

# **PREDICTION OF WORST COMBINATION OF VARIABLE SOIL PROPERTIES IN SEISMIC PILE RESPONSE**

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## **Abstract**

Robustness analysis of seismic pile response of a structure-pile-soil system with uncertain soil properties is presented in this paper. The uncertainties of soil properties are extremely large compared to superstructures and inherent. The upper and lower bounds of the bending moment of a pile are investigated by means of the previously proposed uncertainty analysis method (Updated Reference-Point method). Soil stiffnesses and damping ratios as uncertain parameters are treated as interval parameters. The earthquake ground motion defined in the engineering bedrock in the form of a response spectrum is used as the input. An efficient finite-element model of an overall structure-pile-soil system is adopted and a response spectrum method is applied in the evaluation of the seismic pile responses of the system. It is shown that the worst combination of uncertain soil parameters can be determined and this information certainly upgrades the robustness of the structure-pile-soil system.

**Keywords:** Soil uncertainty, Robustness analysis, Structure-pile-soil system, Interval analysis, Response upper bound, Engineering bedrock, Response spectrum method

## **1. Introduction**

Soil-pile-structure systems include various and large uncertainties compared to superstructures (for example, see [1-4]). The main sources of uncertainties are from properties (stiffness and damping) of soil, effective confining pressure, soil-pile interaction, pile-soil-pile interaction, layered soil geometrical irregularity due to lack of measurement data etc. Especially properties (stiffness and damping) of soil seem a central concern of structural designers. The strain dependency of soil properties is investigated through in-situ experiments recently [3]. What the structural designers would like to know is the upper and lower bounds of earthquake responses of piles and superstructures under these uncertainties.

Although soil properties are often explained in terms of probabilistic measures (see for example [1, 2, 4, 5]), the amount of data available in the design stage at a specific site is very limited. In such situation, it may be appropriate to express the uncertainties in terms of possibilistic measures called interval parameters.

In this paper, a soil-pile-structure interaction system subjected to an engineering bedrock input ground motion is considered [6-11] and the soil properties (stiffness and damping ratio) are treated as interval parameters (see [12-15]). Then the upper and lower bounds of its earthquake response are evaluated. This problem is a kind of interval analysis problems. An innovative method for interval analysis for the non-deterministic response has been presented even for large intervals by using second-order Taylor series expansion [15]. The possibility has been taken into account of occurrence of the extreme value of the objective function in an inner feasible domain of interval parameters. The critical combination of uncertain structural parameters has been determined by the approximation using second-order Taylor series expansion.

A response spectrum method due to Kojima et al. [10, 11] is used in this paper for evaluating the maximum seismic pile response. In order to investigate the accuracy of the method used in this paper, the upper bound derived by GA (Genetic Algorithm) is compared with the result by the present method.

## **2. Application of Interval Analysis to Seismic Pile Response for Uncertain Ground Properties**

The estimation of the upper bound of the structural response considering various uncertainties of the structure-pile-soil system is useful in the design procedure. In this section, the Updated Reference Point (URP) method developed by the present authors [15] is applied to the seismic pile response.

The structure-pile-soil system used in the present uncertainty analysis was introduced in the previous research [6-8]. In the pile-soil system, an efficient finite-element model (FEM model) with the Winkler-type springs [6, 7] was used and the FEM model was extended to the model including the strain-dependent soil properties [8]. A 10-story super-structure is investigated in this paper. The fundamental natural period  $T_{B1}$  is 1.0(s) for fixed base. The floor mass of the building for a single pile is  $10 \times 10^3$ [kg] and the mass of the foundation for a single pile is  $30 \times 10^3$ [kg]. The building model is simplified to two-mass models (floor masses are transformed into two masses). A cast-in-place reinforced concrete pile is used and its pile diameter is 1.5(m). The Young's modulus of concrete is  $2.1 \times 10^{10}$ [N/m<sup>2</sup>] and the concrete mass density is  $2.4 \times 10^3$ [kg/m<sup>3</sup>]. The mass densities of surface soil layers and engineering bedrock are assumed to be  $1.8 \times 10^3$ [kg/m<sup>3</sup>] and  $2.0 \times 10^3$ [kg/m<sup>3</sup>], respectively. Poisson's ratio is 0.45.

A complex-domain response spectrum method (RSM) [10] is employed here in the evaluation of the maximum seismic pile response of the structure-pile-soil system to the ground motion defined at the engineering bedrock surface as an acceleration response spectrum (damage limit level input [10]). The accuracy and reliability of this model and the response spectrum method have been verified through the comparison with the recorded pile response under an actual earthquake and with the multi-input model considering nonlinear soil stress-strain relation [8, 10].

The equivalent shear wave velocity  $\mathbf{V}_e = \{V_{e1}, V_{e2}, \dots, V_{eN}\}$  ( $N$ : number of soil layers) and equivalent damping ratio  $\boldsymbol{\beta}_e = \{\beta_{e1}, \beta_{e2}, \dots, \beta_{eN}\}$  considering the strain dependency in the FEM model are chosen as the interval parameters. The interval ranges of those interval parameters are given by

$$0.7 \leq V_{ei} / V_{ei}^c \leq 1.3, \quad 0.7 \leq \beta_{ei} / \beta_{ei}^c \leq 1.3 \quad (1)$$

where the nominal value  $\mathbf{V}_e^c$  and  $\boldsymbol{\beta}_e^c$  of  $\mathbf{V}_e$  and  $\boldsymbol{\beta}_e$  are given by the equivalent linearization using the RSM for the free-field ground [16]. Although the shear modulus can be used as an interval parameter, the shear wave velocity is employed here because the shear wave velocity is usually obtained directly from the standard penetration test. Two ground models (ground models A, B) used in [10,11] are treated here and those are modeled by using the finite elements divided by 1.0 meter depth. The index in ground model A is  $i = 1, \dots, 37$ , and that in ground model B is  $i = 1, \dots, 29$ .

Two different cases are investigated for ground models A and B (see [10]). Case 1 employs  $\mathbf{V}_e$  and  $\boldsymbol{\beta}_e$  of both the free-field ground and the interaction springs as the uncertain parameters. On the other hand, case 2 adopts  $\mathbf{V}_e$  and  $\boldsymbol{\beta}_e$  of only the

interaction spring as the uncertain parameters. In case 1, the variation of  $V_{ei}$  and  $\beta_{ei}$  influences the free-field ground and the interaction spring at the  $i$ -th soil layer. The forces to the pile as an excitation caused by the free-field ground displacement are influenced greatly. In case 2, the equivalent shear wave velocities and damping ratios of the free-field ground are fixed as the nominal value. In this case, the uncertainty in the evaluation of equivalent stiffnesses is caused mainly by the difference between the seismic response of the free-field and that of the soil near pile.

## 2.1 Uncertainty analysis of bending moment at pile head

Since the maximum bending moment at pile head is generally a principal concern in the pile design, the maximum bending moment  $M_{\max}$  at pile head evaluated by the RSM is defined as the objective function of the uncertainty analysis. A detailed formulation of a complex-domain RSM can be found in [10]. The upper and lower bounds of the maximum bending moment at pile head are estimated by applying the URP method [15] to both ground models A and B.

Table 1 summarizes the results of the uncertainty analysis using the URP method. In this table, the maximum bending moments at pile head in case of the lower limit combination (LLC) for both ground models A and B are compared with those by the URP method. The ratios of the upper bound of the maximum bending moment at pile head for the case 1 to that of the nominal model are 1.97 for ground model A and 2.05 for ground model B. Since the variation of the equivalent natural period of the ground is strongly correlated with the uncertainty of the free-field ground, this result may be caused by high sensitivity of the pile response to the variation of the free-field ground properties. Compared with case 1, the ratios of the upper bounds for the case 2 are 1.15 for ground model A and 1.19 for ground model B. From the comparison of the upper bound by the URP method with that by LLC, LLC is not necessarily the worst combination as seen in ground model A. The worst combination in case 1 for the ground model A and the validity of the URP method will be discussed by using an optimization approach in Section 3.

## 2.2 Variation of kinematic and inertial effect caused by uncertainty of soil profile

The maximum values of the kinematic and inertial interaction effects of the seismic pile response can also be evaluated approximately by applying the RSM [10]. A detailed procedure for evaluating the kinematic and inertial interaction effects in the structure-pile-soil system can be found in [10]. In this section, variations of the kinematic and inertial interaction effects for the lower bound, nominal and upper bound

for the bending moment at pile head are investigated in both cases of case 1 and 2 for the ground model A.

In Figure 1(a), the comparison of the variation of kinematic and inertial interaction effects at pile head are shown as bar plots. The solid line with circles represents the variation of the total pile-head bending moment. The maximum value of the total response described in this figure coincides with the upper bound in ground model A derived by the URP method in Table 1. As seen in Figure 1(a) (left-hand side), in case 1, both kinematic and inertial interaction effects increase in the upper bound compared with the nominal ones and lower bound ones. This is mainly because the displacement of the free-field ground, which is varied by uncertainty of soil profiles, strongly influences the variation of the kinematic interaction effect. On the other hand, as seen in Figure 1(a) (right-hand side), in case 2, the uncertainty of the interaction springs may not cause the large variability of the kinematic interaction effect. For example, when the interaction springs are assumed to be stiffer than the nominal model, the kinematic interaction effect at pile head increases and the inertial interaction effect at pile head decreases. In this case, since the uncertainty of the interaction springs may not increase or decrease both the kinematic and inertial interaction effects simultaneously, the variability of the upper and lower bounds of the bending moment is narrow compared with case 1.

### 2.3 Uncertainty analysis of pile bending moment along whole depth

Given a possibility of soil liquefaction after an earthquake disaster, the bending moment of piles may be amplified in the underground. In this case, the upper and lower bounds of the pile bending moment along the overall depth may be important. The proposed uncertainty analysis method via the URP algorithm is applied here where the objective function  $M_i$  ( $i=1,2,\dots,37$ ), the maximum bending moment at every node, is changed sequentially along the overall depth for the ground model A.

Figure 1(b) shows the comparison of the upper and lower bounds of the pile bending moment with that of the corresponding nominal value. Compared with the results in Section 2.1, the result at pile head is the same as the upper and lower bounds for ground model A shown in Table 1. From this figure, it can be observed that, although the maximum bending moment occurs at pile head in the nominal model, almost the same bending moment can be seen around -26[m] depth in the upper bound in case of setting the objective function along the overall depth of the pile. It can be confirmed that the combination of  $V_e$  which maximizes the bending moment at -26[m] is different from that for pile head, where the shear wave velocities

$V_{ei}$  ( $i=1,2,\dots,20$ ) in the shallow ground attain relatively stiff values compared with the nominal ones.

### 3. Accuracy Check of Upper Bound Estimation by URP Method through Comparison with Result by Optimization Approach

For accuracy validation of the upper bound of the seismic pile response via the URP method, GA is used. It should be noted that, while the URP method is a fast method for estimation of the upper bound, GA requires a great amount of computational task (approximately 10 times).

The objective of the uncertainty analysis method is to find the maximum value of a structural response under constrained design parameters. Since the problem of uncertainty analysis can be regarded as an anti-optimization problem (in the sense of maximization of response) where the design variables as uncertain values are constrained as given intervals, a reliable upper bound as a reference value can be obtained by applying general optimization approaches. In this section, by considering the dependency of the initial values of design parameters in the optimization, an upper bound is evaluated by a genetic algorithm (GA: MOGA-II (Multi-Objective-Genetic-Algorithm) [17]). The description of the optimization problem can be defined as follows for ground model A.

$$\text{Find} \quad \mathbf{V}_e = \{V_{e1}, V_{e2}, \dots, V_{e37}\}, \quad \boldsymbol{\beta}_e = \{\beta_{e1}, \beta_{e2}, \dots, \beta_{e37}\} \quad (2)$$

$$\text{so as to maximize} \quad M_0(\mathbf{V}_e, \boldsymbol{\beta}_e)$$

$$\text{subject to} \quad 0.7V_{ei}^c \leq V_{ei} \leq 1.3V_{ei}^c, \quad 0.7\beta_{ei}^c \leq \beta_{ei} \leq 1.3\beta_{ei}^c \quad (i=1,2,\dots,37) \quad (3)$$

In ground model A, the uncertain design parameters are defined as  $V_{ei}, \beta_{ei}$  ( $i=1,2,\dots,37$ ). The objective function is given by the maximum bending moment  $M_0(\mathbf{V}_e, \boldsymbol{\beta}_e)$  at pile head.

Figure 2(a) presents the comparison of the upper bound of the bending moment at pile head in case 1 derived by the URP method with that by the optimization approach (GA). The upper bound derived by the URP method is almost the same as the GA (difference ratio is less than 1.0%). The large amplification at GL-25m may result from the initial soil profile (sharp change of soil stiffness).

Figure 2(b) presents the comparison of the worst combination  $\{\hat{V}_{ei}\}, \{\hat{\beta}_{ei}\}$  of the

equivalent shear wave velocities and the damping ratios for ground model A. From the investigation on the worst combination shown in Figure 2(b),  $V_e$  and  $\beta_e$  get the lower bounds in the shallow domain. On the other hand, it can be confirmed that the variation of  $V_e$  in ground model A is different from that of the ground deeper than 25[m]. As for  $V_e$ , some differences under the ground level -20[m] can be seen, but an important result can be seen in the proposed method that the lower bound of  $V_e$  especially under 25[m] may not necessarily critical for the pile head response. However, it is shown that the uncertainty of  $V_e$  at a deeper domain will influence the variation of the bending moment at pile head. For instance, when all  $V_e$  and  $\beta_e$  are assumed to get LLC, i.e.  $V_{ei} = 0.7V_{ei}^c$  and  $\beta_{ei} = 0.7\beta_{ei}^c$ , the maximum bending moment at pile head is relatively small compared with that by the URP method (see Table 1). An optimization approach is effective to solve such a problem, but it usually requires a long calculation time. On the other hand, the URP method can be used to estimate the upper bound of the objective function in an allowable evaluation time. From the accuracy validation by the comparison with GA, it can be concluded that the URP method is a useful bound analysis method in the structure-pile-soil system subjected to the damage-limit ground motion.

Table 1 Comparison of upper bound of bending moment at pile head by URP method with that for lower-limit combination

| Maximum bending moment at pile head |             | Ground A           |                    | Ground B           |                    |
|-------------------------------------|-------------|--------------------|--------------------|--------------------|--------------------|
| Nominal [Nm]                        |             | $6.47 \times 10^5$ |                    | $2.73 \times 10^5$ |                    |
|                                     |             | Case 1             | Case 2             | Case 1             | Case 2             |
| Upper bound by URP [Nm]             |             | $1.27 \times 10^6$ | $7.34 \times 10^5$ | $5.60 \times 10^5$ | $3.25 \times 10^5$ |
| Lower-limit combination (LLC) [Nm]  |             | $1.04 \times 10^6$ | $7.00 \times 10^5$ | $5.67 \times 10^5$ | $3.10 \times 10^5$ |
| Ratio                               | URP/Nominal | 1.97               | 1.15               | 2.05               | 1.19               |
|                                     | LLC/Nominal | 1.60               | 1.08               | 2.08               | 1.14               |

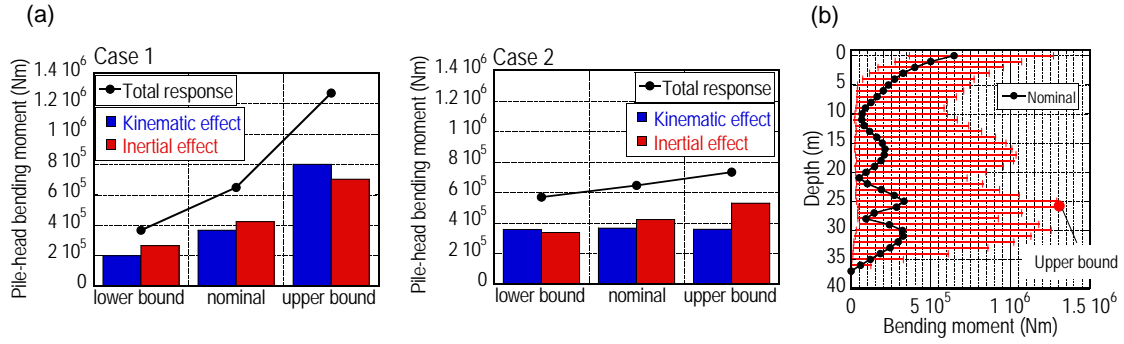


Figure 1 Interval analysis in ground model A: (a) Comparison of variation of total response of bending moment at pile head, kinematic effect and inertial effect (Objective function is bending moment at pile head in URP method), (b) Along whole depth (objective function is maximum bending moment at any depth)

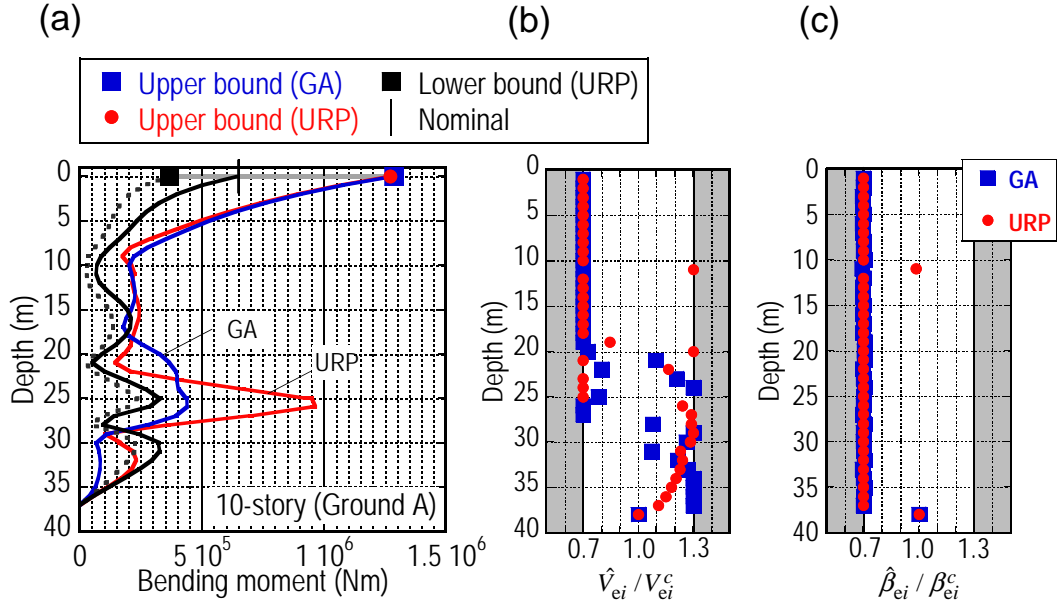


Figure 2 Accuracy validation of upper bound of bending moment at pile head obtained by URP method with that by optimization approach (Case 1, ground model A)  
(a) Maximum bending moment distribution, (b) Combination of  $\hat{V}_e$ , (c) Combination of  $\hat{\beta}_e$



#### **4. Conclusions**

The worst combination of uncertain soil properties to maximize the seismic pile response has been investigated by an uncertainty analysis approach. The conclusions are summarized as follows.

- (1) Under the condition that soil properties (stiffness and damping) are treated as uncertain parameters, the maximum pile bending moment at pile head in a structure-pile-soil system has been evaluated by the complex-value domain response spectrum method considering the modal correlation. The ground motion has been defined at the engineering bedrock surface. It has been shown that the upper and lower bounds of the pile-head bending moment and related bending moments at other depths can be computed effectively by using the advanced uncertainty analysis method called the Updated Reference-Point method (URP method).
- (2) It has been shown that the present method can be applied to the problem in which the pile bending moment at an arbitrary depth is treated as the objective function for maximization.
- (3) Two different ground models (ground model A as a rather soft ground, ground model B as a rather hard ground) have been investigated to find the worst combination of soil properties for the pile response. In the worst combinations of soil parameters, it has been confirmed that the variability of the equivalent shear wave velocities even at a deep underground can influence the bending moment at the pile head and those variables do not necessarily attain their lower bounds.
- (4) The worst combinations of soil parameters have been derived in two cases depending on the setting of uncertain parameters. While case 1 treats equivalent shear wave velocities and equivalent damping ratios of both the free-field ground and the interaction springs as uncertain parameters, case 2 regards the equivalent shear wave velocities and equivalent damping ratios of only interaction springs as uncertain parameters.
- (5) In order to investigate the accuracy of the proposed method, the upper bound derived by an anti-optimization approach using a genetic algorithm (MOGA-II [17]) has been compared with those by the URP method. It has been confirmed that the URP method can be applied to the seismic pile response in terms of the structure-pile-soil system within an acceptable accuracy.

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## References

- [1] AIJ. Seismic Response Analysis and Design of Buildings Considering Dynamic Soil-Structure Interaction. Architectural Institute of Japan 2006 (in Japanese).
- [2] Schneider HR and Schneidaer MA. Dealing with uncertainties in EC7 with emphasis on determination of characteristic soil properties, (eds.) Arnold P et al., Modern geotechnical design codes of practice: Implementation, Application and Development: Advances in Soil Mechanics and Geotechnical Engineering (Vol.1), IOP Press, Netherlands 2012; 87-101.
- [3] Biscontin, G. and Ahn, J. Measuring stiffness of soils in situ, Proc. of the 14th Int. Conf. of the Int. Assoc. for Computer Methods and Advances in Geomechanics (14th IACMAG), Plenary lecture, September 22-25, 2014, Kyoto.
- [4] Kumar, R and Bhargava, K. Review of evaluation of uncertainty in soil property estimates from geotechnical investigation, V.Matsagar (ed.) Advances in Structural Engineering 2015; 2545-2550.
- [5] Dowrick, D. Earthquake risk reduction, John Wiley & Sons, Chichester, UK.
- [6] Nakamura T, Takewaki I and Asaoka Y. Sequential stiffness design for seismic drift ranges of a shear building-pile-soil system, Earthquake Engineering and Structural Dynamics 1996; **25**(12): 405-420.
- [7] Takewaki I. Inverse stiffness design of shear-flexural building models including soil-structure interaction, Engineering Structures 1999; **21**(12): 1045-1054.
- [8] Takewaki I. Frequency domain analysis of earthquake input energy to structure-pile systems, Engineering Structures 2005; **27**(4): 549-563.
- [9] Kishida A and Takewaki I. Response spectrum method for kinematic soil-pile interaction analysis, Advances in Structural Engineering 2010; **13**(1): 181-197.
- [10] Kojima, K., Fujita, K. and Takewaki, I. Unified analysis of kinematic and inertial earthquake pile responses via single-input response spectrum method. Soil Dynamics and Earthquake Engineering 2014; **63**: 36-55.
- [11] Kojima, K., Fujita, K. and Takewaki, I. Simplified analysis of the effect of soil liquefaction on the earthquake pile response, Journal of Civil Engineering and Architecture 2014; **8**(3), 289-301.
- [12] Ben-Haim Y. Information-gap decision theory: decisions under severe uncertainty. Academic Press, San Diego 2001.
- [13] Rao S. and Berke L. Analysis of uncertain structural systems using interval analysis, AIAA Journal 1997; **34** (4): 727-735.
- [14] Chen S., Ma L., Meng G and Guo R. An efficient method for evaluating the natural frequency of structures with uncertain-but- bounded parameters, Comp. Struct. 2009; **87**: 582-590.

- [15] Fujita, K. and Takewaki, I. An efficient methodology for robustness evaluation by advanced interval analysis using updated second-order Taylor series expansion, *Engineering Structures* 2011; **33**(12): 3299-3310.
- [16] Takewaki I. Response spectrum method for nonlinear surface ground analysis, *Advances in Structural Engineering* 2004; **7**(6): 503-514.
- [17] modeFRONTIER, ESTECO, Ver.4.5, (<http://www.esteco.com>).