Numerical data for the generalized slip-flow theory

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In this article, we summarize the numerical data required in the application of the generalized slip-flow theory and explain the contents of the files `knfunc.dat` and `vdf_figures.pdf` uploaded at the same address (http://hdl.handle.net/2433/199811) as this article in Kyoto University Research Information Repository.

A systematic asymptotic theory of the Boltzmann equation [1, 2] provides the complete framework to describe the behavior of a gas for small Knudsen numbers. In the linear case of this theory (we call it the generalized slip-flow theory), where the deviation from an equilibrium state at rest is assumed to be small, the fluid-dynamic-type equations and their appropriate slip/jump boundary conditions which describe the overall behavior of the gas and formulas of the Knudsen-layer correction to the overall solution are given for steady problems up to the second order in the Knudsen number expansion [1, 2]. Also, for unsteady problems, they have been obtained up to the first order by Sone [2]. Recently, the further extension of the generalized slip-flow theory to unsteady problems has been carried out in [3], and the fluid-dynamic-type equations, their appropriate boundary conditions, and correction formulas have been obtained for unsteady problems up to the second order.

In order to apply the results obtained in [3], some numerical data are required. Recently, the data have been completed for a hard-sphere gas under the diffuse reflection boundary condition. Since the data are scattered over the papers [3, 4, 5, 6, 7, 8, 9], we summarize them for easy reference. In applying the theory to the specific problems, one will use the fluid-dynamic-type equations [(41), (42), (44), (46), and (48) of [3]], their slip/jump boundary conditions [(43), (45), and (47) of [3]], and formulas of the Knudsen-layer correction [(28), (29), (37), (38), (39), and (40) of [3]]. Here, the γ's in the fluid-dynamic-type equations are respectively $\gamma_1 = 1.270042427$, $\gamma_2 = 1.922284066$, $\gamma_3 = 1.947906335$, $\gamma_6 = 1.419423836$, $\gamma_{10} = 1.63607346$, and $\gamma_{11} = 2.7931173$. The slip/jump coefficients in the boundary conditions are shown in Table 1. The Knudsen-layer functions in the correction formulas are shown in Tables 2 and 3. For the sake of the coherence of data, the data that have already been

Table 1: Slip/jump coefficients for the Boltzmann equation for a hard-sphere gas under the diffuse reflection condition.

$b_1^{(1)}$	1.25395	$c_1^{(0)}$	2.40014
$b_2^{(1)}$	0.64642	$c_2^{(0)}$	-0.4993
$b_3^{(1)}$	-1.5846	$c_3^{(0)}$	0.00874
$b_4^{(1)}$	-0.90393	$c_4^{(0)}$	4.6180
$b_5^{(1)}$	-0.66012	$c_5^{(0)}$	0.45957
$b_6^{(1)}$	0.24381	$c_6^{(0)}$	-3.1800
$b_7^{(1)}$	0.44728	$\int_0^\infty Y_1^{(1)} dz$	-0.21369
$b_8^{(1)}$	-0.23353	$\int_0^\infty Y_2^{(1)} dz$	-0.47816

obtained in the previous works [4, 5, 6, 7] were computed again by the method in [8] with the standard grid (S1,M1).

For reference, we show the correspondence of the notation in [3, 8, 9] to that in [1, 2], which one may often refer to, in Table 4. Note that the functions corresponding to $H_\beta^{(1)}$ ($\beta = 3, 4, \ldots, 8$) as well as the data of $(c_6^{(0)}, \Omega_6^{(0)}, \Theta_6^{(0)})$ are not found in [1, 2]. In the case of the diffuse reflection condition, the component half-space problem determining the data of $(c_5^{(0)}, \Omega_5^{(0)}, \Theta_5^{(0)})$ is equivalent to that determining the data of $(4/3)(d_6, \Omega_6, \Theta_6)$ in [2, 1] for the complete condensation condition.

`knfunc.dat` is a data file written in the ascii format for those who need the values of the Knudsen-layer functions at much more points than those tabulated in Tables 2 and 3. The data of the position η or one of the Knudsen-layer functions are recorded in each column, where the correspondence is shown in Table 5.

In `vdf_figures.pdf`, we show the velocity distribution functions $\phi_1 E$, $\phi_2 E$, $\phi_3 E$, $\phi_5 E$, $\phi_6 E$, $\psi_1 E$, $\psi_2 E$, $\psi_3 E$, $\psi_4 E$, $\bar{\phi}_4 E$, $\bar{\psi}_5 E$, $\bar{\psi}_6 E$, $\bar{\psi}_7 E$, ϕE, $\psi_A E$, and $\psi_B E$, respectively (Figs. 1–16). Here, instead of $|\boldsymbol{\zeta}|$ and ζ_n in [3], $\zeta = |\boldsymbol{\zeta}|$ and $\mu = \zeta_n/|\boldsymbol{\zeta}|$ are used as molecular velocity variables, and $E = \pi^{-3/2}\exp(-\zeta^2)$. The correspondence of the present notation, the same as in [8, 9], to that in [3] is shown in Table 6. Here, ϕ_1 is the solution of the temperature jump problem and was obtained in [4]; ψ_1 and ψ_2 are respectively the solution of the shear-slip and thermal-slip problems and were obtained in [5]; ψ_3 is the solution of the thermal-stress slip problem and was obtained in [6]; ϕ_6 is the solution of the problem of thermal inertia due to time evolution and was obtained in [7]; ϕ_2, ϕ_3, ϕ_5, and ψ_4 were obtained in [8]; and ϕ, ψ_A, ψ_B, $\bar{\phi}_4$, $\bar{\psi}_5$, $\bar{\psi}_6$, and $\bar{\psi}_7$ were obtained in [9]. For ϕ_1, ϕ_2, ϕ_3, ϕ_5, ϕ_6, ψ_1, ψ_2, ψ_3, ψ_4, $\bar{\phi}_4$, $\bar{\psi}_6$, and $\bar{\psi}_7$, the observed features are similar to those for $\bar{\psi}_5$ explained in [9], while for ϕ and ψ_B, the observed features are similar to those for ψ_A explained in [9]. As shown in [3], the Knudsen-layer functions $\Omega_\alpha^{(0)}$, $\Theta_\alpha^{(0)}$ ($\alpha = 1, 2, \ldots, 6$) are obtained as the moment of $\phi_\alpha^{(0)}$, while $Y_\beta^{(1)}$, $H_\beta^{(1)}$ ($\beta = 1, 2, \ldots, 8$) are obtained as the moment of $\phi_\beta^{(1)}$ (see also Table 6).

Table 2: Knudsen-layer functions $\Omega_\alpha^{(0)}$, $\Theta_\alpha^{(0)}$ ($\alpha = 1, \ldots, 6$), and $\int_\eta^\infty (\Omega_1^{(0)} + \Theta_1^{(0)}) dz$ for the Boltzmann equation for a hard-sphere gas under the diffuse reflection condition.

η	$\Omega_1^{(0)}$	$-\Theta_1^{(0)}$	$-\Omega_2^{(0)}$	$\Theta_2^{(0)}$	$-\Omega_3^{(0)}$	$-\Theta_3^{(0)}$	$\Omega_4^{(0)}$	$-\Theta_4^{(0)}$	$-\Omega_5^{(0)}$	$-\Theta_5^{(0)}$	$-\Omega_6^{(0)}$	$\Theta_6^{(0)}$	$-\int_\eta^\infty (\Omega_1^{(0)} + \Theta_1^{(0)}) dz$
0.00000	0.51641	0.73783	0.07965	0.07610	0.31803	0.04368	0.55644	1.64676	0.65823	0.31749	1.49067	2.26004	0.19433
0.02348	0.47095	0.67662	0.07508	0.07019	0.28692	0.04108	0.49619	1.49739	0.60555	0.29707	1.41236	2.13212	0.18934
0.05165	0.43818	0.63144	0.07153	0.06590	0.26406	0.03872	0.45041	1.38089	0.56413	0.28044	1.34961	2.02891	0.18373
0.09881	0.39827	0.57556	0.06693	0.06064	0.23601	0.03547	0.39296	1.23232	0.51105	0.25858	1.26730	1.89339	0.17501
0.15009	0.36527	0.52873	0.06290	0.05623	0.21276	0.03253	0.34442	1.10501	0.46540	0.23928	1.19448	1.77360	0.16628
0.19315	0.34250	0.49610	0.05999	0.05316	0.19673	0.03038	0.31052	1.01520	0.43311	0.22536	1.14164	1.68687	0.15946
0.27263	0.30805	0.44627	0.05539	0.04846	0.17258	0.02696	0.25885	0.87696	0.38327	0.20339	1.05755	1.54932	0.14788
0.30336	0.29664	0.42966	0.05380	0.04689	0.16463	0.02578	0.24169	0.83072	0.36657	0.19589	1.02857	1.50210	0.14371
0.33632	0.28533	0.41313	0.05220	0.04532	0.15677	0.02460	0.22469	0.78471	0.34992	0.18834	0.99926	1.45444	0.13942
0.40911	0.26314	0.38053	0.04897	0.04220	0.14146	0.02224	0.19136	0.69409	0.31708	0.17323	0.94000	1.35842	0.13050
0.58327	0.22112	0.31832	0.04252	0.03618	0.11301	0.01763	0.12898	0.52281	0.25470	0.14363	0.82115	1.16766	0.11190
0.79673	0.18323	0.26175	0.03628	0.03059	0.08822	0.01340	0.07444	0.37133	0.19900	0.11603	0.70582	0.98535	0.09324
0.98271	0.15789	0.22378	0.03187	0.02674	0.07228	0.01059	0.03963	0.27370	0.16263	0.09728	0.62394	0.85803	0.07986
1.19037	0.13528	0.18989	0.02776	0.02323	0.05863	0.00814	0.01031	0.19072	0.13118	0.08051	0.54734	0.74081	0.06739
1.41884	0.11533	0.16007	0.02400	0.02007	0.04715	0.00606	-0.01360	0.12222	0.10458	0.06582	0.47680	0.63474	0.05608
1.58214	0.10344	0.14238	0.02170	0.01814	0.04061	0.00488	-0.02669	0.08419	0.08938	0.05719	0.43335	0.57044	0.04926
1.84260	0.08760	0.11895	0.01856	0.01554	0.03230	0.00340	-0.04235	0.03766	0.07008	0.04592	0.37368	0.48356	0.04015
2.02594	0.07826	0.10526	0.01667	0.01399	0.02764	0.00259	-0.05037	0.01298	0.05931	0.03945	0.33754	0.43184	0.03481
2.51495	0.05872	0.07700	0.01263	0.01067	0.01860	0.00109	-0.06324	-0.03017	0.03855	0.02654	0.25944	0.32267	0.02389
2.61756	0.05540	0.07227	0.01193	0.01009	0.01717	0.00087	-0.06476	-0.03612	0.03530	0.02445	0.24580	0.30402	0.02208
3.04221	0.04379	0.05592	0.00947	0.00807	0.01245	0.00019	-0.06800	-0.05292	0.02469	0.01748	0.19729	0.23879	0.01598
3.26234	0.03889	0.04913	0.00842	0.00720	0.01059	-0.00004	-0.06820	-0.05785	0.02060	0.01472	0.17638	0.21126	0.01353
3.48717	0.03451	0.04314	0.00748	0.00642	0.00901	-0.00022	-0.06766	-0.06096	0.01716	0.01236	0.15750	0.18674	0.01141
3.94956	0.02713	0.03321	0.00589	0.00510	0.00653	-0.00045	-0.06482	-0.06301	0.01187	0.00865	0.12519	0.14556	0.00805
4.06758	0.02553	0.03110	0.00554	0.00481	0.00603	-0.00049	-0.06383	-0.06285	0.01082	0.00790	0.11813	0.13672	0.00736
4.91724	0.01667	0.01961	0.00362	0.00319	0.00345	-0.00059	-0.05503	-0.05702	0.00565	0.00412	0.07827	0.08787	0.00386
5.04166	0.01567	0.01836	0.00340	0.00300	0.00319	-0.00058	-0.05361	-0.05575	0.00515	0.00374	0.07375	0.08246	0.00351
6.06059	0.00956	0.01081	0.00207	0.00186	0.00172	-0.00050	-0.04205	-0.04428	0.00245	0.00171	0.04556	0.04948	0.00160
7.11725	0.00580	0.00636	0.00126	0.00114	0.00094	-0.00038	-0.03144	-0.03298	0.00118	0.00075	0.02789	0.02956	0.00069
8.07087	0.00372	0.00399	0.00081	0.00074	0.00056	-0.00028	-0.02364	-0.02459	0.00063	0.00034	0.01801	0.01876	0.00030
10.06348	0.00149	0.00155	0.00032	0.00030	0.00020	-0.00013	-0.01245	-0.01269	0.00018	0.00005	0.00732	0.00742	0.00004
13.81166	0.00028	0.00028	0.00006	0.00006	0.00003	-0.00003	-0.00338	-0.00335	0.00002	-0.00001	0.00139	0.00137	-0.00001
15.00856	0.00017	0.00016	0.00004	0.00003	0.00002	-0.00002	-0.00220	-0.00216	0.00001	-0.00001	0.00083	0.00081	-0.00001
20.05147	0.00002	0.00002	0.00000	0.00000	0.00000	0.00000	-0.00034	-0.00033	0.00000	0.00000	0.00009	0.00009	0.00000
25.14669	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.00005	-0.00005	0.00000	0.00000	0.00001	0.00001	0.00000
30.05092	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.00001	-0.00001	0.00000	0.00000	0.00000	0.00000	0.00000
35.11181	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 3: Knudsen-layer functions $Y_\beta^{(1)}$, $H_\beta^{(1)}$ ($\beta = 1,\ldots,8$), $\int_\eta^\infty Y_p^{(1)} dz$, and $\int_\eta^\infty H_p^{(1)} dz$ ($p = 1,2$) for the Boltzmann equation for a hard-sphere gas under the diffuse reflection condition.

η	$-Y_1^{(1)}$	$H_1^{(1)}$	$-Y_2^{(1)}$	$H_2^{(1)}$	$Y_3^{(1)}$	$-H_3^{(1)}$	$Y_4^{(1)}$	$-H_4^{(1)}$	$Y_5^{(1)}$	$-H_5^{(1)}$
0.00000	0.34516	0.13382	0.44515	1.38951	0.77230	1.86180	0.62894	0.56938	0.89334	1.09051
0.02348	0.30638	0.12693	0.42003	1.29424	0.72481	1.75797	0.58003	0.53685	0.84419	1.03017
0.05165	0.27874	0.12131	0.39951	1.21801	0.68673	1.66972	0.54166	0.51032	0.80432	0.98148
0.09881	0.24556	0.11380	0.37231	1.11903	0.63691	1.55078	0.49254	0.47522	0.75158	0.91763
0.15009	0.21864	0.10703	0.34807	1.03273	0.59302	1.44380	0.45030	0.44399	0.70457	0.86124
0.19315	0.20038	0.10204	0.33041	0.97100	0.56132	1.36566	0.42042	0.42128	0.67030	0.82045
0.27263	0.17332	0.09399	0.30226	0.87451	0.51119	1.24105	0.37426	0.38510	0.61554	0.75576
0.30336	0.16454	0.09119	0.29255	0.84179	0.49402	1.19817	0.35876	0.37265	0.59662	0.73353
0.33632	0.15592	0.08834	0.28274	0.80901	0.47671	1.15489	0.34332	0.36006	0.57748	0.71110
0.40911	0.13930	0.08255	0.26293	0.74370	0.44194	1.06777	0.31279	0.33467	0.53874	0.66587
0.58327	0.10903	0.07081	0.22343	0.61705	0.37323	0.89561	0.25459	0.28411	0.46115	0.57572
0.79673	0.08336	0.05936	0.18563	0.50032	0.30822	0.73339	0.20221	0.23582	0.38637	0.48902
0.98271	0.06724	0.05126	0.15929	0.42163	0.26333	0.62219	0.16769	0.20222	0.33389	0.42799
1.19037	0.05370	0.04374	0.13517	0.35153	0.22251	0.52189	0.13752	0.17148	0.28550	0.37133
1.41884	0.04251	0.03692	0.11351	0.29027	0.18612	0.43328	0.11167	0.14392	0.24175	0.31957
1.58214	0.03622	0.03279	0.10050	0.25425	0.16437	0.38076	0.09672	0.12738	0.21530	0.28793
1.84260	0.02834	0.02724	0.08312	0.20709	0.13546	0.31152	0.07747	0.10530	0.17973	0.24482
2.02594	0.02400	0.02395	0.07292	0.17993	0.11855	0.27138	0.06657	0.09234	0.15869	0.21893
2.51495	0.01569	0.01711	0.05184	0.12517	0.08384	0.18987	0.04506	0.06560	0.11479	0.16369
2.61756	0.01440	0.01597	0.04832	0.11621	0.07807	0.17645	0.04162	0.06114	0.10739	0.15417
3.04221	0.01017	0.01202	0.03626	0.08594	0.05837	0.13096	0.03014	0.04586	0.08182	0.12065
3.26234	0.00854	0.01039	0.03132	0.07373	0.05032	0.11252	0.02560	0.03959	0.07121	0.10643
3.48717	0.00716	0.00896	0.02699	0.06316	0.04330	0.09654	0.02171	0.03412	0.06188	0.09371
3.94956	0.00503	0.00664	0.01996	0.04620	0.03191	0.07077	0.01558	0.02522	0.04651	0.07230
4.06758	0.00461	0.00615	0.01849	0.04269	0.02955	0.06544	0.01433	0.02336	0.04328	0.06769
4.91724	0.00248	0.00358	0.01074	0.02444	0.01708	0.03758	0.00795	0.01356	0.02592	0.04231
5.04166	0.00227	0.00331	0.00993	0.02255	0.01578	0.03469	0.00731	0.01254	0.02407	0.03952
6.06059	0.00111	0.00175	0.00525	0.01179	0.00830	0.01817	0.00370	0.00662	0.01321	0.02264
7.11725	0.00055	0.00091	0.00274	0.00611	0.00431	0.00942	0.00186	0.00345	0.00716	0.01276
8.07087	0.00029	0.00051	0.00153	0.00340	0.00241	0.00526	0.00101	0.00193	0.00415	0.00762
10.06348	0.00008	0.00015	0.00046	0.00103	0.00072	0.00159	0.00029	0.00058	0.00135	0.00260
13.81166	0.00001	0.00002	0.00005	0.00011	0.00008	0.00017	0.00003	0.00006	0.00017	0.00035
15.00856	0.00000	0.00001	0.00003	0.00006	0.00004	0.00009	0.00002	0.00003	0.00009	0.00018
20.05147	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001
25.14669	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
30.05092	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
35.11181	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 3: (continued from the previous page)

η	$Y_6^{(1)}$	$-H_6^{(1)}$	$-Y_7^{(1)}$	$-H_7^{(1)}$	$Y_8^{(1)}$	$-H_8^{(1)}$	$-\int_\eta^\infty Y_1^{(1)} dz$	$-\int_\eta^\infty Y_2^{(1)} dz$	$\int_\eta^\infty H_1^{(1)} dz$	$\int_\eta^\infty H_2^{(1)} dz$
0.00000	0.26440	0.52114	0.13408	0.55360	0.35551	0.82712	0.21369	0.47816	0.15297	1.27656
0.02348	0.25697	0.49034	0.12204	0.47299	0.36279	0.85993	0.20611	0.46804	0.14991	1.24519
0.05165	0.24826	0.46490	0.11350	0.41545	0.36798	0.88355	0.19789	0.45650	0.14642	1.20984
0.09881	0.23478	0.43116	0.10319	0.34719	0.37368	0.90999	0.18557	0.43833	0.14088	1.15484
0.15009	0.22146	0.40119	0.09471	0.29286	0.37744	0.92837	0.17370	0.41988	0.13523	1.09974
0.19315	0.21118	0.37946	0.08886	0.25677	0.37929	0.93838	0.16469	0.40528	0.13073	1.05662
0.27263	0.19403	0.34503	0.08000	0.20477	0.38048	0.94786	0.14989	0.38017	0.12295	0.98342
0.30336	0.18795	0.33322	0.07705	0.18834	0.38032	0.94915	0.14470	0.37103	0.12010	0.95705
0.33632	0.18172	0.32133	0.07413	0.17248	0.37985	0.94935	0.13942	0.36155	0.11714	0.92985
0.40911	0.16896	0.29743	0.06838	0.14267	0.37784	0.94624	0.12869	0.34171	0.11093	0.87339
0.58327	0.14297	0.25031	0.05739	0.09170	0.36901	0.92453	0.10723	0.29949	0.09761	0.75542
0.79673	0.11774	0.20590	0.04734	0.05289	0.35324	0.88155	0.08686	0.25600	0.08376	0.63676
0.98271	0.10013	0.17539	0.04054	0.03135	0.33694	0.83648	0.07293	0.22401	0.07350	0.55130
1.19037	0.08407	0.14777	0.03442	0.01549	0.31729	0.78240	0.06044	0.19352	0.06366	0.47130
1.41884	0.06978	0.12326	0.02899	0.00439	0.29495	0.72169	0.04951	0.16519	0.05447	0.39826
1.58214	0.06127	0.10867	0.02574	-0.00085	0.27897	0.67881	0.04310	0.14774	0.04879	0.35387
1.84260	0.05003	0.08934	0.02142	-0.00616	0.25400	0.61276	0.03474	0.12391	0.04099	0.29403
2.02594	0.04351	0.07807	0.01887	-0.00836	0.23705	0.56858	0.02995	0.10963	0.03631	0.25862
2.51495	0.03027	0.05505	0.01361	-0.01064	0.19528	0.46183	0.02041	0.07945	0.02637	0.18493
2.61756	0.02809	0.05123	0.01272	-0.01071	0.18722	0.44156	0.01887	0.07431	0.02467	0.17255
3.04221	0.02073	0.03824	0.00967	-0.01023	0.15655	0.36544	0.01371	0.05648	0.01877	0.12999
3.26234	0.01776	0.03294	0.00841	-0.00969	0.14234	0.33068	0.01166	0.04906	0.01631	0.11245
3.48717	0.01519	0.02833	0.00730	-0.00905	0.12897	0.29825	0.00990	0.04252	0.01414	0.09709
3.94956	0.01106	0.02086	0.00547	-0.00762	0.10486	0.24051	0.00711	0.03175	0.01056	0.07203
4.06758	0.01021	0.01931	0.00509	-0.00726	0.09939	0.22753	0.00654	0.02948	0.00981	0.06678
4.91724	0.00579	0.01116	0.00303	-0.00494	0.06708	0.15180	0.00363	0.01737	0.00578	0.03902
5.04166	0.00534	0.01031	0.00281	-0.00465	0.06326	0.14297	0.00334	0.01608	0.00535	0.03609
6.06059	0.00276	0.00543	0.00153	-0.00277	0.03888	0.08708	0.00169	0.00861	0.00286	0.01921
7.11725	0.00141	0.00283	0.00082	-0.00158	0.02321	0.05169	0.00085	0.00454	0.00150	0.01009
8.07087	0.00078	0.00158	0.00047	-0.00094	0.01447	0.03212	0.00046	0.00256	0.00085	0.00568
10.06348	0.00023	0.00048	0.00015	-0.00031	0.00530	0.01175	0.00013	0.00078	0.00026	0.00174
13.81166	0.00002	0.00005	0.00002	-0.00004	0.00077	0.00172	0.00001	0.00009	0.00003	0.00020
15.00856	0.00001	0.00003	0.00001	-0.00002	0.00042	0.00093	0.00001	0.00004	0.00001	0.00010
20.05147	0.00000	0.00000	0.00000	0.00000	0.00003	0.00007	0.00000	0.00000	0.00000	0.00001
25.14669	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
30.05092	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
35.11181	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 4: Correspondence of the notation to [1, 2].

present notation	notation in [1, 2]	present notation	notation in [1, 2]
$(c_1^{(0)}, \Omega_1^{(0)}, \Theta_1^{(0)})$	$(d_1, \Omega_1, \Theta_1)$	$(b_3^{(1)}, Y_3^{(1)})$	$d_1(K_1, \frac{1}{2}Y_1) - (a_4, Y_{a4})$
$(c_2^{(0)}, \Omega_2^{(0)}, \Theta_2^{(0)})$	$-(d_3, \Omega_3, \Theta_3)$	$(b_4^{(1)}, Y_4^{(1)})$	$-(a_1, Y_{a1})$
$(c_3^{(0)}, \Omega_3^{(0)}, \Theta_3^{(0)})$	$(d_4, \Omega_4, \Theta_4)$	$(b_5^{(1)}, Y_5^{(1)})$	$-(a_2, Y_{a2})$
$(c_4^{(0)}, \Omega_4^{(0)}, \Theta_4^{(0)})$	$(d_5, \Omega_5, \Theta_5)$	$(b_6^{(1)}, Y_6^{(1)})$	$-(a_3, Y_{a3})$
$(b_1^{(1)}, Y_1^{(1)}, H_1^{(1)})$	$(-k_0, -Y_0, H_A)$	$(b_7^{(1)}, Y_7^{(1)})$	$d_1(K_1, \frac{1}{2}Y_1) - (a_6, Y_{a6})$
$(b_2^{(1)}, Y_2^{(1)}, H_2^{(1)})$	$(-K_1, -\frac{1}{2}Y_1, H_B)$	$(b_8^{(1)}, Y_8^{(1)})$	$-(a_5, Y_{a5})$

Table 5: The list of the quantities recorded on the n-th column of `knfunc.dat`.

n		n		n		n	
1	η	10	$\Theta_3^{(0)}$	19	$Y_6^{(1)}$	28	$H_7^{(1)}$
2	$\Omega_1^{(0)}$	11	$\Theta_4^{(0)}$	20	$Y_7^{(1)}$	29	$H_8^{(1)}$
3	$\Omega_2^{(0)}$	12	$\Theta_5^{(0)}$	21	$Y_8^{(1)}$	30	$\int_\eta^\infty [\Omega_1^{(0)}(z) + \Theta_1^{(0)}(z)]dz$
4	$\Omega_3^{(0)}$	13	$\Theta_6^{(0)}$	22	$H_1^{(1)}$	31	$\int_\eta^\infty Y_1^{(1)}(z)dz$
5	$\Omega_4^{(0)}$	14	$Y_1^{(1)}$	23	$H_2^{(1)}$	32	$\int_\eta^\infty Y_2^{(1)}(z)dz$
6	$\Omega_5^{(0)}$	15	$Y_2^{(1)}$	24	$H_3^{(1)}$	33	$\int_\eta^\infty H_1^{(1)}(z)dz$
7	$\Omega_6^{(0)}$	16	$Y_3^{(1)}$	25	$H_4^{(1)}$	34	$\int_\eta^\infty H_2^{(1)}(z)dz$
8	$\Theta_1^{(0)}$	17	$Y_4^{(1)}$	26	$H_5^{(1)}$		
9	$\Theta_2^{(0)}$	18	$Y_5^{(1)}$	27	$H_6^{(1)}$		

Table 6: Correspondence of the notation for the velocity distribution functions to that in [3].

present notation	notation in [3]	present notation	notation in [3]
ϕ_1	$\phi_1^{(0)}$	ψ_2	$\phi_2^{(1)}$
ϕ_2	$\phi_2^{(0)}$	ψ_3	$\phi_3^{(1)}$
ϕ_3	$\phi_3^{(0)}$	ψ_4	$\phi_4^{(1)}$
$\bar{\phi}_4 + \phi$	$\phi_4^{(0)}$	$\bar{\psi}_5 + \frac{1}{2}\psi_A$	$\phi_5^{(1)}$
ϕ_5	$\phi_5^{(0)}$	$\bar{\psi}_6 + \frac{1}{2}\psi_A$	$\phi_6^{(1)}$
ϕ_6	$\phi_6^{(0)}$	$\bar{\psi}_7 + \frac{1}{2}\psi_B$	$\phi_7^{(1)}$
ψ_1	$\phi_1^{(1)}$	$\frac{1}{2}\psi_B$	$\phi_8^{(1)}$

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