

# Multi-disformal invariance of nonlinear primordial perturbations

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We study disformal transformations of the metric in the cosmological context. We first consider the disformal transformation generated by a scalar field  $\phi$  and show that the curvature and tensor perturbations on the uniform  $\phi$  slicing, on which the scalar field is homogeneous, are non-linearly invariant under the disformal transformation. Then we discuss the transformation properties of the evolution equations for the curvature and tensor perturbations at full non-linear order in the context of spatial gradient expansion as well as at linear order. In particular, we show that the transformation can be described in two typically different ways: one that clearly shows the physical invariance and the other that shows an apparent change of the causal structure. Finally we consider a new type of disformal transformation in which a multi-component scalar field comes into play, which we call a “multi-disformal transformation”. We show that the curvature and tensor perturbations are invariant at linear order, and also at non-linear order provided that the system has reached the adiabatic limit.

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## I. INTRODUCTION

Cosmic inflation [1–5] is now a widely accepted paradigm of the very early Universe. It provides a mechanism of generating primordial scalar and tensor perturbations, both of which are almost scale-invariant, adiabatic, and Gaussian: The quantum vacuum fluctuations of a scalar field and those of gravitons on microscopic scales inside the causal horizon (*i.e.* on scales smaller than the Hubble horizon radius) are amplified and stretched to super-horizon scales by the inflationary expansion of the Universe, leading to primordial scalar and tensor perturbations, respectively. The scalar perturbation is equivalent to the so-called comoving curvature perturbation (which we call simply the curvature perturbation in the rest of this paper) since it describes the perturbation in the spatial curvature at linear order in the comoving slicing.<sup>1</sup> The curvature perturbation develops into density inhomogeneities after it re-enters the Hubble horizon, and seeds the formation of galaxies and galactic clusters [6]. The tensor perturbation gives rise to a gravitational wave background [7–9].

As for the curvature perturbation, the above mentioned features have been confirmed by the full sky observations of cosmic microwave background (CMB) anisotropies [10, 11]. In particular, the anti-correlation of the temperature and E-mode polarization detected on angular scales of  $50 \lesssim \ell \lesssim 200$  [10, 12] strongly supports the presence of the super-horizon scale curvature perturbation at the time of decoupling of CMB photons ( $z \approx 1090$ ). Moreover, detection of the B-mode polarization on angular scales of  $\ell \lesssim 150$ , which may become possible in the near future, will imply the presence of the super-horizon scale tensor perturbation at the decoupling time, and will give further strong evidence for inflation.

Since present and planned-future cosmological observations, including those of the galaxy distribution, CMB and 21-cm fluctuations on various scales are getting precise enough to probe non-linear features such as the bispectra and trispectra of scalar and tensor perturbations, there is a possibility to identify or constrain the origin of these perturbations observationally. Although all of the single-field slow-roll inflation models with a canonical kinetic term predict unobservably small non-linear effects on the primordial fluctuations [13–15] (see [16] for a gravitational enhanced friction model), any deviation from canonical single-field slow-roll inflation can predict detectably large non-Gaussianity of the curvature perturbation, including models with non-canonical kinetic terms [17–23], plural light fields [24], *e.g.*, curvaton model [25–29], modulated reheating [30, 31], and multi-brid inflation [32].

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<sup>1</sup> The comoving slicing is defined as a slicing on which the hypersurface orthogonal vector coincides with the timelike eigen-vector of the energy momentum tensor (four-velocity in the case of a fluid), under the assumption that the vector-type perturbation is negligible.

Among those non-canonical models, many of them are constructed in extended scalar-tensor theories of gravity, such as generalized G-inflation [33–36] and its extensions [37–41]. Therefore, studies of non-linear cosmological perturbations within the framework of extended gravity are necessary for distinguishing models of inflation. Such studies are also equally important for testing alternative theories of inflation, such as bouncing cosmology [42].

A formalism has been developed on the basis of spatial gradient expansion to study the classical evolution of the curvature perturbation to full non-linear order on super-horizon scales. The zeroth order truncation of gradient expansion corresponds to the separate universe approach [13, 43, 44], which is the basis of the  $\delta N$  formalism [43, 45, 46] that allows us to study the nonlinear evolution of the curvature perturbation. Moreover, at the next leading order, namely the second order in the gradient expansion we can take into account the effect of a mode closely related to the cosmic shear, which usually decays rapidly on super-horizon scales, but which may become important in a model where the slow-roll evolution may be violated. Studies on gradient expansion to second order were carried out for single-field k-essence [47], for multi-field k-essence [49], and for the kinetic gravity braiding (KGB) model [48].

Creminelli *et al.* [50] have recently shown that the primordial linear tensor power spectrum from inflation can be always cast into the standard form, i.e.,  $P_\gamma(k) = H^2/(2M_{\text{Pl}}^2 k^3)$ , at leading order in derivatives with suitable conformal and disformal transformations in the context of the effective field theory (EFT) of inflation [51] where the speed of propagation of gravitational waves is not necessarily unity and hence the theory of gravity can be modified from general relativity (GR).

Thus, regardless of the theory of gravity, we can formally obtain a prediction for linear tensor modes, which strongly indicates a close relation between modifying gravity and a disformal plus conformal transformation from GR. Moreover, invariance of the curvature perturbation under disformal transformations has been shown at linear order [52, 53].

In this paper, we extend the invariance of the curvature and tensor perturbations to fully non-linear order. In Sec. II, we show the invariance under the disformal transformation to fully non-linear order by exploiting the effect of the transformation on the form of the metric. We then consider the evolution equations for the curvature and tensor perturbations, and show explicitly that the invariance holds not only in linear perturbation theory but also full non-linearly up through second order in spatial gradient expansion. In Sec. III, we consider a new type of disformal transformation, dubbed multi-disformal transformation, generated by a multi-component scalar field. We show that the curvature and tensor perturbations are still invariant under the multi-disformal transformation at linear order, and the curvature and tensor perturbations remain invariant again up through the second order in gradient expansion, provided that the system has reached the adiabatic limit. In doing so, we also show that the curvature and tensor perturbations in a wide class of extended scalar-tensor theories can be described in terms of those in GR with redefinitions of background quantities.

## II. INVARIANCE OF NON-LINEAR PERTURBATIONS AND EQUATIONS

### A. Invariance of non-linear perturbations

Let us employ the Arnowitt-Deser-Misner (ADM) formalism and the metric is expressed as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \hat{\gamma}_{ij} (dx^i + \beta^i dt) (dx^j + \beta^j dt), \quad (2.1)$$

where  $\alpha$  is the lapse function,  $\beta^i$  is the shift vector and Latin indices run over 1, 2 and 3. In addition to the standard ADM decomposition, the spatial metric are further decomposed so as to separate trace and unimodular parts as

$$\hat{\gamma}_{ij} = a^2(t) e^{2\psi} \gamma_{ij}, \quad \det \gamma_{ij} = 1, \quad (2.2)$$

where  $a(t)$  is the scale factor of a fiducial flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime with the metric,

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j. \quad (2.3)$$

It is sometimes convenient to use the conformal time  $\eta$ ,

$$d\eta = \frac{dt}{a(t)}, \quad (2.4)$$

in place of the cosmic proper time  $t$ . Below we denote the proper Hubble expansion rate of the background FLRW universe by  $H$  and the conformal Hubble expansion rate by  $\mathcal{H}$ . That is,  $H = \dot{a}/a$  and  $\mathcal{H} = a'/a$ , where a dot denotes  $d/dt$  and a prime  $d/d\eta$ .

The general form of disformal transformation is defined as [54]

$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X) \partial_\mu \phi \partial_\nu \phi, \quad X \equiv -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2. \quad (2.5)$$

To focus on the effect of disformal transformation, we concentrate on the case  $A = 1$ , that is, a pure disformal transformation. For simplicity, we simply call it the disformal transformation throughout this paper unless confusion may arise. As clear from the above form, the general disformal transformation consists of a disformal transformation ( $A = 1, B \neq 0$ ) and a conformal transformation ( $A \neq 1, B = 0$ ). In the end of this subsection, we also discuss the invariance under the conformal transformation.

As a choice of the time slicing, we take the uniform  $\phi$  slicing on which the scalar field is given as a function of time only, that is,

$$\phi = \phi(t). \quad (2.6)$$

Under the assumption that the scalar field  $\phi$  dominates the universe, this slicing is equivalent to the comoving slicing. We denote the scalar component of  $\gamma_{ij}$  by  $\chi$ , which is extracted from  $\gamma_{ij}$  as

$$\chi \equiv -\frac{3}{4} \Delta^{-1} \left\{ \partial^i \left[ e^{-3\psi} \partial^j \left( e^{3\psi} (\gamma_{ij} - \delta_{ij}) \right) \right] \right\}, \quad (2.7)$$

where  $\Delta$  is the flat 3-dimensional Laplacian and  $\partial^i = \delta^{ij} \partial_j$ , and denote  $\psi + \chi/3$  on the comoving slicing by  $\mathfrak{R}_c$  ( $\mathfrak{R}_c \equiv \psi_c + \chi_c/3$ ) and its linearized version by  $\mathcal{R}_c$ . Namely,  $\mathcal{R}_c$  is the (comoving) curvature perturbation at linear order and  $\mathfrak{R}_c$  its non-linear generalization, which is the variable of our interest.<sup>2</sup> As for the tensor perturbation, we define its non-linear generalization as the transverse part of  $\gamma_{ij}$  with respect to the flat background metric  $\delta_{ij}$ . We denote it by  $\gamma_{ij}^{\text{TT}}$ . That is,  $\partial^j \gamma_{ij}^{\text{TT}} = 0$ .<sup>3</sup> We note that  $\gamma_{ij}^{\text{TT}}$  is independent of the time-slicing condition at linear order but is slice-dependent at higher orders. Thus at non-linear order our discussion applies only to  $\gamma_{ij}^{\text{TT}}$  defined on the uniform  $\phi$  slicing.

An immediate consequence here is that the disformal transformation affects only the  $(0, 0)$ -component of the metric:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B \dot{\phi}^2 \delta_\mu^0 \delta_\nu^0, \quad (2.8)$$

and hence only the lapse function,

$$\tilde{\alpha}^2 = \alpha^2 - B \dot{\phi}^2. \quad (2.9)$$

For completeness, let us write down the transformed metric,

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = -\tilde{\alpha}^2 dt^2 + \hat{\gamma}_{ij} (dx^i + \beta^i dt) (dx^j + \beta^j dt), \quad (2.10)$$

and the corresponding FLRW background,

$$d\tilde{s}^2 = -\tilde{\alpha}_0^2 dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad (2.11)$$

where  $\tilde{\alpha}_0$  is the background value of  $\tilde{\alpha}$ .

Thus, with the choice of the uniform  $\phi$  slicing, the disformal transformation only induces the change in the lapse function and hence the spatial metric as well as the shift vector are trivially invariant to fully non-linear order. This fact implies the invariance of the curvature perturbation on the uniform  $\phi$  slicing as well as that of the tensor perturbation which is contained in the unimodular part of the spatial metric.

Next let us discuss the invariance of the curvature and tensor perturbations under the conformal transformation. It is widely known that when  $A$  in (2.5) is a function of  $\phi$  only, the curvature and tensor perturbations on the uniform  $\phi$  slicing are invariant even at non-linear level [55]. Here we shall extend their discussion by introducing the dependence on  $X$  in  $A$ . First as for the tensor perturbation, it is manifestly invariant even under this extended conformal transformation because the conformal factor does not affect the unimodular part of the spatial metric. On the

<sup>2</sup> We note that the non-linear generalization of the curvature perturbation is not unique, but it is unique up through second order in spatial gradient expansion [47, 48].

<sup>3</sup> Note that  $\gamma_{ij}^{\text{TT}}$  is traceless only at linear order, in spite of the fact that we use the superscripts TT for convenience, which could mean transverse and traceless.

other hand, the curvature perturbation is no more invariant under this transformation in general. By perturbatively expanding the transformation equation,

$$\tilde{g}_{\mu\nu} = A g_{\mu\nu} = \bar{A} g_{\mu\nu} + \delta A \bar{g}_{\mu\nu} + \delta A \delta g_{\mu\nu}, \quad (2.12)$$

one easily notices that the second (as well as third) term on the RHS implies the change in the curvature perturbation as  $\mathcal{R} \rightarrow \mathcal{R} + \delta A/2 + \dots$ , while the first term only affect the (background) scale factor. However there is a case where the invariance of curvature perturbation still holds, namely when  $\delta A$  vanishes. Since in the uniform  $\phi$  slicing  $\phi$  is a function of time,  $\delta A$  is sourced by the perturbation of  $X$ , which in turn comes from that of the lapse function because

$$X_c = \frac{1}{2} \frac{\dot{\phi}^2(t)}{\alpha_c^2(t, \mathbf{x})} = \frac{1}{2} \dot{\phi}^2(t) - \delta\alpha_c(t, \mathbf{x}) \dot{\phi}^2(t) + \dots, \quad (2.13)$$

where the subscript  $c$  is for the comoving (or uniform  $\phi$ ) slicing. In the context of single-field inflation, it is well known that the curvature perturbation on the uniform  $\phi$  slicing is conserved on large scales.<sup>4</sup> This fact may be regarded as a consequence of the vanishing of the lapse function perturbation on large scales,  $\delta\alpha_c = \mathcal{O}(\varepsilon^2)$  where  $\varepsilon$  represents terms of first order in spatial derivatives, because the evolution of the curvature perturbation is solely induced by the lapse function in the uniform  $\phi$  slicing,  $\mathcal{R}_c \propto H\delta\alpha_c$ , which is valid not only in GR but also in Horndeski [33–36] and GLPV [37] theories.

To summarize we conclude that the invariance of the curvature perturbation holds under the pure disformal transformation ( $A = 1$ ) to full non-linear order, and is effectively realized to full non-linear order on superhorizon scales (or at leading order in gradient expansion) under the most general disformal transformation given by (2.5), while the tensor perturbation defined on the comoving slicing is invariant under the most general disformal transformation to full non-linear order.

## B. Transformation of non-linear equations

Here the effects of the disformal transformation on the evolution equations for the scalar and tensor perturbations are discussed for linear theory and for fully non-linear theory but in the context of gradient expansion.

In linear theory, the equation for  $\mathcal{R}_c$  in the case of a canonical scalar field in GR is

$$\frac{1}{z^2} \frac{1}{\alpha_0} \frac{d}{d\eta} \left( \frac{z^2}{\alpha_0} \frac{d}{d\eta} \mathcal{R}_c \right) + c_s^2 k^2 \mathcal{R}_c = 0, \quad z \equiv a \frac{\phi'}{\mathcal{H}}, \quad (2.14)$$

where  $\alpha_0$  is the background value of the lapse function and  $c_s$  is the sound velocity, both of which are unity in the present case,  $\alpha_0 = 1$  and  $c_s = 1$ . However, we keep them here for convenience.

Noting (2.9), the disformal transformation gives

$$\frac{1}{\tilde{z}^2} \frac{1}{\tilde{\alpha}_0} \frac{d}{d\eta} \left( \frac{\tilde{z}^2}{\tilde{\alpha}_0} \frac{d}{d\eta} \mathcal{R}_c \right) + \tilde{c}_s^2 k^2 \mathcal{R}_c = 0, \quad \tilde{z} \equiv a \frac{\phi'}{\mathcal{H}}. \quad (2.15)$$

Comparing (2.14) and (2.15), it is readily apparent that both of them are in the same form if the derivatives are expressed in terms of the proper time defined in respective frames,  $d\tau = \alpha_0 dt$  and  $d\tilde{\tau} = \tilde{\alpha}_0 dt$  ( $dt = a d\eta$ ). There is no change in  $z$  nor in  $c_s$ .

On the other hand, sometimes it is convenient to write the transformed equation by redefining  $z$  and  $c_s$ ,

$$\frac{1}{\tilde{z}^2} \frac{1}{\alpha_0} \frac{d}{d\eta} \left( \frac{\tilde{z}^2}{\alpha_0} \frac{d}{d\eta} \mathcal{R}_c \right) + \tilde{c}_s^2 k^2 \mathcal{R}_c = 0, \quad \tilde{z} \equiv a \frac{\phi'}{\mathcal{H}}, \quad (2.16)$$

where we have defined

$$\tilde{c}_s \equiv \tilde{\alpha}_0 c_s, \quad \tilde{z} \equiv \frac{z}{\sqrt{\tilde{\alpha}_0}}. \quad (2.17)$$

With this redefinition of the background quantities, the equation for  $\mathcal{R}_c$  can be reinterpreted as the one in a modified theory of gravity. Conversely, this implies that the equation for the curvature perturbation in a theory of modified gravity may be put in the form of the one in GR by a suitable disformal transformation.

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<sup>4</sup> Strictly speaking, the argument cannot be applied to the so-called ultra slow-roll model [56, 57].

Now we turn to the non-linear case. In the context of spatial gradient expansion, the full non-linear equation for  $\mathfrak{R}_c$  is given by [47]

$$\frac{1}{z^2} \frac{1}{\alpha_0} \frac{\partial}{\partial \eta} \left( \frac{z^2}{\alpha_0} \frac{\partial}{\partial \eta} \mathfrak{R}_c \right) + \frac{c_s^2}{4} {}^{(3)}R [e^{2\psi} \gamma_{ij}] = \mathcal{O}(\varepsilon^4), \quad (2.18)$$

with the same  $z$  as in (2.14). Here  $\psi = \mathfrak{R}_c + \mathcal{O}(\varepsilon^2)$  and  ${}^{(3)}R$  is the spatial scalar curvature. It was later extended so as to include the KGB term in [48]. By the same reasoning as in the linear case, the nonlinear equation for  $\mathfrak{R}_c$  remains invariant if expressed in terms of the proper time in each frame, but it can also be interpreted as the one in a modified theory of gravity with modified  $z$  and  $c_s$  as defined by (2.17).

As for the tensor perturbation, one can find the full-nonlinear equation for  $\gamma_{ij}^{TT}$ :

$$\frac{1}{z_t^2} \frac{1}{\alpha_0} \frac{\partial}{\partial \eta} \left( \frac{z_t^2}{\alpha_0} \frac{\partial}{\partial \eta} \gamma_{ij}^{TT} \right) + \frac{1}{4} \left( e^{-2\psi} {}^{(3)}R_{ij} [e^{2\psi} \gamma_{ij}] \right)^{TT} = \mathcal{O}(\varepsilon^4), \quad (2.19)$$

where  $z_t \equiv a$  and  $(\dots)^{TT}$  denotes the transverse-traceless projection,<sup>5</sup>  $\psi = \mathfrak{R}_c + \mathcal{O}(\varepsilon^2)$  and  ${}^{(3)}R_{ij}$  is the spatial Ricci tensor. Thus the same argument as the scalar case applies to the tensor case as well. Namely, the equation takes exactly the same form if expressed in terms of the proper time. Also, the equation after the disformal transformation may be expressed as the one in a modified gravity theory,

$$\frac{1}{\tilde{z}_t^2} \frac{1}{\alpha_0} \frac{\partial}{\partial \eta} \left( \frac{\tilde{z}_t^2}{\alpha_0} \frac{\partial}{\partial \eta} \gamma_{ij}^{TT} \right) + \frac{\tilde{c}_t^2}{4} \left( e^{-2\psi} {}^{(3)}R_{ij} [e^{2\psi} \gamma_{ij}] \right)^{TT} = \mathcal{O}(\varepsilon^4), \quad (2.20)$$

where we have defined

$$\tilde{c}_t \equiv \tilde{\alpha}, \quad \tilde{z}_t \equiv \frac{z_t}{\sqrt{\tilde{\alpha}}} = \frac{a}{\sqrt{\tilde{\alpha}}}. \quad (2.21)$$

To conclude this subsection, we have explicitly shown that the evolution equations for the curvature and tensor perturbations are invariant under the disformal transformation not only in linear perturbation theory but also non-linearly up through second order in gradient expansion. We have also shown that the transformed equations can also be cast in the form of those in a theory of modified gravity.

### III. MULTI-DISFORMAL TRANSFORMATION

In this section, we consider a new type of disformal transformation generated by a multi-component scalar field, dubbed ‘‘multi-disformal transformation’’. Suppose there are  $\mathcal{N}$  component scalar field,  $\phi^I$  ( $I = 1, \dots, \mathcal{N}$ ). Extending the general disformal transformation, we consider

$$\tilde{g}_{\mu\nu} = A(\phi^I, X^{IJ}) g_{\mu\nu} + B_{KL}(\phi^I, X^{IJ}) \partial_\mu \phi^K \partial_\nu \phi^L \quad (3.1)$$

where

$$X^{IJ} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J. \quad (3.2)$$

Again we focus on the pure disformal part,  $A = 1$ . An immediate consequence is that the linear curvature and tensor perturbations are invariant even in this general transformation because the contribution to the spatial section of the metric is of second order in perturbation, i.e.,  $B_{IJ} \partial_i \phi^I \partial_j \phi^J = \mathcal{O}(\delta^2)$  where  $\delta$  represents the order of linear perturbation. Furthermore because it is also of second order in spatial derivatives, the curvature and tensor perturbations are invariant to full non-linear order at leading order in gradient expansion. However apparently this result cannot be extended to second order in gradient expansion, nor to second order in perturbation.

Nevertheless, there exists a situation where the non-linear invariance still holds, namely, when the system has converged to a state where all the components are determined by a unique scalar function, say  $\varphi$ . Conventionally it is said that a system has reached the adiabatic limit when such a state is realized. Now let us consider the adiabatic-limit,

$$\phi^I = \phi^I(\varphi). \quad (3.3)$$

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<sup>5</sup> Note that the time derivative of  $\gamma_{ij}^{TT}$  is of second order in gradient expansion and is traceless, though  $\gamma_{ij}^{TT}$  itself is not so.

Restricting to  $A = 1$ , the above transformation in this limit reduces to

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_{KL} \left[ \phi^I(\varphi), X^{IJ}(\varphi, \partial\varphi) \right] (\phi^K)' (\phi^L)' \partial_\mu \varphi \partial_\nu \varphi, \quad (\phi^I)' \equiv \frac{\partial \phi^I}{\partial \varphi}. \quad (3.4)$$

Here a prime denotes  $d/d\varphi$ , not to be confused with the conformal time derivative. Then by taking a uniform  $\varphi$  slicing, this equation further reduces to

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_{KL} \left[ \phi^I(\varphi), X^{IJ}(\varphi, \dot{\varphi}/\alpha) \right] (\phi^K)' (\phi^L)' \dot{\varphi}^2 \delta_\mu^0 \delta_\nu^0. \quad (3.5)$$

Since the disformal transformation only affects the lapse function, we can apply the same argument as in Sec. II and conclude again that the curvature and tensor perturbations defined on the uniform  $\varphi$  slicing are invariant under the disformal transformation to full non-linear order.

Finally, as for the conformal transformation, the invariance holds in the adiabatic limit if  $A$  is a function of only  $\phi^I$ . The invariance does not hold in general if  $A$  contains  $X^{IJ}$  dependence. Nevertheless, if we focus on the adiabatic limit, we can again apply the same argument as in Sec. II and show the invariance on superhorizon scales at leading order in gradient expansion.

#### IV. SUMMARY

In this paper we have discussed the disformal transformation in the context of cosmology, particularly of inflationary cosmology. We have shown that the curvature and tensor perturbations on the uniform  $\phi$  slicing are fully non-linearly invariant under the disformal transformation by exploiting the form of the transformed metric. We have also shown explicitly to second order in gradient expansion that the evolution equations for the curvature and tensor fluctuations are invariant under the disformal transformation. In doing so, we have shown another aspect of the disformal transformation, namely the transformed equations may be expressed in the form which are identical to those in a modified gravity theory.

Then we have discussed a new type of disformal transformation, dubbed “multi-disformal transformation,” which is an extension of the disformal transformation to the multi-component field case. In this case assuming the system has reached the adiabatic limit where all the components of the scalar field depend only on a single scalar function,  $\varphi$ , we have also shown the full nonlinear invariance of the curvature and tensor perturbations on the uniform  $\varphi$  slicing.

It seems there are several important or interesting applications of the results obtained in this paper. First as we have seen, utilizing disformal and conformal transformations one can modify the background quantities appearing in the perturbation equations such as the speed of propagation of a perturbation. Thanks to this property, once a second order differential equation is obtained either from a concrete theory like Galileon theory or EFT approach to inflation, one can map it to the form same as the one in GR by a suitable disformal transformation without solving the equations. We have shown that this property can be extended to non-linear equations up to second order in gradient expansion. In fact, the full nonlinear invariance of the curvature and tensor perturbations suggests that this mapping may be extended to full non-linear order without resorting to gradient expansion [63]. This will make a very useful tool for studying the cosmological perturbations in otherwise complicated models. Definitely further studies in this direction seems fruitful.

As for the multi-disformal transformation, it might play an important role in uncovering a new theory. After the (re-)discovery of the generalized Galileon or Horndeski theory [36], which is the most general scalar-tensor theory with the second-order field equations, it has been further extended to a more general class of theories [37]. The relations among those theories under disformal plus conformal transformations have been partially studied in the literature, but they have not been fully understood yet. The relation between Galileon/Horndeski theory and DBI theory under the disformal transformation has been also discussed [58, 59], but restricted to a very special case.

For multi-field extensions of Galileon/Horndeski theory, there are so far two attempts in the literature: one is a (not necessarily general) multi-field extension of the single-field Galileon/Horndeski theory [40], and the other is a multi-scalar version of DBI Galileon theory [60–62] obtained by a special form of the multi-disformal transformation applied to the DBI theory. It has been found that thus obtained multi-DBI Galileon theory contains terms which are absent in the above-mentioned multi-field Galileon/Horndeski theory [41]. This fact indicates the existence of a more general class of multi-scalar-tensor theories with second-order field equations. Thus studies of the multi-disformal transformation in its most general form would shed light on new theoretical possibilities and lead us to new cosmological models.

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