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Kyoto University
The year 2013 was designated a special year for the Mathematics of Planet Earth [MPE2013; http://mpe2013.org/], supported by the United Nations Educational, Scientific and Cultural Organization (UNESCO). MPE2013 is dedicated to the study of the challenges facing our planet, which is the setting for dynamic processes that determine our weather and climate. These challenges are multidisciplinary and multifaceted, and the mathematical sciences play a central role in the efforts to understand and address them. The Research Institute for Mathematical Sciences (RIMS) of Kyoto University is one of the participants in MPE2013, and it sponsored this conference as part of the RIMS 2013 Research Project: “Fluid Dynamics of Large-Scale Flows.”

Since the pioneering work by Lorenz (1963), the study of chaos and predictability has been of keen interest to mathematicians and atmospheric scientists (e.g., Yoden 2007). This conference builds on the 50-yr history of innovative mathematical research on atmospheric predictability. Its main objectives were to review recent progress in our understanding of the variability and predictability of weather and climate systems, to enrich the exchange of information within the communities of atmospheric and climate sciences, and to attract researchers with a wide range of expertise in mathematical sciences.
Supérieure (ENS), France, and University of California, Los Angeles (UCLA), United States] and Professor Tim Palmer (University of Oxford, United Kingdom) discussed the role and importance of mathematics in the study of weather and climate. The recorded videos of Ghil’s (2013) and Palmer’s (2013) lectures can be viewed online (see the webpage of the MOSIR1 video/material archive project of Center for Planetary Science, Kobe University). The scientific highlights presented in this conference are summarized below.

**Variability of Weather and Climate.** The presentations in this session spanned a wide spectrum; particularly worthy of mention are three relatively new mathematical tools to describe the variability and the predictability of weather and climate.

The first tool is the Boolean delay equation (BDE), which is an intermediate category between ordinary/partial differential equations and cellular automata. In BDEs, the values of the dependent variables are discretized, while the independent variable, the time, remains continuous. A simple model of ENSO using BDE successfully describes its chaotic variation and contributes to a deeper mathematical understanding of its mechanism. The methodology of BDEs may also be used to model economic cycles and to study the economic impacts from extreme events. For example, the model suggests natural disasters produce worse impacts in the recession phase of the economic cycle.

The second tool is the stochastic differential equation (SDE), in which atmospheric variations are described as a combination of deterministic and stochastic processes. Estimates of error growth in ensemble forecasts based on SDE were compared to those from traditional deterministic equations. The dependence of theoretically estimated error growth on the phase of extratropical low-frequency variability in 500-hPa height variations in Northern Hemisphere winter was compared with ensemble forecast spread. This comparison showed the SDE estimate provided a better correspondence with the forecast spread than the estimate from traditional deterministic equations, suggesting that stochastic processes exert a significant control on the dynamics of extratropical tropospheric low-frequency variability and on its predictability.

The third tool is the conditional nonlinear optimal perturbation (CNOP), which gives the perturbation whose nonlinear evolution increases the value of a specified cost function most rapidly. Compared with traditional approaches that use tangent linear equations, CNOP has an advantage that it is more directly applicable to strongly nonlinear systems. The CNOP approach may be used to detect optimal precursors (OPR) and optimally growing initial error (OGE) for weather or climate events, such as ENSO, atmospheric blocking, or large meanders in the Kuroshio. When applied to these phenomena, OPR and OGE identified similar localized structures, which may help to identify regions where improvements to the observational networks would be most advantageous.

**Predictability of Weather and Climate.** The presentations in this session encouraged us to revisit the theory and practice of numerical weather prediction (NWP) and to apply novel techniques and knowledge in predictability research using rich ensemble forecast datasets now available. Several remaining challenges in the predictability of weather and climate were presented in this session, including the formulation of dynamical cores and physics schemes in global convection-permitting models, the prediction of severe weather events on global and regional scales, and forecasting on monthly to decadal time scales. Three recent developments presented in this session are discussed below.

The first development is that computing power has increased to the point where global NWP models have begun to resolve convective motions. The assumptions of some subgrid-scale parameterizations, such as the type of subgrid-scale moist convection that may be in need of parameterization, are no longer valid at grid sizes on the order of 5–15 km. This inadequacy requires us to revisit the traditional modeling distinction between dynamics and physics (Fig. SB1). Stochastic parameterization is a promising alternative approach that has had some success in operational forecasting. Increasing horizontal resolution also raises the issue of predictability on such fine scales. One of the most important limitations to the predictability of such mesoscale motions has been thought to arise from the upscale propagation of errors in initial conditions at very small scales. On the other hand, the large scales, which have relatively long predictability thresholds, have generally been assumed to enhance mesoscale predictability. Yet, results from large ensembles of mesoscale forecasts and from Lorenz (1969)’s original spectral turbulence model were presented, suggesting that small relative errors in the initial large-scale state can degrade

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1 MOSIR is a word of Ainu, an indigenous people in Japan and Russia, that means literally the “land,” and hoping our archive to be a fundamental knowledge base.
mesoscale forecasts more rapidly than gross initial errors in the smallest resolved scales.

A second key development is the improvement in the quality and availability of ensemble forecasts. The identification of optimal initial perturbations that most strongly influence the forecast had generally required access to a data assimilation system with an adjoint model. New formulations now allow the use of data from prerun ensemble forecasts to approximate adjoint and singular vector sensitivities. Examples were presented showing that ensemble-based formulations can successfully identify sensitive regions of midlatitude and tropical cyclones. The Observing System Research and Predictability Experiment (THORPEX) Interactive Grand Global Ensemble (TIGGE) is a database of weekly ensemble forecasts from major operational centers that can be used to verify the forecast skill during high-impact weather regimes. As an example, the predictability of the weather regimes over Europe and the Atlantic Ocean was investigated using a clustering technique. The results show excellent skills in forecasting North Atlantic Oscillation (NAO) persistence in winter and the transition from Atlantic ridge to NAO in summer, but the lowest skill predicting the development of Euro-Atlantic blocking in winter and summer.

The third key development involves efforts to forecast weather and climate variations beyond weekly time scales. Studies of troposphere–stratosphere coupling using one-month forecasts from the Japan Meteorological Agency (JMA) reveal downward influences from the stratosphere to the troposphere due to Rossby wave reflection and extended predictability during some stratospheric sudden warming events. Such a mechanism implies a possibility of improved forecast skill beyond two weeks if a stratospheric sudden warming is predictable. Attempts at decadal prediction using the Model for Interdisciplinary Research on Climate (MIROC) by the Atmosphere and Ocean Research Institute (AORI;...
University of Tokyo)/Japan Agency for Marine-Earth Science and Technology (JAMSTEC)/National Institute for Environmental Studies (NIES) show that initial conditions remain important even on the decadal time scale, and that the removal of large systematic errors remains an important challenge.

**DATA ASSIMILATION OF WEATHER AND CLIMATE.** Data assimilation is widely used to optimize initial conditions for weather forecasts, but it can also be used to determine optimal boundary conditions and other parameters in weather and climate models. The data assimilation session considered several of the major contemporary problems in this field.

One important problem arises from the need to properly treat a wide range of different scales when assimilating data for high-resolution global models. We have successfully improved synoptic-scale weather forecasting in the past several decades through improvements in observing capabilities and increases in computing power. These technological developments are likely to continue for the foreseeable future. It is therefore important to understand how to design suitable data assimilation algorithms for increasingly higher-resolution weather and climate models and increasingly dense observational datasets.

A key question is how data assimilation should include multiscale energy transfers for better predictability of both convective and synoptic scales. This question can be restated in an idealized framework by asking how data assimilation should account for the difference in error propagation through the small scales (the regime where the kinetic energy spectrum decays in proportion to the \(-5/3\) power of the wavenumber) and the large scales (the \(-3\) regime). Idealized data assimilation experiments were presented showing that small-scale information is produced by the dynamics of the \(-3\) regime (large scales), but the \(-5/3\) regime (small scales) requires observations at each scale because only limited information is generated by the small-scale dynamics themselves. A second study tackling the problem of multiscale data assimilation demonstrated a practical approach to obtain longer-range error covariances for the ensemble Kalman filter using a limited ensemble size.

Another important problem is data assimilation for longer time scales, in which the ocean plays a significant role. A few groups have developed four-dimensional variational data assimilation (4D-Var) systems for global coupled atmosphere-ocean simulations. A key question is how to cope with different time scales present in climate prediction. A newly developed master-slave formulation can extract slow signals in 4D-Var. With this formulation, slow modes are constrained to observations, while fast modes are tied to a reference state, so that we can have a seasonal-scale time window in 4D-Var without contamination by the fast modes.

**CONCLUDING REMARKS.** Ever since Lorenz’s early 1960s discovery of deterministic chaos, the connection between mathematics and atmospheric science has been growing closer and closer. The three days of presentations in this conference are manifestations of this connection, and they remind us that continued cooperation between mathematicians and atmospheric scientists is required to better understand the variability and predictability of weather and climate. Such understanding is important both for day-to-day weather forecasting, which provides substantial societal and economic benefits, and for those extreme situations where accurate forecasts or climate change assessments can allow us to better anticipate and mitigate extreme events and natural disasters that have an immense impact on human society.

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**REFERENCES**


