# An $\mathrm{X}_{0}$ shim coil for precise magic-angle <br> adjustment 

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#### Abstract

A new method for precise setting of the spinning angle to the magic angle by using a saddle coil is described. The coil, which is referred to as an $\mathrm{X}_{0}$ shim coil, is wound to produce a uniform static magnetic field $B_{x}$ perpendicular to the main magnetic field $B_{0}$. The magnetic field felt by a sample is a vector sum of the main field $B_{0}$ and the transverse field $B_{x}$ produced by the $\mathrm{X}_{0}$ shim coil. Hence the angle between the spinner axis and the effective magnetic field can be controlled by current $I$ supplied to the $\mathrm{X}_{0}$ shim coil, leading to precise angle adjustment without backlash accompanied with a mechanical system conventionally used. It is shown that the angle range achieved is $\pm 0.05^{\circ}$ for $I= \pm 5 \mathrm{~A}$ at $B_{0}=7 \mathrm{~T}$.


Key words: $\mathrm{X}_{0}$ shim coil, MAS, high resolution solid state NMR

[^0]
## 1 Introduction

In high-resolution solid-state NMR, magic angle spinning (MAS)[1] is an indispensable technique to remove anisotropic interactions. Under MAS, the anisotropic spin interactions such as the dipolar and chemical shielding interactions are modulated by sample spinning, and their secular parts are scaled by $\frac{1}{2}\left(3 \cos ^{2} \theta-1\right)$, where $\theta$ is the angle between the spinner axis and the external main magnetic field $B_{0}$. By setting $\theta$ to the "magic angle" $\theta_{\mathrm{MA}}=\arccos \frac{1}{\sqrt{3}} \approx 54.74^{\circ}$, one can remove the secular part of an anisotropic interaction. Hereafter, we refer $\theta$ to as the spinning angle.

Conventionally, the spinning angle $\theta$ is adjusted by a mechanical gear system, and, with the backlash of a gear, precise adjustment is difficult. Small deviation from the exact magic angle may be appreciable in some experiments. A remarkable case is ST-MAS for half-integer quadrupolar nuclei. It was shown that the angle deviation over $0.008^{\circ}$ would bring appreciable broadening to the ${ }^{23} \mathrm{Na}$ ST-MAS line shape of $\mathrm{Na}_{2} \mathrm{SO}_{4}$, whose quadrupoler coupling constant is $2.560 \mathrm{MHz}[2]$. Fine setting of the spinning angle by a mechanical way is also a troublesome process to observe a 1D spectrum of a nuclear which has a large anisotropic interaction, for example, ${ }^{2} \mathrm{H}$ with a large quadrupole interaction[3] and ${ }^{195} \mathrm{Pt}$ with a large chemical shift anisotropy[4]. Hence the precise setting of the spinning angle by a non-mechanical way should be secured in many situations of solid state NMR.

In this work, we develop a new method for precise setting of the spinning angle, that is, tilting the static magnetic field felt by a sample by applying an additional static field perpendicular to the main magnetic field $B_{0}$. Throughout this work, we define the $z$ axis parallel to $B_{0}$, and the $x$-axial additional field vertical to $B_{0}$ is hence written as $B_{x}$. It's produced by a saddle coil attached to a MAS probe, which we shall refer to as an $X_{0}$ shim coil[5].

In this work, we estimate spacial inhomogeneity of $B_{x}$ and also an additional "redundant" magnetic field along the $z$ axis $\left(B_{z}\right)$ produced by the $\mathrm{X}_{0}$ shim coil. The geometry and the size of the $\mathrm{X}_{0}$ shim coil are then determined to keep line broadening caused by $B_{x}$ and $B_{z}$ less than 0.05 ppm at $B_{0}=7 \mathrm{~T}$. As for the range of adjustable angle achieved by the $\mathrm{X}_{0}$ shim coil, we thought ca. $\pm 0.05^{\circ}$ would be enough, as we are going to use the classical gear system for coarse adjustment in conjunction with the $\mathrm{X}_{0}$ shim coil. With the coil dimensions given in this work, we show that the current for the $\mathrm{X}_{0}$ shim coil to achieve the range of $\pm 0.05^{\circ}$ for $B_{0}=7 \mathrm{~T}$ becomes $\pm 5 \mathrm{~A}$. Application of $\pm 5 \mathrm{~A}$ is feasible, however, the current would bring some practical problems due to Joule heat and Lorentz force. We show that temperature rising due to the Joule heat can be suppressed by air cooling and further by turning off the current during the relaxation delay of a NMR pulse sequence. For the Lorentz force, we show that the tilting of the $\mathrm{X}_{0}$ shim coil by the Lorentz force can be reduced largely by fabricating the $\mathrm{X}_{0}$ shim coil on a firmly built and fixed mounting retainer, which is used as a cap for a MAS probe. We further
show that accumulation of residual slight tilting of the $\mathrm{X}_{0}$ shim coil during repeated signal acquisition can be avoided by applying opposite current after each signal acquisition.

## 2 The $\mathrm{X}_{0}$ shim coil

In section 2.1, the configuration of the $\mathrm{X}_{0}$ shim coil is illustrated to define the effective field and its tilt angle from the static field. In section 2.2, for an ideal one-turn $\mathrm{X}_{0}$ shim coil, we derive equations for the magnetic field produced at a general point. In section 2.3, its optimum geometry is determined by examining the magnetic fields at the position of the sample. In section 2.4, a 50 -turn $\mathrm{X}_{0}$ shim coil actually used is examined, and in section 2.5 , the performance of the $\mathrm{X}_{0}$ shim coil is discussed. Finally in section 2.6, we examine the practical problems due to the Joule heat and the Lorentz force.

### 2.1 The configuration of the $X_{0}$ shim coil

Fig. 1a illustrates the configuration of the $\mathrm{X}_{0}$ shim coil. In the figure, $2 a, 2 l$, and $2 \alpha$ designate the diameter, the height, and the gap angle between two opposite arcs, respectively. The angle between the spinner axis in the $x z$ plane and the static field along the $z$ axis is written as $\theta$. In Fig. 1b, we define two angles, $\varepsilon$ and $\Delta \theta$. The angle $\varepsilon$ in the $x z$ plane represents misalignment of the $\mathrm{X}_{0}$ shim coil from the $x$ axis, which may be written as

$$
\begin{equation*}
\varepsilon=\varepsilon_{0}+\varepsilon(I) \tag{1}
\end{equation*}
$$

where $\varepsilon_{0}$ denotes the static misalignment, and $\varepsilon(I)$ represents tilting of the coil axis by the Lorentz force. We shall refer $\varepsilon$ to as the coil-tilt angle. The tilt angle $\Delta \theta$ of the effective field $B_{\text {eff }}$ is written as

$$
\begin{equation*}
\left.\Delta \theta=\frac{180}{\pi} \arctan \left[\frac{B_{\mathrm{eff}}^{2}+B_{x}^{2}-B_{0}^{2}}{2 B_{0} B_{\mathrm{eff}}}\right] \approx \frac{180}{\pi} \frac{B_{x}}{B_{0}} \text { [degree }\right], \tag{2}
\end{equation*}
$$

where $B_{\text {eff }}$ is the size of the effective field given by

$$
\begin{equation*}
B_{\mathrm{eff}}=\sqrt{B_{0}^{2}+B_{x}^{2}-2 B_{0} B_{x} \sin \varepsilon} \approx B_{0}+\frac{B_{x}^{2}}{2 B_{0}}-\frac{\pi}{180} B_{x} \varepsilon \tag{3}
\end{equation*}
$$

Note here that $B_{\text {eff }}$ causes a signal shift $\delta$ written as

$$
\begin{equation*}
\delta=\frac{B_{\mathrm{eff}}-B_{0}}{B_{0}} \approx \frac{B_{x}^{2}}{2 B_{0}^{2}}-\frac{\pi}{180} \frac{B_{x}}{B_{0}} \varepsilon, \tag{4}
\end{equation*}
$$

and the spinning angle becomes $\theta-\Delta \theta$.

### 2.2 Magnetic fields produced by the $X_{0}$ shim coil

In this section, we evaluate the magnetic field produced by the $\mathrm{X}_{0}$ shim coil. A saddle coil with an infinite height $(2 l=\infty)$ does not produce magnetic field along $z$. For a coil with a finite height, however, the $z$-magnetic field is zero only at the coil center designated as $\boldsymbol{O}$ in Fig. 1, and the $z$-magnetic field at a general position $\boldsymbol{P}\left(B_{z}(\boldsymbol{P})\right)$ would result in line broadening. In other words, the height of the $\mathrm{X}_{0}$ shim coil should be optimized to reduce spacial variation of $B_{z}(\boldsymbol{P})$ over the sample.

The $y$-magnetic field produced by the $\mathrm{X}_{0}$ shim coil is small at the sample position, and, moreover, its effect on the linewidth would be averaged out by the sample spinning. Therefore, we ignore the $y$-magnetic field in this work.

The $x$-magnetic field $B_{x}$ is the main field produced by the $\mathrm{X}_{0}$ shim coil. As shown below, $B_{x}$ is spacially distributed, and the distributed $B_{\text {eff }}$ and $\Delta \theta$ cause line broadening. As broadening due to spacial variation of $\Delta \theta$ would be quite small, it is ignored in this work. We thus examine only line broadening due to spacially distributed $B_{\text {eff }}$ through Eq. (4). The task is therefore to obtain a coil geometry, which maximizes $B_{x}(\boldsymbol{O})$ without increasing $\Delta B_{x}(\boldsymbol{P}) \equiv$ $B_{x}(\boldsymbol{P})-B_{x}(\boldsymbol{O})$ and $B_{z}(\boldsymbol{P})$ appreciably.

The magnetic field produced by the $\mathrm{X}_{0}$ shim coil is calculated separately for the arc parts and for the vertical-line parts. We shall refer the magnetic fields produced by the former and the latter to as $\boldsymbol{B}^{\boldsymbol{A}}$ and $\boldsymbol{B}^{\boldsymbol{L}}$, respectively. As discussed above, the $y$-components of $\boldsymbol{B}^{\boldsymbol{A}}$ and $\boldsymbol{B}^{\boldsymbol{L}}$ are ignored. The $z$ magnetic field produced from the line parts $\left(B_{z}^{L}\right)$ can be negligible when the lines are almost parallel to the $z$ axis. Thus, we evaluate $B_{x}^{A}, B_{x}^{L}$, and $B_{z}^{A}$ in the following. For simplicity, we assume one-turn coil with infinitely thin wire.

For the arc parts, the differential magnetic field $d \boldsymbol{B}^{\boldsymbol{A}}$ at $\boldsymbol{P}$ produced by current at a point $\boldsymbol{Q}$ in the arc is described by the Biot-Savart law written as

$$
\begin{equation*}
d \boldsymbol{B}^{\boldsymbol{A}}=\frac{\mu_{0} I}{4 \pi} \int_{C} \frac{d s \times \overrightarrow{Q P}}{|Q P|^{3}} \tag{5}
\end{equation*}
$$

Here, $d s$ represents the line element on $\boldsymbol{Q}$ of the $\mathrm{X}_{0}$ shim coil. The $z$ - and $x$ -
elements of $\boldsymbol{B}_{\boldsymbol{A}}$ are given by

$$
\begin{align*}
B_{z}^{A}(\boldsymbol{P})= & \frac{\mu_{0}}{4 \pi} \int_{C_{\mathrm{up}}} \frac{a R \cos \left(\phi^{\prime}-\phi\right)-a^{2}}{\left[a^{2}+R^{2}+(z-l)^{2}-2 a R \cos \left(\phi^{\prime}-\phi\right)\right]^{3 / 2}} d \phi^{\prime} \\
& +\frac{\mu_{0} I}{4 \pi} \int_{C_{\text {low }}} \frac{a R \cos \left(\phi^{\prime}-\phi\right)-a^{2}}{\left[a^{2}+R^{2}+(z+l)^{2}-2 a R \cos \left(\phi^{\prime}-\phi\right)\right]^{3 / 2}} d \phi^{\prime}, \tag{6}
\end{align*}
$$

and

$$
\begin{align*}
B_{x}^{A}(\boldsymbol{P})= & \frac{\mu_{0} I}{4 \pi} \int_{C_{\mathrm{up}}} \frac{-a(z-l) \cos \phi^{\prime}}{\left[a^{2}+R^{2}+(z-l)^{2}-2 a l R \cos \left(\phi^{\prime}-\phi\right)\right]^{3 / 2}} d \phi^{\prime} \\
& +\frac{\mu_{0} I}{4 \pi} \int_{C_{\mathrm{low}}} \frac{-a(z+l) \cos \phi^{\prime}}{\left[a^{2}+R^{2}+(z+l)^{2}-2 a l R \cos \left(\phi^{\prime}-\phi\right)\right]^{3 / 2}} d \phi^{\prime}, \tag{7}
\end{align*}
$$

respectively. Here, $(R, \phi, z)$ and $\left(a, \phi^{\prime}, l\right)$ are the polar coordinates of $\boldsymbol{P}$ and $\boldsymbol{Q}$, and $C_{\text {up }}$ and $C_{\text {low }}$ represent integral paths for the upper- and lower-arc pairs, respectively, whose explicit forms are explicitly given in the supplementary materials (Fig. S1 and Eq. S1).

For the line part, the differential magnetic field $d \boldsymbol{B}^{\boldsymbol{L}}$ at $\boldsymbol{P}$ is described as

$$
\begin{equation*}
d \boldsymbol{B}^{L}=\frac{\mu_{0} I}{4 \pi} \frac{a d z \sin \xi}{|Q P|^{2}}, \tag{8}
\end{equation*}
$$

where $\xi$ is the angle between current $I$ on $\boldsymbol{Q}$ and $\overrightarrow{Q P}$. As have discussed above, we ignore $B_{z}^{L}$ and $B_{y}^{L}$, and the integration of $B_{x}^{L}$ can be done analytically for each line part. The explicit integral paths and the corresponding $B_{x}^{L}$ are given in the supplementary materials (Fig. S2 and Eq. S2).

### 2.3 Optimization of the height of the $X_{0}$ shim coil

At the center of the coil $\boldsymbol{O}$, Eqs. (7) and (8) (and Eq. (S2-5) in supplementary materials) lead the following $x$-magnetic fields,

$$
\begin{equation*}
B_{x}^{A}(\boldsymbol{O})=\frac{2 \mu_{0} I}{\pi} \frac{a l \cos \alpha}{\left(a^{2}+l^{2}\right)^{3 / 2}} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
B_{x}^{L}(\boldsymbol{O})=\frac{2 \mu_{0} I}{\pi} \frac{l \cos \alpha}{a \sqrt{a^{2}+l^{2}}} \tag{10}
\end{equation*}
$$

respectively. The total $x$-magnetic field produced by the $\mathrm{X}_{0}$ shim coil $B_{x}(\boldsymbol{O})=$ $B_{x}^{A}(\boldsymbol{O})+B_{x}^{L}(\boldsymbol{O})$ becomes

$$
\begin{equation*}
B_{x}(\boldsymbol{O})=\frac{2 \mu_{0} I}{\pi} \frac{\left(2 a^{2}+l^{2}\right) l \cos \alpha}{a\left(a^{2}+l^{2}\right)^{3 / 2}} \tag{11}
\end{equation*}
$$

Eq. (11) implies that the strongest $B_{x}(\boldsymbol{O})$ is achieved with the height $2 l$ being equal to $2 \sqrt{2} a$ and the coil diameter $2 a$ and the gap angle $2 \alpha$ being as small as possible. Practically, the gap angle $\alpha$ can not be zero, and we assume $\alpha \sim 15^{\circ}$ for our $\mathrm{X}_{0}$ shim coil. The $\alpha$ value does not affect magnetic homogeneity around the sample appreciably as compared to the other two parameter ( $2 a$ and $2 l$ ), and $\alpha$-dependent inhomogeneity is ignored. The coil diameter $2 a$ is set to 66.4 mm as we are to wind the $\mathrm{X}_{0}$ shim coil around the MAS probe whose diameter is ca. 66 mm . The optimal coil height for the maximum $B_{x}(\boldsymbol{O})$ is therefore $l=\sqrt{2} a \sim 47 \mathrm{~mm}$. However, in the following, we show that the unwanted $z$-field $B_{z}(\boldsymbol{P})$ is also large at $l=\sqrt{2} a$.

To appreciate the spacial distribution of $B_{x}$ and $B_{z}$ along the axis of
the sample, we introduce a new coordinate $d$ to designate position in the sample (Fig. 2a). The center $\boldsymbol{O}$ is denoted by $d=0 \mathrm{~mm}$, and the both ends of the sample, since the sample length of the 5 mm rotor we use is ca. 4.5 mm , correspond to $d= \pm 2.3 \mathrm{~mm}$. We shall refer to $d=0 \mathrm{~mm}$ and $d=2.3$ mm as $d_{0}$ and $d_{\text {end }}$, respectively, and use $B_{z}\left(d_{\text {end }}\right)$ as a measure of the spacial inhomogeneity of $B_{z}(\boldsymbol{P}) . B_{x}\left(d_{0}\right)$ and $B_{z}\left(d_{\text {end }}\right)$ are calculated for various coil heights $2 l$ with $I=1$ A using Eqs. (11) and (6), and the results are shown in Figs. 2b and 2c, respectively. As indicated by Eq. (11), $B_{x}\left(d_{0}\right)$ becomes maximum at $l=\sqrt{2} a \sim 47 \mathrm{~mm}$. However, at $l=47 \mathrm{~mm}$, the redundant magnetic field $B_{z}\left(d_{\text {end }}\right)$ becomes ca. -9.5 nT , which corresponds to $1 \times 10^{-3}$ ppm at 7 T. In Fig. 2c, it is notable that at $l \simeq 2 a \sim 66.4, B_{z}\left(d_{\text {end }}\right)$ become ca. 0 T . We therefore chose $l \simeq 2 a \sim 66.4 \mathrm{~mm}$ as a compromised height for large $B_{x}\left(d_{0}\right)$ and small $B_{z}\left(d_{\text {end }}\right)$.

### 2.4 The geometry of the $X_{0}$ shim coil

So far, we considered an ideal coil (an one-turn $\mathrm{X}_{0}$ shim coil with infinitely thin wire) for ease of calculation. In practice, the larger turn number is better as it produces larger $B_{x}$. For the $\mathrm{X}_{0}$ shim coil, wire with a round section (round wire) is not suitable as there are large Arbelos-shaped gaps among the adjacent turns of the wire (Fig. 3a), which causes low electricconductivity per turn and poor effluence of the Joule heat. In addition, it is mechanically fragile against the Lorentz force because of pliability of round wire in any direction. Thus, we wound the $\mathrm{X}_{0}$ shim coil by using wire with a
rectangular section (rectangular wire) with the longer side of the wire section standing perpendicularly to the coil surface ('flatwise' winding). It has smaller gaps among the adjacent turns of the wire than that wound with the round wire (Fig. 3b). The larger contact areas between the adjacent turns and also that between the wire and the coil bobbin improve the effluence of the Joule heat. Furthermore, it is robust against the plastic deformation by the Lorentz force.

An illustration of one of the pair of the flatwise-wound saddle coil is shown with a part of its outmost turn in Fig. 3c. The width and the thickness of the wire are 2.6 mm and 0.3 mm , respectively. We wound a 50-turn coil, and the interval between the adjacent turns of wire is 0.34 mm , hence the actual size parameters of the $\mathrm{X}_{0}$ shim coil, the diameter, the length and the gap angle, becomes $2 a=66.3-71.5 \mathrm{~mm}, 2 l=123.6-157.2 \mathrm{~mm}$ and $2 \alpha=10-70^{\circ}$, respectively.

To maintain the coil against the Lorentz force, the $\mathrm{X}_{0}$ shim coil is wound in grooves cut on the surface of a duralumin body of a mounting retainer. The pictures of the $\mathrm{X}_{0}$ shim coil on the retainer and the MAS probe are given in the supplementary materials (Fig. S3). Fig. 4 shows a cross-section sketch of an $\mathrm{X}_{0}$ shim coil and a MAS probe in a magnet with a room-shim unit. The $\mathrm{X}_{0}$ shim coil is used as the "shielding cap" of the MAS probe, which is fixed by the bolts with the room-shim unit. The retainer with the $\mathrm{X}_{0}$ shim coil is fixed at the top of the main magnet and by the O-ring to the main magnet
at the flank. Further, the bottom of the flank has a cut to couple closely with the top of the room-shim unit.

In the following calculation, the actual width of the wire is taken into account approximately by expressing the wire as a bundle of 10 separated wires with a width of 0.26 mm . In other words, we assume that the magnetic field produced by 1 A current flowing through one flat wire with its width of 2.6 mm can be calculated as a sum of 10 magnetic fields produced by 0.1 A current flowing through 10 thin wires aligned with the interval of 0.26 mm perpendicularly to the coil surface.
2.5 The calculated performance of the $X_{0}$ shim coil

The magnetic fields $B_{x}^{A}(\boldsymbol{O})$ and $B_{x}^{L}(\boldsymbol{O})$ obtained by integrating Eqs. (9-11) for the 50 -turn coil with $2 l=123.6-157.2 \mathrm{~mm}$ and $2 \alpha=10-70^{\circ}$ become

$$
\begin{equation*}
B_{x}^{A}(\boldsymbol{O}) \simeq 1.89 \times 10^{-4} I[\mathrm{~T}] \tag{12}
\end{equation*}
$$

and

$$
\begin{equation*}
B_{x}^{L}(\boldsymbol{O}) \simeq 9.59 \times 10^{-4} I[\mathrm{~T}], \tag{13}
\end{equation*}
$$

respectively, and the total $x$ magnetic field is

$$
\begin{equation*}
B_{x}(\boldsymbol{O})=B_{x}^{A}(\boldsymbol{O})+B_{x}^{L}(\boldsymbol{O}) \simeq 1.15 \times 10^{-3} I[\mathrm{~T}] . \tag{14}
\end{equation*}
$$

For $B_{0}=7 \mathrm{~T}$, Eq. (14) leads the angle $\Delta \theta$ (Eq. (2) and Fig. 1) written as

$$
\begin{equation*}
\Delta \theta \simeq 9.33 \times 10^{-3} I \text { [degree] }, \tag{15}
\end{equation*}
$$

and the additional shift $\delta$ (Eq. (4)) is given by

$$
\begin{equation*}
\delta \simeq 1.32 \times 10^{-2} I^{2}-2.84\left(\varepsilon_{0}+\varepsilon(I)\right) I[\mathrm{ppm}] . \tag{16}
\end{equation*}
$$

Eq. (15) shows that we can adjust the angle within $\pm 0.05^{\circ}$ for $I= \pm 5 \mathrm{~A}$.

Here, we examine the redundant magnetic fields $B_{x}(d)$ and $B_{z}(d)$. The calculated $B_{x}(d)$ for $I=1 \mathrm{~A}$ (Fig. 5a) shows that the difference $\Delta B_{x} \equiv$ $B_{x}\left(d_{0}\right)-B_{x}\left(d_{\text {end }}\right)$ is ca. $1.5 \mu \mathrm{~T}$ for $I=1 \mathrm{~A}$. Hence it would be ca. $7.5 \mu \mathrm{~T}$ for $I=5 \mathrm{~A}$. Line broadening $\Delta_{x}$ due to the variation of the $x$-field $\Delta B_{x}$ can be estimated by using Eq (4) as

$$
\begin{equation*}
\Delta_{x} \approx\left(\frac{B_{x}\left(d_{0}\right)}{B_{0}^{2}}-\frac{\pi}{180} \frac{\varepsilon}{B_{0}}\right) \Delta B_{x} \tag{17}
\end{equation*}
$$

For $\varepsilon=0, \Delta B_{x}$ for $I=5$ A corresponds to line broadening of $9 \times 10^{-4} \mathrm{ppm}$ at 7 T . In the following, we show that the experimentally obtained $\varepsilon$ is about $-0.1^{\circ}$ to $-0.2^{\circ}$. Hence line broadening due to $\Delta B_{x}$ is less than 0.005 ppm at 7 T , and broadening due to $\Delta B_{x}$ is negligible.

Fig. 5b shows the calculated $B_{z}(d)$ for $I=1 \mathrm{~A}$. As the $z$ component acts directly to the resonance frequency, its distribution is of most concern. The estimated variation of $B_{z}(d)$ over $\pm d_{\text {end }}$ is 0.12 nT , which corresponds to $2 \times 10^{-5} \mathrm{ppm}$ line broadening at 7 T . Hence, this can also be ignored. Practically, however, these broadening effects may become appreciable due to deformation of the coil shape from the ideal one and displacement between the sample and the coil center. Hence, we examine a way to reduce line broadening
in advance.

It is notable that both $B_{x}(d)$ and $B_{z}(d)$ can roughly be expressed by a function of $d^{2}$ for the range of $\left[-d_{\text {end }}, d_{\text {end }}\right]$. In the Cartesian coordinate $(x, y, z), d^{2}$ can be written by

$$
\begin{equation*}
d^{2}=z^{2}+x^{2}=\frac{3 \sqrt{2}}{2} x z \tag{18}
\end{equation*}
$$

Thus, line broadening due to the inhomogeneous magnetic field produced by the $\mathrm{X}_{0}$ shim coil can be reduced by adjusting the $x z$ term of the room-shim coil $\left(C_{x z}\right)$. Therefore, we firstly obtain optimal $C_{x z}$ values for several $I$ values by observing the ${ }^{1} \mathrm{H}$ MAS NMR signal of $\mathrm{H}_{2} \mathrm{O}$. The observed $C_{x z}$ values are leastsquare fitted to $C_{x z}=a I^{2}+b I+c$, with $a, b$, and $c$ as adjustable parameters (Fig. S4 in the supplementary materials) and used for further experiments. To conclude, the line broadening caused by the redundant magnetic field may be appreciable but can be removed by the $C_{x z}$ correction.

## 2. 6 Joule heat and Lorentz force

Since the resistance of the $\mathrm{X}_{0}$ shim coil is ca. $1.2 \Omega, I=5$ A produces 30 W Joule heat. In fact, we found that the Joule heat increases the temperature of the $\mathrm{X}_{0}$ shim coil by $11^{\circ} \mathrm{C}$. However, the Joule heat problem can be overcome mostly by flowing cooling air from the bottom of the probe with the rate of $70 \mathrm{~L} / \mathrm{min}$, which reduces the temperature increment of the $\mathrm{X}_{0}$ shim coil from $11^{\circ} \mathrm{C}$ to $4^{\circ} \mathrm{C}$ for $I=5 \mathrm{~A}$. Further reduction was made by turning off the $\mathrm{X}_{0}$-shim current during the relaxation delay. An example of the on/off timing
sequence is given for a single-pulse experiment as shown in Fig. 6. The explicit temperature data of the $\mathrm{X}_{0}$ shim coil and the probe are shown in Fig. S5 of the supplementary materials.

For the given coil dimensions, we estimated the Lorentz force to be 220 N at the top and bottom of the coil, which acts to bring the " $x$-axis" of the $\mathrm{X}_{0}$ shim coil parallel to the main magnetic field $B_{0}$. As described above, the $\mathrm{X}_{0}$ -shim-coil body (the retainer) has a role to sustain the $\mathrm{X}_{0}$ shim coil against the Lorentz force (Fig.4). Even with the retainer, we found that a small deviation of the angle caused by the Lorentz force is accumulated during repeated signal acquisition. To avoid such accumulation, we put opposite current after each signal acquisition. A schematical timing chart is given in Fig. 6. We shall refer this procedure to as the $-I$ procedure in the following. The effect of the opposite current is described in section 4.

## 3 Experimental

NMR measurements were done for $B_{0}=7.05 \mathrm{~T}$ (JASTEC Co. Ltd.) using an OPENCORE[6] spectrometer operating at the resonance frequencies of 301.4 MHz for ${ }^{1} \mathrm{H}, 75.8 \mathrm{MHz}$ for ${ }^{13} \mathrm{C}$, and 42.3 MHz for ${ }^{2} \mathrm{H}$ with a MAS probe (Doty Scientific, Inc.) for a 5 mm rotor. We measured ${ }^{1} \mathrm{H}$ MAS NMR of $\mathrm{H}_{2} \mathrm{O}$ doped with $5 \mathrm{mM} \mathrm{CuSO} 4,{ }^{13} \mathrm{C}$ CPMAS NMR of adamantane, and ${ }^{2} \mathrm{H}$ MAS NMR of partially deuterated dimedone-2,3- $d_{2}$. In ${ }^{1} \mathrm{H}$ MAS NMR, 5 mM $\mathrm{CuSO}_{4}$ aqueous solution was packed in a homemade sealed cell. Adamantane was purchased from Wako Pure Chem. Ind. Ltd. Dimedone was purchased
from Tokyo Chem. Ind. Ltd., and the dimedone-2,3- $d_{2}$ sample for ${ }^{2} H$ NMR was obtained by recrystallizing dimedone from $99.9 \%{ }^{2} \mathrm{H}_{2} \mathrm{O}$ (Sigma-Aldrich Co. Ltd.) solution for 3 times[3]. The ${ }^{13} \mathrm{C}$ CPMAS spectra of adamantane were obtained under ${ }^{1} \mathrm{H}$ CW decoupling, while the ${ }^{2} \mathrm{H}$ MAS spectra was obtained without decoupling. For all experiments, 5 dummy scans were applied before starting each accumulation, and the $-I$ procedure (Fig. 6) was used unless otherwise denoted. The spinning frequency was ca. 500 Hz for ${ }^{1} \mathrm{H}$ and 10 kHz for ${ }^{13} \mathrm{C}$ and ${ }^{2} \mathrm{H}$, respectively. The current through the $\mathrm{X}_{0}$ shim coil was controlled by using a bipolar power supply (PBX20-20, KIKUSUI ELECTRONICS Corp.), whose resolution is 1 mA .

## 4 Results and discussions

In this section, we firstly examine line broadening caused by the $\mathrm{X}_{0}$ shim coil by observing the ${ }^{1} \mathrm{H}$ spectrum of $\mathrm{H}_{2} \mathrm{O}$, and secondly, the tilting of the $\mathrm{X}_{0}$ shim coil due to the Lorentz force is examined by monitoring the shift of the ${ }^{13} \mathrm{C}$ peak of $\mathrm{CH}_{2}$ of adamantane. Then the precise setting of the spinning angle is demonstrated for the ${ }^{2} \mathrm{H}$ MAS spectrum of partially deuterated dimedone.

Fig. 7 shows the $I$ dependence of full width at half maximum (FWHM) of ${ }^{1} \mathrm{H}$ MAS NMR spectra of $\mathrm{H}_{2} \mathrm{O}$ of $5 \mathrm{mM} \mathrm{CuSO}_{4}$ aqueous solution spinning at 500 Hz with and without the $C_{x z}$ correction. For experiments with the $C_{x z}$ correction, the linewidth varies only from 0.012 ppm to 0.014 ppm for $I=-5$ A to 5 A, while for those without the $C_{x z}$ correction, it varies from 0.010 to 0.021 ppm . The additional linewidth without the $C_{x z}$ condition is thus 0.011
ppm, which is larger than that calculated $(\sim 0.005 \mathrm{ppm})$ in section 2.5. The larger broadening may be caused by displacement between the sample center and the center of the $\mathrm{X}_{0}$ shim coil and/or deformation of the coil shape from the ideal one. Since the line broadening of 0.011 ppm is small enough for most of solid state NMR experiments, the $C_{x z}$ correction is not a mandatory procedure.

Fig. 8 shows ${ }^{13} \mathrm{C}$ CPMAS NMR spectra of adamantane at various current $I$ for the $\mathrm{X}_{0}$ shim coil. It is notable that the signal-to-noise ratio deteriorated with varying the current $I$ from 5 A to -5 A , which is ascribed to CP mis-matching due to the peak shift $\delta[7]$. The apparent peak shift $\delta$ is ascribed to the size change of the effective field $B_{\text {eff }}$ and the tilting of the coil $\varepsilon(I)$ by the Lorentz force (Eqs. (4) and (16)). We obtained $\varepsilon(I)$ from the observed $\delta(I)$ by using Eq. (16) and plotted in Fig. 9. The observed $\varepsilon(I)$ can be least-squares fitted to a straight line, and the result is

$$
\begin{equation*}
\varepsilon=\varepsilon_{0}+\varepsilon(I)=-0.135-0.0082 I \text { [degree]. } \tag{19}
\end{equation*}
$$

This shows that the present $\mathrm{X}_{0}$ shim coil was built with static misalignment of $\varepsilon_{0}=-0.135^{\circ}$, and the Lorentz force further changes the angle by $0.0082^{\circ}$ per 1 A. Practically, as long as the angle stays constant during the NMR experiment, the $I$ dependence of the coil-tilt angle $\varepsilon$ is not a serious problem. In adjusting process of the spinning angle, however, hysteresis of $\varepsilon$ against current $I$ is cumbersome. We therefore examined hysteresis of $\varepsilon$ by changing current $I$ in
three deferent ways; (1) increasing from -5 A to 5 A , (2) starting from 0 A , and increased to 5 A and jumped back to 0 A followed by decreasing to -5 A , and (3) changing rather randomly as $0,5,-0.5,4.5 \cdots$ A. The observed $\varepsilon(I)$ for the three experiments (Fig. S6 in the supplementary materials) can also be described by Eq. (19) with the standard deviation of $\left(1.4 \times 10^{-3}\right)^{\circ}$. This small hysteresis ensures high reproductivity even in the setting process of the magic angle.

During long signal accumulation, the coil-tilt angle $\varepsilon$ may change gradually. To examine the long-time behavior with and without the $-I$ procedure in the pulse sequence, we examined $\varepsilon$ for 16 hours at $I= \pm 5 \mathrm{~A}$. The pulse repetition time and the accumulation number for one measurement were 4.8 sec and 50 times, respectively. With adding 5 dummy scans, the total experimental time for one measurement was 250 sec . The measurement was repeated 240 times, and variation of $\varepsilon$ obtained for each measurement is plotted in Fig. 10. For $I=5 \mathrm{~A}$ with the $-I$ procedure (represented by the open circles), $\varepsilon$ increased by $\left(2.2 \times 10^{-3}\right)^{\circ}$ in the first one hour and stayed there with the standard deviation of $\left(3 \times 10^{-4}\right)^{\circ}$. Similarly, for $I=-5 \mathrm{~A}, \varepsilon$ increased by $\left(2.3 \times 10^{-3}\right)^{\circ}$ in the first 1 hour and stayed there with a standard deviation of $\left(2 \times 10^{-4}\right)^{\circ}$. In both cases, $\varepsilon$ changes only a few $10^{-3}$ degrees, which does not deteriorate spectral resolution in most of the MAS experiments. When angle stability of a few $10^{-4}$ degrees is required, one should wait for an hour with applying the pulse sequence before starting signal accumulation. Time
variation of $\varepsilon$ without the $-I$ procedure is also shown in Fig. 10 (the crosses); $\varepsilon$ kept changing even after 16 hours.

Fig. 11 shows the ${ }^{2} \mathrm{H}$ MAS NMR spectra of partially-deuterated dimedone observed at various current $I$. Due to the quadrupolar interaction of ${ }^{2} \mathrm{H}$, the spinning sidebands spread over 6600 ppm , and their peak heights are very sensitive to deviation of the spinning angle from the exact magic angle. Before precise adjustment by using the $\mathrm{X}_{0}$ shim coil, we coarsely adjusted the angle of the spinner axis $\theta$ by using a mechanical gear system with monitoring ${ }^{79} \mathrm{Br}$ MAS NMR of $\mathrm{KBr}[8]$. Note that we used different spinners for KBr and dimedone, and the sample changing would caused a slight change in orientation of the stator. Fig. 11 indicates that the spinning angle set by the KBr method $(I=0 \mathrm{~A})$ is slightly deviated from the exact magic angle. Also we found that the current range ( -5 to 5 A ) is wide enough to bring $\theta-\Delta \theta$ to $\theta_{\mathrm{MA}}$, which is realized by current $I \sim-1 \mathrm{~A}$ in this case. For precise setting of the spinning angle, we plotted the sum of the peak intensities of the sidebands at various $I$ in Fig. 12, which shows that the best matching to the magic angle is obtained at $I=-0.8 \mathrm{~A}$. Fig. 12 also shows that the precision/stability required for current is ca. $\pm 0.01 \mathrm{~A}$, which is well satisfied in the present equipment.

Lastly, we would like to point out that a fully automated "magic-angle stabilizer" may be realized when one combines the $\mathrm{X}_{0}$ shim coil and a device that detects the spinning angle. As for the latter device, one may use a Hall sensor developed by S. Mamone et al.[9] or an optical detection scheme
developed by E. Mihaliuk and T. Gullion[10].

## 5 Summary

In this work, we presented the $\mathrm{X}_{0}$ shim coil for precise setting of the spinning angle for MAS NMR. The $\mathrm{X}_{0}$ shim coil produces an additional static magnetic field $B_{x}$ perpendicular to the main magnetic field $B_{0}$ and tilts the effective magnetic field.

We examined the coil configuration for a larger magnetic field $B_{x}$ without introducing appreciable magnetic-field inhomogeneity over the sample. We found that the optimal coil geometry is the coil height being about twice of the coil diameter, $l \sim 2 a$. Under the practical geometrical restriction such as the bore size of the main magnet and the MAS probe, the turn number of the coil became 50 and its diameter, the length, and the gap angle were decided to be $66.3-71.5 \mathrm{~mm}, 123.6-157.2 \mathrm{~mm}$, and $10-70^{\circ}$, respectively. With these values, we estimated that the range of the adjustable angle by the $\mathrm{X}_{0}$ shim coil is $\pm 0.05^{\circ}$ at $B_{0}=7 \mathrm{~T}$ for $I= \pm 5 \mathrm{~A}$, and line broadening due to the redundant magnetic fields is less than 0.05 ppm .

To overcome the Joule heat and particularly the Lorentz force problems, we wound the coil using rectangular wire with flatwise winding. In addition to this, cooling air and the turning-off step of current in a pulse sequence helped to reduce temperature increment by the Joule heat only $1^{\circ} \mathrm{C}$. To resist the Lorentz force, we fixed the mounting retainer with the $\mathrm{X}_{0}$ shim coil firmly to
the room-shim unit and the main magnet. Furthermore, we showed that the $-I$ procedure in the pulse sequence reduces accumulation of the tilting of the $\mathrm{X}_{0}$ shim coil during long signal acquisition.
${ }^{1} \mathrm{H}$ NMR measurement of $\mathrm{H}_{2} \mathrm{O}$ showed that the line broadening due to the $\mathrm{X}_{0}$ shim coil is only 0.011 ppm . It can be reduced further down to 0.002 ppm by the $x z$-term correction of the room shim. ${ }^{13} \mathrm{C}$ NMR measurement of adamantane showed high stability against the Lorentz force. ${ }^{2} \mathrm{H}$ spectra of demedone- $2,3-d_{2}$ showed that the spinning angle can be adjusted straightforwardly to the magic angle.

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## Figure Captions

## Fig. 1

Schematic of the $\mathrm{X}_{0}$ shim coil (a) and the effective magnetic field produced by the coil (b). The spinner axis of the sample lies in the $x z$ plane. $2 a, 2 l$ and $2 \alpha$ are the coil diameter, the height and the gap angle between the two opposed arcs, respectively. The angle $\varepsilon$ in (b) represents misalignment of the $\mathrm{X}_{0}$ shim coil from the $x$ axis.

Fig. 2
Schematic of a coordinate $d$ along the spinner axis (a) and the calculated magnetic field $B_{x}$ at $d=0 \mathrm{~mm}(\mathrm{~b})$ and $B_{z}$ at $d=2.3 \mathrm{~mm}$ (c). It was assumed that $I=1[\mathrm{~A}]$ flows through the one-turn surface coil with the diameter of $2 a=66.4 \mathrm{~mm}$ and the gap angle of $2 \alpha=30^{\circ}$. The height $2 l$ was used as a variable. The vertical dashed lines are drawn at $l=\sqrt{2} a \simeq 46 \mathrm{~mm}$ (left) and $l=69 \mathrm{~mm}$ (right) for eye guidance.

## Fig. 3

Schematics of a coil section wound by round wire (a) and rectangular wire (b). Note that, in (a) and (b), the cross-section areas of each wire are almost equal. (c) illustrates one of the pair of a flatwise-winding coil wound with the rectangular wire and a part of its outermost turn.

Fig. 4
Sketch of an $\mathrm{X}_{0}$ shim coil on a mounting retainer attached to a MAS NMR probe in a main magnet with a room-shim unit.

## Fig. 5

Calculated $B_{x}$ (a) and $B_{z}(\mathrm{~b})$ along the spinner axis $d$ for the 50 -turn $\mathrm{X}_{0}$ shim coil. The details of the coil dimensions are given in text.

## Fig. 6

Schematic of pulsed current $I$ synchronized to a single-pulse sequence. A rectangle followed by FID (top) represents a $90^{\circ}$ pulse. To stabilize current $I$ and the tilt-angle of the $\mathrm{X}_{0}$ shim coil $\varepsilon(I)$, rf pulsing starts 400 ms after $I$ reaches the target value. After acquisition of FID, the current is linearly decreased to the opposite current $-I$ in 200 ms and stays there for 100 ms to reduce accumulation of tilting of the $\mathrm{X}_{0}$ shim coil due to the Lorentz force. The current is finally returned linearly back to 0 A in 300 ms .

## Fig. 7

Observed full width at half maximum (FWHM) of ${ }^{1} \mathrm{H}$ MAS spectra of 5 mM $\mathrm{CuSO}_{4}$ solution at various current $I$; the filled and open circles indicate with and without the correction of the $x z$-term of the room shim $\left(C_{x z}\right)$, respectively.

## Fig. 8

Observed dependence of the ${ }^{13} \mathrm{C}$ CPMAS spectra of adamantane on current $I$ for the $\mathrm{X}_{0}$ shim coil. The spectra are plotted on the same amplitude scale, and they can be directly compared.

## Fig. 9

Dependence of the coil-tilt angle $(\varepsilon(I))$ of the $\mathrm{X}_{0}$ shim coil on current $I . \varepsilon(I)$ was calculated from the observed shifts in Fig. 8 using Eq. (16). The straight line through the data points is the least-squares fitted one (Eq. (19)). The
dashed lines at $I=0 \mathrm{~A}$ and $\varepsilon(I=0)$ are for eye guidance.

Fig. 10
Long-time stability of the coil-tilt angle $\varepsilon$ at $I= \pm 5 \mathrm{~A}$ with (the open circles) and without (the crosses) the $-I$ procedure in the pulse sequence (Fig. 6). In each series, the first $\varepsilon$ value is set to be 0 .

Fig. 11
Dependence of ${ }^{2} \mathrm{H}$ MAS spectra of dimedone-2,3- $d_{2}$ (inset) on current $I$.

## Fig. 12

Dependence of the sum of the sideband peak-intensities of the ${ }^{2} \mathrm{H}$ MAS spectra of dimedone- $2,3-d_{2}$ on current $I$. The peak intensities for those within the $\pm 9$ th sidebands were added.
(a)
$\mathrm{X}_{0}$ shim coil

(b)


Fig. 1

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(b)



Fig. 2

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Fig. 3

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Fig. 4

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Fig. 5

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Fig. 6

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Fig. 7

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Fig. 8

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Fig. 9

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Fig. 10

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Fig. 11

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Fig. 12

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# An $\mathrm{X}_{0}$ shim coil for precise magic-angle adjustment 

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## Supplementary materials

## Contents

S1. Explicit integral paths for $\boldsymbol{B}^{\boldsymbol{A}}$
S2. Explicit integral paths for $\boldsymbol{B}^{\boldsymbol{L}}$
Fig. S3. Photographs of a MAS probe and the $\mathrm{X}_{0}$ shim coil
S4. $C_{x z}$ correction
Fig. S5. $I$-dependent temperature of the $\mathrm{X}_{0}$ shim coil due to the Joule heat Fig. S6. Dependence of the angle of the $\mathrm{X}_{0}$ shim coil on $I$

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## S1. Explicit integral paths for $B^{A}$



Fig. S1
Schematic of the arc parts of the $\mathrm{X}_{0}$ shim coil (blue solid line).

Fig. S1 illustrates the direction of current $I$ in the arc parts of the $\mathrm{X}_{0}$ shim coil for $B_{x}^{A}>0$ (red arrow). The arc parts is separated into the upper two parts ( $A_{1}$ and $A_{2}$ ) and lower two $\left(A_{3}\right.$ and $\left.A_{4}\right)$, respectively, and the corresponding integral paths are given as follows,

$$
\begin{align*}
& A_{1}=\left\{Q\left(a, \phi^{\prime}, l\right): \phi^{\prime}=\left[\frac{\pi}{2}-\alpha \rightarrow-\frac{\pi}{2}+\alpha\right]\right\}  \tag{S1-1}\\
& A_{2}=\left\{Q\left(a, \phi^{\prime}, l\right): \phi^{\prime}=\left[\frac{\pi}{2}+\alpha \rightarrow \frac{3 \pi}{2}-\alpha\right]\right\}  \tag{S1-2}\\
& A_{3}=\left\{Q\left(a, \phi^{\prime},-l\right): \phi^{\prime}=\left[-\frac{\pi}{2}+\alpha \rightarrow \frac{\pi}{2}-\alpha\right]\right\}, \tag{S1-3}
\end{align*}
$$

and

$$
\begin{equation*}
A_{4}=\left\{Q\left(a, \phi^{\prime},-l\right): \phi^{\prime}=\left[\frac{3 \pi}{2}-\alpha \rightarrow \frac{\pi}{2}+\alpha\right]\right\} . \tag{S1-4}
\end{equation*}
$$

The integral paths in Eqs. (6) and (7) in the text are given by $C_{\text {high }}=A_{1}+A_{2}$ and $C_{\text {low }}=A_{3}+A_{4}$, respectively.

## S2. Explicit integral paths for $B^{L}$



## Fig. S2

Schematic of the line parts of the $\mathrm{X}_{0}$ shim coil (bule solid line).

Fig. S2 illustrates the direction of current $I$ in the line parts of the $\mathrm{X}_{0}$ shim coil for $B_{x}^{L}>0$ (red arrow). The corresponding integral paths are given as follows,

$$
\begin{align*}
& L_{1}=\left\{Q\left(a, \phi_{1}^{\prime}, z^{\prime}\right): \phi_{1}^{\prime}=\frac{\pi}{2}-\alpha, z^{\prime}=[-l \rightarrow l]\right\},  \tag{S2-1}\\
& L_{2}=\left\{Q\left(a, \phi_{2}^{\prime}, z^{\prime}\right): \phi_{2}^{\prime}=\frac{\pi}{2}+\alpha, z^{\prime}=[-l \rightarrow l]\right\},  \tag{S2-2}\\
& L_{3}=\left\{Q\left(a, \phi_{3}^{\prime}, z^{\prime}\right): \phi_{3}^{\prime}=-\frac{\pi}{2}-\alpha, z^{\prime}=[l \rightarrow-l]\right\}, \tag{S2-3}
\end{align*}
$$

and

$$
\begin{equation*}
L_{4}=\left\{Q\left(a, \phi_{4}^{\prime}, z^{\prime}\right): \phi_{4}^{\prime}=-\frac{\pi}{2}+\alpha, z^{\prime}=[l \rightarrow-l]\right\} . \tag{S2-4}
\end{equation*}
$$

With these, we can analytically evaluate the integral of Eq. (8), and the result is

$$
\begin{align*}
B_{x}^{L_{i}}(\boldsymbol{P})=\frac{\mu_{0} I}{4 \pi} & \frac{a \sin \phi_{i}^{\prime}-R \sin \phi}{a^{2}+R^{2}-2 a R \cos \phi_{i}^{\prime}} \\
& \times\left\{\frac{l-z}{\sqrt{a^{2}+R^{2}-2 a R \cos \left(\phi_{i}^{\prime}-\phi\right)+(l-z)^{2}}}\right. \\
& \left.\quad+\frac{l+z}{\sqrt{a^{2}+R^{2}-2 a R \cos \left(\phi_{i}^{\prime}-\phi\right)+(l+z)^{2}}}\right\} . \tag{S2-5}
\end{align*}
$$



Fig. S3
A photograph of a MAS probe (Doty Science, Inc.) without its original cap and the $\mathrm{X}_{0}$ shim coil on the retainer (a), which is used as a cap of the MAS probe (b).

S4. $C_{x z}$ correction


Fig. S4
Dependence of the optimum $x z$-term value $C_{x z}$ of our room-shim unit on current $I$ for the $\mathrm{X}_{0}$ shim coil. $C_{x z}$ was obtained by observing ${ }^{1} \mathrm{H}$ MAS NMR spectra of $\mathrm{H}_{2} \mathrm{O}$ doped $\mathrm{CuSO}_{4}$ (the measuring condition is same as that shown in the text).

Fig. S4 shows that the magnetic-field homogeneity around the sample was disturbed by the redundant magnetic field produced by the $\mathrm{X}_{0}$ shim coil. The $I$-dependent $C_{x z}$ values can be least-squares fitted to a quadric curve (red line) described as

$$
\begin{equation*}
C_{x z}=-0.0022 I^{2}+0.0636 I+4.44 \tag{S4-1}
\end{equation*}
$$



Fig. S5
$I$-dependent temperature due to the Joule heat at the $\mathrm{X}_{0}$-shim-coil surface (a) and at the sample space (b). In both figures, the triangles, circles, and the crosses represent temperatures obtained without air flow, with the cooling air flowing from the bottom of the probe, and with both of the cooling air and the turning off of current $I$ during relaxation time (Fig. 6), respectively. The temperatures were measured by chromel-copper thermocouples. The pulse interval was 5 s ( 1 s acquisition time and 4 s relaxation time).


Fig. S6
Dependence of the tilting angle of the $\mathrm{X}_{0}$ shim coil on current $I$. $I$ was (1) increased from -5 A to 5 A (black circles), (2) decreased from 5 A to -5 A (green squares), (3) started from 0 A to 5 A , and jumped back to 0 A followed by decreasing to -5 A (red triangle), and (4) changed rather randomly as 0 , $5,-0.5,4.5, \cdots$ A (blue crosses). The solid line through the data points is the least-squares fitted one described in the text (Eq. (19)).


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