:

$$
\arg \min_{} \left\| \begin{bmatrix} G \\ \kappa^2 \frac{d}{dt} \end{bmatrix} \phi_{inc} - \begin{bmatrix} \phi_{obcr} \\ 0 \end{bmatrix} \right\|
$$

(3.1)

where $G$ represents the designed matrix constructed from the temporal distribution of SAR acquisitions, and $\phi_{inc}$ are the vector composed of the incremental displacements between SAR acquisitions and that composed of the displacements measured by the differential interferogram, and the coefficient $\kappa^2$ is a weighting coefficient which describes the degree of temporal smoothing. This weighting coefficient $\kappa^2$ was determined by Akaike’s bayesian information criterion (ABIC) [Akaike, 1980], and I used 0.47 for the coefficient, which is the average value for randomly selected 10% of all the pixels. In estimating the coefficient using ABIC, I assumed that the covariance of the observed data had a Gauss distribution. Because this is an over-determined problem, I used the least-squares method to obtain a solution. For all obtained displacement maps, the radar incidence angle of PALSAR was approximately 39 degree and the horizontal component of the line of sight vector was approximately 80 degree eastward from north. Therefore, the measurements were sensitive to surface displacements mostly in the vertical and partly in E-W directions. Moreover, considering that the soft clay is covered with the surface in the analyzed area [Sinsakul, 2000], and the poisson ratio of unsaturated clay is generally 0.1-0.3 [Bowles, 1996], 84-94% of observed displacements would be in vertical direction. Thus, I finally converted the estimated displacements in the line-of-sight direction to vertical direction based on the simple calculation that the displacement in the horizontal direction is smaller.
3.3 Spatio-temporal pattern of the surface rebound

Figure 3.4 shows the vertical surface displacement between the periods of 45 interferometric pairs. Every pair shows the similar displacement pattern: ground uplift around the center of Bangkok, and the spatial pattern is more noticeable with increasing the time span of interferometric pairs. Therefore, the estimated displacement time-series, obtained according to Equation 4.1, generally shows ground surface uplift around the center of Bangkok during 3 years of ALOS observation (magenta in Figure 3.4), in addition to ground subsidence at the northeast and southeast margin of the analyzed area (yellow in Figure 3.4). The western and eastern boundaries of the uplift area approximately correspond to the west bank of the Chao Phraya River and the international airport in Bangkok, respectively. The northern and southern boundaries correspond roughly to the extent of Bangkok city in each direction. The observed uplift areas are approximately covered with areas over 250 km². The northwest part of the uplift area (A in Figure 3.6) has rebounded a total of about 2.0-3.0 cm during 3 years, whereas the southeast (C in Figure 3.6) and central (B in Figure 3.6) parts rebounded 3.0 and 1.0 cm, respectively. On the other hand, ground subsidence of 1 cm during the period has been detected at the northeast of analyzed area where an industrial estate is located (D in Figure 3.6). Aobpaet et al. (2013) conducted a InSAR time-series analysis on RADARSAT-1 data acquired between 2005 and 2010 and found that most of the area of Bangkok still subsided during the corresponding period. However, during this period, the groundwater levels in three main pumped aquifers (i.e., the second, the third, and the forth shallowest aquifers) were found recovered [Giao, 2010], which can justify well the surface rebound found by InSAR time-series analysis results in this study.

I used data acquired by SAR system with smaller central frequency (L-band) compared with other SAR system onboard on satellite. Phase in data acquired by SAR system with smaller central frequency is more sensitive to decorrelation-induced phase noise compared with that obtained by SAR system with larger central frequency (X- and C-band), though it is known that interferometric pairs provided by L-band SAR system tend to have larger coherent pixels especially at non-urban area. To check the reliability of the estimated displacement time-series, I evaluated the standard deviations of the residual phase of displacement time-series in each PS, after Equation 4.1. The standard deviations were about 0.5 cm in most areas, but they were about 1.5 cm around the southeast part of the uplift area (around C in Figure 3.6). Because the magnitudes of the observed surface displacement were large compared with the standard deviation, the differential interferograms were in agreement. Thus, I consider these results to be reliable.

I validated the estimated displacement time-series by comparing our results with observations of vertical displacement obtained at surface settlement measurement points in monitoring wells in Bangkok (Figure 3.1 a) [DGR, 2012]. During October 2010 and August 2011, displacement in the monitoring wells along the Chao Phraya River indicated an uplift rate of 0.2 cm/year (Figure 3.6, dashed line in A) [DGR, 2012]. Thus, despite the difference in the observation period between the SAR data and the in situ measurements, these results are consistent.
3.4 Secular and seasonal changes associated with the characteristics of the aquifer system

To understand the source of the observed displacements during the ALOS observation period, I examined the temporal displacement by considering seasonal and long-term factors, and determined the combination of the factors statistically (i.e., I determined whether temporal model consists of (a) seasonal factor, (b) long-term factor, or (c) both seasonal and long-term factors). I modeled the seasonal factor by a sinusoidal curve with a period of one year, and the long-term factor, which I attributed to the increase in the hydraulic head, as an exponential function of time:

\[
U_{se}(t) = a_{se} \left( \sin \left( \frac{2\pi t}{365} \right) - \sin(b_{se}) \right) \\
U_{lt}(t) = a_{lt} \left( \exp \left( b_{lt} t \right) - 1 \right)
\]  

where \( U_{se} \) and \( U_{lt} \) are seasonal and long-term displacement, \( a_{se} \) and \( a_{lt} \) are coefficients which describes the magnitude of each factor, and \( b_{se} \) is the time shift of the sinusoidal function. This time shift means the starting period of uplifting for seasonal displacement with respect to the first SAR image. \( b_{lt} \) is a coefficient controlling the decay rate. When \( b_{lt} \) is larger, the decay is faster, and when \( b_{lt} \) is smaller, the fitted curve is closer to a line. \( \sin(b_{se}) \) in the seasonal term and \( -1 \) in the long-term term are introduced to be \( U(0) = 0 \). This sinusoidal model for seasonal displacement has often been
applied to describe seasonal displacement (e.g., Bell et al. (2008)). For long-term displacement, the aquifer deformation can be described with a diffusive equation based on classical poro-elastic theory [Coussy, 1995] and the general solution can be described with an exponential term. This exponential term physically stands for the decay in pore water invasion due to increase in pore pressure. In other words, at the beginning phase of natural groundwater recovery, the difference of groundwater head between extraction area and non-extraction area would be considerable, and this difference would become smaller over time. In fact, it has been observed that the amount of natural surface rebound after subsidence due to preceding groundwater extraction decayed over time [Chen et al., 2007]. To determine the optimal combination of the temporal models, I used Akaike information criterion (AIC) [Akaike, 1974]. From a statistical point of view, the model having the minimum AIC is the optimum. After that, I estimated unknown parameters of the optimum model. To fit this function to the data, I performed a grid search for $b_{se}$ within the range of 0 to $2\pi$ rad at intervals of $\frac{\pi}{18}$ rad and for $d_{lt}$ within the range of 0 to 1 at intervals of $10^{-4}$. To determine the coefficients $a_{se}$ and $a_{lt}$, I applied the least-squares method during the grid search for $b_{se}$ and $b_{lt}$.

I mapped the spatial trends in the amplitude of the seasonal variation ($a_{se}$), the time shift ($b_{se}$) of seasonal uplift, the average rate of the long-term uplift, and the exponential decay constant ($b_{lt}$) of the long-term uplift (Figure 3.7 a-d, respectively). For $a_{se}$, $b_{lt}$, and the average rate of the non-seasonal uplift, I plotted the values in areas where $a_{lt}$ and $b_{lt}$ were negative (uplift area) and the standard deviation of the residual displacement was less than 1 cm, whereas for $b_{se}$, I plotted the values in areas where $a_{se}$ was over 0.5 cm. Long-term uplift occurred uniformly over the whole area at the rate
Figure 3.6: Examples of temporal evolution of surface displacement. The top figure is the annual average of surface displacement. Black squares and lines indicate the observation and the best-fit curve of temporal displacement model (Equation 3.2 and 3.3), respectively. The red dashed line at (a) indicate the ground-based observation by using surface settlement point.
Figure 3.7: The parameters of the best fit model for observed surface displacement. (a) The magnitude of seasonal displacement ($a_{se}$ in Equation 3.2) [cm], (b) The time shift of sinusoidal function with reference to November 2007 ($b_{se}$ in Equation 3.2) [rad], (c) The annual average of secular uplift [cm/year], (d) The decay coefficient of exponential function for unseasonal uplifting ($b_{ui}$ in Equation 3.3)
Figure 3.8: The amount of monthly precipitation in Bangkok city (blue bar). Black dots and line indicate an example of displacement time-series inferred from InSAR time-series analysis and a best-fit temporal model (C in Figure 3.6). Gray area indicates the positive gradient of sinusoidal curve representing seasonal uplift season.

of 0.5-1.5 cm/year (Figure 3.7 c). On the other hand, the values of $a_{se}$, $b_{se}$ and $b_{lt}$ showed considerable spatial variation (Figure 3.7 a,b,d).

Seasonal displacement (Figure 3.7 a) was not found in all areas showing long-term uplift (Figure 3.7 c), and the time shift of the seasonal displacement differed even in those areas where it occurred (Figure 3.7 b). The discrepancy in the areas of the seasonal and long-term uplifts can be attributed to spatial connections between aquifers. The multi-aquifer system beneath the Bangkok plain consists of several interbedded aquifers and aquitards (see section 3.1). Groundwater was overdrafted mainly from the second shallowest (Phra Pradang aquifer, 60-80 m depth) and the third shallowest aquifers (Nakorn Luang aquifer, 100-140 m depth). Seismic reflection studies in Bangkok have shown that the stratum of the second aquifer is horizontal and laterally continuous, whereas the uppermost aquifer (Bangkok aquifer, 16-30 m depth) is not continuous laterally [Whiteley et al., 1998]. Therefore, the spatially uniform long-term displacement can be explained mainly by the connectivity of the second and the third aquifer, whereas the spatial distribution of the observed seasonal displacement likely reflects horizontal connections in the uppermost aquifer. From the time shift of seasonal displacement, the largest seasonal uplift started occurring at the margins and river, and flow toward the surrounding areas (Figure 3.7 b). This observation implies that the major sources of groundwater in the Bangkok plain include percolation of runoff from the adjacent highlands and the Chao Playa River. However, I note the possibility that swelling and shrinkage of soft clay layer at the surface may also have induced the seasonal surface displacement. Compared with monthly precipitation, I can infer that
this seasonal uplift have occurred due to seasonal precipitation, because the gradient of seasonal
displacement become positive from the middle of rainy season (Figure 3.8). Additionally, since the
time shift in the southwestern part of the seasonal displacement area differed significantly from that
in the rest of the area, the uppermost aquifer in this area might not connect hydraulically with the
northwestern and southeastern parts of the uplift area (Figure 3.7 b).

The spatial pattern of the decay coefficient $b_n$ (Figure 3.7 d) can be interpreted to reflect the dis-
tribution of aquifer properties (compressibility and permeability) or the timing of the start of uplift.
I cannot rule out either of these two possibilities. First, spatial differences in the properties of an
aquifer may mean that its elastic properties differ depending on the depositional environment and the
magnitude of preceding subsidence. Most of the Bangkok plain sediments were deposited in a tidal
environment, but fluvial processes dominated along the Chao Phraya River [Sinsakul, 2000]. And,
higher values of $b_n$ may correspond to the floodplain shifted in response to sea level fluctuations. Ad-
ditionally, about 50 years before the present, groundwater extraction began and resulted in the largest
cumulative subsidence near the center of Bangkok city, that is, in the northwest part of the observed
uplift area, where the value of $b_n$ is comparatively large [Whiteley et al., 1998; Phien-wej et al., 2006;
Taniguchi et al., 2011]. Thus, the greater total subsidence in this area would correspond to longer and
greater compression of the aquitard, which could account for the difference in the decay coefficient $b_n$. Regarding the second possibility, if the assumption that the surface rebound can be described by
an exponential function is valid, then fitting the function to the beginning phase of the surface rebound
might result in a larger value of $b_n$, whereas fitting it after the uplift is largely completed would result
in smaller $b_n$, even if the elastic properties of the aquifer are uniformly distributed.

3.5 Conclusions

To map surface displacement around Bangkok, Thailand, I performed InSAR time-series analysis
of ALOS/PALSAR images acquired from November 2007 to December 2010. Our results showed
that the ground around Bangkok generally uplifted from 0.5 to 3.0 cm during the observation period.
Because the groundwater level had been recovering for about 10 years in 2009, the observed uplift
was likely caused by natural groundwater recovery in the previously subsided area. Secular uplift
occurred in most of the area around Bangkok, but seasonal displacement controlled by rainfall ex-
hibited a different spatial pattern and occurred only in parts of the uplift area. This result reflects
the significant spatial variation in the hydraulic connectivity of individual interbedded aquifer layers
under the Bangkok plain. The magnitude of the decay coefficient $b_n$ of the exponential function de-
scribing the secular uplift might reflect floodplain changes or the subsidence history. Our results show
that temporal model interpretation is useful for investigating the structure of an aquifer and its elastic
properties, information that cannot be inferred from a simple examination of the estimated surface
displacement.
Chapter 4

Heterogeneous surface displacement pattern at the Hatchobaru geothermal field

4.1 Introduction

Surface subsidence at some geothermal fields, which is known to be induced by pressure changes in the geothermal reservoir and surrounding media, has been measured by geodetic observations such as leveling data and the global positioning system (GPS) [Mossop and Segall, 1997; Allis, 2000; Allis et al., 2009]. Recently, InSAR time-series analysis has made possible estimates of displacement with millimeter to sub-millimeter accuracy and high spatial density [Ferretti et al., 2000; Ferretti et al., 2001; Werner et al., 2003; Hooper et al., 2004; Hooper et al., 2007]. However, this technique has seen limited use for geothermal fields, because they present few targets that reflect coherent backscattered waves and small displacements. One study used InSAR time-series analysis to map subsidence along with plate boundary deformation around geothermal fields on the Reykjanes Peninsula, Iceland [Keiding et al., 2010], and another used it to map and model past subsidence and subsequent uplift due to fluid production and injection in the Geysers geothermal field, California [Vasco et al., 2013].

Surface displacement patterns at a geothermal field may be expected to be heterogeneous owing to the anisotropy of reservoir permeability and the role of faults as pathways or partitions of fluid migration. Thus, the spatially dense measurements made possible by InSAR time-series analysis offer advantages for understanding the subsurface structures and reservoir characteristics controlling fluid migration. However, efforts to directly correlate heterogeneous fluid migrations with the heterogeneity of surface displacement distribution and its temporal evolution around geothermal fields using this technique have been rare.

InSAR time-series analysis uses high-quality phase information about coherent targets, referred to as persistent scatterers (PSs), to estimate surface displacement. A high density of PSs is required to delineate the spatial patterns of surface displacement. However, PSs typically correspond to artificial structures, which are rare in geothermal fields. Most surface features there are distributed scatterers
Recently, algorithms that combine PSs and DSs have been proposed [Hooper, 2008; Ferretti et al., 2011] and shown to increase the density of sufficiently accurate pixels (measureable pixels) for accurate estimates of surface displacement. One of these is space adaptive filtering, in which spatial filtering is applied to pixels with statistically similar features [Ferretti et al., 2011; Ishitsuka et al., 2014b]. To limit filtering to DSs and preserve the high-quality data of PSs, the filtering method classifies pixels on the basis of noise levels and imposes filtering upon pixels that are classified in the same category.

In this study, I mapped more than 3 years of surface displacements in the Hatchobaru geothermal field using InSAR time-series analysis processed with InSAR time-series analysis and the space adaptive filtering. The Hatchobaru geothermal field is located west of Kuju volcano on Kyushu Island, southwest Japan (Figure 4.1), and lies in the Beppu-Shimabara volcanic graben comprising andesitic lava domes and stratovolcanoes. The Hatchobaru No. 1 unit has generated 55 MW of power since 1977, and the Hatchobaru No. 2 unit has generated 55MW of power since 1990. The Ohtake geothermal power station, located about 2 km north from the Hatchobaru geothermal field has gen-
erated 12.5 MW of power since 1967. In the Hatchobaru geothermal field, previous studies reported decreases in gravity as great as 200 μgal during 1990-2002 at the location where the reservoir pressure had decreased [Nishijima et al., 2005; Saito et al., 2006]. And, ground subsidence of about 20 mm/year by GPS during November 1998 and November 1999 has been reported around production area [Motoyama et al., 1999]. However, the detail spatial pattern of surface displacement has not been clear yet. I first estimate surface subsidence pattern using the InSAR time-series processing, and then illustrate heterogeneity of the surface displacement pattern.

4.2 Methodology

The synthetic aperture radar (SAR) technique is based on backscatter of microwave signals sent from a satellite to the Earth’s surface. Displacement along the line of sight is estimated from the phase changes between two overlapping images from similar viewing geometries after subtracting terrain and orbital contributions by using the differential SAR interferometry (DInSAR) technique [Bamler and Hartl, 1998]. This technique has been shown to be effective for surface displacement monitoring at an oil field [Khakim et al., 2013] and a geothermal field [Massonnet et al., 1997]. The resulting differential interferograms reveal phase changes caused not only by surface displacements but also by digital elevation model (DEM) errors, atmospheric effects, and decorrelation-induced phase noise.

PSs are defined at points where the phase is stable and minimally affected by decorrelation. The InSAR time-series technique extracts high-quality phase information using PSs from differential interferograms created from a large number of SAR images. Moreover, because the phases in PSs are stable, the other nuisance terms, including DEM error and atmospheric effects, can be estimated and subtracted to estimate the surface displacement with greater accuracy.

Physically, a PS can be interpreted as a scattering point, such as a manmade structure that produces stable microwave reflections. For that reason, urban areas are favorable targets for analysis. Recently, several algorithms have been proposed for application to areas with scatterers of low reflectivity, of DSs [Hooper, 2008; Ferretti et al., 2011]. To make use of both PSs and DSs, I need to filter DSs to increase the signal-to-noise ratio (SNR) while preserving PSs by applying less filtering to them. Therefore, I adopted the space adaptive multi-looking filter, which is applied to pixels sharing statistically similar features [Ferretti et al., 2011; Goel and Adam, 2012; Ishitsuka et al., 2014b] (Figure 4.2). This method enables DSs with similar SNR to be filtered, while preserving the high-quality phase information of PSs. Space adaptive filtering enables us to map surface displacement around geothermal fields, where bare surfaces and vegetated terrain present many DSs and few PSs [Touzi, 2002; Lee et al., 2003].
4.2.1 Application of a space adaptive multi-looking filter to synthetic differential interferograms

I confirmed the effectiveness of the space adaptive multi-looking filter using synthetic differential interferograms. First, I created wrapped differential interferograms with dimensions of 100 × 100 pixels, using (i) phase information arising from surface displacement and (ii) decorrelation-induced Gaussian noise (Figure 4.3 a), by assuming that DEM errors and atmospheric effects are properly removed during InSAR time-series analysis. For the surface displacement pattern, I assumed sharp displacement boundaries (Figure 4.3 b). I added phase noise with a Gaussian distribution to the complex signal. The standard deviation of this phase noise was classified into five categories, which were randomly distributed in the synthetic differential interferograms as an analog of differences in surface cover (Figure 4.3 c).

For comparison with the space adaptive multi-looking filter, I applied standard multi-looking using a window size of 7 × 7 pixels. This method detected phase changes at the center of the analyzed area where surface displacement existed (Figure 4.3 d). However, the boundaries of the surface displacement became obscured owing to averaging of phase information. Moreover, artifacts appeared outside the area of displacement, possibly because noisy pixels were filtered (Figure 4.3 d). When phase information was extracted only from pixels with low noise magnitudes (Figure 4.3 e), the displacement trend retained its sharp boundaries but phases were degraded by the high noise levels. At this point, I applied multi-looking to pixels with similar SNR within 7 × 7 pixel windows (space adaptive multi-looking). To avoid bias arising from the pixel classification, I used the pixel classifications determined when I simulated the synthetic differential interferograms (Figure 4.3 c). The resulting displacement pattern (Figure 4.3 f) was spatially more continuous compared with the pattern depicted...
Figure 4.3: A synthetic differential interferogram and filtering results. (a) Phase of synthetic differential interferograms composed of surface displacement and decorrelation-induced phase noise. (b) Phase due to synthetic surface displacement. (c) Pixel classification based on SNR of pixels as follows: 1, 0.01; 2, 0.05; 3, 0.1; 4, 0.5; 5, 1. (d) Phase after standard multi-looking filtering using a window size of $7 \times 7$. (e) Extracted phases in pixels of high SNR (0.5 or greater). (f) Phase after space adaptive multi-looking filtering using a window size of $7 \times 7$. Extracted phases are the same as in e of this Figure.

In Figure 4.3 e and had sharper boundaries compared with that in Figure 4.3 d.

In summary, space adaptive multi-looking retained fine spatial resolution by relying on pixels with less decorrelation-induced noise, whereas noisy pixels were filtered at the cost of resolution. Thus, space adaptive filtering increased the number of pixels available to measure surface displacement at a lower cost in spatial resolution. This advantage was effective in applications to a geothermal field, where the surface displacements may be heterogeneous.

4.2.2 Analysis of ALOS/PALSAR images

In this study, I processed 18 ALOS/PALSAR images acquired from July 2007 to December 2010 from an ascending orbit (Figure 4.4). The images had radar incidence angles of approximately 39° and a line-of-sight vector of approximately N80°E. Thus, a unit vector of data acquired from an ascending orbit was approximately $[EW, NS, ND] = [-0.62, -0.088, 0.78]$. The resolution of the images corresponds to a polygon of about $4.68 \times 3.18$ m in slant-range and azimuthal directions.

I chose possible coherent pixels using the amplitude dispersion index to reduce the number of pixels to be processed [Ferretti et al., 2001]. Then I selected 53 interferometric data pairs with the criterion that the perpendicular component of the geometric baseline is less than 800 m and created
Figure 4.4: Baseline conditions of ALOS/PALSAR images and interferometric pairs. Bperp is the perpendicular component of the geometric baseline. Squares represent the acquisition date and Bperp of SAR images used in this study, and lines connect the selected interferometric pairs, with Bperp less than 800 m.

53 differential interferograms (blue lines in Figure 4.4).

I applied space adaptive multi-looking at this point. Using a two-sample F test and a t test of the SAR amplitude time series, I first classified pixels around candidate coherent pixels within a 14 × 20 pixel window as to whether the amplitude of each surrounding pixel was statistically similar to that of the candidate pixel [Goel and Adam, 2012]. Next I applied multi-look processing to candidate pixels in differential interferograms classified in the same category (space adaptive multi-looking). Figure 4.5 shows the topography of the analyzed area and the number of statistically similar pixels in each possible coherent pixel. The number is larger (and spatial resolution was greater) on the right side of the image, an area of mountainous slopes, and smaller on the left side, to the west of this mountainous area. Because the radar illumination was from the west, microwave reflections from the western side of the mountains were more stable than those from the eastern side.

I selected coherent pixels that exceeded a temporal coherence threshold of 0.5 [Ferretti et al., 2001]. I then carried out a phase unwrapping procedure using a minimum-cost flow algorithm [Costantini and Rosen, 1999; Werner et al., 2002]. Finally, I estimated a surface displacement time series by inverting the unwrapped interferograms according to Equation 4.1 [Schmidt and Bürgmann,
where $G$ is the matrix denoting the temporal span of interferometric data pair 0 and 1, $\phi_{\text{inc}}$ is a vector expressing the displacement between SAR acquisitions, and $\phi_{\text{obcr}}$ is the surface displacement obtained from each interferometric data pair. The $\frac{d}{dt}$ term is introduced to compensate for residual atmospheric effects, and $\kappa^2$ is a weighting coefficient that determines the magnitude of the compensation. I determined the optimal value of the weighting coefficient using Akaike’s Bayesian information criterion (ABIC) [Akaike, 1980]. With the Bayesian approach, a priori knowledge about the model parameters can be described as a probabilistic model. In this study, I introduced the a priori assumption that the temporal change of surface displacement is smooth, an assumption whose correctness should be evaluated. ABIC is defined as follows:

$$ABIC(\kappa^2) = -2\log M(\kappa^2) + 2N_{\text{damp}}$$

where $M(\cdot)$ is the marginal likelihood and $N_{\text{damp}}$ is the number of weighting coefficients. The maximum marginal likelihood can be derived from the probabilistic model of data $p(\phi_{\text{obcr}}|\phi_{\text{inc}}, \sigma^2)$ and model parameters $p(\phi_{\text{inc}}, \rho^2)$:

$$M(\kappa^2) = \int p(\phi_{\text{obcr}}|\phi_{\text{inc}}, \sigma^2)p(\phi_{\text{inc}}, \rho^2)d\phi_{\text{inc}}$$

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Figure 4.6: (a) Spatial distribution of the surface displacement rate using both PSs and DSs. The square is the location of the Hatchobaru geothermal field, and the circle indicates Mt. Hossho, chosen as a reference location. (b) Spatial distribution of surface displacement using only PSs.
where $\sigma^2$ and $\rho^2$ are the variance of corresponding probabilistic distributions and the weighting coefficient is defined as $\kappa^2 = \sigma^2 \rho^2$. As I assumed a Gaussian distribution for the probabilistic model of data, ABIC is

$$ABIC(\kappa^2) = (N_{intf} + P - N_{mdl}) \log(s(\phi_{inc})) - P \log \kappa^2 + \log |G^T G + \kappa^2 H_d|$$  \hspace{1cm} (4.4)$$

with

$$s(\hat{\phi}_{inc}) = (U - G\hat{\phi}_{inc})^T (U - G\hat{\phi}_{inc}) + \kappa^2 \hat{\phi}_{inc}^T H_d \hat{\phi}_{inc}$$  \hspace{1cm} (4.5)$$

where $H_d$ is the designed matrix of $\frac{d}{dt}$ in Equation 4.1, $\hat{\phi}_{inc}$ is the set of optimal increment displacements, $N_{intf}$ is the number of data, $P$ is the rank of matrix $H_d$, and $N_{mdl}$ is the number of model parameters. By minimizing Equation 4.4, we can determine the optimal value of $\kappa^2$ numerically.

I selected a total of 16,587 coherent pixels in our SAR images, among which I classified PSs and DSs by the number of statistically similar pixels [Ferretti et al., 2011]. I considered a pixel as part of a PS if the number of statistically similar pixels was below 20, and I considered a pixel as part of a DS if the number of similar pixels was over 21 [Ferretti et al., 2011]. In our results, 2256 pixels were PSs and 14,331 pixels were DSs. Therefore, by incorporating DSs in addition to PSs, I realized a sevenfold gain in the number of pixels, and the spatial pattern of surface displacement was greatly clarified (Figure 4.6).

### 4.3 Interpretation and discussion

#### 4.3.1 Surface displacement rate and its temporal evolution

I detected surface displacement around the Hatchobaru geothermal field as great as about -15 mm/year (Figure 4.6 a). The displacement was opposite to the line-of-site direction, so I interpreted it as subsidence. The amount of subsidence was greatest around the area of geothermal production. The main subsidence area corresponds to the area of decreasing gravity during 1994-2000, where geothermal fluids were exploited [Saito et al., 2006]. Thus, I attribute this surface displacement to pressure and mass changes in the geothermal reservoir. However, I cannot rule out the possibility of superficial mass movements, as several landslide features have been mapped around the geothermal area [NIED website]. To characterize displacement trends around the geothermal field, I modeled the surface displacement time-series. To cancel out fluctuations caused by displacements of Kuju volcano, I measured displacement with reference to Mt. Hossho, east of the geothermal field (Figure 4.6 a). I also modeled long-term displacement using both an exponential function of time representing a decaying displacement velocity (Equation 4.6) and a linear function representing a constant displacement velocity (Equation 4.7). I assessed the two models statistically from:

$$U(t) = a_{t1} (\exp(b_{t1}t)) - 1)$$  \hspace{1cm} (4.6)$$

$$U(t) = a_{t2} t$$  \hspace{1cm} (4.7)$$
Figure 4.7: Time-series surface displacement around the Hatchobaru power plant (The square in Fig. 6). Black squares indicate surface displacements estimated by InSAR time-series analysis, and the line is the best-fit curve of the exponential model (AIC=28.0) (a) and linear model (AIC=29.5) (b) (Eq. 4.6).

where $U(t)$ is the surface displacement time-series and $a_{lt1}$ and $a_{lt2}$ are the magnitudes of the respective temporal models. In the exponential function, the coefficient $b_{lt1}$ controls the decay rate. The absolute value of $b_{lt1}$ is greater as the decay of displacement is faster. The factor $-1$ in the exponential function is introduced so that $U(0) = 0$. I determined the value of $b_{lt1}$ using a grid search method and determined $a_{lt1}$ and $a_{lt2}$ by the least-squares method during the grid search. To determine the optimal temporal model, I used the Akaike information criterion (AIC) [Akaike, 1974]. The AIC measures the fit of a model to observations using the maximum likelihood method and enables us to compare models with different numbers of model parameters. The optimal model has the minimum AIC value. The AIC for model $m$ is defined by

$$AIC(m) = -2\log(L(m)) + 2N_{mdl} \quad (4.8)$$

where $L(m)$ is the likelihood for the model $m$ and $N_{mdl}$ is the number of model parameters. The likelihood can be written as

$$L(m) = \left(2\pi\sigma_{ts}^2\right)^{-\frac{N_{ts}}{2}} \exp\left(-\frac{1}{2\sigma^2} (U_{obs} - U_{mdl})^T (U_{obs} - U_{mdl})\right) \quad (4.9)$$

where $N_{ts}$ is the number of displacement time series, $\sigma_{ts}$ is the standard deviation of the observed displacements, and $U_{obs}$ and $U_{mdl}$ are vectors of observed and modeled surface displacement, respectively. Equations 4.8 and 4.9 lead to

$$AIC(m) = N_{ts}\log\left(2\pi\sigma_{ts}^2\right) + \frac{1}{\sigma_{ts}^2} (U_{obs} - U_{mdl})^T (U_{obs} - U_{mdl}) + 2N_{mdl} \quad (4.10)$$
Our calculations showed that the exponential model produces a better fit to surface displacement around the geothermal field than the linear model (Figure 4.7), which means that the surface displacement rate has decreased over time. This agrees with gravity changes of up to 200 μgal observed around the production area during 1990-2003, the period soon after the start of commercial production of the second geothermal unit, a trend that also has decayed over time [Nishijima et al., 2005].

4.3.2 Sharp boundaries of surface displacement

I mapped the slope of annual surface displacement (Figures 4.8 A and 4.8 B) and plotted north-south and east-west cross sections of surface displacement together with the topography of the study area (profiles a-c in Figure 4.8). Cross sections were plotted by interpolating estimated surface displacement rate. According to the elastic displacement of a homogeneous medium, slopes of the center of displacement are bigger, and the magnitude of displacement is prone to decreasing toward the margin. On the other hands, in the margin of the center of displacement area, I found lineaments of steep surface displacement slope as shown by white dashed lines in Figure 4.8A, B and black dashed lines in Figure 4.8 a-c. Since these lineaments of steep slope indicate boundaries of dynamic properties of the crust, these steep slope lines can be interpreted as fault traces. Published simulations of land subsidence due to groundwater pumping around faults showed that the faults cause steep changes in subsidence gradients [Burbey, 2002], and previous InSAR studies of areas of groundwater pumping mapped fault traces by examining such sharp displacement pattern changes [Lu and Danskin, 2001; Chauassard et al., 2014].

Around the geothermal field, it has been reported that strike-slip faults are oriented mainly in NW-SE direction, which act as a geothermal fluid reservoir [Hayashi et al., 1985]. Moreover, fault traces oriented in NE-SW and E-W directions have been reported [Hayashi et al., 1985]. An analysis of the stress regime in this region has suggested that these faults create a compressional field in the ENE-WSW direction [Hayashi et al., 1985]. Consistent with that analysis, strike-slip faults trending NW-SE direction and NE-SW direction have been reported extending from the surface to deeper than 1500 m depth, whereas, normal faults oriented E-W direction found throughout the geothermal area are limited to be shallower crust [Hayashi et al., 1985].

Although many faults have been documented in this region, our study of surface displacement gradients has detected some sharp boundaries that may correspond to previously unmapped fault traces. Displacement on these faults may be the result of fluid injection from deeper parts of faults in the geothermal area. Also, fluid intrusion can destabilize faults by decreasing the effective normal stress or increasing pore pressure [Lachenbruch, 1980; Byerlee, 1990]; thus, fault slip can be triggered by ground subsidence. Indeed, aseismic slip that is controlled by fluid intrusion has been detected by InSAR time-series analysis in the Asal rift in Djibouti [Doubre and Peltzer, 2007] and by InSAR analysis in the Onikobe geothermal area of Japan [Takada et al., 2010].
Figure 4.8: Maps showing the relative gradient of surface displacement rate in the study area (A, B) and cross sections of surface displacement rate (a-c). Cross sections are plotted by interpolation of estimated surface displacement rates. Dashed lines (white in A and B, black in a-c) correspond to sharp boundaries of surface displacement. Black areas in a-c show topography of the cross sections indicated in A and B by white lines, and blue lines are surface displacement rates along the cross sections.
4.4 Conclusions

I estimated surface displacement around the Hatchobaru geothermal field in southern Japan from July 2007 to December 2010 using InSAR time-series processing of ALOS/PALSAR images. By incorporating distributed scatterers in addition to persistent scatterers through the use of the space adaptive multi-looking technique, I increased the number of measurable pixels by about seven times. As a result, I detected displacements that I interpret as ground subsidence. Because the displacement area corresponds to the area of gravity decrease measured during 1994-2000, the estimated surface displacement is likely due to mass changes beneath the geothermal field. Our analysis of the pattern of surface displacement using temporal models showed that surface displacement has likely decayed over time consistent with an exponential function. The spatial pattern of displacement enables us to infer sharp boundaries of displacement areas that correspond to faulting. Faults in this setting might create paths for fluid migration that would decrease effective normal stress on the fault surface. Therefore, the displacement boundaries detected from our analysis may be useful for monitoring the development of geothermal fields. This study shows the effectiveness of InSAR time-series processing with the space adaptive technique to monitor surface displacement around a geothermal field.
Chapter 5

Detection and mapping of soil liquefaction in the 2011 Tohoku earthquake

5.1 Introduction

The 2011 Tohoku earthquake (Mw 9.0) occurred northeast of the Japan Trench on 11 March with a rupture area as large as 500 × 200 km [Ozawa et al., 2011], and was the fourth largest earthquake in the instrumental record. It was followed by numerous large aftershocks along that trench. In the Kanto region including Tokyo, about 350 km from the mainshock, seismic intensities of 5-lower to 6-lower were recorded (Figure 5.1 a) [Hoshiba et al., 2011]. There were large areas of soil liquefaction, especially along Tokyo Bay and the Tone River, causing extensive damage to residential buildings and infrastructure [Yasuda and Harada, 2011; Bhattacharya et al., 2011; Senna et al., 2012]. According to the geomorphologic classification of the Kanto region [Wakamatsu et al., 2005], areas around Tokyo Bay are mainly covered by filled land. Back marsh and natural levees are widely distributed along the upper Tone River, and delta and coastal lowland extend from the middle to lower part of the river (Figure 5.1 c).

Soil liquefaction is usually investigated by field reconnaissance and aerial photography. Here I successfully apply synthetic aperture radar (SAR) interferometry (InSAR), derived from satellite data, to identify and map soil liquefaction. Satellite-based remote sensing methods hold promise for providing broad and dense information on the earth's surface [Massonnet and Feigl, 1998; Bürgmann et al., 2000; Simons and Rosen, 2007] and have shown a potential advantage for investigating soil liquefaction [Atzori et al., 2012]. To our knowledge, this study is the first application of InSAR analysis to mapping soil liquefaction caused by the 2011 Tohoku earthquake.

Soil liquefaction can cause decorrelation because the ground surface becomes wetter and structures may be tilted. I derive such changes of ground surface properties and thereby identify soil liquefaction areas by quantifying the degree of decorrelation, using phase-corrected coherence information based on SAR amplitude data. Previous studies have considered decorrelation as a geophysical phe-
Table 5.1: SAR datasets used in this study. Master and slave indicate period of SAR data for performing SAR interferometry. Displacement from master to slave is estimated via InSAR analysis. Off-nadir and Bperp show representative values of off-nadir angle and perpendicular component of baseline, respectively.

<table>
<thead>
<tr>
<th>Type</th>
<th>ID</th>
<th>Master</th>
<th>Slave</th>
<th>Cycles</th>
<th>Direction</th>
<th>Off-nadir</th>
<th>Bperp</th>
<th>Covered area</th>
</tr>
</thead>
<tbody>
<tr>
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<td>I1</td>
<td>20110104</td>
<td>20110219</td>
<td>1 cycle</td>
<td>Ascending</td>
<td>34.3°</td>
<td>709m</td>
<td>Tokyo Bay</td>
</tr>
<tr>
<td>Preseismic</td>
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<td>20101120</td>
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</tr>
<tr>
<td>Coseismic</td>
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<td>20110406</td>
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<td>34.3°</td>
<td>394m</td>
<td>Tokyo Bay</td>
</tr>
<tr>
<td>Coseismic</td>
<td>I4</td>
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<td>20110407</td>
<td>3 cycles</td>
<td>Descending</td>
<td>34.3°</td>
<td>914m</td>
<td>Tone River</td>
</tr>
</tbody>
</table>

nomenon and integrated it with field surveys, for instance in mapping surface ruptures and estimating areas of major building damage caused by earthquakes [Simons et al., 2002; Fielding et al., 2005].

I also estimated surface displacement using InSAR, which can produce a surface displacement map over wide areas using phase information from paired satellite observations, without the need for ground-based observations. InSAR analysis has been successfully applied to understanding synoptic surface displacement caused by the 2011 earthquake [Kobayashi et al., 2011; Feng et al., 2011]. These studies mainly focus on deformation over a few to several tens of kilometers. Here, the focus is on detection of small-scale surface displacement produced by soil liquefaction, using InSAR analysis.

5.2 Method

5.2.1 Interferometric processing

I obtained single-look complex (SLC) data after preprocessing SAR data from a Level-1.0 product. The SAR data were acquired by the Phased Array Type L-band SAR (PALSAR) instrument aboard the Japanese Advanced Land Observing Satellite (Table 5.1; Figure 5.1 b and 5.2). SLC data is composed of a regular grid with complex values ($C$), which is decomposed into amplitude and phase information:

$$C = A \exp(j\nu)$$

(5.1)

where $A$ and $\nu$ represent amplitude and phase, respectively. Interferometric processing uses two SLC datasets, called master and slave. After repositioning and resampling each slave pixel to its corresponding master, I apply the following equation at each pixel:
Figure 5.1: (a) The study area (black rectangle) overlaid on the Japan Meteorological Agency seismic intensity of the 2011 earthquake (based on Hoshiba et al., 2011). (b) The topography of the Kanto region and the analyzed scene. Two rectangles located on waterfront along Tokyo Bay and the Tone River covered by the SAR data, with ascending and descending orbits. Main branch of the Tone River is shown by a black line. (c) A geomorphologic classification map of the Kanto region with about 1 km mesh (based on Wakamatsu et al., 2005).
Figure 5.2: Epochs of the SAR data used in this study. I used four SAR data pairs (I1–I4). Two pairs (preseismic pair I1; coseismic pair I3) include waterfront along Tokyo Bay, and the other two pairs (preseismic pair I2; coseismic pair I4) include waterfront along midstream and downstream of the Tone River. All data pairs (I1–I4) are used to estimate the change in the surface scattering property using the phase-corrected coherence, while two coseismic pairs (I3, I4) are used to estimate the local surface displacement associated with the soil liquefaction. A pink star represents the occurrence of the 2011 Tohoku earthquake.

\[ C_1 C_2^* = A_1 A_2 \exp(j(u_1 - u_2)) \]  

(5.2)

where \( C_1 \) and \( C_2 \) represent master and slave SLC data, and * indicates complex conjugation. Equation 5.2 yields amplitude \((A_1, A_2)\) and phase \((u_1, u_2)\) information that describe the relationship between the SAR data in the pair.

I sought to detect the distribution of soil liquefaction using change in coherence of amplitude \( A \), and to estimate ground displacement in the InSAR analysis using phase information \( u \). Data processing was done using GAMMA software [Wegmüller and Werner, 1997]. By averaging three pixels in the range direction and five in the azimuthal direction with multi-look processing, per-pixel resolution of the interferogram corresponds to a 14 \( \times \) 16 m polygon on the ground.

### 5.2.2 Damage area detection using decorrelation

The SAR system transmits microwaves and receives backscatter signal from illuminated surfaces. Generally, various scatterers (e.g., bare soil, buildings or crops) exist within each SAR data pixel. Reflective intensity (amplitude) and phase in a pixel is measured as the result of coherent summation of all returns from individual scatterers. When scatterers change between observations, the sum of scatter returns varies, which leads to decorrelation. To quantify decorrelation, I calculated phase-corrected coherence \( \gamma_{pc} \) [Hagberg et al., 1995; Guarnieri and Prati, 1997]:

\[
\gamma_{pc} = \frac{\left| \sum_{n=1}^{N_{pix}} C_1^{(n)} C_2^{(n)*} \exp(-j(u_1^{(n)} - u_2^{(n)})) \right|}{\sqrt{\sum_{n=1}^{N_{pix}} \left| C_1^{(n)} \right|^2 \sum_{n=1}^{N_{pix}} \left| C_2^{(n)} \right|^2}}
\]

(5.3)
where $C_1^{(n)}$ and $C_2^{(n)}$ indicate complex signals in each pixel and $(v_1^{(n)} - v_2^{(n)})$ is the phase difference between master and slave. $N_{pix}$ is the number of adjacent pixels used to compute coherence as a spatial average, usually called window size. This size should be large enough to estimate coherence, because of computing the expectation as spatial averages over a number of pixels in an interferometric pair [Touzi et al., 1999; Zebker and Chen, 2005]. However, spatial resolution decreases with window size.

To estimate coherence, I chose $N = 7 \times 7$, corresponding to an area roughly $98 \times 112$ m. This is almost the same as the number used by Fielding et al. (2005). Generally, coherence including both amplitude and phase is used in InSAR analysis. However, this coherence is influenced by phase variation not only from change of noise in the signal, but also from systematic phase variation caused by topographic, atmospheric, or deformation gradients.

To quantify and classify changes of surface properties attributable to liquefaction, I used phase-corrected coherence calculated based on amplitude. Amplitude in a SAR pixel represents backscatter intensity at a surface location. This intensity is affected by water content and surface roughness, given the same incidence angle, polarization, and radar wavelength. Hence, sand boils on the surface or buildings tilted by soil liquefaction alter amplitude relationships among adjacent pixels, generating decorrelation. The amplitude relationship among adjacent pixels was smoothed by filtering.

Although these surface changes are a source of decorrelation resulting from the earthquake, such decorrelation includes other effects that must be taken into account. Coherence comprises contributions from three effects [Zebker and Villasenor, 1992]:

$$\gamma = \gamma_{\text{therm}}\gamma_{\text{geom}}\gamma_{\text{temp}}$$  \hspace{1cm} (5.4)

where, $\gamma_{\text{therm}}$ represents noise in the radar system and processing approach, $\gamma_{\text{geom}}$ is geometric coherence proportional to the perpendicular component of the baseline, and $\gamma_{\text{temp}}$ is the influence of temporal backscatter change, e.g., from surface cover change or vegetation.

Because soil liquefaction appears as a decrease in $\gamma_{\text{temp}}$, I need to extract $\gamma_{\text{temp}}$ from total coherence. Therefore, I made a coherence difference map. This used the result of subtracting the coherence of preseismic data pairs (dataset I1 covers Tokyo Bay and I2 covers Tone River; Table 5.1 and Figure 5.2) from that of coseismic pairs, acquired before and after the earthquake (dataset I3 covers Tokyo Bay and I4 covers Tone River; Table 5.1 and Figure 5.2):

$$\gamma_{\text{diff}} = \gamma - \gamma_p = \gamma_{\text{therm}}\gamma_{\text{geom}}(\gamma_{\text{temp}} - \gamma_{\text{temp}}^p)$$  \hspace{1cm} (5.5)

where $\gamma$ and $\gamma_p$ are coherence of the coseismic pair and preseismic pair, respectively. This procedure is based on the assumption that components $\gamma_{\text{therm}}$ and $\gamma_{\text{geom}}$ are identical or nearly identical in both preseismic and coseismic pairs, for the following reasons. Since SAR data are obtained by the same sensor (PALSAR), $\gamma_{\text{therm}}$ is identical. Supposing observation in a flat area, $\gamma_{\text{geom}}$ of the datasets used can be calculated from the value of perpendicular baselines and critical baseline of PALSAR (about 23000 m). As a result, I obtained $\gamma_{\text{geom}}$ values 0.97 for I1, 0.97 for I2, 0.98 for I3, and 0.96 for I4.
Figure 5.3: Differential interferograms representing the regional trend of the surface displacement caused by the 2011 Tohoku earthquake, in each line of sight (LOS) direction. (a) A differential interferogram calculated from the coseismic pair I3 in Table 5.1 and Figure 5.2. (b) A differential interferogram calculated from the coseismic pair I4 in Table 5.1 and Figure 5.2. (c) and (d) are the enlarged areas represented by the insets (dashed lines) in (a) and (b), in which the local phase disturbance is evident.
Table 5.2: SAR datasets used to estimate general trends of temporal coherence change before earthquake.

<table>
<thead>
<tr>
<th>Master</th>
<th>Slave</th>
<th>Cycles</th>
<th>Bperp</th>
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</thead>
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<td>20060923</td>
<td>1 cycle</td>
<td>576m</td>
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</tr>
<tr>
<td>20071227</td>
<td>20080211</td>
<td>1 cycle</td>
<td>945m</td>
<td>Tokyo Bay</td>
</tr>
<tr>
<td>20080211</td>
<td>20080328</td>
<td>1 cycle</td>
<td>96m</td>
<td>Tokyo Bay</td>
</tr>
<tr>
<td>20081113</td>
<td>20081229</td>
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<td>22m</td>
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<td>20081229</td>
<td>20090213</td>
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<td>740m</td>
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<td>20100403</td>
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<td>144m</td>
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<td>20110219</td>
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<tr>
<td>20080329</td>
<td>20080514</td>
<td>1 cycle</td>
<td>146m</td>
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<tr>
<td>20080629</td>
<td>20080814</td>
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<td>1507m</td>
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</tr>
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<td>236m</td>
<td>Tone River</td>
</tr>
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<td>1 cycle</td>
<td>237m</td>
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</tr>
<tr>
<td>20100520</td>
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<td>1 cycle</td>
<td>139m</td>
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</tr>
<tr>
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<td>20100820</td>
<td>1 cycle</td>
<td>173m</td>
<td>Tone River</td>
</tr>
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<td>1 cycle</td>
<td>593m</td>
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<tr>
<td>20071228</td>
<td>20080514</td>
<td>3 cycles</td>
<td>616m</td>
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<td>20090517</td>
<td>20091002</td>
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<td>654m</td>
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</tr>
<tr>
<td>20091002</td>
<td>20100102</td>
<td>3 cycles</td>
<td>587m</td>
<td>Tone River</td>
</tr>
</tbody>
</table>

These results show that the effect of $\gamma_{geom}$ is almost negligible after filtering.

Although soil liquefaction decreases $\gamma_{temp}$, so does ordinal surface cover change and vegetation. To obtain significant change of $\gamma_{temp}$ caused only by the earthquake, I determined a coherence change threshold from the average and standard deviation of ordinal change in temporal coherence (Equation 5.6). This estimation was made using datasets acquired before the earthquake, between 2006 and 2011 (Table 5.2)

$$
\gamma_{diff}^{thre} = \gamma_{diff}^{ave} - k_\gamma \gamma_{diff}^{std}
$$

where $\gamma_{diff}^{thre}$ is the threshold of significant coherence difference caused by the earthquake at a pixel, and $\gamma_{diff}^{ave}$ and $\gamma_{diff}^{std}$ are average and standard deviation of coherence difference calculated by preseismic datasets (Table 5.2). $k_\gamma$ is a coefficient that represents a weighting factor for standard deviation.
Suppose that the coherence difference at each pixel obeys a normal distribution (random surface cover change), I assumed that the coherence difference beyond 3-sigma ($k_\gamma = 3$) represents significant surface change owing to liquefaction.

### 5.2.3 InSAR analysis

From interferometric phase information, InSAR analysis enables us to obtain surface displacements. I used the coseismic pair datasets I3 (Tokyo Bay) and I4 (Tone River) in this analysis. To remove the effect of the difference in satellite positions for the two acquisitions of a pair, I used 50-m mesh digital elevation models provided by the Geospatial Information Authority of Japan. To retain the high resolution of the data, I used an adaptive filter whose performance magnitude depends on coherence [Baran et al., 2003]. Then, from phase information over the coherence threshold (0.30), I carefully conducted a phase unwrapping procedure using a branch-cut algorithm [Goldstein et al., 1988]. A variety of displacement scales may be induced by soil liquefaction, and some spatially small displacements may be filtered out or be eliminated by the coherence threshold. This filtering and unwrapping effect gives a spatially continuous displacement, which is difficult to obtain from field survey only. Finally, I obtained a differential interferogram that represents total surface displacement caused by the 2011 earthquake along the line of sight (LOS) direction (Figure 5.3). To display local ground subsidence associated with soil liquefaction, I removed long-wave ground displacement caused by the earthquake via application of a best-fit quadratic function.

### 5.3 Results and discussion

#### 5.3.1 Temporal decorrelation

Coherence maps (Figure 5.4 and I1, I2, I3 and I4 in Figure 5.6) show coherence reduction from three effects (Equation 5.4) between each pair. In the coherence difference maps of Figure 5.5a and b, zero values indicate areas where coherence did not change between preseismic and coseismic pairs (that is, coherence in both interferograms was either high or low). Negative values indicate areas where coherence decreased in the coseismic pair (I3 or I4) relative to the preseismic pair (I1 or I2). Positive values indicate areas where coherence of the preseismic pair was lower than that of the coseismic pair. Negative values are distributed over the ground with artificial fill along Tokyo Bay (Figure 5.5 a), and near the banks of the Tone River (Figure 5.5 b). These observations suggest that physical properties of the ground surface, such as water content or surface roughness, were changed by the earthquake in these areas. Although negative values in the coherence difference map are possibly associated with the earthquake, I must discriminate scatter change caused by the 2011 earthquake from other temporal effects (e.g., ordinal change in surface cover or decorrelation from vegetation). Therefore, I estimated the coherence change threshold at each pixel using the average and standard deviation of coherence.
Figure 5.4: (a) Coherence maps from the preseismic pair, covering Tokyo Bay (I1 in Table 1), (b) the coseismic pair, covering Tokyo Bay (I3 in Table 1), (c) the preseismic pair, covering the Tone River (I2 in Table 1), (d) the coseismic pair, covering the Tone River (I4 in Table 1).
change caused by other temporal effects, which appears in the coherence difference maps.

To reveal effects of the temporal and spatial baseline conditions of our analyzed area, I studied datasets before the earthquake within a perpendicular baseline of 2000 m (Table 5.2): one satellite-cycle between 2006 and 2011 covering waterfront areas along Tokyo Bay (seven datasets), and one cycle (seven datasets) and three cycles (three datasets) for the same period along the Tone River. Coherence maps of each preseismic pair showed that most of coherence in areas along Tokyo Bay was stable with time, while coherence along the Tone River was low and variable (Figure 5.6). This can be interpreted as a result of surface cover in each area. Land-cover classification derived from optical satellite data [Takahashi et al., 2010; Takahashi et al., 2012] show areas along the bay to be urban, mainly covered by buildings and asphalt roads. In contrast, areas along the Tone River are classified as paddy, crops and other vegetation, where backscatter signals tend to be unstable (Figure 5.7). Ordinal coherence change in paddy and crop areas along the river were discovered in the analysis (Figure 5.6 B). Along the river, three-cycle pair coherences are lower than those of one-cycle pairs, because surface cover likely changes with duration of observation (Figure 5.6 B). Considering the physical reason for decorrelation in vegetated and agriculture areas (i.e., surface cover change), it may be inferred that there are annual trends of coherence reduction at each pixel. However, since the observation data are limited, I used all one-cycle and three-cycle pairs from areas along the Tone River for further analysis.

Next, average and standard deviation of coherence difference values were calculated using all
Figure 5.6: Coherence maps of each dataset used for areas along (A) Tokyo Bay, and (B) the Tone River.
dataset combinations for the Tokyo Bay area (21 total combinations), and one-cycle pair and three-cycle pair combinations for the Tone River area (21 total combinations) (Figure 5.8a, b, d and e). Then, according to Equation 5.6, I calculated the coherence change threshold at each pixel. I considered a coherence difference below the threshold as significant decorrelation caused by surface changes of the 2011 earthquake (Figure 5.8c and f). The processing indicated that the threshold in areas along the bay is nearly zero, because coherence is stable over time, as described previously (Figure 5.6A). On the other hand, the threshold in agricultural areas along the river is lower than in the urban area (Figure 5.8f). This means that it is difficult to detect small surface changes caused by the earthquake in these areas. However, in the urban area along the Tone River, the coherence remains stable enough to detect surface changes (Figure 5.8d, e and f).

To confirm this association of surface scatter changes with soil liquefaction, I compared our result with surveys conducted on foot or based on aerial photography soon after the earthquake [KRDB and JGS, 2011]. The survey clarified soil liquefaction areas in the Kanto region, checking the existence of sand boils or water spouts associated with the liquefaction. The walking survey was mainly performed along roads, and inaccessible areas such as industrial plants were covered by aerial photography. These are plotted in Figure 5.9a2 and b2, using red and blue to indicate liquefaction and non-liquefaction locations, respectively. Although the survey results showed soil liquefaction areas with high reliability, all areas were not investigated because of a wide variation of soil liquefaction occurrence. Yellow and orange areas in Figure 5.9a1, a2, b1 and b2 indicate negative values of phase-corrected coherence, under the estimated threshold. Significant decorrelation due to disaster in areas where coherence of preseismic pair is lower than the threshold value cannot be detected in principle. These areas are plotted by gray in Figure 5.9a1, a2, b1 and b2. Comparison shows that 85 and
61% of liquefaction locations detected by the field survey of *KRDB and JGS* (2011) in areas along Tokyo Bay and the Tone River, respectively, are consistent with the present results. Because of this consistency, negative coherence corresponds to surface changes from soil liquefaction caused by the 2011 earthquake. Agreement in areas along the bay is slightly greater than along the river, probably because coherence was temporally stable in those areas (Figure 5.6), resulting in easy detection of coherence change.

### 5.3.2 InSAR-derived surface displacement

Figure 5.10 and 5.11 show local surface displacement inferred by InSAR, after removal of global surface displacement. Magnitudes between about -30 to 3 cm were detected in soil liquefaction areas along Tokyo Bay and the Tone River. Although local changes in the positive range are also found outside soil liquefaction areas, I cannot completely determine whether these changes reflect surface displacement or other effects (e.g., quadratic fitting error or atmospheric artifacts) (Figure 5.10). Nevertheless, the magnitude of these changes (less than 3 cm) is much smaller than that of local surface displacement in soil liquefaction areas. Previous studies have shown that decorrelation leads to an increase of phase variance [*Just and Bamler*, 1994; *Rodriguez and Martin*, 1992]. In soil liquefaction areas, there are certainly decorrelation-induced phase discontinuities (e.g., A in Figure 5.11a1, C in Figure 5.11b1); however, I partly obtained spatially continuous displacement (e.g., B in
Figure 5.9: Estimated liquefaction areas along (a1) Tokyo Bay and (b1) the Tone River. Observed liquefaction through field surveys conducted by KRDB and JGS (2011) overlaid on the estimated liquefaction for the waterfronts along (a2) Tokyo Bay and (b2) the Tone River. Red and blue indicate liquefaction and non-liquefaction areas, respectively, detected by field survey. Yellow and orange areas indicate the negative coherence areas below the coherence reduction threshold value. Significant coherence reduction cannot be estimated in gray areas where coherence in preseismic pair is less than the threshold value. Bold black lines indicate the Tone River.
Figure 5.10: Local surface displacements after the removal of the regional displacement trend obtained from (a) I3 (covering the waterfront along Tokyo Bay), and (b) I4 (covering the waterfront along the midstream and downstream of the Tone River).

Figure 5.11a1). Since most of SAR data in liquefaction areas were collected in only one direction (not both ascending and descending orbits), I cannot explicitly determine whether these local surface displacements were subsidence. I inferred that local displacement may be ground subsidence, because soil liquefaction can cause soil grains to rearrange into a more dense packing due to the phenomena that a mass of soil loses its shear resistance and pore water pressures increase, and such displacement is at liquefaction locations where the ground surface is flat.

Estimated local ground displacements demonstrate characteristics of local displacement in each area. Along Tokyo Bay, this displacement was particularly widespread and continuous in the Narashino area (B in Figure 5.11a1). A continuous fringe pattern predominates in the Narashino (B in Figure 5.11a1) and Chiba areas, whereas a discontinuous fringe pattern dominates in the Urayasu area (A in Figure 5.11a1). These observations suggest that continuous fringe pattern areas are uniformly deformed over a relatively wide area, but this is not true of discontinuous areas.

5.4 Conclusions

I detected changes of surface scattering properties caused by the 2011 Tohoku earthquake, using the difference of phase-corrected coherence between preseismic and coseismic SAR data pairs. To retrieve the significant decorrelation signal attributable to the earthquake, I proposed use of a coherence change threshold estimated from preseismic datasets. I also revealed local surface displacements us-
Figure 5.11: Local surface displacements near the waterfront along (a1) Tokyo Bay and (b1) the Tone River. Local surface displacements in the liquefaction areas along (a2) Tokyo Bay and (b2) the Tone River.

The results are consistent with field surveys. Therefore, these surface changes are associated with soil liquefaction. Areal extent of soil liquefaction inferred from our analysis is difficult to obtain by ground-based measurements. This study shows that SAR data is effective for investigating soil liquefaction extent and detail.

In selecting SAR data, Massonnet and Feigl (1998) reported that the maximum detectable deformation gradient is one fringe per pixel. Thus, long-wavelength, high-resolution SAR data are better for detecting high deformation gradients associated with soil liquefaction. This means that L-band SAR data are superior to C-band and X-band data, given the same spatial resolution. I performed SAR interferometry analysis and then compared the result to field surveys. However, InSAR analysis done immediately after an earthquake shows promise in effectively guiding subsequent field surveys.
Chapter 6

A method using co-polarized components of polarimetry based on equivalent stacking

6.1 Introduction

InSAR time-series analysis is a powerful technique of mapping surface displacement and estimating a target’s scatterer height [Ferretti et al., 2000; Ferretti et al., 2001; Kampes, 2006]. Results obtained from differential synthetic aperture radar (SAR) interferometry (DInSAR) analysis have sometimes suffered from temporal and geometrical decorrelation. However, since InSAR time-series analysis makes use of high-quality information obtained from persistent scatterers (PS) with less decorrelation, values of estimated parameters such as the displacement rate or scatterer height obtained by InSAR time-series analysis are more accurate than values obtained from DInSAR results. Generally, InSAR time-series analysis adopts only single-polarization data; however, I propose a method using the data of the co-polarized components of multi-polarimetric SAR acquisition (HH and VV) based on InSAR time-series analysis. In recent years, several satellites equipped with SAR, which can acquire multi-polarimetric data, have been launched. These include ALOS, ENVISAT, Radarsat-2, and TerraSAR-X. Additionally, upcoming SAR satellite missions such as ALOS-2 and Sentinel-1 will operate in multi-polarimetric acquisition mode.

To enhance the ability to estimate surface displacement and a target’s scatterer height from multi-polarimetric data, several methods incorporating multi-polarimetric data have been proposed [Pipia et al., 2009; Navarro-Sanchez and Lopez-Sanchez, 2012; Navarro-Sanchez et al., 2013]. In this study, I improve estimation accuracy by making use of twice the number of interferograms using both HH-HH and VV-VV data pairs. In other words, I equally stack HH-HH and VV-VV interferograms (the weighting of each co-pol observation is the same) by considering both co-polarized interferograms as constraining the same surface displacement with a different magnitude of noise.

Besides PS, distributed scatterers (DS), defined by pixels more affected by decorrelation than PS but with decorrelation still low enough to estimate parameters of interest, have been recently
incorporated into InSAR time-series analysis to increase the number of pixels in InSAR time-series analysis and estimate parameters of interest [Ferretti et al., 2011]. The idea of considering DS and PS would be important to the analysis of multi-polarimetric SAR data because of the coarser slant-range resolution of data. Thus, I also incorporate an analysis for DS into the time-series multi-polarimetric data analysis proposed in this study.

In this paper, I first present the theoretical consideration for the improvement of the estimation accuracy using the method proposed in this study and a method of considering DS in addition to PS. I then apply our method to real SAR data acquired by ALOS/PALSAR in full polarimetric observation mode for a target area of ground subsidence in Japan to demonstrate the ability of the proposed methodology.

6.2 Equivalent stacks

I use both HH and VV observations equivalently in the InSAR time-series technique based on the scattering characteristics of PS as follows. PS are generally manmade structures such as residences and buildings or bare rocks, which are often represented by odd-bounce scattering from roof or plane surfaces (single bounces) and buildings with a courtyard internal angle (triple bounces), or by even-bounce scattering from the bottom side of a manmade structure and the ground surface (double bounces) [Perissin and Ferretti, 2007]. Theoretically, odd-bounce scattering can be modeled as scattering from a flat plate, a spherical surface or trihedral corner reflector, and even-bounce scattering can be modeled as scattering from a dihedral corner reflector. Because the contributions of the magnitudes of these scatterings from ideal targets to the co-polarized components are the same, it is reasonable to use both HH and VV observations to estimate the parameter of interest.

InSAR time-series analysis finds pixels with a small residual between the observed wrapped phase ($\phi_{\text{obs}}$) and modeled phase ($\phi_{\text{model}}(m)$) obtained by maximizing the objective function often referred to as the temporal coherence [Ferretti et al., 2000; Ferretti et al., 2001; Kampes, 2006].

$$m_{\text{est}} = \arg\max \left\{ \sum_{i=1}^{N_{\text{inf}}} \exp(j(\phi_{\text{obs},i} - \phi_{\text{model},i}(m))) \right\}$$

where $N_{\text{inf}}$ and $j$ are respectively the number of interferograms and the imaginary unit, while $m$ denotes model parameters such as the displacement rate and scatterer height. Since pixels that fit the modeled phase are recognized as PS, the estimated parameters of InSAR time-series analysis are constrained by this modeled phase. Note that the problem of finding optimum model parameters from the observed phase wrapped by modulo $2\pi$ is a nonlinear optimization; however, the accuracy of the estimated parameter can be discussed as a linear problem at PS because of the small magnitude of the phase noise. Thus, from the viewpoint of basic InSAR time-series analysis theory, an increase in the number of interferograms helps improve the estimation accuracy as follows. According to least-
square theory, the variance of the estimated parameters ($\sigma_m^2$) is linearly correlated with the phase noise variance ($\sigma_{dcr}^2$) [Ferretti et al., 2001; Aster et al., 2013].

$$\sigma_m^2 \approx \frac{\sigma_{dcr}^2}{K_m} \quad (6.2)$$

where $K_m$ is a coefficient that describes the relationship between the variance of the estimated parameter and the phase noise variance. If the displacement rate is constant over time ($\phi_i = \frac{4\pi}{\lambda}T_iv$), then the coefficient $K_p$ becomes $(4\pi/\lambda)^2 \sum_i (T_i - \bar{T})^2$, where $\phi$ is the absolute phase increment due to displacement, $\lambda$, $T$, and $v$ are the wavelength, temporal baseline length, and displacement velocity, respectively, and $\bar{T}$ is the mean temporal baseline value. If both HH-HH and VV-VV interferograms are used in the data analysis, the value of the coefficient $K_m$ would be up to doubled, and, correspondingly, the variance of the errors of the estimated displacement would be up to halved [Aster et al., 2013].

The improvement of estimation accuracy described above is not always true for the estimation of the scatterer height, because the scatterer height might depend on the polarization. Because there is a possibility that the scatterer height differs for each polarization, a procedure to check whether there is a dependence of scattering height on polarization is required in order to estimate the scatterer height properly. As a method of such checking, I propose to measure the electromagnetic width from the critical baseline in each co-pol interferogram according to the following theory. If the signals from scattering targets are assumed to be uncorrelated and uniformly distributed within a certain area on the surface, the electromagnetic width ($W_e$) can be theoretically described as a function of the critical baseline $B_{\perp,e}$ [Ferretti et al., 2001; Li and Goldstein, 1990; Zebker and Villasenor, 1992].

$$W_e = \pm \frac{4R\tan\theta}{2B_{\perp,e}} \quad (6.3)$$

where $R$ and $\theta$ are the sensor-target distance and incidence angle, respectively, and $B_{\perp,e}$ is the critical baseline of the pixel of interest. Theoretically, the coherence linearly decreases with increasing normal baseline owing to the spectral separation between interferometric datasets [Li and Goldstein, 1990; Zebker and Villasenor, 1992]. The critical baseline is defined by the normal baseline whose coherence becomes zero following this geometrical decorrelation model. If scatterers are uniformly distributed within a pixel, the electromagnetic width equals the slant-range resolution. However, in real data, the critical baseline and electromagnetic widths depend on the target. Thus, I estimated the critical baseline of HH-HH and VV-VV interferograms from the spatial coherence in each interferogram as a function of the normal baseline. The scatterer height was then considered to differ with polarization when pixels had a significant difference in the critical baseline of HH-HH and VV-VV interferograms.

As stated above, the spatial coherence is used to estimate the critical baseline. To measure the spatial coherence properly, a number of neighboring pixels are required; otherwise, the estimated coherence is biased and unreliable. One possible way to properly estimate the coherence is to estimate
statistically homogeneous pixels (SHP) [Ferretti et al., 2011] whose noise magnitude is similar to that of the target pixel and then to calculate the coherence using the SHP. SHP can be selected by statistically testing amplitude data to check whether two pixels are drawn from similar amplitude statistics. Because properly selected SHP have similar noise magnitude, coherence can be properly estimated in general. Pixels with a small number of SHP are empirically known as PS, and they can thus be reliably measured. Note that one disadvantage of this method for determining whether the scatterer height depends on the polarization is that I cannot differentiate scatterer heights of different polarizations when the target signal originates from different scatterer heights in each polarization and both targets behave as PS.

6.3 Space adaptive filtering for DSs

The slant-range resolution of multi-polarimetric SAR data is generally twice as coarse as that of single-polarimetric data. Thus, PS pixels may behave as DS, because more scatterers are likely to be included in a pixel. As a matter of fact, it has been reported that the number of PS increases as the spatial resolution of the data becomes finer [Prati et al., 2010]. Therefore, if polarimetric data do not have enough PS to reveal the spatial extent of displacement and scatterer height, an analysis incorporating both PS and DS is required. To this aim, I propose to apply space-adaptive filtering within statistically homogeneous areas to improve the signal-to-noise ratio (SNR) [Ferretti et al., 2011; Goel and Adam, 2012]. Space-adaptive filtering is a method of applying filtering to SHP that makes it possible to improve the SNR of DS even though the spatial resolution is degraded, while obtaining high-quality information from PS. In this study, I applied the coherent average (multi-look) as a spatial filtering technique.

One way to select SHP is to apply the Kolmogorov-Smirnov (KS) test [Kvam and Vidakovic, 2007], introduced in [Ferretti et al., 2011] for this purpose. The KS test, of course, can be effectively applied as a non-parametric test for selecting pixels. However, if I have only a small number of SAR data, the classification results of the KS test may be more robust than those of parametric tests. Thus, it is likely to be better to apply a parametric test when there are few SAR data, because it tends to show more rigid statistical classification. In this study, I apply both a two-sample F test and a two-sample t test [Bulmer, 1979; Lomax and Hahs-Vaughn, 2011] as a parametric test. SAR amplitudes are known to be described by the Rice distribution [Ferretti et al., 2001; Papoulis, 1984]; the Rice distribution tends to become a Gaussian distribution when the SNR is high and a Rayleigh distribution when the SNR is low. Therefore, by applying the F test and t test, I focus on high-SNR pixels whose amplitude can be described by a Gaussian distribution. The F test checks the null hypothesis that there is no difference in the variances of two populations with a Gaussian distribution, while the t test checks the null hypothesis that there is no difference in the mean values of two populations with a Gaussian distribution. Therefore, by applying both an F test and t test, I test whether two sampled amplitudes
6.4 Experimental results

6.4.1 Data analysis and results

The application area was the city of Itayanagi in Aomori Prefecture, northern Japan. Subsidence due to groundwater extraction for snow melting has been reported in this area. I used six ALOS/PALSAR images acquired in full-polarimetric observation mode from July 2006 to September 2009. Full polarimetric acquisition of an ALOS/PALSAR image results in the image having a single-look resolution of about 9.4 and 3.5 m in the slant-range and azimuth directions, respectively. I selected 255 × 950 pixels, corresponding to the area of a rectangular region of about 2.4 × 3.3 km. The central and lower parts of the analyzed area are urban areas with higher amplitude backscatter signals, where even- and odd-bounce scattering are rather dominant according to Pauli-based decomposition [Lee and Pottier, 2009] (red and blue in Figure 6.1 a), while the upper and side parts of the analyzed area are rural with lower-amplitude backscatter signals and odd-bounce and diffusive scattering (blue and green in Figure 6.1 a).

In the following, I performed the equivalent stacking of HH-HH and VV-VV interferograms and space-adaptive filtering for DS based on the processing flow proposed in [Ferretti et al., 2001]. The first step is to select coherent pixel candidates (candidates of PS and DS). As discussed in sections 2 and 3, our aim is to choose pixels that have a high SNR and contain ideal even or odd-bounce scat-
To extract pixels with these characteristics, I used three criteria: (1) the amplitude dispersion index [Ferretti et al., 2001] (smaller than 0.35) to detect pixels with a high SNR, (2) the absolute phase difference between HH and VV SAR images [Inglada et al., 2006; Samsonov and Tiampo, 2011] (less than 0.3 rad for odd-bounce scatterers and more than 2.8 rad for even-bounce scatterers), and (3) the amplitude ratio between HH and VV SAR images (from 0.8 to 1.2) to detect pixels containing ideal even or odd-bounce scatterers.

For each candidate, I then searched for SHP using a statistical test for the amplitude time-series. I applied the two-sample F test and t test with a 95% confidence level to the amplitude time-series of coherent pixel candidates and each pixel within a rectangular window of 14 × 30 pixels around a candidate. The number of SHP extracted from the F test and t test ranged from 0 to 366 pixels and averaged 110 pixels. For comparison with the results of the F test and t test, I also applied the KS test and obtained the number of SHP ranging from 0 to 420 pixels and averaging 250 pixels. Compared with the number of SHP per candidate obtained by applying the KS test, fewer were obtained by applying the F test and t test, probably owing to the more rigid classification.

I next created differential interferograms using phases in coherent pixel candidates. I created 14 single-look differential interferograms (seven from HH-HH datasets and seven from VV-VV datasets) with a normal baseline less than 900 m (Figure 6.1 b). Subsequently, I created 14 single-look differential interferograms (seven from HH-HH datasets and seven from VV-VV datasets), and calculated the spatial coherence in each interferogram using SHP. In this study, I removed coherent pixel candidates having average spatial coherence less than the threshold of 0.3 and having a standard deviation of

Figure 6.2: Differential interferograms (in radians) at the measurement candidates (a) before the space adaptive filtering and (b) after the space adaptive filtering. The upper seven interferograms are derived from the HH-HH data sets, whereas the lower seven interferograms are derived from the VV-VV data sets.
Figure 6.3: Geometrical decorrelation in the HH-HH and VV-VV interferograms. The blue and green squares indicate the observed coherence of the HH-HH and VV-VV interferograms, and the blue and green lines indicate the best fit decorrelation rate by means of minimizing the L1 norm.

spatial coherence more than the threshold of 0.2, because these candidates are not likely to have high phase stability.

In the next step, the coherent average was taken for each of the SHP in each differential interferogram as the space-adaptive filtering. Figure 6.2 shows the differential interferogram phase of the HH-HH and VV-VV datasets before and after space-adaptive filtering was applied. There are no significant changes in the urban area around the center of the analyzed scene where the number of SHP was rather small, whereas there was improvement in the rural area where the number of SHP was large, suggesting that the magnitude of the filtering differed depending on the noise level of pixels. Consequently, space-adaptive filtering enhanced the SNR and clarified the boundaries of a physically reasonable interferometric phase.

I next checked whether the scatterer height, one of the model parameters, differs between HH and VV observations. To check the polarization dependence of scatterer height according to Equation 6.3, I calculated the critical baseline from all available datasets (15 datasets for each polarization). The critical baseline was estimated by minimizing the L1 norm of the difference between the theoretical line and the observed coherence value as a function of the normal baseline. In Figure 6.3, I show examples of geometrical decorrelation. Some pixels show a similar decorrelation rate in the HH-HH and VV-VV differential interferograms (Figure 6.3 a), but others show a different decorrelation rate in the HH-HH and VV-VV differential interferograms (Figure 6.3 b). If a measured critical baseline differed between HH-HH and VV-VV differential interferograms, I considered different values for scatterer heights in each interferogram. In contrast, some pixels did not follow the geometrical decorrelation model, probably because of temporal decorrelation or possibly inaccurate selection of SHP. In these pixels, I assumed that the scatterer height differed between the polarizations.

The next step is to select coherent pixels (PS and DS) and estimate parameters from measurement candidates by maximizing the temporal coherence (Equation 6.1). To avoid atmospheric effects, I cal-
Figure 6.4: Estimated scatterer heights (in meters) from (a) HH-HH interferograms and (b) VV-VV interferograms overlaid on the temporal average of the single-look amplitude image.

culated the wrapped phase gradient at neighboring coherent pixel candidates connected by Delaunay triangulation between pixels with a spacing of less than 150 m [Kampes, 2006]. For the parameter of interest, I considered surface displacement with a constant rate ($\phi_{\text{disp}} = \frac{4\pi}{\lambda} T v$), and the scatterer height with respect to a reference surface ($\phi_{\text{height}} = \frac{4\pi B_{\perp} \Delta h}{\lambda K_{\text{stg}}}$), where $B_{\perp}$ is the normal baseline. The displacement rate and scatterer height were simultaneously estimated using 14 interferograms. Phase gradients whose temporal coherence was below the threshold of 0.75 were then removed.

Unwrapping was then performed by minimizing the L0 norm of the phase difference between the wrapped and unwrapped phase gradients using the minimum Lp norm algorithm [Ghiglia and Romero, 1996]. Finally, the surface displacement rate ($v$) and scatterer height ($\Delta h$) were estimated by least-squares linear regression.

In our analysis results, scatterer heights, referenced to a digital elevation model, were the same in HH and VV observations at most pixels in the center of the analyzed area, but in the lower part of the analyzed area, heights of VV observations were higher than HH observation heights (Figure 6.4). From scatterer height estimation, I found that 78% of higher scatterers found over 10 m are odd-bounce scatterers, implying the response from the roof or top of a building. From the surface displacement rate, ground subsidence was detected at a rate of about 15 mm/year in the center of the analyzed area.

6.4.2 Comparison with single-polarimetric data analysis

Figure 6.5 shows the estimated surface displacement derived from the described method (Figure 6.5 a) and from the analysis of single-polarimetric data of HH observations (Figure 6.5 b) and VV observa-
Figure 6.5: Estimated displacement rate (in millimeters per year) derived from the equivalent stack of the (a) multipolarimetric interferogram that is overlaid on the temporal average of a single-look amplitude image, (b) HH-HH interferograms with the space adaptive filtering, and (c) VV-VV interferograms with the space adaptive filtering.

These results were obtained from the same coherent pixel candidates, statistically homogeneous pixels, and the combined datasets. The comparison was performed for a variance of the estimated displacement rate of less than 0.4 (mm/year)$^2$. The variance of the displacement rate was calculated using Equation 6.2, and the phase noise variance needed for Equation 6.2 was obtained from the expectation of the square of the residual phase ($|e|^2$) compensated for the number of interferograms ($N_{intf}$) and parameters of interest ($N_{mdl}$), according to least squares theory.

$$
\sigma_{dcr}^2 = \frac{E(|e|^2)}{N_{intf} - N_{mdl}}
$$

As a result, we found that 86% of the selected coherent pixels had increased accuracy of the estimated displacement rate resulting in 1182 coherent pixels when employing the proposed method, whereas there were 659 and 797 coherent pixels in the single-polarimetric data analyses of HH and VV observations. As predicted by our theoretical consideration, the number of the selected coherent pixels increased as a result of the improvement of the estimation accuracy. When we did not apply the space-adaptive filtering, there were only 50 and 64 selected coherent pixels in the single-polarimetric analysis of HH and VV observations. This is possibly because our data did not originally have many ideal persistent scatterers owing to the coarse spatial resolution and long temporal interval.

### 6.5 Conclusions

In this study, I used co-polarized interferograms of multi-polarimetric SAR observations to estimate the displacement rate and scatterer height. Because this method increases the number of in-
interferograms, the estimation accuracy is improved. I applied the method to real data acquired by ALOS/PALSAR in full-polarimetric mode. As a result, the number of selected coherent pixels increased by 79 and 48% compared with single-polarimetric data analysis results, because the variance of the estimated displacement rate was improved at 86% of the selected coherent pixels and became less than half at 72% of coherent pixels. Moreover, I demonstrated the effectiveness of space-adaptive filtering in improving the SNR of distributed scatterers. Using multi-polarimetric data, the proposed method makes it possible to estimate displacement trends or scatterer heights from a smaller number of data over a shorter time span compared with the case when using single-polarimetric data.
Chapter 7

A method using co- and cross-polarized components of polarimetry based on maximum likelihood theory

7.1 Introduction

Using synthetic aperture radar (SAR) data acquired during repeated satellite passes, InSAR time-series analysis enables the estimation of surface displacement from the high-quality interferometric phase information of persistent scatterers (PSs) [Ferretti et al., 2000; Ferretti et al., 2001; Hooper et al., 2004; Kampes, 2006; Hooper et al., 2007]. Recently, several studies have extended the advantages of InSAR time-series analysis for accurate high-spatial-density surface displacement mapping by means of multi-polarimetric SAR data acquisition [Navarro-Sanchez et al., 2012; Navarro-Sanchez et al., 2014; Ishitsuka et al., 2014b], though the standard InSAR time-series analysis is generally performed using single-polarimetric data. InSAR time-series analysis with single-polarimetric data can detect coherent pixels in a certain single-polarimetry, whereas the simultaneous analysis of multi-polarimetry has an advantage to identify coherent pixels either of all of polarimetry. Since a scatterer’s coherence is influenced by polarimetry depending on scatterer’s properties (e.g., geometry or dielectric characteristics), the analysis with multi-polarimetry would be useful for more detailed mapping of surface displacement.

Some of previous studies incorporated multi-polarimetric SAR data into PS candidate selection [Navarro-Sanchez et al., 2012; Navarro-Sanchez et al., 2014]. On the other hand, another [Ishitsuka et al., 2014b] and I incorporated multi-polarimetric SAR interferograms into PS selection for improving the accuracy of PS detection and corresponding physical parameter estimation. In [Ishitsuka et al., 2014b], it was demonstrated that the number of interferograms increases by using multi-polarimetric SAR data using co-polarized components of multi-polarimetric SAR acquisitions; this is simply due to the theoretical consideration that the number of interferograms can be regarded to increase using...
ideal PSs that equally reflect signals from the co-polarized components. Consequently, the number of coherent pixels to map surface displacement increase compared with the results of the analysis of single-polarimetric interferograms.

However, the application of the algorithm in [Ishitsuka et al., 2014b] is limited to co-polarized components. In this study, I developed a method for simultaneous process of interferograms created from co- and cross-polarized components based on maximum likelihood theory. I aim to consider greater variety of coherent pixels in terms of polarimetry and map larger number of coherent pixels; specifically, the standard InSAR time-series analysis using single-polarimetric data considers coherent pixels in a certain polarization used for the analysis, and the previous study using co-polarized component [Ishitsuka et al., 2014b] considered pixels that are coherent in both of co-polarized components. On the other hand, the targets of our method are coherent pixels in either and all of polarimetry used for the analysis.

Maximum likelihood estimation for InSAR time-series analysis of single-polarimetric SAR data has been proposed in [Shanker and Zebker, 2007]. In this study, I used maximum likelihood theory for the simultaneous analysis of multi-polarimetric SAR interferograms and considering the difference of decorrelation amount in different polarimetry. The decorrelation amount of differential interferograms differs in polarimetry, reflecting target’s scattering characteristics. The maximum likelihood estimation proposed in this study accounts for the different reliability of phase, which depends on polarimetry and a target, by introducing the weighting coefficient described in Section 7.2.

Properties of PSs (or coherent scatterers) have been examined in terms of carrier frequency, acquisition geometry or baseline distribution [Perissin et al., 2006; Ferretti et al., 2006; Perissin et al., 2007]. These properties are useful for interpreting the estimated displacement mechanism. In principal, it becomes more useful to recognize scatterers’ properties as more features can be exploited. However, the effects of polarimetry in terms of phase stability have not been examined yet. Since our analysis with maximum likelihoods analyzes phase stability of each single-polarimetric interferograms, the scattering properties of scatterers depending on polarimetry were empirically derived in this study.

7.2 Maximum likelihood estimation of surface displacement

The processing flow for InSAR time-series analysis consists of the following steps [Kampes, 2006]: (1) computation of differential interferograms, (2) selection of PS candidates, (3) PS selection, (4) phase unwrapping, (5) filtering to remove the atmospheric phase, and (6) final estimation of surface displacement. In step (1), I assume that differential interferograms are created using data acquired with same polarimetry. The processing avoid considering the dependency of phase information on target’s scattering characteristics in terms of polarimetry. In step (2), PS candidates are selected based on a criteria such as the amplitude dispersion index [Ferretti et al., 2001]. Then, starting with
step (3), multi-polarimetric SAR interferograms are incorporated using maximum likelihood theory; this PS selection is important for determining the accuracy of the estimation. Section 7.2.1 describes a stochastic model for the phase of differential interferograms, and Section 7.2.2 presents the maximum likelihood method that is used for PS selection and parameter estimation. Once I select the same coherent pixels among different polarimetric SAR interferograms and corresponding physical model parameters, I could perform the following step (4)-(6).

7.2.1 Stochastic model for the interferometric phase

Phase in differential interferograms consists of the phase due to surface displacement ($\phi_{\text{disp}}$), scatterer height with respect to a reference surface ($\phi_{\text{height}}$), atmospheric phase ($\phi_{\text{atm}}$) and interferometric decorrelation-induced noise ($\phi_{\text{dcr}}$),

$$\phi_{i,j} = \phi_{\text{disp},i,j} + \phi_{\text{height},i,j} + \phi_{\text{atm},i,j} + \phi_{\text{dcr},i,j}$$  (7.1)

where $\phi_{i,j}$ indicates the interferometric phase, wrapped by modulo $2\pi$, at the $i$th pixel in the $j$th differential interferogram. Once the interferometric phase is unwrapped, it can be modeled as follows. The unwrapped phase due to surface displacement can be modeled as a displacement with a constant velocity ($v$), whereas the unwrapped phase due to scatterer height can be theoretically modeled as a function of the normal baseline ($B_\perp$) and scatterer height with respect to a reference surface ($\Delta h$) [Ferretti et al., 2001]:

$$\psi_{\text{disp},i,j} = \frac{4\pi}{\lambda} T_j v_i$$  (7.2)

$$\psi_{\text{height},i,j} = \frac{4\pi}{\lambda R_{i,j} \sin \theta_{i,j}} B_{\perp,i,j} \Delta h_i$$  (7.3)

where $\psi$ is the unwrapped phase, and the variables $\lambda$, $\theta$, $B_\perp$, $T$ and $R$ are the wavelength corresponding to the central frequency of SAR acquisition, the incidence angle, the geometric baseline, the temporal baseline, and the distance between satellite and scatterers, respectively. Depending on the SAR acquisition geometry, additional parameters can be introduced, e.g., the sub-pixel position of scatterers as a function of carrier frequency and Doppler centroid [Perissin and Ferretti, 2007]. It is well known that phase unwrapping is a delicate part of InSAR time-series analysis because interferometric decorrelation may induce phase unwrapping error. Such error is likely if interferograms suffer severe decorrelation. Therefore, InSAR time-series analysis generally selects coherent pixels with a small amount of decorrelation before the phase unwrapping procedure. Thus, InSAR time-series analysis uses the rewrapped phase of a modeled phase, as shown in Equations 7.2 and 7.3, to estimate each modeled phase contribution. In a general InSAR time-series analysis formulation, the observed wrapped phase ($\phi_{i,j}$) can be described as:

$$\phi_{i,j} = G(m_i) + \phi_{\text{dcr},i,j}$$  (7.4)

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where \( m \) is a model parameter such as the displacement rate (\( v \)) or scatterer height (\( \Delta h \)). \( G() \) represents the relationship between the model parameters and wrapped modeled phases. For example, when I assume the phase due to scatterer height and surface displacement with constant velocity to be the modeled phase, \( G() \) becomes:

\[
G(m_i) = W \left[ \frac{4\pi}{\lambda R_{i,j} \sin \theta_{i,j}} B_{i,j} \Delta h_i \right] + W \left[ \frac{4\pi}{\lambda} T_j v_i \right] (7.5)
\]

where \( W[ ] \) is the wrapping operator. If I have \( N_{\text{intf}} \) differential interferograms, the size of the matrix is \( N_{\text{intf}} \times 1 \).

In InSAR time-series processing, the interferometric decorrelation is often assumed to be drawn from a Gaussian distribution [Ferretti et al., 2001; Kamps, 2006]. I considered an ideal coherent pixel that shows stable phase behavior over time, and for which the magnitude of phase noise is small and Gaussian, having zero mean and a variance of \( \sigma^2_{\text{dcr}} \). The stochastic model then can be described as:

\[
p(\phi_{i,\text{pol}}|m_i, \sigma^2_{\text{dcr},i,\text{pol}}) = \left(2\pi \sigma^2_{\text{dcr},i,\text{pol}}\right)^{-\frac{N_{\text{intf}}}{2}} \exp\left(-\frac{1}{2\sigma^2_{\text{dcr}}} (\phi_{i,\text{pol}} - G(m_i))^T (\phi_{i,\text{pol}} - G(m_i)) \right) (7.6)
\]

where \( \text{pol} \) indicates the value obtained from a certain type of polarimetric SAR interferogram (e.g., HH-HH interferograms). In this stochastic model, I assumed that the difference between observed and modeled phase consists of interferometric decorrelation. A constant value of \( \sigma^2_{\text{dcr}} \) can be used for the variance of the interferometric decorrelation, without depending on temporal and geometrical baselines, as long as I assume the analysis of coherent pixels. For multi-polarimetric SAR data, the stochastic model of phase in differential interferograms of other polarimetry can be described as the same stochastic model with a different noise variance magnitude. Therefore, for cases that involve the three different polarimetric SAR interferograms, i.e., HH-HH, HV-HV and VV-VV, the posterior probability distribution, which is the probability distribution of model parameters and the variance of noise given interferometric phase, can be described as follows:

\[
p(m_i, \sigma^2_{\text{dcr},i,\text{HH}}, \sigma^2_{\text{dcr},i,\text{HV}}, \sigma^2_{\text{dcr},i,\text{VV}}) = \frac{p(\phi_{i,\text{HH}}|m_i, \sigma^2_{\text{dcr},i,\text{HH}}) p(\phi_{i,\text{HV}}|m_i, \sigma^2_{\text{dcr},i,\text{HV}}) p(\phi_{i,\text{VV}}|m_i, \sigma^2_{\text{dcr},i,\text{VV}})}{\int p(\phi_{i,\text{HH}}|m_i, \sigma^2_{\text{dcr},i,\text{HH}}) p(\phi_{i,\text{HV}}|m_i, \sigma^2_{\text{dcr},i,\text{HV}}) p(\phi_{i,\text{VV}}|m_i, \sigma^2_{\text{dcr},i,\text{VV}}) \, \text{dm}_i} (7.7)
\]

Here, I assumed that the probability distribution of model parameters and noise variance \( p(m_i, \sigma^2_{\text{dcr},i}) \) is uniformly distributed (i.e., there is no priori assumption for model parameters), thus this term is not included in Equation 7.7. The denominator of the distribution is introduced so that the integral can be one according to the definition of a probability distribution.

Substituting Equation 7.6 into Equation 7.7, and introducing a weighting coefficient (\( \alpha_i, \beta_i \)), results in

\[
p(m_i, \sigma^2_{\text{dcr},i,\text{HH}}, \alpha_i, \beta_i|\phi_{i,\text{HH}}, \phi_{i,\text{HV}}, \phi_{i,\text{VV}}) = Z \left(2\pi \sigma^2_{\text{dcr},i,\text{HH}}\right)^{-\frac{N_{\text{intf}}}{2}} \alpha_i^{\frac{N_{\text{intf}}}{2}} \beta_i^{\frac{N_{\text{intf}}}{2}} \exp\left(-\frac{1}{2\sigma^2_{\text{dcr},i,\text{HH}}} s(m_i, \alpha_i, \beta_i) \right) (7.8)
\]
with

$$\alpha_i = \frac{\sigma_{dcr,i,HH}}{\sigma_{dcr,i,HV}}$$

(7.9)

$$\beta_i = \frac{\sigma_{dcr,i,HH}}{\sigma_{dcr,i,VV}}$$

(7.10)

$$s(m_i, \alpha_i, \beta_i) = (\phi_{i,HH} - G(m_i))^T (\phi_{i,HH} - G(m_i))$$

$$+ \alpha_i^2 (\phi_{i,HV} - G(m_i))^T (\phi_{i,HV} - G(m_i))$$

$$+ \beta_i^2 (\phi_{i,VV} - G(m_i))^T (\phi_{i,VV} - G(m_i))$$

(7.11)

$$Z = \frac{1}{\int p(\phi_{i,HH}|m_i, \sigma^2_{dcr,i,HH}) p(\phi_{i,HV}|m_i, \sigma^2_{dcr,i,HV}) p(\phi_{i,VV}|m_i, \sigma^2_{dcr,i,VV}) \, dm_i}$$

(7.12)

The problem is to find an optimal value of $m_i$ as well as weighting coefficients $\alpha_i$ and $\beta_i$ that maximize the posterior probability distribution.

### 7.2.2 Estimation algorithm using maximum likelihood

I determined the optimal values of the weighting coefficient by maximizing the posterior probability distribution in Equation 7.8.

$$\hat{m}_i, \hat{\alpha}_i, \hat{\beta}_i = \text{argmax} \left( p(m_i, \sigma^2_{dcr,i,HH}, \alpha_i, \beta_i|\phi_{i,HH}, \phi_{i,HV}, \phi_{i,VV}) \right)$$

(7.13)

Since the denominator is a constant, maximizing the posterior distribution corresponds to maximizing the logarithm of the numerator of the posterior probability distribution function. The necessary condition for maximizing the posterior distribution is

$$\frac{\partial \log(p(m_i, \sigma^2_{dcr,i,HH}, \alpha_i, \beta_i|\phi_{i,HH}, \phi_{i,HV}, \phi_{i,VV}))}{\partial \sigma^2_{dcr,i,HH}} = 0$$

(7.14)

Solving Equation 7.14, I obtain

$$\sigma^2_{dcr,i,HH} = \frac{s(\hat{m}_i, \hat{\alpha}_i, \hat{\beta}_i)}{3N_{inf}}$$

(7.15)

Since the maximum value solution of Equation 7.13 is non-linear and it is difficult to obtain the solution analytically, I propose that the logarithm of the posterior distribution be maximized numerically. Substituting Equation 7.15 into Equation 7.8, I obtain

$$\log(p(m_i, \sigma^2_{dcr,i,HH}, \alpha_i, \beta_i|\phi_{i,HH}, \phi_{i,HV}, \phi_{i,VV})) = -\frac{3N_{inf}}{2} \log(s(m_i, \alpha_i, \beta_i)) + N_{inf} \log \alpha_i + N_{inf} \log \beta_i + Const$$

(7.16)

where $Const$ is a constant. Without calculating $Const$, the optimal values of the model parameter and weighting coefficient $(\hat{m}_i, \hat{\alpha}_i, \hat{\beta}_i)$ are obtained by maximizing Equation 7.16.
7.3 Validation using synthetic data

I validated our method using synthetic data created based on [Kampes, 2006]. I assumed the case of three different polarimetric SAR interferograms, and that each interferometric phase consists of the displacement with linear velocity, the scatterer height and the decorrelation-induced noise. The values I simulated are the displacement velocity of -60 mm/year, the scatter height of 0 m in the first polarimetry, 2 m in the second polarimetry and 6 m in the third polarimetry. As for decorrelation-induced phase noise, I simulated on SLC. The standard deviation of the gaussian noise in master acquisition is set to 5 degree for the first and the second polarimetry and 10 degree for the third polarimetry. On the other hand, the noise magnitude in the slave acquisitions is set slightly higher to account coregistration error. The standard deviation of gaussian noise in slave acquisition has uniform distribution of 0 to 5 degree in addition to that of master acquisition. All of phase contribution (displacement velocity, scatterer height and decorrelation noise) was summed and wrapped by modulo $2\pi$. The number of differential interferograms was set to be 50 in each polarization (totally 150), and temporal and geometrical baselines were uniformly distributed with the range of -600–600 days and -2000–2000 m, respectively. SAR system was assumed to be ALOS/PALSAR full-polarimetric mode; I have SAR images acquired with the wave length corresponding to central frequency of about 23.6 cm, and incidence angle of about 24.0 degree. With the synthetic differential interferograms time-series of a pixel, I estimated displacement velocity, scatterer heights of each polarization and two weighting coefficients according to Equation 7.16. This synthetic interferogram simulation and parameter estimation was performed 100 times, and then I obtained the mean and the standard deviation of estimated parameters. For comparison, I also performed the estimation of displacement velocity and scatterer height using time-series phases assuming a single polarimetry, and obtained the mean and the standard deviation as well.

Table 7.1 shows estimation result using synthetic data. Since the mean and the standard deviation of estimated parameters were consistent with the simulated value, the algorithm was shown to be correct. Compared with the same analysis using single-polarimetric interferograms (2, 3 and 4 in Table 7.1), the standard deviation of estimated displacement velocity decreased by performing the simultaneous processing of multi-polarimetric interferograms, likely because of the increase in the number of interferograms. In addition, I note that the mean value of the scatterer height in the third polarimetry is slightly different from the synthetic value, because phase information of the polarimetry was not significantly used for parameter estimation owing to the larger amount of phase noise.

7.4 Application to ALOS/PALSAR data

I processed six ALOS/PALSAR images covering Aomori prefecture, northern Japan, acquired in full-polarimetric mode from an ascending orbit. The incidence angle of the SAR image is about
Table 7.1: Simulation results using synthetic differential interferograms. A: The simultaneous analysis using three polarimetric synthetic interferograms, and B-D: the analysis using single-polarimetric synthetic interferograms. Sim denotes simulated values, and Std denotes the standard deviation of estimated parameters.

<table>
<thead>
<tr>
<th></th>
<th>Displacement rate [mm/year]</th>
<th>Scatterer height 1 [m]</th>
<th>Scatterer height 2 [m]</th>
<th>Scatterer height 3 [m]</th>
<th>Weighting coefficient 1</th>
<th>Weighting coefficient 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sim</td>
<td>-60</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>A (3 pol) Mean</td>
<td>-60.02</td>
<td>0.07</td>
<td>1.89</td>
<td>4.36</td>
<td>1.05</td>
<td>0.48</td>
</tr>
<tr>
<td>A (3 pol) Std</td>
<td>0.48</td>
<td>2.38</td>
<td>2.20</td>
<td>3.55</td>
<td>0.51</td>
<td>0.46</td>
</tr>
<tr>
<td>B (1 pol) Mean</td>
<td>-59.97</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B (1 pol) Std</td>
<td>1.55</td>
<td>2.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C (1 pol) Mean</td>
<td>-60.12</td>
<td></td>
<td>2.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C (1 pol) Std</td>
<td>1.41</td>
<td></td>
<td>2.33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D (1 pol) Mean</td>
<td>-60.33</td>
<td></td>
<td></td>
<td>4.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D (1 pol) Std</td>
<td>2.03</td>
<td></td>
<td></td>
<td>3.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

24.0°, and the resolution in slant-range and azimuth direction is about 9.4 and 3.5 m, respectively. Figure 7.1 a and 7.1 b shows the SAR amplitude and Pauli decomposition image of the site. The central and south part of the analyzed area shows large amplitudes. The VV minus HH component (possible double-bounce scattering) in this region corresponds to urban areas, mainly consisting of residential buildings. The other margin of the area exhibits lower amplitudes and is characterized by high HH plus VV (possible surface scattering) and 2HV (possible volumetric scattering) components that mostly correspond to vegetated and agriculture areas. On the other hand, there are targets that show greater amplitudes for the HV component in urban areas, probably because target buildings were not orthogonal to each other, but rather oriented obliquely with respect to satellite illumination [Yamaguchi et al., 2011].

I performed analysis based on standard InSAR time-series processing [Ferretti et al., 2001], as shown below, and applied the method to the simultaneous processing of HH-HH, HV-HV and VV-VV interferograms (Figure 7.2). First, coherent pixel candidates were selected based on the amplitude dispersion index [Ferretti et al., 2001] for each polarization. In this study, I considered candidates of coherent pixels if either one of the indices derived from multi-polarimetric SAR data satisfied the threshold of 0.35.

Interferometric data pairs were selected based on a small baseline approach with a maximum geometrical baseline of 900 m; seven data pairs were selected at the Aomori site (Figure 7.3). I used multiple master images for selecting interferometric data pairs to increase the number of differential interferograms, which were then created using candidates of coherent pixels.

Since the SAR images used in this study have relatively long temporal and spatial baselines and
coarse spatial resolution, the data tend to be noisier. Thus, I applied spatially adaptive filtering in order to enhance the signal-to-noise ratio (SNR) [Ishitsuka et al., 2014; Ferretti et al., 2011; Goel and Adam, 2012] to consider PSs but also distributed scatterers (DSs). To this aim, statistically homogeneous pixel groups were selected for coherent candidate pixels sharing the same amplitude statistics using the Kolmogorov-Smirnov test [Kvam and Vidakovic, 2007], a non-parametric test that determines whether or not two statistical samples are drawn from same statistical distribution. I applied this statistical test to candidate pixels and pixels within a rectangular area of 14 × 20 pixels around a candidate pixel. Then, I averaged the complex data for pixels classified in the same category. Although I accounted for DSs, the processing step I used was the same as InSAR time-series analysis except this space adaptive filtering.

Subsequently, according to Equation 7.16, I simultaneously estimated an optimal weighting coefficient, as well as the surface displacement velocity and scatterer heights, by the simultaneous processing of HH-HH, HV-HV and VV-VV interferograms using the interior point algorithm [Byrd et al., 2000; Waltz et al., 2006]. I assumed that different polarimetric SAR interferograms have different scattering heights, though surface displacement velocity was assumed to be same in all polarization. This maximum likelihood estimation corresponds to PS selection in the standard InSAR time-series algorithm; coherent pixels were selected for this. The atmospheric effect sometimes cannot be modeled because of severe atmospheric disturbances during observations. Thus, in order to neglect the atmospheric effect, I estimated parameters from the difference of differential interferogram phases between neighboring pixels connected by Delaunary triangulation [Kampes, 2006]. As well as standard InSAR time-series analysis, coherent pixels are evaluated using temporal coherence $\gamma_i$ modified in terms of multi-polarimetric SAR interferogram analysis as described here:

$$\gamma_i = w_{iHH}^2 \left| \frac{1}{N_{int}} \sum_{i=1}^{N_{int}} \exp(j(\phi_{iHH} - G(m_j))) \right|$$
Figure 7.2: The flow chart used in this study. Rectangles indicate products, and circles indicate processing steps.

\[
\begin{align*}
&+ w^2_{i,HV} \left| \frac{1}{N_{int}} \sum_{i=1}^{N_{out}} \exp(j(\phi_{i,HV} - G(m_i))) \right| \\
&+ w^2_{i,VV} \left| \frac{1}{N_{int}} \sum_{i=1}^{N_{out}} \exp(j(\phi_{i,HV} - G(m_i))) \right|
\end{align*}
\]

(7.17)

with

\[
\begin{align*}
 w^2_{i,HH} &= \frac{\sigma^2_{der,i,HH}}{\sigma^2_{der,i,HH} + \sigma^2_{der,i,HV} + \sigma^2_{der,i,VV}} \\
 w^2_{i,HV} &= \frac{\sigma^2_{der,i,HV}}{\sigma^2_{der,i,HH} + \sigma^2_{der,i,HV} + \sigma^2_{der,i,VV}} \\
 w^2_{i,VV} &= \frac{\sigma^2_{der,i,VV}}{\sigma^2_{der,i,HH} + \sigma^2_{der,i,HV} + \sigma^2_{der,i,VV}}
\end{align*}
\]

(7.18)  
(7.19)  
(7.20)
Figure 7.3: Baseline conditions of SAR images used in this study. Squares indicate SAR images and lines indicate SAR data pairs.

Since both the temporal coherence function and the likelihood function measures the magnitude difference between observed and modeled phases, these two functions are maximized by the same parameters. Thus, the magnitude of decorrelation is correctly evaluated using a temporal coherence function (Equation 7.17), although I estimate parameters by maximizing the likelihood function (Equation 7.16). Eventually, in order to examine the rather precious characteristics of the algorithm, I selected coherent pixels based on a modified temporal coherence over 0.8 to account for the coherence of every polarimetric interferogram, according to Equation 7.17. Finally, the surface displacement velocity and scatterer height were estimated for each coherent pixel after phase unwrapping using the minimum cost flow algorithm [Costantini and Rosen, 1999; Werner et al., 2002] by estimating modeled parameters in a weighted least square sense. Specifically, I estimated modeled parameters and weighting coefficients by maximizing the modified version of Equation 7.16

\[
\log(p(\hat{m}, \hat{\alpha}, \hat{\beta}|\psi_{i,HH}, \psi_{i,HV}, \psi_{i,VV})) = -\frac{3N_{\text{inf}}}{2} \log \left( \hat{s}(\hat{m}, \hat{\alpha}, \hat{\beta}) \right) + N_{\text{inf}} \log \hat{\alpha} + N_{\text{inf}} \log \hat{\beta} + \text{Const} \quad (7.21)
\]

where

\[
\hat{s}(\hat{m}, \hat{\alpha}, \hat{\beta}) = (\psi_{i,HH} - \hat{G}\hat{m}_i)^T (\psi_{i,HH} - \hat{G}\hat{m}_i) + \hat{\alpha}_i^2 (\psi_{i,HV} - \hat{G}\hat{m}_i)^T (\psi_{i,HV} - \hat{G}\hat{m}_i) + \hat{\beta}_i^2 (\psi_{i,VV} - \hat{G}\hat{m}_i)^T (\psi_{i,VV} - \hat{G}\hat{m}_i) \quad (7.22)
\]

I used · for the accent in order to clarify that the values are different from that of Equation 7.16. The
Figure 7.4: Histograms of optimal weighting coefficients derived from (a) HH-HH and HV-HV interferograms and (b) HH-HH and VV-VV interferograms.

The difference between Equation 7.11 and 7.16 is that the relationship between the data kernel \( \hat{G} \) and model parameters \( \hat{m} \) is linear in Equation 7.22, because phases are unwrapped. It may be redundant that I performed maximum likelihood estimation again, one can use the optimal weighting coefficients obtained in Equation 7.16 for this processing, if Equation 7.16 is calculated pixel by pixel.

Histograms were plotted of optimal weighting coefficients between HH-HH and HV-HV interferograms (\( \alpha \) in Equation 7.9) (Figure 7.4 a), and between HH-HH and VV-VV interferograms (\( \beta \) in Equation 7.10) (Figure 7.4 b). Histograms consisted of using 3403 arcs. When the noise magnitudes of differential interferograms derived from different polarization is identical, the optimal weighting coefficient is 1. The weighting coefficient becomes smaller than 1 if the HH-HH interferograms have less noise than the interferograms obtained from an alternative polarization. From Figure 7.4a, most of the optimal weighting coefficients between the HH-HH and HV-HV interferograms were less than one and close to 0, suggesting that HH-HH interferograms were generally more weighted by a smaller noise magnitude. On the other hand, most of the optimal weighting coefficients between the HH-HH and VV-VV interferograms were almost 1 (Figure 7.4b), suggesting that the magnitude of noise in co-polarized interferograms was almost constant in the analyzed area.

### 7.5 Properties of optimal weighting coefficients

Four-component decomposition method classifies pixels in terms of a polarimetric covariance matrix decomposition, which consists of second-order statistics of the target’s polarimetric scattering vector. The covariance matrix of arbitrary statistically homogeneous pixels is defined as:

\[
Cov = \langle z \cdot z^\dagger \rangle
\]

(7.23)
where $\dagger$ is the conjugate transpose operator, and

$$z = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} \\ 2S_{HV} \\ S_{VV} \end{bmatrix}$$  \hspace{1cm} (7.24)$$

where $S_{HV}$ is the complex scattering coefficient for transmitted (V) and observed (H) polarization. It is known that the covariance matrix is a Hermitian, positive semidefinite matrix. I obtained a covariance matrix for each coherent pixel by averaging its statistically homogeneous pixels both spatially and temporally. The four-component decomposition method decomposes a polarimetric covariance matrix into four covariance models: surface scattering, double-bounce scattering, volume scattering, and helix scattering [Yamaguchi et al., 2005],

$$Cov = f_s \cdot Cov_s + f_d \cdot Cov_d + f_v \cdot Cov_v + f_h \cdot Cov_h$$  \hspace{1cm} (7.25)$$

where $f_s$, $f_d$, $f_v$, and $f_h$ are the power contributions of the respective scattering models. The method was used to model surface and double-bounce covariance matrices from the scattering matrix of a metallic flat plate and a dihedral corner reflector. Also, the volumetric covariance matrix was derived from a randomly oriented dipole with an a priori probability density function [Yamaguchi et al., 2005].

The decomposition results of coherent pixel candidates at the Aomori site, derived from four-component decomposition show that the central part of the analyzed area are characterized by volumetric scattering (red in Figure 7.5). In contrast, the northern and western areas are characterized by double-bounce scattering, and other surrounding areas are characterized by surface scattering (green and blue in Figure 7.5). No parts of the analyzed area are characterized that helix scattering is the most dominant scattering characteristics with four-component decomposition, possibly because helix scattering, if it does exist, is generally weak compared with other scattering mechanisms.

I then examined the relationships between optimal weighting coefficients with a coherence threshold exceeding 0.8 and classified by their scattering characteristics. In this study, I estimated optimal weighting coefficients between neighboring pixels (arcs), not pixel by pixel, as described in section 7.3. Thus, I assigned certain scattering characteristic to the optimal weighting coefficients, where both of the connected pixels were assigned the same category of scattering. A total of 2,143 coherent arcs were classified at the site.

Regarding weighting coefficients between HH-HH and HV-HV interferograms, histograms of both surface and double-bounce scattering have a peak around a weighting coefficient of 0, however weighting coefficients classified into double-bounce scattering were more likely between 0.2 and 1 for double-bounce scattering (Figure 7.6 Aa and 7.6 Ab). This result suggests that not only HH-HH but also HV-HV interferograms contribute to estimate surface displacements at pixels classified into double-bounce scattering. Moreover, the histogram of volumetric scattering shows that smaller percentage of pixels have weighting coefficient around 0, suggesting that more complicated scattering likely bear co-polarized components (Figure 7.6 Ac).
The blue, green and red colors indicate surface, double-bounce and volumetric scattering, respectively.

On the other hand, weighting coefficients for surface and double-bounce scattering between HH-HH and VV-VV interferograms were mostly 1 (Figure 7.6 Ba and 7.6 Bb), although those of HH-HH and HV-HV interferograms were almost 0 (Figure 7.6 Aa and 7.6 Ab). These observations mean that displacement velocities of arcs classified into surface scattering were estimated mainly from HH-HH and VV-VV interferograms, rather than HV-HV interferograms. The histogram of weighting coefficients for volumetric scattering also has a peak around weighting coefficients of 1, although weighting coefficients were more widely distributed compared with those of surface and double-bounce scattering (Figure 7.6 Bc).

In this section, I examined properties weighting coefficients with the four-components decomposition method. The comparison with Entropy-alpha decomposition [Cloude and Pottier, 1996; Cloude and Pottier, 1997], an unsupervised decomposition method based on eigenvalues and eigenvectors of polarimetric covariance matrix, is described in Ishitsuka et al. (2014c).

### 7.6 Surface displacement velocity map

I have mapped and compared surface displacement velocities derived from multi-polarimetric (i.e., HH-HH, HV-HV and VV-VV combined) and single-polarimetric (i.e., one of HH-HH, HV-HV and VV-VV) interferograms from both the Aomori and Niigata sites. The rigidity of the threshold used to map displacement is known to differ depending on the number of unknown parameters [Ishitsuka et al., 2014b]. In our processing, I estimated two parameters (displacement velocity and scatterer height) when analyzing the single-polarimetric interferograms. In contrast, I estimated six parameters (displacement velocity, scatterer heights for the three polarization states, and two weighting coeffi-
Figure 7.6: Histograms for the Aomori site showing the relationship between the decomposition result and optimal weighting coefficients between (A) HH-HH and HV-HV interferograms, and (B) HH-HH and VV-VV interferograms.

cients) when analyzing the three multi-polarimetric interferograms simultaneously. To map surface displacement with a threshold that accounts for the difference in the number of model parameters, I used the variance of the displacement velocity compensated by the number of model parameters and interferograms as the threshold to map displacement velocity [Ishitsuka et al., 2014].

As a result, 10,613 coherent pixels were mapped for the analysis of multi-polarimetric interferograms at the Aomori site, whereas 7,420 pixels were used in the HH-HH interferogram analysis, 6,377 pixels were used in the HV-HV interferogram analysis, and 7,345 pixels were used to analyze the VV-VV interferograms (Figure 7.7). The number of coherent pixels increased by 1.4–1.6 times by analyzing multi-polarimetric interferograms compared with single-polarimetric interferograms.

I compared our results with annual velocity derived from leveling survey that had been conducted by Geospatial Information Authority of Japan in November 2007, April 2008 and May 2009 around the subsidence area. For comparison of data with sufficient density, displacement velocities were derived with the threshold of 15 mm in the analysis of HV-HV interferograms and 5 mm in the analysis of other polarimetric data (Figure 7.8). Then, InSAR time-series results at leveling location were derived as the average and the standard deviation of pixels within 200 m circle area surrounding a leveling location. Our results were consistent with leveling data; standard deviation of difference between InSAR time-series results and leveling data was about 5 mm in the analysis of multi-polarimetric, HH-HH interferograms, and VV-VV interferograms, while that was about 15 mm
Figure 7.7: Surface displacement velocities [mm/year] at the Aomori site derived from (a) multi-polarimetric and (b)-(d) single-polarimetric interferograms.
Figure 7.8: Comparison between InSAR time-series results and leveling data. Bars are annual displacement velocities and error bars indicate standard deviations of the displacement velocities. The location of measurement numbers of leveling survey is in Figure 7.7.
in the analysis of HV-HV interferograms. Moreover, the standard deviations of InSAR time-series results of multi-polarimetric interferograms were minimal in most of leveling location, suggesting that the better accuracy of the analysis compared with the analysis of single-polarimetric interferograms (Figure 7.8).

7.7 Conclusions

I have proposed a method for estimating surface displacement using multi-polarimetric SAR interferograms based on PSI analysis. Since the magnitude of decorrelation-induced noise for a certain scatterer differs in polarization, this method objectively determines weighting coefficients, representing noise variance ratios, by maximizing the posterior probability distribution. The properties of optimal weighting coefficients were then examined using the four-component model decomposition method.

This method has been applied to multi-polarimetric SAR interferograms obtained from PALSAR images acquired in full polarimetric mode. From the histograms of weighting coefficients, I examined the contribution of multi-polarimetry for estimating surface displacements. The optimal weighting coefficients between HH-HH and HV-HV interferograms showed that HH-HH interferograms were less affected by decorrelation than that of HV-HV interferograms. Nevertheless, the weighting coefficients were not 0 for most pixels, meaning that the HV-HV interferograms still contribute to the estimation of surface displacement. In addition, most of the weighting coefficients between HH-HH and VV-VV interferograms were almost identical, which correspond to our understanding that the ideal PSs equally reflect the co-polarized component of polarimetry.

The objective determination of weighting coefficients allows mapping a variety of coherent pixels that are identified from (1) one polarimetry, whose weighting coefficients are far from 1, and (2) multiple polarimetry, whose weighting coefficients are almost 1. Accordingly, the number of coherent pixels that measured displacement increased 1.4–1.6 times.

Since more and more SAR satellites have operated in polarimetric mode (e.g., ALOS, Radarsat-2, TerraSAR-X), InSAR time-series analysis with polarimetry becomes realistic. Our results demonstrate that the simultaneous analysis of multi-polarimetric interferograms produces a better understanding of surface displacement phenomena.
Chapter 8

Conclusions

8.1 Conclusions

This thesis investigated InSAR time-series analysis for the monitoring of surface displacements. The main topics were the interpretation of the map of estimated surface displacement and improvement of the accuracy of the estimation of surface displacement.

Chapter 2 first reviewed InSAR and InSAR time-series analyses and examined the performance of InSAR time-series analysis using simulation data. InSAR analysis estimates surface displacements in a certain period using phase differences of two repeat-pass SAR acquisitions. High coherence is an important condition in InSAR processing, because phases are deterministic values as long as signals are coherent. However, an InSAR image is not often coherent (decorrelation) and phases become random values, because SAR data of a pixel are the sum of the signals of several scatterers. In addition to the decorrelation problem, it is known that residual phases remaining after InSAR processing make it difficult to interpret resulting phases. Besides displacement phases, phases relating to the scatterer height and atmospheric effects are main residual phases that cannot be subtracted in InSAR processing. InSAR time-series analysis has been developed to overcome the decorrelation and residual-phase problems. Even though several InSAR time-series analyses have been proposed, all algorithms focus on solving an ill-posed problem of extracting the surface displacement phase using the temporal phase trend. The chapter then examined processing steps in InSAR time-series analysis used in the thesis using simulation data. This examination quantified the estimation accuracy of parameters of interest under certain decorrelation conditions of the number of interferograms and the size of the analyzed area. The importance of coherent pixel selection in avoiding phase unwrapping error was also shown. The results of this examination revealed that the surface displacement velocity can be estimated with accuracy of millimeter order or a few centimeter even when using L-band SAR data. The next research questions are how can we use this method for displacement monitoring and how can we improve the estimation accuracy, which were addressed in the following chapters.

Chapter 3 estimated the spatiotemporal pattern of recent surface displacement in Bangkok em-
ploying InSAR time-series analysis, and characterized aquifer connectivity. Ground subsidence due to groundwater extraction has been a severe problem in this area. However, it has been reported that the groundwater level has recovered recently, and ground subsidence has thus been mitigated. As a result of processing, ground surface rebound was detected during the ALOS observation period. Since groundwater use has decreased in Bangkok owing to the regulation of groundwater pumping, and the groundwater levels of productive aquifers have been rising in the area of estimated uplift following regulation, the uplift can be attributed to an increase in the hydraulic head due to the preceding depression in pore pressure. The lateral connectivity of aquifers was then estimated using a temporal model. The discrepancy in seasonal and non-seasonal spatial distributions might be the result of lateral hydraulic connectivity within individual aquifers of the multi-aquifer system under the Bangkok plain. The properties of the aquifers and the seasonal flow of groundwater were evaluated using temporal evolutions of seasonal and long-term surface displacements.

Chapter 4 estimated the details of the surface displacement pattern around the Hatchobaru geothermal field employing InSAR time-series analysis. A geothermal field is a difficult target for InSAR time-series analysis because the dominant surface coverings are natural targets and there are fewer coherent pixels. Spatial adaptive filtering was applied in the study area and increased the number of pixels used to map surface displacement by a factor of approximately 7. Processing revealed ground subsidence at a rate of approximately 15 mm/year around the geothermal field. Examination of the temporal surface displacement pattern using statistical models inferred that ongoing surface displacement has reduced over time. This temporal trend agrees with the temporal trend of decreasing gravity measured during 1990–2003. The spatial pattern of surface displacement suggested lineaments as there are steep gradients of surface displacement; such results would be useful information for geothermal development. The results demonstrate that surface displacements obtained in time-series InSAR analysis provide a synoptic view of the area of interest that would be useful for displacement monitoring.

Chapter 5 developed a method of detecting an area damaged in a disaster from temporal changes in decorrelation. Ordinal coherence changes (apart from disaster damage) associated with vegetation were evaluated using time-series data acquired prior to the disaster. Important changes due to disaster damage were then detected by examining whether the change in coherence was statistically significant. The method was applied to the Kanto region where there was soil liquefaction following the 2011 Tohoku earthquake. The proposed method accurately detected the area of soil liquefaction.

Chapter 6 developed a method of incorporating co-polarized components of polarimetry into InSAR time-series analysis. Ideal PSs are generally artificial structures, such as residences and buildings, or bare rocks, which often correspond to odd-bounce scattering from roof or plane surfaces (single bounces), triple-bounce scattering from buildings with a courtyard (triple bounces), or even-bounce scattering from the bottom side of an artificial structure and the ground surface (double bounces). The stability of phase from these scatterers would be almost identical in co-polarized components. Application of the method to ALOS/PALSAR data acquired in full-polarimetric mode
resulted in improved accuracy of the estimated surface displacement velocity because of the greater number of interferograms used. Correspondingly, displacement areas were more clearly detected compared with the standard analysis of single polarimetry.

Chapter 7 developed a method of incorporating co- and cross-polarized components of polarimetry into InSAR time-series analysis. Since the magnitude of decorrelation is generally dependent on polarization, the proposed method estimates the surface displacement velocity by considering the decorrelation magnitude of each polarimetry. The decorrelation magnitude was obtained by estimating weighting coefficients, which are the ratio of phase noise of two different polarimetric interferograms based on maximum likelihood theory. The method was applied to HH-HH, HV-HV and VV-VV interferograms acquired by ALOS/PALSAR in full-polarimetric mode. As a result, more coherent pixels could be used to estimate surface displacement, and the spatial pattern of surface displacement was clearer compared with the standard analysis with single polarimetry. Weighting coefficients of HH-HH/HV-HV interferograms were almost zero, while those of HH-HH/VV-VV interferograms were mostly 1. This results of weighting coefficients indicate that the decorrelation magnitudes of co-polarized interferograms were almost the same, whereas cross-polarized interferograms suffered more from decorrelation compared with co-polarized interferograms.

In summary, the thesis examined and improved the performance of InSAR time-series analysis in terms of interpretation and accuracy. Results showed the advantage of the method in terms of obtaining accurate time-series displacements with high spatial density and were used to improve this advantage. The results are thus useful for operational displacement monitoring and understanding the Earth’s dynamics.

### 8.2 Future direction

InSAR time-series analysis enables to estimate both spatial and temporal pattern of surface displacement with fine accuracy. This advantage is useful for understanding complex displacement mechanism. For instance, as shown in chapter 3 and 4, the spatio-temporal analysis of surface displacements would reveal the mechanisms of surface displacements that has not been detected before.

The first operational SAR satellite has been launched in about 25 years ago. Since then, SAR data has been continuously acquired in the world. The analysis and interpretation with long-term SAR data would be promising for revealing the evolution of a variety of surface displacements.

Although we detected spatio-temporal pattern of ground uplift in the Bangkok plain, the evolution from subsidence to uplift has not been intensively analyzed by InSAR analysis, which would be important to understand mechanism of ground uplift after groundwater extraction. Comparison with geological and other geophysical data would be helpful to infer undergoing physical phenomena.

InSAR time-series analysis using polarimetry would be promising techniques for increasing the estimation accuracy and the number of coherent pixels to measure surface displacements. More inten-
sive studies is necessary to establish the analysis. Scatterer heights of HH and VV polarimetry were shown to be different in most of pixels in chapter 6. More detail examination would clarify properties of targets that has the same height and different height, which would be useful for interpretation of scatterers we measure.

InSAR time-series analysis using co- and cross-polarized components of polarimetry was shown in chapter 7 assuming Gaussian distribution for interferometric phase. This assumption is valid for a coherent pixel, whereas the distribution of interferometric phase becomes uniform as the coherence decreases. Thus, more general assumption could be incorporated into the analysis. One notion is to assume the complex Gaussian distribution for a complex signal including both amplitude and phase.

Recently, more and more SAR satellite has been launched and operated. Correspondingly, the temporal resolution increases as the acquisition span becomes shorter. The instantaneous displacements such as the trigger of landslide occurrence might be detected by increasing the temporal resolution. Moreover, the studies for SAR acquisition mode is now one of the active research fields. Although this thesis used SAR data acquired with the strip map mode, data obtained by new acquisition mode would improve both spatial and temporal resolution of SAR data. These developments for SAR acquisitions definitely improve the monitoring capability of InSAR time-series analysis.
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Ph.D. Dissertation

Synthetic Aperture Radar Interferometry Time-series
for Surface Displacement Monitoring:
Data interpretation and improvement in accuracy

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