Stochastic Analysis For Water Pipeline System Management

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Executive Summary

Pipeline system is an important infrastructure to supply purified water for maintenance and development of city. It has been expanded with rapid urban development and improvement in standards of living and now asset of pipeline system has become enormous amount but, the pipeline system was built at the high economic growth intensively and now, it has emerged as a major problem to the water authorities worldwide to renew these enormous aging pipelines. Thus, it is big issue to maintenance of aging pipeline which is laid under the ground intensively at period of rapid economic growth. However, pipelines are placed in underground due to the limitation of land-use and social requirements, it is difficult to understand deterioration process with inspection and monitoring. So far, to reduce high repair and social costs in case of damage occurrences, pipelines within a certain period time from building point have been replaced regardless of deterioration condition. These inefficient management methods that depend on manager's experience cause waste of budget. Thus, it has been required to introduce asset management system for pipeline system. Decision making for pipeline system management that is policy variables for determining a significant strategy for the timing of rehabilitation is greatly affected by deterioration prediction model. It is therefore important to know how the deterioration of the system proceeds. This research has proposed probabilistic deterioration forecasting models based on statistical methods with inspection data for optimal rehabilitation strategy of the pipeline system. The deterioration process of pipeline is formulated by a hazards model. In addition, the author also shows model determining optimal rehabilitation timing with least life cycle cost analysis.

A pipeline is deteriorated with the lapse of time after installation and along with the deterioration, pipe bursts can occur by various types of failure, and the choice of a maintenance and repair strategy will depend on the type of failure. It is therefore important to know how these types of failure proceed. In chapter 3, the study addresses a competing deterioration-hazard model that permits modelling of deterioration by multiple types of failure and focus on the bursts which occur in pipe body or connection. The Weibull hazard model is used to address the lifetime of each pipeline, measured from when it was buried, and the model takes into account the competing nature of various types of failure by using a competing hazard model. The competing deterioration-hazard model allows us to determine the probability of

deterioration in pipe body and connection. The model is estimated by Bayesian inference using a Markov chain Monte Carlo method. The applicability of the method to data for an existing pipeline system is examined. The competing deterioration-hazard model allows us to determine the probability density of bursts in pipe body and connection. In comparison with the conventional Weibull deterioration hazard, the competing deterioration hazard model can improve the quality of deterioration forecasting. Therefore, the proposed model is hoped to bring in innovative academic contributions. Among the failure types which are in competition, the competing deterioration-hazard model prevents overestimating the occurrence probability of interest failure due to the presence of competing failures. It enables us to formulate optimal rehabilitation model.

In chapter 4, the study targeted mainly on development of methodology to apply deterioration hazard model proposed in chapter 3. Failure of aging pipeline leads to greater social and economic damage such as traffic control by flooding of neighborhood, restoration of damaged pipelines, water quality degradation due to the influx of pollutants and so on. Thus, through the proper rehabilitation and replacement, the pipeline systems must be managed by safe water quality and structure performance. This ideal consequently leads to the demand of determining the optimal rehabilitation time based on the principle of minimizing the overall life cycle cost. The pipeline is deteriorated with the lapse of time after installation and along with the deterioration, leakage occurs due to cracking or partial break and eventually, the pipeline reaches complete burst. The important point in this study is that the author considers the occurrence of leakage in pipeline and develops optimal rehabilitation model considering repair of leakage during life time of pipeline. The time to burst and leak are used as random variables and explained by using Weibull distribution and exponential distribution, respectively. The deterioration procedure of burst and leakage is forecasted by using competing deterioration hazard model which is proposed in chapter 3. Estimation for optimal replacement time and expected life cycle cost are carried out in the second phase after estimating the competing deterioration hazard model. The occurrence probability of pipe leakage and burst are predicted and then, least life cycle cost analysis is conducted on the basis of maintenance strategy that repair of leakage and pipe replacement due to burst are carried out on an as needed basis. The empirical application of the proposed model was carried out to the real pipeline system, S city in Korea. We could obtain the optimal life cycle cost and optimal replacement time of each pipe type and diameter. The estimation results demonstrated that the DCIP is more beneficial type of pipe than CIP in asset management of the pipeline system. It is expected that this study assists government agencies in implementing a comprehensive pipeline system management to further assist in making satisfactory financial decisions.

The proposed deterioration forecasting model in chapter 3 portrayed the deterioration progress only by using binary condition state. However, the working status or condition state of pipeline is not just binary expression but often in a wide range of discrete numbers. In chapter 5, the deterioration process among the discrete condition state of pipeline is expressed by markov deterioration hazard model. Deterioration process of pipeline is complex phenomenon consisting of corrosion of inner surface and pipe body degradation. The existing studies have been carried out forecasting the deterioration process of each surface corrosion and pipe body degradation but there is no study about deterioration prediction considering the interaction of the corrosion of inner surface and the degradation of pipe body. But for maintenance of pipeline, deterioration forecasting model considering deterioration process of inner surface and pipe body at the same time is required. In this study, the author formulates compound deterioration process considering interaction of corrosion of inner surface and pipe body degradation with systematic loss of data using compound hidden markov deterioration model. The compound hidden markov deterioration model is estimated using Bayesian estimation method. The empirical study was carried out by using an inspection dataset of real pipeline system. We could verify that the degradation of pipe body and corrosion of inner surface influence each other with complex interaction. It is expected that the compound hidden markov deterioration model can be applied to establishing optimal maintenance strategy for rehabilitation of inner surface and replacement of pipeline.

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1. Introduction

1.1. General introduction

Pipeline system is an important infrastructure which maintains the civic life and economic activity. It has been expanded with rapid urban development and improvement in standards of living and now asset of pipeline system has become enormous amount. According to statistics of Korea's ministry of environment, the total length of water pipeline is about 180,000km and over 20years old pipeline of them is about 48,000km, 26.6% of all. Pipeline system was built at the high economic growth intensively and now, it has emerged as a major problem to the water authorities worldwide to renew these enormous aging pipelines. Thus, it is big issue to maintenance of aging pipeline which is laid under the ground intensively at period of rapid economic growth.

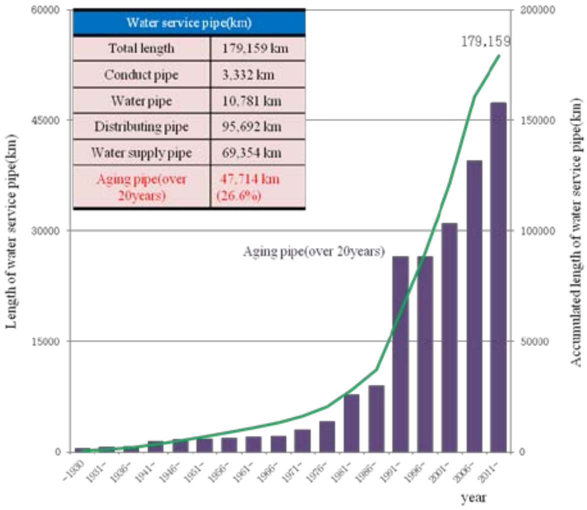

Figure 1.1 Distribution of water service pipe and trend of aging pipeline
(Source: 2011 waterworks statistics[1])

In case of other developed country, infrastructure systems become deteriorated from 1980s and it caused rapid increase of maintenance costs and then, around the 90s, the maintenance cost for infrastructure takes about 40% of total budget. Thus, many studies have been carried out with great interest in asset management of aging infrastructures.

Even though paradigm moved from optimal design and construction of pipeline system to optimal maintenance in Korea, most water authorities not only do not establish maintenance plan such as methodical inspection, repair and renewal strategy but also not have basic information of buried asset. Thus, it has been required to introduce asset management system for pipeline system. Infrastructure asset management is a method that is a optimal allocation of the scare budget between the new arrangement of infrastructure and rehabilitation/maintenance of the existing infrastructure to maximize the value of the stock of infrastructure and to realize the maximum outcomes for the citizen.[2-4] It is not independent method from existing maintenance way. The asset management is a decision making approach about optimal maintenance plan with collecting and analyzing data obtained from existing activity for maintenance of facilities.

From an asset management point of view, it is important to carefully consider some critical questions, “How can we allocate the limited budget optimally?”, “What is the optimal maintenance strategy for long-term pipeline systems management”, and “How can we use the pipeline system to maximize the value of stocks?”. In order to answer these questions, many researches about stochastic deterioration hazard model, reliability analysis and optimization methods have been carried out. The stochastic hazard model which is developed to forecast deterioration process of pipeline can be extended to methodology of optimal pipeline system management, optimal allocation of budget and optimal maintenance strategy, by incorporating cost evaluation techniques. The development and application of deterioration hazard model rely heavily on the mechanism of structural deterioration and inspection data of pipeline system. The deterioration process of pipeline shows a wide difference according to environment of the buried pipelines and operating conditions. Thus, it is important to develop a proper deterioration hazard model and apply the model to actual water pipeline systems.

A lot of studies about physical mechanism of pipe deterioration have been proposed[5-6]. However, the existing studies have some limitations to describe the process of deterioration of the pipeline in a variety of embedded environments and operating conditions. In recent decades, many studies on statistical method [7-9] have been developed but in the conventional researches, the type of pipe failure is not classified, and all failures are considered as a single type of failure. In real pipeline system, however, pipe failures caused by deterioration appear in various forms and the choice of a maintenance and repair strategy will depend on the type of failure. It is therefore important to know how these types of failure proceed.

The condition of inner surface of pipeline directly affect on the level of service to user on the other hand, the condition of pipe body affect on the structure performance. Thus, A great deal of past research had paid attention to the deterioration process of inner surface and pipe body [10-12]. However, the deterioration process of pipeline is complex phenomenon consisting of deterioration of inner surface and pipe body and the deterioration processes affect each other. Therefore, for maintenance of pipeline, deterioration forecasting model is required considering deterioration process of inner surface and body of pipe at the same time.

In this study, we propose some deterioration forecasting models on the basis of observed inspection data. Asset management forecast the demand of future rehabilitation based on life cycle cost analysis and establish required budget plan. Thus, it largely depends on deterioration forecasting model. Although many studies have elucidated the deterioration prediction method, only some of them realized as a tool working in management system. For realization of deterioration forecasting model in asset management system, it is required to develop a estimation methodology of deterioration forecasting model based on information obtained from inspection activity. In addition, In case of deterioration process of infrastructure, because there are many uncertainty, it is impossible to predict future deterioration exactly. Furthermore, the factors affecting on structure deterioration are diverse and largely depend on service condition and environment. Therefore, we suggest deterioration model that explain the deterioration process using stochastic process and predicts deterioration process with accumulated inspection data. We also show model determining optimal rehabilitation timing based on the principle of minimizing the overall life cycle cost (LCC)

1.2. Objective of Research

The objectives for development of this paper can be categorized into three concrete items as follows:

For pipe failure characteristics in competition, we establish a competing deterioration hazard model to predict the probability of occurrence of each failure characterization using a competing hazard model.

Developing optimal rehabilitation model considering pipe repair and replacement with least life cycle cost analysis. Particularly, the prediction of occurrence probability of leakage and burst is carried out with competing deterioration hazard model.

The compound deterioration process of inner surface corrosion and pipe body degradation is predicted with incomplete data set using compound hidden markov deterioration hazard model.

1.3. Scope of Research

Outlines of scopes are given as follows

Chapter 3 discusses a competing deterioration-hazard model that permits modelling of deterioration by multiple types of failure and focus on the failures which occur in pipe body or connection. The Weibull hazard model is used to address the lifetime of each pipeline, measured from when it was buried, and the model takes into account the competing nature of various types of failure by using a competing hazard model. The applicability of the method to data for an existing pipeline system is examined.

The scope of chapter 4 is to formulate an optimal rehabilitation model which consider pipe replacement and repair. The deterioration procedure of burst and leakage is forecasted by using competing deterioration hazard model and least life cycle cost analysis is conducted on the basis of maintenance strategy that repair of leakage and pipe replacement due to burst are carried out on an as needed basis. The empirical application of the proposed model was carried out to the real pipeline system, S city in Korea.

In chapter 5, in case of incomplete data caused by temporal mismatch in the data of the inner surface corrosion and pipe body condition, the compound deterioration process of pipeline is explained with compound hidden markov deterioration model. The compound hidden markov deterioration model is estimated using Bayesian estimation method. The empirical study was carried out by using an inspection dataset of real pipeline system.

Conclusions and recommendations on models and empirical studies are given at every last section of respective chapters.

1.4. Expected Contribution

It is very positively that, after completing the research, the paper and the knowledge of this research will contribute to some extend as follows:

The competing deterioration hazard model in chapter 3 is hoped to bring in innovative academic contributions. Among the failure types which are in competition, the proposed model prevents overestimating the occurrence probability of interest failure due to the presence of competing failures. It enables us to formulate optimal rehabilitation model.

The optimal rehabilitation model in chapter 4 is a new analytical methodology for optimal replacement timing considering pipe leakage. The proposed model enables us to apply to the practical management of pipeline system.

In chapter 5, we formulate compound deterioration process considering interaction of inner surface corrosion and pipe body degradation with incomplete data using a compound hidden markov deterioration model. The compound hidden markov deterioration model can be applied to establishing optimal maintenance strategy for rehabilitation of inner surface and replacement of pipeline.

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2. Pipeline Management System

2.1. General introduction

Asset management for pipeline system is hierarchical management system dividing by three steps as long-term plan, mid-term plan, and short-term plan. Figure 2.1 indicates the hierarchy of the asset management system.

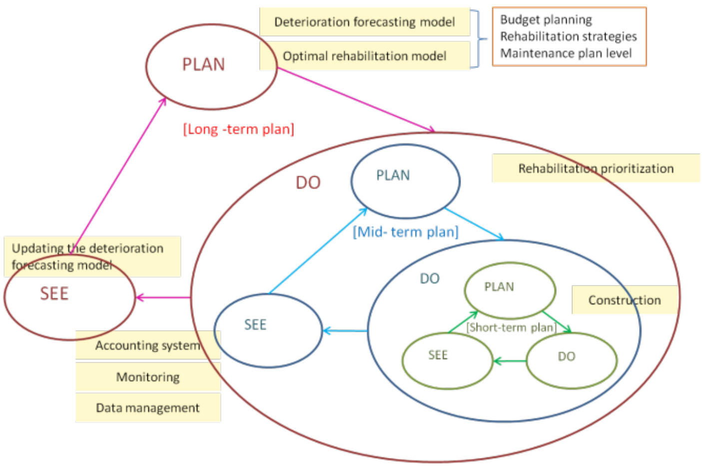

Figure 2.1 Hierarchical Management Cycle[1]

Long-term plan is a step to set up long-term budgets and service levels. This step needs life cycle cost analysis to decide optimal rehabilitation strategy. In other words, when a pipeline is used semi-permanently, it is required to get least life cycle cost including rehabilitation costs. As the life-cycle cost assessment methods, some models have been proposed, such as minimum average cost model or minimum net present value model and so on. Optimal maintenance strategy is not determined uniformly to all of the pipeline system components. These are grouped according to their characteristics, such as the current condition state of the soundness and environment conditions. For every group that is classified, for example, the optimal strategy for proactive or reactive strategy is applied. The deterioration process of the pipeline in the long-term plan is usually represented by a Markov transition probability matrix. And the estimation of the probability of transition is required historical inspection data. Thus the long-term goals of budget and service level is derived from long-term planning as a result, it is used in the work of the next step. In addition, decisions of the inspection intervals, the introduction of a new maintenance method and so on are carried out in long-term planning step.

Mid-term plan is a step of selecting a target in the mid-term maintenance to be carried out using the results derived from long-term planning. In this step, the regular inspections are carried out, and the performance indicators are updated. The mid-term rehabilitation target pipeline and priorities are determined with considering budget service level, and risk according to inspection results. The priority of the rehabilitation will be comprehensively judged by a variety of indicators such as, the importance of the facility and risk of management flaw as well as damage risk indicators.

Finally, short-term plan is a step to carry out maintenance in accordance with the priorities determined in the mid-term plan under the budget constraints of the short-term. At the time of determination of the pipe rehabilitation, performed in consideration of the timing of the compensation considering the properties as a component of the function and structure as the pipeline network examines the priority. The rehabilitation priority of pipeline is determined with considering the function as a network and properties as a component of the structure when determining the maintenance timing. The performed maintenance activities are recorded in management accounting system, and the renewed pipelines are removed from the list of mid-term plan, and then the next year plan is updated.

Thus, in the pipeline management system, the required information and output are different in each decision making step. The deterioration prediction model for pipeline is required to determine the necessity of rehabilitation and rehabilitation timing. Deterioration prediction methods can be classified as: 1) Statistical deterioration prediction models based on historical inspection data, etc. 2) Deterioration prediction models based on dynamical mechanism.

Both deterioration prediction methods are important information to the decision making of the pipeline management system. Generally, the deterioration prediction model based on the mechanical mechanism is used for micro-level management, such as review of the response to the specific Damage parts and the prediction of remaining life, etc. in mid-term plan. On the other hand, the statistical deterioration forecasting models are useful for establishing the maintenance strategy and the budget management for the entire facility in the long-term planning. Thus the deterioration prediction models play different roles according to the management steps.

In this study, we developed model for estimating the statistical deterioration forecasting model by using the inspection. In addition, by using the deterioration prediction model, we propose a model for deriving the optimal maintenance strategy.

2.2. Deterioration forecasting model

A pipe segment is subject to various effects such as embedded environment and operating conditions over a long period of time. Each pipeline in the system is under different condition such as design conditions, operating environment, and each pipeline must be properly managed depending on conditions. A pipe segment is deteriorated with age and in other to maintain the function as water pipe, a proper rehabilitation is required.

In mid-term plan, for aging pipelines, it is required to indentify deterioration factors and condition state and to predict pipe deterioration and risk and to recover its function with proper rehabilitation. Thus, in mid-term plan, the deterioration prediction model based on the mechanical mechanism is important. A variety of studies about the mechanical deterioration prediction model based on the specific material and the deterioration factor of the structure have been accumulated up to now [2-7].

On the other hand, the statistical deterioration forecasting model is used when the pipeline system managers to derive a rehabilitation strategy in the overall budget planning and long-term asset management plan. In long –term plan for pipeline system management, because the pipe system has a complicated structure and there are a lot of components, we faced problem for managing the all components at the same time. And since the pipeline deterioration is caused by various factors as previously mentioned, it comprises a number of uncertainties. Thus, it is not easy to establish the optimum maintenance strategy for entire pipeline system with considering the status of individual pipeline. It is not possible to express the pipe deterioration process as a definitive one curve, the facility generated more predictable results in fast deterioration uncertainties that exist in reality and the pipelines that deteriorate faster than predicted result with the uncertainties exist in reality. Thus, the pipe system administrators will need to allow the "risk" relates to the deterioration of the pipe, it is necessary to make a decision in a long-term plan to minimize the risk of deterioration under a variety of conditions. In order to cope with the deterioration risk, it is required to define the risk properly and evaluate the risk quantitatively. Since the risk is caused by the uncertainty on the deterioration of the pipeline, it can be expressed as the probability. In this study, it is assumed that the deterioration process of the pipeline follows the probability distribution and then we consider the problem of estimating the deterioration statistical prediction model to approximate the distribution of the actual deterioration process of the pipeline based on the actual observed inspection data.

Decision making in the long-term plans for pipeline system that is policy variables for determining a significant strategy for the timing of rehabilitation is greatly affected by deterioration prediction model. The inspection data derived by empirical knowledge of the manager and the comprehensive judgment by experts is an important judgment information when making decisions in asset management systems. Inspection work is a component constituting the major subsystems of a cycle of an asset management, and observed data obtained by the inspection provides useful information when the long-term decision making. Pipes are used in a complex environment, it is difficult to determine the factors that deteriorate the process. However, data obtained by actual inspection of the pipeline which is used under such a complex environment, contains a wide range of conditions due to the uncertainty and is a simplified information describing the deterioration process of the pipeline. With the actual inspection data, to explain the deterioration process of pipeline and derive the optimal maintenance strategy, it is important to develop a statistical model with high prediction accuracy as a pipeline deterioration forecasting model in the asset management system.

In this regard, this research propose a methodology for estimating the statistical pipeline deterioration forecasting model with inspection data for decision making support of long-term planning of the pipeline management system. There are some researches in statistical methods about deterioration forecasting model. Deterioration prediction model by statistical methods can be classified as: 1) deterministic methods 2) probabilistic method according to whether or not considering the uncertainty of the deterioration. The deterministic method is a method which does not take into account the uncertainty of deterioration and it is commonly used in instances where the relationships between components are certain but the applicability of model is restricted to a specific condition [8]. Probabilistic method deals with the probability or relative frequency of a deterioration occurring[9].

This study represents the deterioration process of the pipelines by a hazard model. Hazard model has been developed to predict the life time of the facilities and machines in the field of reliable analysis but now it has been applied not only reliable analysis but many fields[10-11]. In asset management field, Shin and Madanat [12] proposed Weibull deterioration hazard model to expect the start time of road pavement crack. The Deterioration process of the structure contains a lot of uncertainties, and it is impossible to predict the future deterioration accurately. The Deterioration or failure of the structure occurring in future can be expressed as a risk which is due to the uncertainties and it is common to express the magnitude of the risk with probabilistic methodology. In the deterioration prediction model according to the hazard model, the probability of occurrence of all observed data is defined and from estimating the model, it can be expected to quantify the risk of deterioration. Because it is possible to quantitatively express the probability of occurrence of deterioration risk, the deterioration prediction model can be used as a useful technique for risk management.

In this study, we propose a probabilistic deterioration forecasting model of pipeline based on statistical methods from the point of view of long-term planning of asset management. The deterioration process of pipeline is formulated by a hazards model. In addition, the optimal maintenance strategy model for the rehabilitation of long-term plans, such as correlation of various management risk and life cycle costs is proposed using pipeline deterioration forecasting model.

2.3. Optimal maintenance model with probabilistically Deterioration prediction

In the field of operation research, studies on the optimal rehabilitation strategy of machinery systems have been well carried out [13]. The studies proposed optimal rehabilitation polices which minimize life cycle cost with formulating the deterioration process of infrastructures and occurrence probability of failures. Aoki[14-15] proposed a methodology to determine optimal inspection and repair timing with binary condition state of asset and analyzed the trade-off relationship between the life cycle cost and failure risk of the system. In addition, a lot of Markov decision models which represent the deterioration condition state into discrete state variable have been accumulated to cope with optimal maintenance and repair problem. [16-18]

In general, the maintenance policy of infrastructure can be divided into two maintenance policy, situation-dependent way and time-dependent way. Situation-dependent way is a method to determine the repair and maintenance strategy according to deterioration state of infrastructure obtained from regular inspection. Thus, it is suitable if there is a lot of uncertainty in the deterioration process of the infrastructures. On the other hand, time-dependent way is a method for performing maintenance of infrastructure every regular interval. The buried infrastructures such as pipeline system which are required a huge budget for inspection activity is suitable to implement the time-dependent way, rather than the situation-dependent manner.

Pipeline system is not easy to check for deterioration. Thus, the pipelines within a certain period time from building point have been replaced regardless of deterioration. However, it is impossible to completely ignore the possibility of occurrence of pipe failure up to the replacement time point of pipeline. Therefore, it is necessary to determine optimal pipe replacement time which minimizes the expected life cycle cost defined as aggregate of social costs and replacement cost due to pipe failure.

Optimal Maintenance Policy to consider the optimal rehabilitation strategy and inspection interval at the same time has been proposed[14,19]. In recent years in Korea, regular inspection of pipeline system has been carried out, and the observed data about pipe condition have been increasingly accumulated. Because the inspection activity of pipe condition takes huge budget and manpower, the study on the optimal inspection interval has also been the subject of interest. In addition, in order to maintain the inner surface of pipe which is directly connected to the level of service of pipeline, the optimization method about timing of lining and pipe replacement is also an important task for pipeline system management.

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3. Estimating Burst Probability of Water Pipelines with a Competing Hazard Model

3.1. General Introduction

Water-supply pipelines, which form important components of the infrastructure of cities, require huge annual maintenance budgets. Consequently, the establishment of optimal regimes for maintaining water pipeline systems has become a major issue for water-utility managers throughout the world. In the management of infrastructure assets, optimal maintenance strategies are frequently based on lifecycle-cost analysis, which is dependent on the deterioration model [1].

It is therefore important to know how the deterioration of the system proceeds. In the field of water-supply systems, many studies have been conducted to assess the condition of pipeline systems and to predict their deterioration process. Shamir and Howard [2] and Marvin [3] assumed that breaks in pipelines increase exponentially with their age, and they obtained break-prediction models by using regression analysis. Clark *et al.* [4] reported a method for estimating the expected failure time of pipelines, whereas Shinstine [5] examined the relationship between pipeline breaks and the diameters of pipes. Because water pipeline systems are usually buried underground, monitoring and inspection of such systems is difficult and it is hard to accumulate adequate observational data for use as a basis for deterioration forecasting analysis. Because of the difficulties in observing deteriorations of pipelines directly, we decided to predict the deterioration of pipelines by examining failures caused by the deterioration process.

Marks *et al*. [6], Constantine and Darroch [7], and Park [8] used proportional hazards models, based on the failure-prediction model proposed by Cox [9], to predict the risk of a pipeline break. Many probabilistic models that use various probability functions have been developed with the average annual number of pipe breaks on the pipeline systems as an indicator of the structural state and the times to failure between pipe breaks are considered as random variables (Le Gat and Eisenbeis[10], Mailhot *et al.*[11][12]; Pelletier *et al.*[13]). These models have overcome challenges that observation data typically show properties, right censored observations(Eisenbeis, et al.[14], Mailhot *et al.*[11]), left truncation(Mailhot *et al.*[11]) and selective survival bias(Scheidegger *et al*.[15]). By setting the deterioration state as a binary condition, ‘failure’ or ‘normal operation’, it is possible to predict the service life of a pipeline by using a conventional hazard model. There are numerous reports of studies in which this type of deterioration prediction method has been applied to other types of system. Aoki [16] proposed a method in which a Weibull hazard model is used to predict the lifetime of tunnel lighting equipment. Tanaka *et al*. [17] similarly used Weibull hazard models to predict the deterioration of pipelines.

In general, the major cause of interruption of water pipeline systems is deterioration of the pipes. In the conventional models for the prediction of pipeline deterioration, the type of pipe failure is not classified, and all failures are considered as a single type of failure. In real pipeline system, however, pipe failures caused by deterioration appear in various forms. We therefore classified pipeline failures as ‘B-burst’, which occur in the pipe body, or ‘C-burst’, which occur in pipe-connection parts. The lifetime of a given part is defined as the period from its installation to burst. And in this study, it is assumed that the burst is regarded as major damage and the damaged pipeline is replaced immediately. The Weibull deterioration-hazard model is used to address the lifetime of each pipeline, and takes into account the nature of the competition between several types of failure by using a competing deterioration-hazard model. The deterioration of the pipeline is predicted by developing a competing deterioration-hazard model that considers competition between C- burst and B- burst. The proposed competing deterioration-hazard model allows us to determine the probability density of bursts in pipe body and connection.

The competing hazard model assumes that competing causes of failure are independent of one another and that the incidence of each cause of failure can be analyzed from lifetime data. Such methods have been used in many fields, including medicine, economics, and engineering. The competing hazard model is widely used in accelerated lifetime testing (ALT) to estimate the lifetime distribution of components. Nelson [18] discussed an analysis of typical competing hazard models for constant stress ALT data. Kim and Bai [19] reported a competing hazard model that considered only two competing causes of failure by using ALT data.

Because the pipeline systems are underground, system administrators face difficulties of insufficient amount of observation data. Thus, the insufficiency of data interrupts the practical application of the statistical model. In order to overcome this problem, in this study, the competing deterioration-hazard model is estimated by a Bayesian technique based on the Metropolis–Hasting method (M-H method), a Markov chain Monte Carlo method for obtaining a sequence of random samples from a probability distribution for which direct sampling is difficult.

3.2. Competing Deterioration-Hazard Model

3.2.1. Pre-assumption of the model

Competing hazards appear in cases in which two or more events can occur. The main idea of a competing hazard model is that the occurrence of an event of interest has to be taken into account while considering the occurrence of competing events. In pipeline systems, the case in which a pipeline is replaced because of a B-burst can be considered. However, there might be the events that could lead to replacement of the pipeline, such as C- bursts. It is therefore possible to assume that a C-burst is a competing event, if we assume that a B-burst is the event of interest, because a B-burst interrupts the occurrence of C-burst. To introduce the competing hazard among the pipe failure types, the major damage which requires pipe replacement is considered, in this study, we focus on the B-burst and C-burst.

In this study, we classify the state of a pipeline as being one of two condition levels: a 'healthy condition' and a 'burst' resulting from B-burst or C-burst. It is assumed that the burst denotes a state in which major damage is found and replacement is required immediately. On the other hand, the healthy condition reflects not only a normal operation but a condition which no major damage is found. In addition, it is also assumed that the records of past repaired incidents, leakages or breaks, are not considered as burst because these incidents would be not major damage.

3.2.2. Competing hazard model

Each pipeline is represented by $i\,(i=1,\ldots,n)$, and the elapsed time from laying pipeline i to the present is expressed as τ. In addition, we assume that more than one type of pipe burst $j\,(j=1,\ldots,J)$ is possible for pipeline i. The life span of pipeline until burst in pipe i is expressed by the random variable ζ_i and this is subject to the probability-density function $f_j(\zeta_i)$ and the distribution function $F_j(\zeta_i)$ for each type of burst type *j*. Here, the domain of life span ζ_i is $[0,\infty)$. In addition, the probability that pipe burst will not occur until time τ is defined as [$\tilde{F}(\tau)$] and is known as the survival probability. This can be expressed as follows:

$$\tilde{F}(\tau) = 1 - \mathrm{F}(\tau) \tag{3.1}$$

When competing hazards exit, the conditional probability that pipe burst does not occur in pipe i

until an arbitrary time t_i and that pipe burst occurs by burst type j during the time span $[\tau,\ \tau+\Delta\tau)$ can be represented by the following equation:

$$\lambda_j(\tau) = \lim_{\Delta\tau\to 0} \frac{\Pr(\tau < \zeta \le \tau+\Delta\tau, TF = j \mid \zeta > \tau)}{\Delta\tau} = \frac{f_j(\tau)}{\tilde{F}(\tau)} \tag{3.2}$$

where $\lambda_j(\tau)$ is the hazard function for each burst type j and TF denotes type of burst. Note that in this competing hazard model, to obtain the hazard function, the density function $f_j(\tau)$ should be divided by $\tilde{F}(\tau)$ rather than by $\tilde{F}_j(\tau)$. For example, dividing $f_c(\tau)$ by $\tilde{F}_c(\tau)$ gives the conditional probability that a C-burst will not occur before an arbitrary time τ and that a C-burst will occur at τ. However, in this case the probability that a pipe burst will occur through C-burst would be overestimated, because the occurrence of a pipe B-burst, which is a competing risk of pipe C-burst, is not considered. It is therefore reasonable that the hazard function for C-burst has to be defined as the probability of a C-burst occurring when no pipe burst occurs until the arbitrary time τ.

The overall survival function $\tilde{F}(\tau)$ can be defined as follows:

$$\tilde{F}(\tau) = \exp\left\{-\int_0^{\tau} \lambda(u)\,du\right\} \tag{3.3}$$

where, $\lambda(\tau)$ is overall hazard function and is defined by $\lambda(\tau) = \sum_{j=1}^{J} \lambda_j(\tau)$. The overall survival function $\tilde{F}(\tau)$ is the probability that any burst type does not occur; it can therefore be represented by the joint probability of the partial survival distribution function for each burst type $\tilde{F}_j(\tau),\ (j = 1,\ldots,J)$, as follows:

$$\tilde{F}(\tau) = \prod_{j=1}^{J} \tilde{F}_j(\tau) \tag{3.4}$$

The partial survival function $\tilde{F}_j(\tau)$ also can be defined as follows:

$$\tilde{F}_j(\tau) = \exp\left\{-\int_0^{\tau} \lambda_j(u)\,du\right\} \tag{3.5}$$

Accordingly, from equation (3.2), the partial density function for each type of burst $f_j(\tau)$ can be expressed as follows:

$$\begin{aligned} f_j(\tau) &= \lambda_j(t_i)\tilde{F}(\tau) \\ &= \lambda_j(\tau)\prod_{j=1}^{J} \exp\left\{-\int_0^{\tau} \lambda_j(u)\,du\right\} \end{aligned} \tag{3.6}$$

3.2.3. Weibull Deterioration Hazard Model

Pipe burst depends largely on the duration of use of the pipeline. The hazard function should therefore consider the elapsed time. In this study, the Weibull hazard model, which is suitable for addressing this process, is applied with the assumption that the probability of pipe burst increases with time, as follows:

$$\lambda_j(\tau) = \gamma_j m_j \tau^{m_j - 1} \tag{3.7}$$

where m_j is the acceleration parameter that represents the time dependency of the hazard function and γ_j is the parameter expressing the arrival rate of pipe burst. It is assumed that γ_j depends on the characteristics of the pipeline, and that it can be expressed as follows:

$$\gamma_j = \exp\left(\boldsymbol{x}_i \boldsymbol{\beta}_j{}'\right) \tag{3.8}$$

where $\boldsymbol{x}_i = (x_i^1, \cdots, x_i^k)$ is the characteristic vector that represents the observed value for pipeline i and $\boldsymbol{\beta}_j = (\beta_j^1, \cdots, \beta_j^k)$ represents the unknown parameter vectors. In addition, k is total number of covariates and the sign ' denotes transposition. By using the Weibull hazard model, the probability-density function $f_j(\tau)$ and survival function $\tilde{F}_j(\tau)$ can be expressed as follows:

$$f_j(\tau) = \gamma_j m_j \tau^{m_j - 1} \exp\left(-\gamma_j \tau^{m_j}\right) \tag{3.9}$$

and

$$\tilde{F}_j(\tau) = \exp\left(-\gamma_j \tau^{m_j}\right) \tag{3.10}$$

3.3. Estimation Method

3.3.1. Estimation Approach For Competing Deterioration Hazard Model

Let us discuss the estimation method for the competing deterioration-hazard model based on inspection data. The time at which the pipe was buried is set as $t = 0$ and $\boldsymbol{t_i}$ denotes the observed duration of use of pipeline i (i = 1, …, n). If a pipe burst occurs and the life span of the pipeline ends, its duration of use is equal to the life span, $\zeta_i = t_i$. On the other hand, it is assumed that a pipeline for which no burst has been reported until the inspection time still survives. In other words, if a pipeline's life span has not ended, this exceeds its duration of use, $\zeta_i > t_i$. Then, let us introduce the dummy variable ε_i, which denotes whether pipe burst has occurred or not.

$$\varepsilon_i = \begin{cases} 0, & \text{survival} \\ 1, & \text{burst} \end{cases} \tag{3.11}$$

In addition, the reported pipe burst type, in this study, the burst type is classified as either a C-burst $(j = c)$ or a B-burst $(j = b)$, can also be represented by the dummy variable d_i.

$$d_i = \begin{cases} 0, & \text{B-burst} \\ 1, & \text{C-burst} \end{cases} \tag{3.12}$$

The observation information for pipeline i can be represented as follows: $\boldsymbol{\xi}_i = (d_i, \varepsilon_i, t_i, \boldsymbol{x}_i)$. Here, we define the unknown parameter vector for the competing deterioration-hazard model as $\boldsymbol{\theta} = (\boldsymbol{\beta}, \boldsymbol{m})$. The parameters, $\boldsymbol{\beta}$ and $\boldsymbol{m}$ denote $\boldsymbol{\beta} = (\boldsymbol{\beta}_c, \boldsymbol{\beta}_b)$ and $\boldsymbol{m} = (m_c, m_b)$, respectively. If we suppose that there is observed information for pipeline i, $\boldsymbol{\xi}_i = (d_i, \varepsilon_i, t_i, \boldsymbol{x}_i)$, the conditional probability that the observed information occurs in pipeline i can be represented by the following equation:

$$\begin{aligned} &\ell\left(d_i,\, \varepsilon_i,\, t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \\ &= \Big[\left\{ \lambda_c\left(t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \tilde{F}_c\left(t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \tilde{F}_b\left(t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \right\}^{d_i} \\ &\quad \cdot \left\{ \lambda_b\left(t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \tilde{F}_b\left(t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \tilde{F}_c\left(t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \right\}^{1-d_i} \Big]^{\varepsilon_i} \\ &\quad \cdot \left[\tilde{F}_c\left(t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \tilde{F}_b\left(t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \right]^{1-\varepsilon_i} \end{aligned} \tag{3.13}$$

This assumes that the pipe burst of each of the n pipelines is mutually independent from that of other parts of the pipeline system. The simultaneous probability density of the pipe deterioration can therefore be expressed by the following likelihood function:

$$L\left(\theta \mid \xi\right) = \prod_{i=1}^{n} \ell\left(d_i,\, \varepsilon_i,\, t_i \mid \boldsymbol{x}_i, \boldsymbol{\theta}\right) \tag{3.14}$$

where, $\boldsymbol{\xi}$ represents $\boldsymbol{\xi} = (\boldsymbol{\xi}_1, \cdots, \boldsymbol{\xi}_n)$. The unknown parameter θ can be estimated by the maximum likelihood estimation method, which provides an estimate of parameter θ that maximizes the likelihood function.

3.3.2. Bayesian Estimation Method for Competing Deterioration-Hazard Model

Because, in the MLE method, huge amount of data are required in order to secure precision but it is not always possible to accumulate a great number of data especially in the pipeline systems. Pipeline system administrators face difficulties of insufficient amount of observation data because pipeline systems are underground. The Bayesian estimation method can provide estimation results by fusing prior information, such as human experience and expert knowledge, with insufficient amount of observation data [20]. In addition, the Bayesian estimation method is easy in comparison with MLE method because in the Bayesian estimation method, it is not required to derive the Jacobian and Hessian matrices. Furthermore, in the nonlinear equation problem, defining the optimal condition may have multiple zero point. In this case, a poor choice of starting point in the MLE method can cause converging to a local optimum that is not the global optimum, or fail to converge entirely. The competing deterioration hazard model is high dimensional nonlinear expression of parameter θ and the optimization problem may have a large number of solutions including complex valued solutions. Thus, in this case, using Bayesian estimation method instead of maximum likelihood estimation method can solve the high dimensional nonlinear multinomial expression. In this section, we present a methodology for estimating the unknown parameter vector θ of the competing deterioration-hazard model by means of a Bayesian estimation method using observed data.

The Bayesian approach permits the estimation of θ on the basis of the inspection data $\boldsymbol{\xi}$ and prior information regarding θ. By using the M-H method, the estimation is carried out by sampling a large number of values of θ from its posterior distribution, which can be expressed as follows:

$$\pi(\theta \mid \xi) \propto L(\theta \mid \xi)\pi(\theta) \tag{3.15}$$

where $\pi(\theta \mid \xi)$ is the posterior probability density function of θ, $L(\theta \mid \xi)$ is the likelihood function, and $\pi(\theta)$ is the prior probability density function of θ. The newly obtained data are denoted by $\xi = (\xi_1, \cdots \xi_n)$. By substituting the Weibull hazard model (3.9) and (3.10) into equation (3.14), the likelihood function can be expressed as follows:

$$L\left(\boldsymbol{\theta}\middle|\xi\right) = (m_c)^{d \cdot \varepsilon}(m_b)^{(1-d)\cdot\varepsilon} \exp\left\{\sum_{i=1}^{n}\Big(d_i\varepsilon_i \boldsymbol{x}_i\boldsymbol{\beta}_c' + d_i\varepsilon_i(m_c - 1)\ln t_i + (1-d_i)\varepsilon_i\boldsymbol{x}_i\boldsymbol{\beta}_b' \right.$$
$$\left. + (1-d_i)\varepsilon_i(m_b - 1)\ln t_i - \exp(\boldsymbol{x}_i\boldsymbol{\beta}_c')t_i^{m_c} - \exp(\boldsymbol{x}_i\boldsymbol{\beta}_b')t_i^{m_b}\Big)\right\} \tag{3.16}$$

where, $\bar{d} = \sum_{i=1}^{n} \bar{d}_i$ and $\bar{\varepsilon} = \sum_{i=1}^{n} \bar{\varepsilon}_i$.

In this study, we assume that the prior probability density function of parameter, $\boldsymbol{m}$ and $\boldsymbol{\beta}$ follow a gamma distribution and a conjugate multidimensional normal distribution, respectively, $\boldsymbol{m} \sim g(m_0, k_0)$, $\boldsymbol{\beta} \sim \mathcal{N}_K(\boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o)$. With this assumption, the probability density function of the gamma distribution function $g(m_0, k_0)$ and the K-dimensional normal distribution $\mathcal{N}_K(\boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o)$ can be further expressed as follows:

$$f(\boldsymbol{m} \mid m_0, k_0) = \frac{1}{\Gamma(m_0)} k_0^{m_0} \boldsymbol{m}^{m_0 - 1} e^{-k_0 \boldsymbol{m}} \tag{3.17}$$

and

$$g(\boldsymbol{\beta} \mid \boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o) = \frac{1}{\sqrt{(2\pi)^K |\boldsymbol{\Sigma}_o|}} \cdot \mathrm{ex} \left\{ -\frac{1}{2} (\boldsymbol{\beta} - \boldsymbol{\mu}_o) \Sigma_o^{-1} (\boldsymbol{\beta} - \boldsymbol{\mu}_o)' \right\} \tag{3.18}$$

where, $\Gamma(m_0)$ denotes the gamma function and $\boldsymbol{\mu}_o$ and $\boldsymbol{\Sigma}_o$ represent the prior expectation vector and the prior variance-covariance matrix of $\mathcal{N}_K(\boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o)$, respectively. On the basis of equation (3.15), the posterior probability density function $\pi(\boldsymbol{\theta} \mid \boldsymbol{\xi})$ is defined as follows:

$$\begin{aligned} \pi(\boldsymbol{\theta} \mid \boldsymbol{\xi}) &\propto L(\boldsymbol{\theta} \mid \boldsymbol{\xi}) f(\boldsymbol{m} \mid m_0, k_0) g(\boldsymbol{\beta} \mid \boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o) \\ &\propto (m_c)^{d \cdot \varepsilon} (m_b)^{(1-d) \cdot \varepsilon} \boldsymbol{m}^{m_0 - 1} \exp \left\{ \sum_{i=1}^{n} \Big(d_i \varepsilon_i x_i \boldsymbol{\beta}_c' + d_i \varepsilon_i (m_c - 1) \ln t_i \right. \\ &\quad + (1 - d_i) \varepsilon_i \boldsymbol{x}_i \boldsymbol{\beta}_b' + (1 - d_i) \varepsilon_i (m_b - 1) \ln t_i - \exp(x_i \boldsymbol{\beta}_c') t_i^{m_c} \\ &\quad \left. - \exp(x_i \boldsymbol{\beta}_b') t_i^{m_b} - k_0 \boldsymbol{m} - \frac{1}{2} (\boldsymbol{\beta} - \boldsymbol{\mu}_o) \Sigma_o^{-1} (\boldsymbol{\beta} - \boldsymbol{\mu}_o)' \Big) \right\} \end{aligned} \tag{3.19}$$

The M-H method is used to perform sampling from an empirical distribution that is similar to $\pi(\boldsymbol{\theta} \mid \boldsymbol{\xi})$ and accordingly obtains samples from the original distribution [21]. Furthermore, a random walk is used to improve the efficiency of sampling. The M-H method is described below.

Step 1. Initial Establishment

The initial value of parameters $\boldsymbol{\theta} = (\boldsymbol{\beta}, m)$, the number of iterations for parameter sampling $\overline{N}$, and the burn-in period $\underline{N}$ are established. In addition the stride of the random walk is set.

Step 2. Sample Extraction for Estimation the Parameter $\boldsymbol{\theta}$

When the number of simulations is $n+1$, the parameter estimation $\boldsymbol{\theta}^{n+1}$ is generated as described in Steps 2-1 to 2-3.

Step 2-1

The stride of the random walk ν is assumed to follow a normal distribution with a mean of 0 and a variance of $(\sigma)^2$. The new candidate value $\boldsymbol{\theta}'$ is then calculated as follows:

$$\boldsymbol{\theta}' = \boldsymbol{\theta}^n + \nu \tag{3.20}$$

Step 2-2

The acceptance probability is calculated as follows:

$$\alpha_{n+1} = \min\left\{ \frac{\pi(\boldsymbol{\theta}' \mid \xi)}{\pi(\boldsymbol{\theta}^{(n)} \mid \xi)}, 1 \right\} \tag{3.21}$$

Step 2-3

The uniform distribution $u_n \sim U(0,1)$ is generated, and then the sample is determined by applying the following condition:

$$\boldsymbol{\theta}^{(n+1)} = \begin{cases} \boldsymbol{\theta}', & if \quad u_n \leq \alpha_n \\ \boldsymbol{\theta}^{(n)}, & if \quad u_n > \alpha_n \end{cases} \tag{3.22}$$

If the acceptance probability is greater than u_n, the candidate value is accepted; otherwise, the original value is retained.

Step 3. Final Judgment of the Algorithm

Step 2 is repeated until the number of samplings reaches *N*.

The samples are then accumulated except for those that were generated during the burn-in period. If the number of samples *N* is sufficiently large, the parameters estimated by using the above algorithm will converge on the estimated value of the posterior distribution. Geweke test statistics [22] are used to test whether the sampling process of the M-H method reaches a steady state and the number of samplings *N* is appropriate or not.

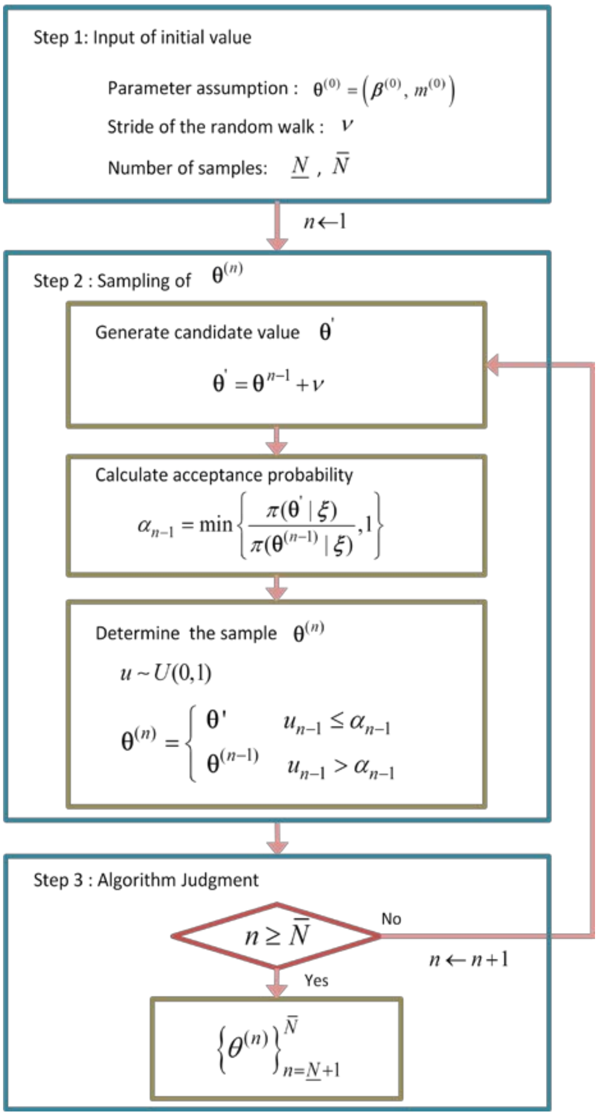

Figure 3.1 Flowchart of Bayesian Estimation for Competing Deterioration Hazard Model

3.4. EMPIRICAL STUDY

3.4.1. Overview of the Empirical Study

To analyze the deterioration of a real pipeline, we focused on the water distribution system of S city in South Korea. The pipe material, ductile cast iron pipe (DCIP), is regarded as the target for this study. The whole data of DCIPs comprise approximately 26,500 pipelines, 850km in length, with an average age of around 13 years. Inspection data were obtained from historical records for pipe bursts in S city during the nine-year period 2001–2009. During this period, 1405 cases of pipe replacement caused by B and C-burst were recorded. Here, in this study, it is assumed that the replaced pipelines had major damage and its condition state is classified as burst. On the other hand, the historical records of past repair are not considered as burst because a repair is not associated with major damage. Table 3.1 shows the basic information of the data used in this study.

The inspection data contain information on whether or not pipe burst occurred and the type of burst for each damaged pipeline. In this study, the type of burst is classified as either a B-burst in a pipe body or a C-burst in pipe connections. Accidents that occurred in other subcomponents, such as valves, rubber packings, and so on, are neglected. On the other hand, the pipe diameter and length are used as characteristic information that affects pipe burst. On the basis of this information, the duration of survival before burst of a pipe is expressed by using the Weibull hazard model, and the competing deterioration-hazard model is used to consider the competition between C-bursts and B-bursts in the pipeline. The model is then estimated by using the Bayesian estimation method.

Table 3.1 Features of target pipelines

Features	value		
Material	Ductile cast iron		
Years laid(average age)	From 1957 to 2009(13years)		
Diameter/mm	75~900		
Number of pipes	26,577		
Total length/km	848.1		
Number of failures	1,405	C-failure	833
		B-failure	572

3.4.2. Estimation Results

The competing deterioration hazard model used for the Bayesian estimation is specified as follows:

$$\lambda_j(t_i) = \exp(\beta_{j0} + \beta_{j1}x_{i1} + \beta_{j2}x_{i2})m_j t_i^{m_j-1} \quad (i = 1, \cdots, n;\ j = c, b) \tag{3.23}$$

The unknown parameter β_{j0} is a constant term, β_{j1} and β_{j2} represent the pipe diameter and pipe length, respectively. In this study, other characteristic variables that reflect the influence of outer and inner rust, soil unit weight, top traffic volume, and so on were neglected, either because of their small impacts or because data were unavailable. The unknown parameters can be expressed as follows:

$$\boldsymbol{\theta} = (\beta_{c0}, \beta_{c1}, \beta_{c2}, \beta_{b0}, \beta_{b1}, \beta_{b2}, m_c, m_b) \tag{3.24}$$

We assume that the prior probability density function of the unknown parameters, $\boldsymbol{m}$ and $\boldsymbol{\beta}$ follow $\boldsymbol{m} \sim g(m_0, k_0)$, $\boldsymbol{\beta} \sim \mathcal{N}_K(\boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o)$. Unfortunately, in this empirical study, because of the absence of detailed substantive knowledge, it is difficult to get information about the expectations and the variance of unknown parameters. But the number of observed data is enough large, the influence of prior distribution can be ignored. Thus, a non-informative prior distribution is applied for the Bayesian estimation. The non-informative prior distribution can be obtained by setting the variance of the prior distribution to be sufficiently large, as follows:

$$\boldsymbol{m} \sim g(1, k_m^{-1}) \tag{3.25}$$

$$\boldsymbol{\beta} \sim \mathcal{N}_K(\mathbf{O}, k_\beta \mathbf{I}) \tag{3.26}$$

Where, k_m and k_β are sufficiently large integer. $\mathbf{O}$ and I are a zero vector and a unit matrix, respectively.

In order to improve the precision of estimation, Bayesian updating rule [20] is used. We created three different data groups (D_{2000}, D_{5000}, D_{10000}) which are extracted based on original data set. Here, the subscript numbers denote the number of extracted data. The estimation is performed in the order of the small size of the data and the estimation results (the mean, variance and covariance) are used as prior information of next estimation using Bayesian updating rule. To conduct the M-H method, the number of iteration required to reach a steady state (the burn-in period) was set to $\underline{N} = 10,000$ and the number of iterations for parameter sampling was set to $\bar{N} = 20,000$. The 10,000 burn-in samples were omitted and the remaining 10,000 parameter samples were used to carry out the estimation.

Table 3.2 shows the results of the Bayesian estimation of competing deterioration hazard model for each of the data bases D_{2000}, D_{5000}, D_{10000} and original data set. The estimations obtained by M-H method show the probability distribution of the parameters. In the table 2, the values estimated by Bayesian estimation method are the sample average of parameters, and the values in parentheses refer 95% credible intervals. All the credible intervals of estimated parameters don’t contain zero. The credible intervals not containing zero imply that there is a statistically significant [23]. As shown in table 2, as the amount of observation data increases, the credible intervals become narrower. The absolute value of the Geweke test statistics shown in italic type are all less than 1.96, so the convergent hypothesis cannot be dismissed at a significance level of

5%.

Table 3.2 Results of estimation of parameters for the competing deterioration-hazard model

Parameters	D_{2000}	D_{5000}	D_{10000}	Original data set
	-11.681	-9.955	-9.504	-9.594
β_{c0}	(-12.874, -10.617)	(-10.717, -9.293)	(-9.963, -9.055)	(-9.941, -9.277)
	0.053	*0.086*	*0.036*	*0.069*
	-3.284	-2.929	-0.763	-0.994
β_{c1}	(-5.175, -1.165)	(-4.369, -1.534)	(-1.615, 0.047)	(-1.595, -0.406)
	0.134	*0.035*	*0.060*	*0.011*
	6.344	5.194	2.755	2.657
β_{c2}	(1.065, 10.737)	(2.239, 7.885)	(1.412, 3.800)	(1.941, 3.284)
	0.042	*0.129*	*0.124*	*0.038*
	-12.461	-10.332	-10.166	-10.094
β_{b0}	(-14.264, -10.842)	(-11.358, -9.374)	(-10.790, -9.644)	(-10.429, -9.768)
	0.139	*0.200*	*0.067*	*0.226*
	-2.489	-3.100	-1.228	-1.884
β_{b1}	(-5.115, 0.287)	(-5.468, -0.822)	(-2.422, -0.189)	(-2.863, -1.022)
	0.391	*0.354*	*0.197*	*0.112*
	4.510	8.126	3.396	3.106
β_{b2}	(-0.797, 7.570)	(5.808, 10.720)	(1.966, 4.409)	(2.442,3.764)
	0.303	*0.036*	*0.145*	*0.001*
	2.954	2.379	2.199	2.256
m_c	(2.600, 3.398)	(2.178, 2.601)	(2.051, 2.338)	(2.161, 2.360)
	0.090	*0.120*	*0.032*	*0.070*
	3.001	2.370	2.306	2.338
m_b	(2.528, 3.537)	(2.078, 2.630)	(2.147, 2.492)	(2.237, 2.449)
	0.083	*0.105*	*0.003*	*0.188*

Notes: Values in $(\cdot)$ show 95% credible intervals and values shown in italic type in each row are the Geweke statistical test.

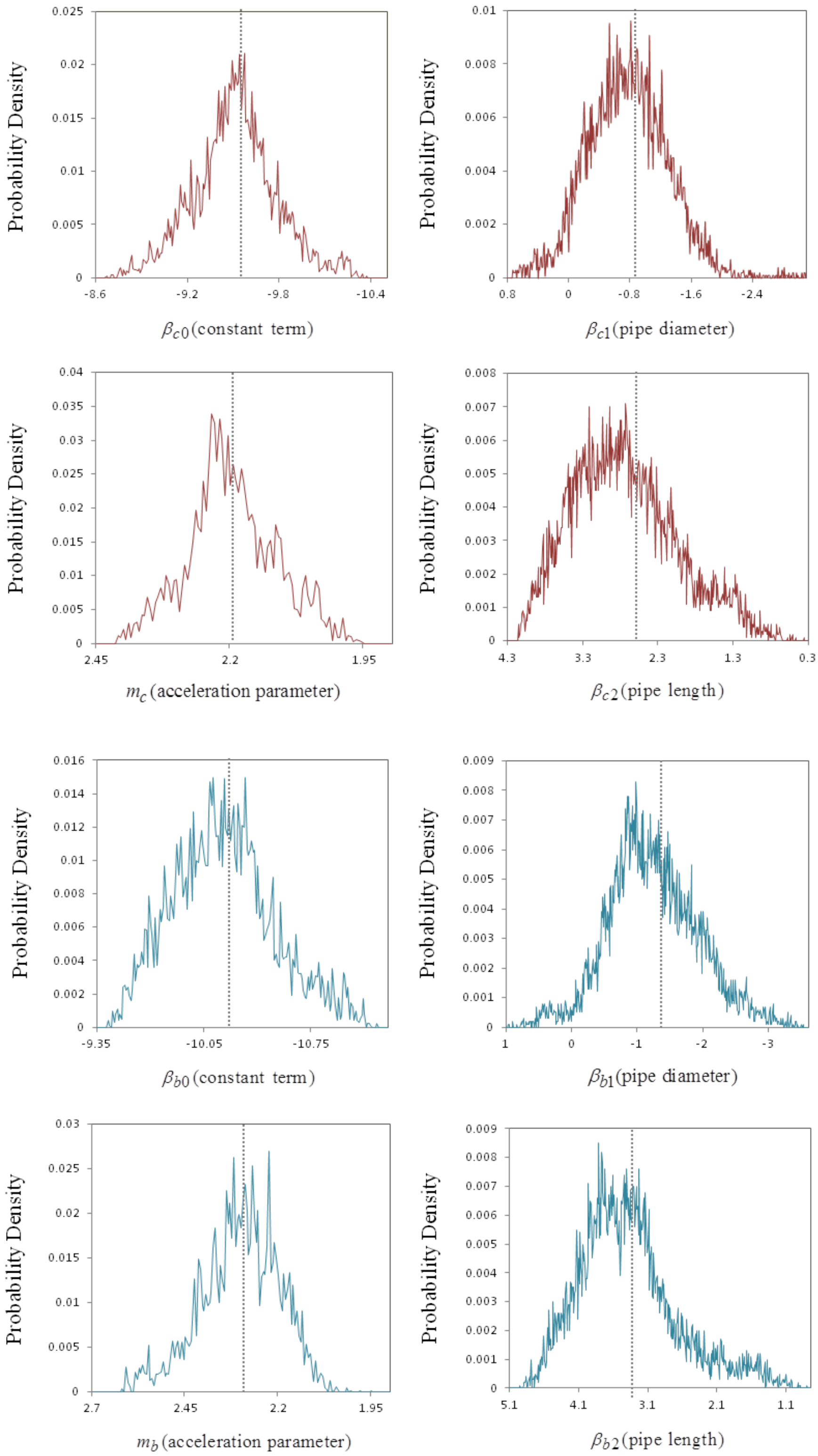

Figure 3.2 Posterior distribution of model parameters of D_{10000}

Figure 3.2 shows the posterior densities of model parameters for D_{10000} database. As shown in Figure 3.2, it shows that the estimation conducted with high confidence because the shapes of most of posterior parameter distributions show normal distributed.

With the estimation results for the competing deterioration-hazard model, it is possible to formulate the survival probability for each type of pipe burst: C-burst or B-burst. Figures 3.3 and 3.4 show the survival probability of DCIP for each of the data bases to B-burst and C-burst, respectively. The survival probability curves of the Bayesian mean estimates are shown. In addition, Figures 3.3 and 3.4 show that as the amount of observation data increases and Bayesian updating is conducted, the survival probability curves approach the survival probability curves obtained from original data base. As shown in Figures 3.3 and 3.4, the survival probability curve for D_{10000} database shows almost same path with result of original data set. It means that the Bayesian updating rule enables to bring the efficiency of model estimation and data acquisition.

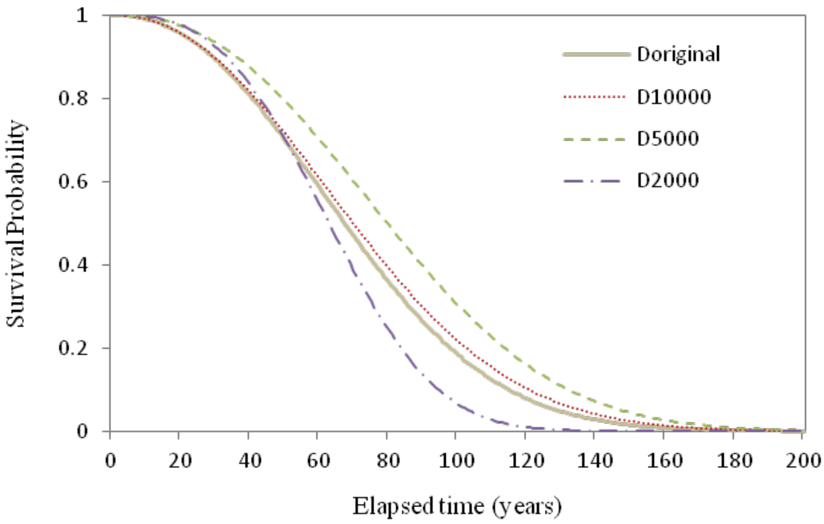

Figure 3.3 Survival probability: C-burst

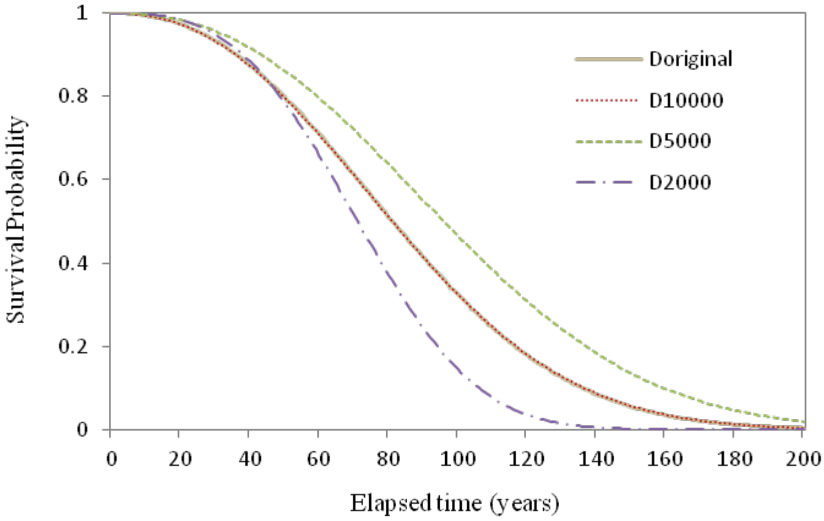

Figure 3.4 Survival probability: B-burst

Figures 3.3 and 3.4 also show that the survival probabilities for both C-burst and B-burst decrease over time and that the survival probability for C-burst decreases more rapidly than that for B-burst. In other words, in ductile cast-iron pipe, bursts in pipe connections (C-bursts) occur at a higher rate than bursts in pipe body(B-bursts).

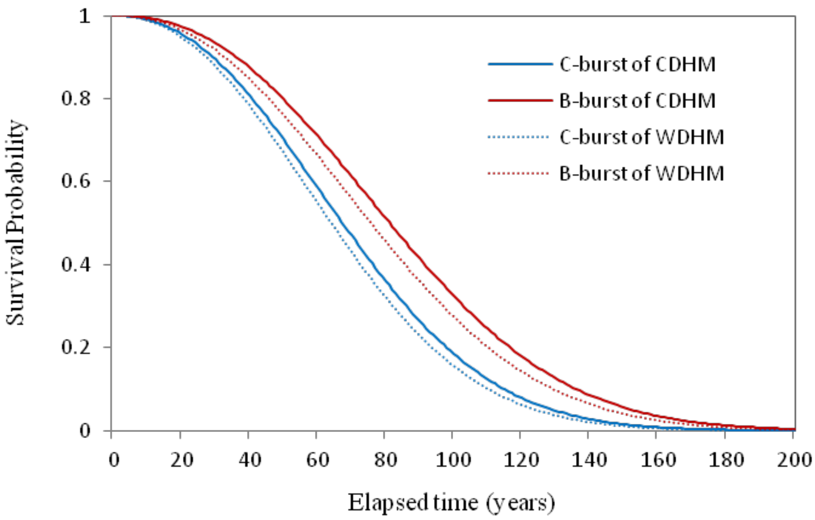

Figure 3.5 Survival probability: Comparison of the competing deterioration-hazard model(CDHM) and the Weibull deterioration-hazard model (WDHM).

Figure 3.5 compares the survival probabilities of C-burst and B-burst obtained from the Bayesian mean estimates of competing deterioration-hazard model and the conventional Weibull deterioration-hazard model. Figure 3.5 also shows that the competing deterioration-hazard model predicts a higher survival probability than does the conventional Weibull deterioration-hazard model. It is noteworthy that the reason why the competing deterioration-hazard model predicts a slower deterioration is that this model considers the occurrence of a competing event when the probability of the event of interest is sought.

3.5. Conclusions

Pipe deterioration model is an important for asset management of pipeline systems. Pipe failures caused by deterioration appear in various forms. Thus, deterioration forecasting model which considers failure types enables the establishment of efficient rehabilitation strategy. We have developed a competing deterioration-hazard model that considers competition among several types of burst in pipeline systems and the proposed model allows us to determine the probability of burst for each type of bursts. The competing deterioration hazard model is estimated using Bayesian estimation method.

The empirical study was carried out by using an inspection dataset of real pipeline system. In the empirical study, because of the absence of detailed substantive knowledge, the competing deterioration-hazard model is estimated with non-informative prior distribution and the Bayesian updating rule is used to improve precision of estimation. The results show that the more estimation results are updated, the more precise estimation results can be obtained. The D_{10000} database shows almost same result with original data set. This result indicates that Bayesian updating rule enables to bring the efficiency of model estimation and data acquisition. In addition, in this study, although we use non-informative prior because of no detailed substantive knowledge, if we can accumulate prior information, the proposed method would be a good way forward.

According to the results of deterioration prediction of C-burst and B-burst obtained by competing deterioration-hazard model, more care is necessary in the pipe connection because we confirmed that the probability of pipe burst in a pipe connection(C-burst) is higher than that in pipe bodies(B-burst). In addition, the results show that the conventional Weibull deterioration-hazard model which does not consider competing properties overestimates pipe burst rates. The bias which arises between the competing deterioration hazard model and the conventional Weibull deterioration hazard model comes from the feature of competing hazard model that considers the occurrence of a competing event when the probability of the event of interest is sought. Even though the prediction accuracy in comparison with conventional Weibull deterioration hazard model is slightly high, it is noteworthy that there is capable of improvement. Thus, if we overcome the problems, left truncated data and high percentage of right censored data, the proposed model can be more useful. Therefore, it is required that much more observed data set and empirical studies are accumulated.

In this study, we classified the pipe burst type into B-burst which occurred in pipe body and C-burst which occurred in pipe connection. As shown in the result, we were able to see that the C-burst and B-burst has a different deterioration rate. Because the choice of a maintenance and repair method will depend on the type of burst, therefore the competing deterioration hazard model enables us to establish an optimum maintenance strategy of pipeline system. In addition, we believe that our new model can be extended to other items of infrastructure and will contribute to advancing asset management.

Our proposed model has not discussed following points, which are considered for future extension of our study.

1) In this paper, most of the failures that typically affect real pipeline systems (i.e. pipe breaks, leakages etc.) are disregarded. To establish optimal maintenance strategy, it is important to consider the repairs due to breaks or leakages.

2) The limited and missing information, left-truncated or survival selection, which are often embedded in observed data have not been mentioned

3) To consider competing hazard model with more than two competing hazards, we could analyze by using synthetic data and considering different amounts of competing failure types.

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4. Optimal Renewal Model for Water Pipeline Systems

4.1. General introduction

In the pipeline system, aging of pipeline due to a variety of internal and external cause reduce the ability of water supply and increase risk of pipe failure. Failure of aging pipeline leads to greater social and economic damage such as traffic control by flooding of neighborhood, restoration of damaged pipelines, water quality degradation due to the influx of pollutants and so on. Thus, through the proper rehabilitation and replacement, the pipeline systems must be managed by safe water quality and structure performance. Pipeline system was built at the high economic growth intensively and now, it has emerged as a major problem to the water authorities worldwide to renew these enormous aging pipelines. In the United States, it was expected to take on the costs of updating aging pipelines and accompanying facilities to $ 250 billion by 2030 [1]. Thus, because an enormous cost is required to rehabilitate the aging pipeline, it is important to establish the economic optimum renewal strategy. In order to establish an economical renewal strategy of aging pipelines, the deterioration state of pipeline and economics of the renewal strategy should be considered. Pipeline system, which occupies 80% of water supply systems, is laid under the ground so it is difficult to detect pipeline condition by inspection and monitoring. So far, to reduce economic and social costs caused by pipe failure, pipelines within a certain period time from building point have been replaced regardless of deterioration. These inefficient management methods that depend on manager’s experience cause waste of budget. Therefore, the optimal renewal model for pipeline system which consider the deterioration of the pipeline is required.

The pipeline is deteriorated with the lapse of time after installation and along with the deterioration, leakage occurs due to cracking or partial break and eventually, the pipeline reaches complete burst. Because pipe failures, leakage and burst cause the enormous social and economic damage, system manager carry out repair the leakage and replace the aging pipeline before reaching complete burst. However, because of frequent rehabilitation of aging pipeline causes increase of maintenance cost, optimal rehabilitation strategy is required to minimize life cycle cost which is summation of total social cost and rehabilitation cost. In this study, we predict the probability of occurrence of leakages and burst in the pipeline, and develop the optimal renewal strategy model considering an as needed basis repair of leakage during the life time of pipeline.

Prediction of deterioration procedure of pipeline system and optimal maintenance strategy based on life cycle cost analysis are necessary for pipeline system management. Until now, a lot of studies about optimal maintenance strategy in pipeline systems have been accumulated. Shamir and Howard[2] estimated the optimal replacement time which minimize the sum of the repair cost and replacement cost. The repair cost was calculated based on pipe break rate. Following Shamir and Howard, many studies have been carried out as similar approaches(e.g., Walski and Pelliccia[3]; Kleiner et al.[4][5], Kleiner and Rajani[6]). Kleiner[7]) forecasted the pipe deterioration using semi-markov model and estimated the optimal schedule of inspection and renewal of large infrastructure asset that minimize the sum of cost of intervention, inspection and failure. Gustafson and Clancy[8] estimated the break order for optimal replacement time which minimizes the economic loss with Monte Carlo simulation. Mailhot et, al.[9] explained the time to failure between pipe breaks by hazard function and defined an optimal replacement criterion involving hazard functions. Minimization of cost function with conditional probabilities to estimate the expected future costs leads to the replacement criterion. Loung and Fujiwara[10] proposed optimal repair strategy which determine the priority of repair that maximize net benefit between repair cost and water saving due to repair in the limited budget. Tanaka[11] proposed a mathematical model to estimate optimal renewal time based on Weibull hazard function and least life cycle cost estimation approach.

These previous studies deal with prediction of deterioration without classifying failure type and with the deterioration model, determined an optimal timing. But in the real pipeline systems, pipe failures occur in the form of diverse type because pipeline system consist of many components and pipe deterioration proceed by diverse factors. Meanwhile, it is the building blocks of asset management for pipeline system that optimal strategy is established depending on pipe failure types. Therefore in this study, the authors briefly classify the pipe failure type due to deterioration into burst which requires replacement and leakage which requires repair and assumed that these types are in competition and then estimated the probability of occurrence of each failure type using competing deterioration hazard model. In addition, the optimal replacement time is determined through a least life cycle cost estimation and the least life cycle cost analysis is conducted on the basis of maintenance strategy that repair of leakage and pipe replacement due to burst are carried out on an as needed basis.

The Weibull deterioration-hazard model and exponential hazard model are used to address the time to burst and leakage of each pipeline and takes into account the nature of the competition between several types of failure by using a competing deterioration-hazard model. The competing deterioration-hazard model is estimated by a Bayesian technique based on the Metropolis–Hasting method (M-H method), a Markov chain Monte Carlo method for obtaining a sequence of random samples from a probability distribution for which direct sampling is difficult. The optimal maintenance model proposed in this study builds on recursive structure which was proposed by Tamura[12] and estimated through the least life cycle cost approach.

Empirical analysis of the model was carried out with the actual data of the pipeline system of S city, Korea.

4.2. Rehabilitation of pipeline system

In general, a pipeline deteriorates with the passage of time as shown in figure 4.1

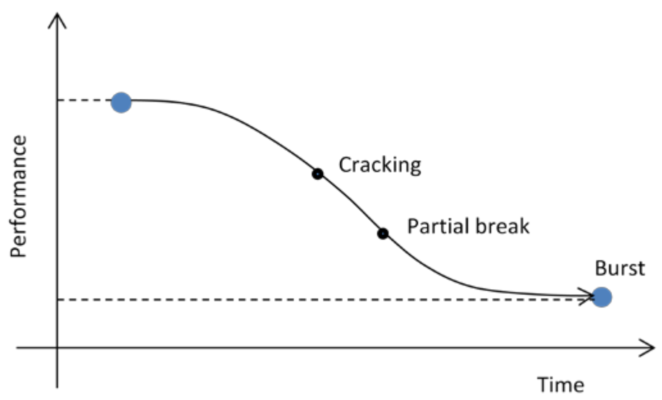

Figure 4.1 Deterioration process of the pipeline

If the resistance of pipeline to the load of the inner and outer surfaces is reduced due to degradation of structure strength, the pipeline reaches a variety of structural damage. Cracks or partial breaks don't directly affect the structural stability of pipeline but bring out leakage. If deterioration progress continues, the pipeline will reach pipe burst. In general, system manager regularly carry out inspection of pipe condition for maintenance of aging pipelines. And, if a leakage due to cracking or partial break is detected, the damaged parts are repaired. In addition, a burst pipeline is replaced immediately. Because the complete failure of pipeline cause enormous social and economic damage, system manager carry out proactive replacement of aging pipeline to avoid pipe failure risk.

4.3. Pre-assumption of the model

In this study, we estimate the optimal renewal interval which minimizes expected life cycle cost in the infinite time base. The maintenance scheme of aging pipeline is set by that whenever pipe leakage is detected, the damaged pipeline will be repaired or complete break is detected the damaged pipeline will be replaced, immediately. In addition, a aging pipeline which has reached a certain operating time is replaced proactively regardless of whether complete failure or not.

In pipeline system, we classify the state of a pipeline as being one of three distinguish level of deterioration, denoting as $E_i\,(i=0,1,2)$. Level E_0 reflects the healthy condition in good level. Level E_1 denotes a state in which leakage due to cracking or partial break is found and repair is required immediately. Level E_2 reflects that a pipeline lost its function as water supply because a state of pipeline reaches complete failure. Thus, whenever the condition level E_2 is detected, the damaged pipeline will be replaced by a new one immediately.

The repairs for leakages are not regarded as a structural reinforcement of whole pipeline, it is just assumed that the repairs are applied at damaged part. In addition, the leakages may not occur even once or may occur many times during life time of a pipeline.

4.4. Pipeline deterioration model

4.4.1. Modeling strategy

What is important for the maintenance of the infrastructure is to predict the procedure of the deterioration. It plays an important role in estimating the expected failure cost and rehabilitation cost over the life cycle of the infrastructure[13]. For this purpose, it is necessary to predict the probability of pipe failure on the basis of available data[14]. In this study, to predict the deterioration of pipe failure types which are in competition, the competing deterioration hazard model proposed in Chapter 3 is used.

Pipe failure depends largely on the duration of use of the pipeline. The hazard function should therefore consider the elapsed time. In this study, the times to failure are used as random variables described by probability density functions. The probability density functions correspond to the probability of occurrence of failures, leakage and burst. The Weibull hazard model and Exponential hazard model, which are suitable for addressing this process, are applied with the assumption that the probability of pipe burst and leakage increase with time, respectively, as follows:

$$\lambda_b\left(\tau\right) = \gamma_b m \tau^{m-1} \tag{4.1}$$

$$\lambda_l\left(\tau\right) = \gamma_l \tag{4.2}$$

where m is the acceleration parameter that represents the time dependency of the hazard function and $\gamma_j\ (j = l, b)$ is the parameter expressing the arrival rate of pipe failure. It is assumed that γ_j depends on the characteristics of the pipeline, and that it can be expressed as follows:

$$\gamma_j = \exp\left(\boldsymbol{x}_i \boldsymbol{\beta}_j\,'\right) \tag{4.3}$$

where $\boldsymbol{x}_i = (x_i^1, \cdots, x_i^k)$ is the characteristic vector that represents the observed value for pipeline i and $\boldsymbol{\beta}_j = (\beta_j^1, \cdots, \beta_j^k)$ represents the unknown parameter vectors. In addition, k is total number of covariates and the sign ' denotes transposition.

By using the Weibull hazard model and Exponential hazard model, each the probability-density function $f_j(\tau)$ and survival function $\tilde{F}_j(\tau)$ can be expressed as follows:

Table 4.1 Equations of probability density, survival and hazard functions of the Exponential and Weibull deterioration model

	Probability density function	Survival function	Hazard function
Exponential	$f_j(\tau) = \gamma_j \exp\left(-\gamma_j \tau\right)$	$\tilde{F}_j(\tau) = \exp\left(-\gamma_j \tau\right)$	$\lambda_l\left(\tau\right) = \gamma_l$
Weibull	$f_j(\tau) = \gamma_j m_j \tau^{m_j - 1} \exp\left(-\gamma_j \tau^{m_j}\right)$	$\tilde{F}_j(\tau) = \exp\left(-\gamma_j \tau^{m_j}\right)$	$\lambda_b\left(\tau\right) = \gamma_b m \tau^{m-1}$

4.4.2. Competing deterioration hazard model

As long as a pipeline is in operation, there is a chance of a pipe failure, leakage or burst. Let us discuss the estimation method for the competing deterioration-hazard model based on inspection data. We assumed that the inspection data is collected by completely observed data scheme. In other words, all failures are recorded within the recording period as shown in figure 4.2. We define T_a as the time at the beginning of the recording period and T_b as the end of the recording period.

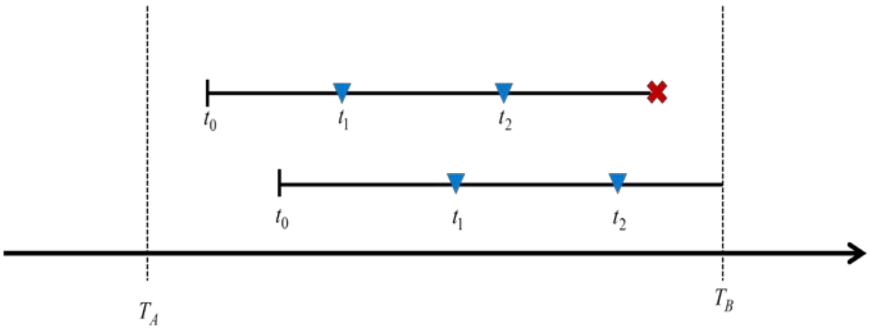

Figure 4.2 Pipe failure information of right censored observation

The time at which the pipe was buried is set as t_0 and the point in time of leakages are denoted by t_l $(l = 1, \cdots, L)$. In addition, the point in time of burst is denoted by t_b. As we mentioned in chapter 3, because pipe replacement due to burst blocks the occurrence of leakage a burst can be regarded as a competing event of leakage. Considering this competition, the conditional probability that the observed information occurs in pipeline i can be represented by the following equation:

$$\ell(d_i, \varepsilon_i, t_i \mid \mathbf{x}_i, \boldsymbol{\theta}) = \left\{ \prod_{j=1}^{n} f_l(t_{ij} \mid \mathbf{x}_i, \boldsymbol{\theta}) \tilde{F}_b\left(t_{ij} \mid \mathbf{x}_i, \boldsymbol{\theta}\right) \right\}^{d_i} \cdot \left\{ \prod_{j=1}^{n-1} f_l(t_{ij} \mid \mathbf{x}_i, \boldsymbol{\theta}) \tilde{F}_b\left(t_{ij} \mid \mathbf{x}_i, \boldsymbol{\theta}\right) f_b(t_{in} \mid \mathbf{x}_i, \boldsymbol{\theta}) \right\}^{1-d_i} \Bigg]^{\varepsilon_i} \cdot \left[\tilde{F}_l\left(t_{in} \mid \mathbf{x}_i, \boldsymbol{\theta}\right) \tilde{F}_b\left(t_{in} \mid \mathbf{x}_i, \boldsymbol{\theta}\right) \right]^{1-\varepsilon_i} \tag{4.4}$$

where, the ε_i and d_i are dummy variables. The ε_i receives a value of 1 when pipe failure was encountered and 0 otherwise. In addition, the reported pipe failure type can be represented by the dummy variable d_i. The d_i is 1 when pipe leakage has occurred otherwise, d_i receives a value of 0 when pipe burst has occurred. Here, we define the unknown parameter vector for the competing deterioration-hazard model as $\theta = (\beta_j, m)$. This assumes that the pipe failure of each of the *n* pipelines is mutually independent from that of other parts of the pipeline system. If the observed information of pipeline i is $\xi_i = (d_i, \varepsilon_i, t_i, \mathbf{x}_i)$, the simultaneous probability density of the pipe deterioration can therefore be expressed by the following likelihood function:

$$L\left(\theta \middle| \xi\right) = \prod_{i=1}^{n} \ell\left(d_i, \varepsilon_i, t_i \mid \mathbf{x}_i, \boldsymbol{\theta}\right) \tag{4.5}$$

where, $\boldsymbol{\xi}$ represents $\boldsymbol{\xi} = (\xi_1, \cdots, \xi_n)$.

4.4.3. Bayesian Estimation Method For Competing Deterioration Hazard Model

In this section, we present a methodology for estimating the unknown parameter vector $\boldsymbol{\theta}$ of the competing deterioration-hazard model by means of a Bayesian estimation method using observed data. The Bayesian approach permits the estimation of $\boldsymbol{\theta}$ on the basis of the inspection data ξ and prior information regarding $\boldsymbol{\theta}$ By using the M-H method, the estimation is carried out by sampling a large number of values of $\boldsymbol{\theta}$ from its posterior distribution, which can be expressed as follows:

$$\pi(\boldsymbol{\theta} \mid \xi) \propto L(\boldsymbol{\theta} \mid \xi)\pi(\boldsymbol{\theta}) \tag{4.6}$$

where $\pi(\boldsymbol{\theta} \mid \xi)$ is the posterior probability density function of $\boldsymbol{\theta}$, $L(\boldsymbol{\theta} \mid \xi)$ is the likelihood function, and $\pi(\boldsymbol{\theta})$ is the prior probability density function of $\boldsymbol{\theta}$. The newly obtained data are denoted by $\xi = (\xi_1, \cdots \xi_n)$.

In this study, we assume that the prior probability density function of parameter, $\boldsymbol{m}$ and $\boldsymbol{\beta}$ follow a gamma distribution and a conjugate multidimensional normal distribution, respectively, $\boldsymbol{m} \sim g(m_0, k_0)$, $\boldsymbol{\beta} \sim \mathcal{N}_K(\boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o)$. With this assumption, the probability density function of the gamma distribution function $g(m_0, k_0)$ and the K-dimensional normal distribution $\mathcal{N}_K(\boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o)$ can be further expressed as follows:

$$f(\boldsymbol{m} \mid m_0, k_0) = \frac{1}{\Gamma(m_0)} {k_0}^{m_0} \boldsymbol{m}^{m_0 - 1} e^{-k_0 \boldsymbol{m}} \tag{4.7}$$

and

$$g(\boldsymbol{\beta} \mid \mu_o, \Sigma_o) = \frac{1}{\sqrt{(2\pi)^K \left|\Sigma_o\right|}} \cdot \mathrm{ex} \left\{ -\frac{1}{2} (\boldsymbol{\beta} - \mu_o) \Sigma_o^{-1} (\boldsymbol{\beta} - \mu_o)' \right\} \tag{4.8}$$

where, $\Gamma(m_0)$ denotes the gamma function and $\boldsymbol{\mu}_o$ and $\boldsymbol{\Sigma}_o$ represent the prior expectation vector and the prior variance-covariance matrix of $\mathcal{N}_K(\boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o)$, respectively.

The M-H method is used to perform sampling from an empirical distribution that is similar to $\pi(\boldsymbol{\theta} \mid \xi)$ and accordingly obtains samples from the original distribution [15]. Furthermore, a random walk is used to improve the efficiency of sampling.

4.5. Optimal rehabilitation model

With the estimated occurrence probability of each pipe failure type over time, the optimal rehabilitation model is established by considering expected failure cost and maintenance cost. The occurrence of a pipe failure j causes failure cost which is denoted by C_j and assumed to be a constant value. When the predetermined time interval of replacement is set by z , the expected failure cost of failure type j is followed in the probabilistic manner via the probability density function $f_j(t)$ shown in table 4.1. Thus, the discounting present value of expected failure cost $EC_j(\mathrm{z})$ calculated during replacement period $[0, z)$ can be expressed by the integral form as follows;

$$EC_j(\mathrm{z}) = \int_0^z C_j f_j(t) \exp\left(-\rho t\right) dt \tag{4.9}$$

where, the coefficient ρ is an instantaneous discounted rate of money over time.

Meanwhile, the cost of replacement activities is denoted by I_b and assumed to be a constant value. It is assumed that a replacement is carried out in case of the occurrence of a burst during $[0, z)$ or the age of pipeline reaching time z . The expected replacement cost is followed in the probabilistic manner via probability density function $f_b(t)$ and the survival probability function $\tilde{F}_b(t)$ shown in table 4.1 when pipe age reaches z . Thus, the present discounted cost of the expected replacement cost for the next pre-determined replacement time $EM(\mathrm{z})$ can be expressed as follows;

$$EM(\mathrm{z}) = \int_0^z I_b f_b(t) \exp\left(-\rho t\right) dt + I_b \tilde{F}_b(\mathrm{z}) \exp\left(-\rho t\right) \tag{4.10}$$

The cost of repair activities is denoted by I_l and assumed to be a constant value. The repair scheme for leakage base on an as needed basis, in other words, it is assumed that when a leakage occurs, it will be repaired.

Suppose that the repair for leakage is carried out in the arbitrary time y . The present discounted cost of the accumulated expected failure and repair cost from time y to z is denoted by $L(y)$. If we consider the possibility of occurrence of next other leakages until time z , when the next repair time is denoted as $y+t$, the $L(y)$ can be calculated by considering $L(y+t)$ caused by next repair as follows;

$$L(y:\tau) = \int_0^{\tau - y} \left\{C_l + I_l + L(y+t)\right\} f_l(t) \exp(-\rho t) dt \tag{4.11}$$

where, τ is a stochastic variable of replacement time ($0 \le y \le \tau \le z$).

Integral equation (4.11) can be simply rearranged as follows through a complex solving process which is further explained in appendix A.

$$L(y:\tau) = \frac{\gamma_l(c_l + I_l)}{\rho}\left[1 - e^{\rho(y-\tau)}\right] \tag{4.12}$$

Consequently, the present discounted cost of the accumulated expected failure and repair cost from buried time $t = 0$ to stochastic replacement time τ is denoted as $\tilde{L}(\tau)$ and can be calculated in the following form;

$$\tilde{L}(\tau) = L(0:\tau) = \frac{\gamma_l(c_l + I_l)}{\rho}\left[1 + e^{\rho\tau}\right] \tag{4.13}$$

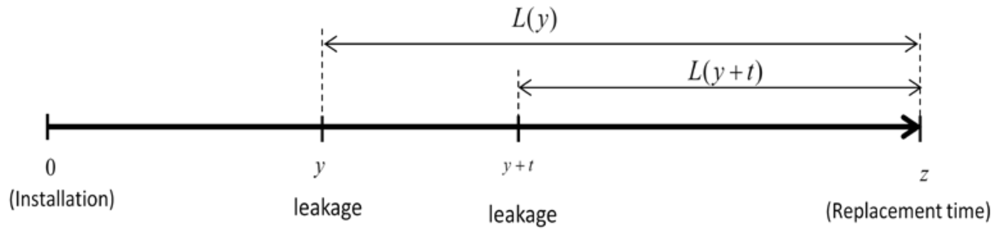

Figure 4.3 A accumulated expected failure and repair cost of leakage

Under a strategy that proactive pipeline replacement in the time interval z , it is assumed that whenever a pipe failure j is detected, it will be repaired or replaced immediately. The expected life-cycle cost(LCC) after the next replacement time is estimated as the net present value of failure costs and rehabilitation(repair and replacement) costs.

As the failure costs and rehabilitation costs are constant value, the expected LCC takes equal value for every replacement time. In order words, the expected LCC estimated at the next replacement time is equal to the expected LCC estimated at the present replacement. The expected LCC denoted as $LCC(0:z)$ can be regulated through the regression estimation expressed in the following form;

$$\begin{aligned} LCC(0:z) = &\int_0^z \left\{\tilde{L}(\tau) + C_b + I_b + LCC(0:z)\right\} f_b(\tau)\exp(-\rho\tau)d\tau \\ &+ \left\{\tilde{L}(z) + I_b + LCC(0:z)\right\} \tilde{F}_b(z)\exp(-\rho z) \end{aligned} \tag{4.14}$$

The following two functions $\Lambda_b(z)$ and $\Gamma_b(z)$ are defined;

$$\Lambda_b(z) = \tilde{F}_b(z)\exp(-\rho z) \tag{4.15}$$

$$\begin{aligned}
\Gamma_b(z) &= \int_0^z f_b(\tau)\exp(-\rho\tau)d\tau \\
&= \int_0^z \gamma_b m\tau^{m-1}\exp(-\gamma_b\tau^m - \rho\tau)d\tau \\
&= -\int_0^z \exp(-\gamma_b\tau^m - \rho\tau)d(\gamma_b\tau^m - \rho\tau) - \rho\int_0^z \exp(-\gamma_b\tau^m - \rho\tau)d\tau \\
&= 1 - \Lambda_b(z) - \rho\int_0^z \Lambda_b(\tau)d\tau
\end{aligned} \tag{4.16}$$

With the functions $\Lambda_b(z)$ and $\Gamma_b(z)$, the integral equation part about $\tilde{L}(\tau)$ can be simply rearranged by as follows,

$$\begin{aligned}
&\int_0^z \tilde{L}(\tau) f_b(\tau)\exp(-\rho\tau)d\tau \\
&= \int_0^z \frac{\gamma_l(c_l + I_l)}{\rho}\{1 + \exp(\rho z)\} f_b(\tau)\exp(-\rho\tau)d\tau \\
&= \frac{\gamma_l(c_l + I_l)}{\rho}\{1 + \exp(\rho z)\}\Gamma_b(z) \\
&= \Omega \cdot \Gamma_b(z), \qquad \Omega = \frac{\gamma_l(c_l + I_l)}{\rho}\{1 + \exp(\rho z)\}
\end{aligned} \tag{4.17}$$

Substituting equations (4.15-17) into equation (4.14), the following explicit form for the expected LCC is obtained:

$$LCC(0:\mathrm{z}) = \frac{(C_b + I_b)\Gamma_b(z) + \Omega\cdot\Gamma_b(z) + (\tilde{L}(z) + I_b)\Lambda_b(z)}{\rho\int_0^z \Lambda_b(t)dt} \tag{4.18}$$

Here, I have to solve the integration of function $\Lambda_b(z)$. But, it is extremely difficult to solve the integration, $\int_0^z \Lambda_b(t)dt$ by analytic method.

The general form of expanding the integration into following discrete series will be accepted.

$$X_k = \int_0^{k\cdot mi} \Lambda(t)dt \tag{4.19}$$

Here, k is number of iteration and mi is the very small amount of time. For example,

value of mi can becomes mi = 0.001 or even smaller.

$$
\begin{aligned}
X_{k+1} &= \int_0^{(k+1)mi} \Lambda(t)dt \\
&= X_k + \int_{k\cdot mi}^{(k+1)mi} \Lambda(t)dt \\
&= X_0 + \frac{\left[\Lambda(0\cdot mi)+\Lambda(\{0+1\}\cdot mi)\right]mi}{2} \cdots \frac{\left[\Lambda(\mathrm{k}\cdot mi)+\Lambda\{(\mathrm{k}+1)\cdot mi\}\right]mi}{2}
\end{aligned}
\tag{4.20}
$$

To this point, the value of integration can be easily estimated by numerical calculation. We substitute equation (4.18) and use Newton method to estimate for the minimum value of $LCC(0:z)$.

Therefore, the optimal rehabilitation model can be formulated as follows;

$$
\Phi(0) = \min_z \left\{ LCC(0:\mathrm{z}) \right\} \tag{4.21}
$$

where, the optimal value function $\Phi(0)$ is denoted as the minimum expected LCC estimated at the initial time.

4.6. Empirical Study

4.6.1. Overview of Empirical study

To analyze the deterioration of a real pipeline, we focused on the water distribution system of S city in South Korea. The total length of distributing pipeline which has 80mm or more diameter is approximately 1,000km. And the entire distribution pipeline system composes of a variety of pipe types, CIP, DCIP, PE, PVC, SP and so on. In this study, we focus on cast iron pipe(CIP) and ductile cast iron pipe(DCIP) which are available to get a statistical significant number of data. In Korea, the CIP was used mainly as a distributing pipe until the 90s, and since then, DCIP has been mainly used. Actually, in S city, now, the about 90% of water distributing pipe is DCIP and the CIP is not being used anymore since 2003. Table 4.2 shows the basic information of target pipe types used in this study.

Table 4.2 Basic information of target pipes

Features	value	value
Material	Ductile cast iron	cast iron
Years laid(average age)	From 1957 to 2010(13years)	From 1944 to 2003(27years)

Diameter/mm	75~900			75~800			
Number of pipes	26,577			4,057			
Total length/km	848.1			72.1			
Number of failures	1,405	leakage	833	403	leakage	297	
		burst	572		burst	106	

In this empirical study, we estimate the optimal replacement time of CIP and DCIP using the optimal rehabilitation model and compared the economical efficiency of two pipe types. To determine optimal rehabilitation strategy, we should consider not only replacement and repair cost of pipeline but damage cost caused by pipe failure. The social cost C, rehabilitation cost I and discounted rate ρ play a major role in establishing the optimal renewal strategy in least LCC analysis.

Because repair cost for leakage is greatly influenced by location, repair method and so on, it is difficult to generalize. Thus, in this study, we assumed that the leakage repair cost is set 30% of replacement cost. In addition, the social cost of each failure was assumed to be equal to five times of replacement costs. The discounted rate ρis assumed by 4% per year. In this study, we focus on the distributing pipes which have 80mm or more diameter were selected for the study because of the critical risk caused by pipe failure. And the unit costs of pipe replacement are shown in Table 4.3.

Table 4.3 Unit costs of each rehabilitation option

Diameter(mm)	CIP ($/m)	DCIP ($/m)
80	276	314
100	311	344
200	488	522
300	687	743
400	900	983
500	1,033	1,247
600	1,243	1,516
700	1,525	1,869
800	1,826	2,251
900	2,213	2,750
1000	2,667	3,306

4.6.2. Estimation results of pipe deterioration model

The deterioration hazard model used for the Bayesian estimation is specified as follows:

$$\text{leakage}: \ \lambda_l\left(t_i\right) = \exp(\beta_{l0} + \beta_{l1}x_{i1} + \beta_{l2}x_{i2}) \ (i = 1, \cdots, n) \tag{4.22}$$

$$\text{burst : } \lambda_b\left(t_i\right) = \exp(\beta_{b0} + \beta_{b1}x_{i1} + \beta_{b2}x_{i2})m \cdot t_i^{m-1} \, (i = 1, \cdots, n) \tag{4.23}$$

The unknown parameter β_{j0} is a constant term, β_{j1} and β_{j2} represent the pipe diameter and pipe length, respectively. In this study, other characteristic variables that reflect the influence of outer and inner rust, soil unit weight, top traffic volume, and so on were neglected, either because of their small impacts or because data were unavailable. The unknown parameters can be expressed as follows:

$$\boldsymbol{\theta} = (\beta_{l0}, \beta_{l1}, \beta_{l2}, \beta_{b0}, \beta_{b1}, \beta_{b2}, m) \tag{4.24}$$

In this study, we assume that the prior probability density function of the unknown parameters, $\boldsymbol{m}$ and $\boldsymbol{\beta}$ follow $\boldsymbol{m} \sim g(m_0, k_0)$, $\boldsymbol{\beta} \sim \mathcal{N}_K(\boldsymbol{\mu}_o, \boldsymbol{\Sigma}_o)$. But unfortunately, because of the absence of detailed substantive knowledge, it is difficult to get information about the expectations and the variance of unknown parameters. Thus, a noninformative prior distribution is applied for the Bayesian estimation. The noninformative prior distribution can be obtained by setting the variance of the prior distribution to be sufficiently large, as follows:

$$\boldsymbol{m} \sim g(1, k_m^{-1}) \tag{4.25}$$

$$\boldsymbol{\beta} \sim \mathcal{N}_K(\mathbf{O}, k_\beta \mathbf{I}) \tag{4.26}$$

Where, k_m and k_β are sufficiently large integer. $\mathbf{O}$ and $\mathbf{I}$ are a zero vector and a unit matrix, respectively.

Where, *O* and *I* are a zero vector and a unit matrix, respectively. To conduct the M-H method, the number of iteration required to reach a steady state (the burn-in period) was set to $\underline{N} = 5,000$ and the number of iterations for parameter sampling was set to $\bar{N} = 20,000$. The 10000 burn-in samples were omitted and the remaining 10,000 parameter samples were used to carry out the estimation.

Table 4.4 shows the results of estimations by the M-H method. The estimated values are the sample average of parameters, and the values in parentheses refer 95% credible intervals. The absolute value of the Geweke test statistics are all less than 1.96, so the convergent hypothesis cannot be dismissed at a significance level of 5%. With the estimation results for the competing deterioration-hazard model, it is possible to formulate the probability density for each type of pipe failure: leakage or burst.

Table 4.4 Results of estimation of parameters for the competing deterioration-hazard model

	CIP	DCIP

	Estimated value	Geweke statistics	Estimated value	Geweke statistics
β_{l0}	–3.789 (–4.257, –3.268)	0.057	–4.201 (–4.547, –3.852)	0.069
β_{l1}	–1.393 (–2.103, –0.717)	0.0448	–1.068 (–2.612, –1.504)	0.021
β_{l2}	2.041 (1.362, 2.808)	0.130	2.325 (1.807, 2.851)	0.224
β_{b0}	–10.917 (–11.437, -10.493)	0.061	–11.177 (–11.431, -10.832)	0.116
β_{b1}	-2.527 (-3.206, -1.894)	0.067	-2.501 (-3.106, -2.014)	0.007
β_{b2}	3.299 (2.686, 3.964)	0.169	3.243 (2.712, 3.726)	0.074
m	2.610 (2.406, 2.814)	0.078	2.524 (2.421, 2.630)	0.011

Notes: Values in $(\cdot)$ show 95% credible intervals.

Figures 4.4 and 4.5 show the cumulative failure probability of cast-iron pipes(CIP) and ductile cast-iron pipes(DCIP) to burst and leakage, respectively. The figures show that the failure probabilities for both leakage and burst increase over time. In this study, the time to leakage and burst are described by exponential distribution and Weibull distribution, respectively. The cumulative distribution curves shown in figures 4.4 and 4.5 are obtained from integration of probability density function, and equal to the difference between 1 and survival function. As can be understood from these figures, leakage shows higher failure probability than burst in both pipe materials. In addition, we could confirm that the probabilities of leakage and burst in CIP increase more rapidly than that for failure probabilities in DCIP.

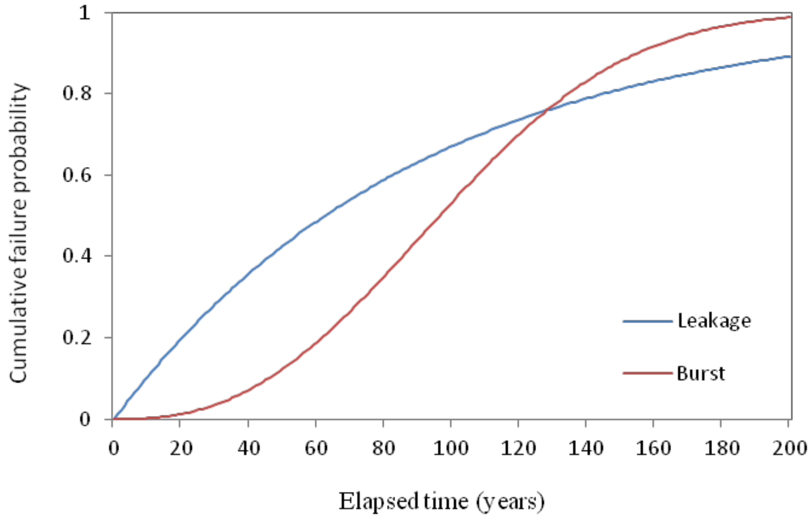

Figure 4.4 Cumulative failure probability in DCIP : Leakage and Burst

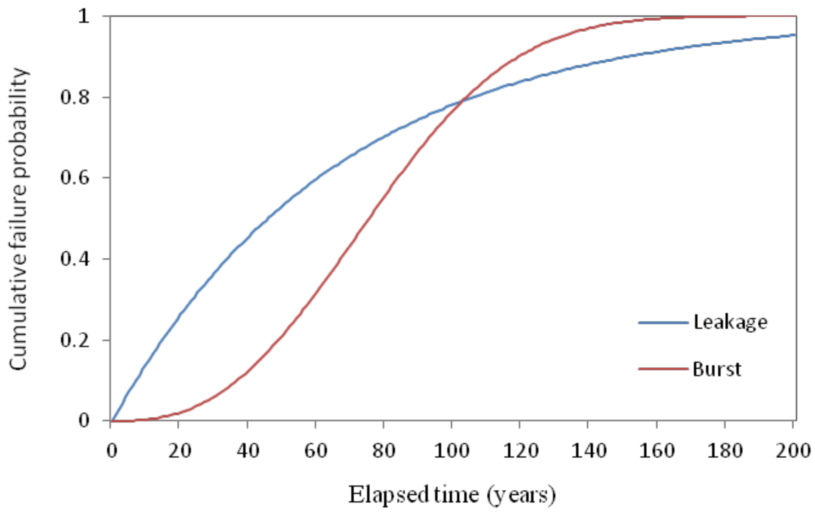

Figure 4.5 Cumulative failure probability in CIP : Leakage and Burst

4.6.3. Optimal replacement time and expected life cycle cost

Estimation for optimal replacement time and expected life cycle cost are carried out in the second phase after estimating the competing deterioration hazard model. The occurrence probability of pipe leakage and burst are predicted and then, least life cycle cost analysis is conducted on the basis of maintenance strategy that repair of leakage and pipe replacement due to burst are carried out on an as needed basis. Minimization problem to seek for the optimal replacement timing z is empirically analyzed by using equation (4.21).

Results of estimation are shown in Figure 4.6 and 4.7 for some pipe diameters, 100, 200, 300, and 500 of CIP and DCIP, respectively. The expected failure costs tend to increase over time due to the increase of failure probability. On the other hand, the expected maintenance costs tend to decrease with time due to discounting. Thus, the total expected life cycle cost forms a convex curve over time as shown in figure 6 and 7.

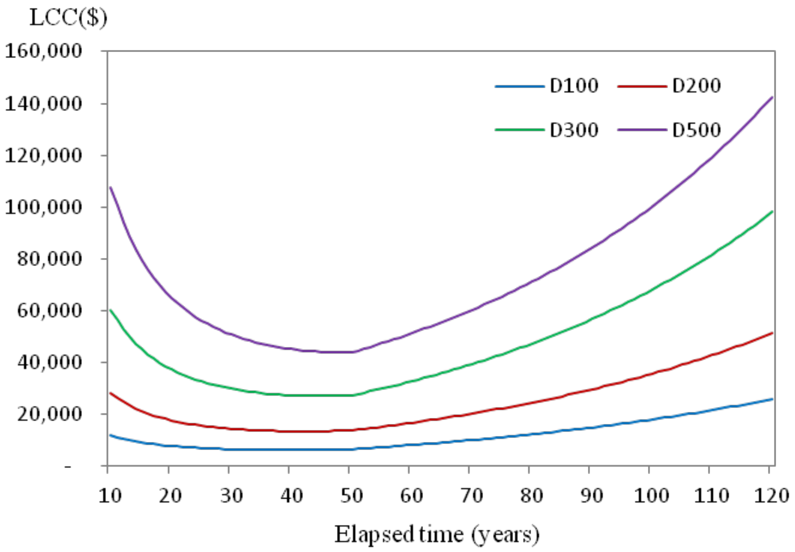

Figure 4.6 Expected life cycle cost comparison by pipe diameter: DCIP

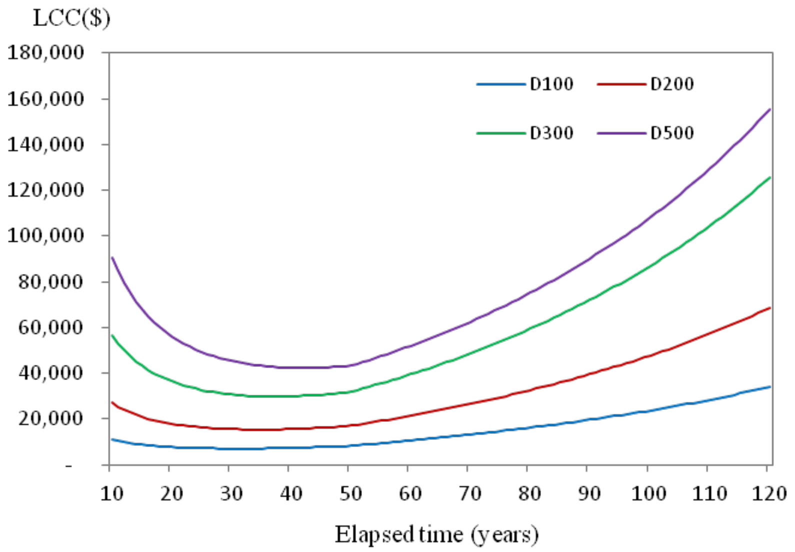

Figure 4.7 Expected life cycle cost comparison by pipe diameter: CIP

As shown in Figure 4.6 and 4.7, the larger size of pipe has high life cycle cost and long optimal replacement interval z. It is because the larger size of pipe shows low pipe failure probability. In addition, comparing Figures 4.6 and 4.7, it is possible to verify that DCIP shows a low life cycle cost for the same diameter than the CIP and the optimal replacement time of DCIP is longer that CIP. The results of least life cycle analysis for each pipe diameter and material are shown in Table 4. From the results, we can confirm that the DCIP is more economical pipe type than CIP.

Table 4.5 Results of least life cycle cost analysis and optimal replacement time

	CIP		DCIP	
	Optimal replacement time(z,year)	Optimal LCC($/m)	Optimal replacement time(z,year)	Optimal LCC($/m)
D100	31	7,002	39	6,370
D200	35	15,339	42	13,546
D300	37	29,777	45	27,168
D500	42	42,233	49	43,977

4.7. Conclusion

Pipe failure cause social and economic loses as well as inconvenience to consumers. Therefore an optimal maintenance strategy is required but because pipeline system is underground, it is difficult to perform inspection and monitoring of pipe condition. Thus, in many case, maintenance of pipeline systems depend on manager's empirical judgment. Therefore it is required to predict the pipe deterioration by accumulated inspection data and to establish optimal maintenance strategy which is minimizing expected life cycle cost. But in the real pipeline systems, there are various types of failure and maintenance strategy depends on these failure types. In this study, pipe failure is briefly classified into burst which requires replacement and leakage which requires repair. And the deterioration procedure of burst and leakage is forecasted by using competing deterioration hazard model. In addition, we suggested optimal maintenance strategy model which consider pipe replacement and repair. The time to burst and leak are explained by using Weibull hazard model and exponential hazard model, respectively. The competing hazard model takes into account the competing nature among the failure types. In addition, The occurrence probability of pipe leakage and burst are predicted and then, least life cycle cost analysis is conducted on the basis of maintenance strategy that repair of leakage and pipe replacement due to burst are carried out on an as needed basis.

The empirical application of the proposed model was carried out to the real pipeline system, S city in Korea. We could obtain the optimal life cycle cost and optimal replacement time of each pipe type and diameter. The estimation results demonstrated that the DCIP is more beneficial type of pipe than CIP in asset management of the pipeline system. From the application view points, we believe that our new model can be extended to other items of infrastructure and will contribute to advancing asset management.

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5. Estimating Compound Deterioration process in Water Pipelines

5.1. General introduction

Pipeline system is an important infrastructure to supply purified water for maintenance and development of city. A huge budget is required to maintain the system annually. Since pipeline system is laid under the ground, it is difficult to inspect and monitor its condition state. Thus, Many water supply authorities have replaced the aging pipelines by depending on experience of managers regardless of pipe condition. Because it is an inefficient management approach that does not get the most out of asset value on a limited budget, an effective maintenance strategy is required. Rehabilitation strategy of pipeline system should satisfy three major requirements of economics, water quality and durability soundness[1]. Aging pipeline lead to decline in water quality caused by influx of contaminant through fractured parts, and if the aging pipeline is completely broken it causes huge social and economic damage. Therefore, aging water pipeline must be maintained by appropriate rehabilitation and replacement plan. For maintenance of infrastructure, optimal rehabilitation strategy should be established by life cycle cost analysis based on deterioration model. Optimal strategy of replacement and repair which makes minimum life cycle cost, is required for economical maintenance on a limited budget. Thus, an accurate prediction of pipe deterioration is the basis for asset management.

Many studies about pipe deterioration prediction have been carried out survival analysis with assuming that a time to failure follows a probability distribution using binary condition state, Good or Failure, because with the limitations of the embedded infrastructures, it is difficult to accumulate condition information of pipelines[2-7]. However, the working status or condition state of pipeline is not just binary expression but often in a wide range of discrete numbers. For the deterioration process of pipeline, the corrosion of surface and pipe body degradation have been at the center of attention. Some studies about the prediction of corrosion rate pipe surface with mechanical deterioration forecasting model [8-13]has been proposed. But these existing mechanical deterioration studies are suitable for the individual pipe but limited to apply to the pipeline system in a complex environment. There are published studies about deterioration of pipe body modeled as a Markov chain process [14-17] with discrete number of states. The existing studies have been carried out forecasting the deterioration process of each surface corrosion and pipe body degradation but there is no study about deterioration prediction considering the interaction of the corrosion of inner surface and the degradation of pipe body. For infrastructure other than the pipeline system, some studies about pavement deterioration forecasting model considering the interaction between the deterioration of road surface and the decrease in the load bearing capacity of pavement have been published [18-19].

The condition of inner surface of pipeline directly affect on the level of service to user. Corrosion of the inner surface of pipe causes red water, scaling, lack of water quantity and pressure by accumulation of foreign matter. In addition, it increases electricity consumption caused by change of water pressure. To recover condition of inner surface of pipe, flushing or lining is carried out and severe internal corrosion pipes have been replaced. Meanwhile, deterioration of structure performance by degradation of pipe body causes pipe failure due to internal and external loads. And pipe failure can cause social and economic damage, such as spilling water over the load, traffic control, water outage and so on.

Forecasting for structural deterioration of the pipeline enables to establish proactive maintenance strategy to prevent pipe failure. Therefore, for maintenance of pipeline, deterioration forecasting model is required considering deterioration process of inner surface and body of pipe at the same time. Deterioration process of pipeline is complex phenomenon consisting of corrosion of inner surface and pipe body degradation. Degradation of pipe body affects on the corrosion rate of inner surface. Therefore, the pipeline which is remarkably degraded in pipe body has a potential to accelerate the corrosion rate of inner surface. In contrast, the corrosion of inner surface also influence on the degradation of pipe body.

This study formulates compound deterioration process considering interaction of corrosion of inner surface and pipe body degradation to compound markov deterioration model. As a result of advances in technology, corrosion of inner surface is able to be check by endoscopic or robotic exploration survey which don’t require excavation work or traffic control. On the other hand, degradation of pipe body is determined by direct physical examination. These two surveys are independent and not always performed at the same time. Specially, because excavation work and traffic control are needed to check condition of pipe body, enormous survey costs and social costs are required. Thus, it is not practical to conduct investigations for all the pipelines. Therefore, it is difficult to obtain data about condition state of inner surface and pipe body at the same time. For these reasons, the cases which either of condition state data of inner surface and pipe body is not observed are far from uncommon depending on the time. Thus, we need to develop compound hidden markove deterioration model considering mechanism, which have missed data of inspection of inner surface and pipe body systemically.

In this study, we formulate compound hidden markove deterioration model which considers systematic loss of data. In addition, based on the data of inspection of inner surface and pipe body at different time point, we suggest estimation method using MCMC(Markov Chain Monte Calro).

5.2. Compound deterioration process

Deterioration of pipeline is caused by complex independent reasons, corrosion of inner surface and degradation of pipe body. This study elucidates deterioration state of pipeline with two indicator, corrosion degree of inner surface and residual thickness showing mechanical characteristics of pipe body. Corrosion of inner surface is mainly influenced by water quality and pressure. Decreased cross-sectional area by tubercle and corrosion products is difficult to secure required water pressure and quantity. It leads to diverse problems such as red water caused by corrosion products and loss of original function as water pipeline. The corrosion degree of inner surface is estimated by endoscopic investigation and Robotic exploration and so on. Meanwhile, degradation of pipe body is affected by various causes such as corrosion, internal and external load, temperature change and so on. The degradation of pipe body causes strength degradation of pipeline and decreases resistance of internal and external load to reach failure as a result. The degree of degradation of pipe body can be checked by residual thickness from direct investigation or ultrasonic examination. The residual thickness becomes a barometer to understand metal loss and an indicator of residual strength.

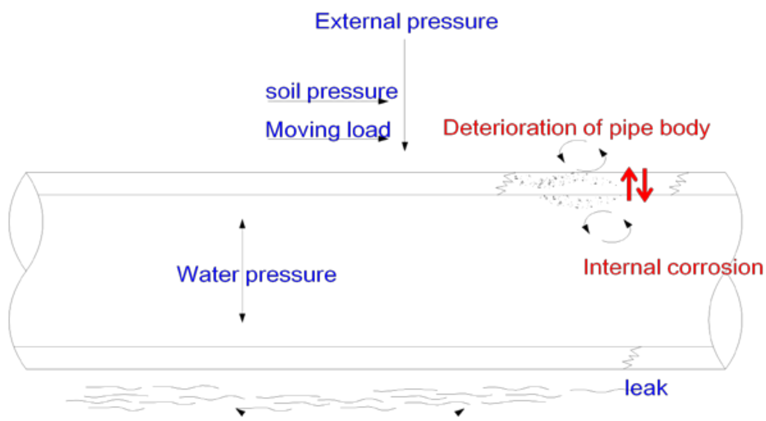

Figure 5.1 Deterioration mechanism of a water pipe[20]

We speculate that degradation of pipe body influences on corrosion rate of inner surface and accelerates corrosion of inner surface. Likewise, we also speculate that corrosion of inner surface affects on degradation rate of pipe body and accelerates degradation of pipe body. Thus, we suppose degradation of pipe body and corrosion of inner surface influence each other with complex interaction as shown in Figure 5.1.

Figure 5.2 shows compound deterioration process of pipeline. Upper and lower shows deterioration process of inner surface and pipe body, respectively. This figure indicates that deterioration process of inner surface is faster than pipe body's process. We empirically know that a pipe degraded pipe body shows fast corrosion rate of inner surface. Likewise, a pipe corroded inner surface may accelerate degradation of pipe body. In this study, we assumed that degradation process of pipe body is relatively slower than corrosion process of inner surface. In addition, the degradation of pipe body accelerates corrosion of inner surface and also the corrosion of inner surface accelerates degradation of pipe body.

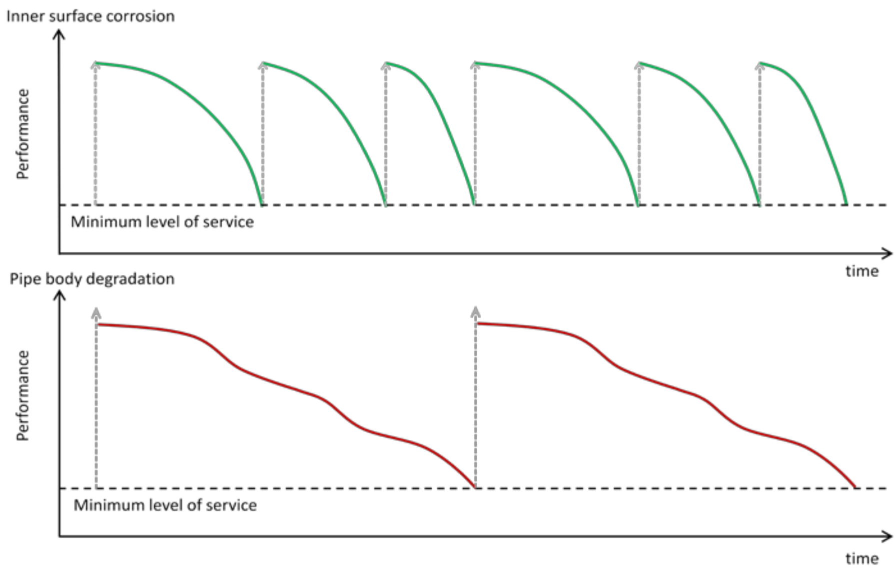

Figure 5.2 Compound deterioration process in the pipeline

5.3. Data implementation

It is an important work for administrator of pipeline system to understand current state of assets and forecasting of deterioration for asset management. Sufficient information about pipe deterioration is required for precise prediction however, it cost heavy charge for a lot of manpower and research cost to check the condition of buried pipeline. The Korea government recognizes the importance of maintenance of aging pipeline and water authorities must carry out technology inspection every five years and then reflect the result on plans for the improvement by 2 of article 55 of Water Supply and Waterworks Installation Act.

Deterioration state of inner surface determining service quality is investigated by endoscope survey and robotic exploration as trenchless method with the development of technology. Deterioration state of pipe body was evaluated by physical examination taking direct sampling or ultrasonic inspection for checking residual thickness. In this study, we handle a problem that data for estimating compound deterioration model is missed systematically. This problem occurs because the investigation of inner surface corrosion and residual thickness are not conducted at the same time. In the practice of the actual pipeline condition survey, inner surface corrosion and the residual thickness inspection are not always carried out at the same time, and there are many case that we can just get either of the two inspection data. Therefore, we need to develop a deterioration prediction model, which considers time discrepancy of inner surface corrosion and the residual thickness inspection, and establish a decision making process from the deterioration prediction results. Although it is not able to check data of corrosion of inner surface and residual thickness at the same time, it is possible to build a model of compound deterioration model with the partial information when we could get either of two inspection data. For example, to forecast compound deterioration process of inner surface, data of residual thickness is required but we can’t get the information of residual thickness at inspection time of inner surface. Meanwhile, if we can get the residual thickness data at the recent past of inspection time of inner surface, we can a get complemented information that “the residual thickness is almost same as recent state or worse”. Similarly, for compound deterioration process of pipe body, we can a get complemented information that “the inner surface corrosion is almost same as recent state or worse” with the inner surface corrosion data at the recent past of inspection time of residual thickness. These complemented information increase the accuracy of estimate of compound markov deterioration model.

Therefore, in this study, we propose a method to estimate compound markov deterioration model using the complemented information.

5.4. Compound markov deterioration model

5.4.1. A prerequisite for modeling

Let us think about the maintenance problem of water pipeline after installation(or renewal) at calendar time a_0 and introduce a discrete time axis which regards a_0 as initial time $t=0$, $t=0,1,\cdots,T$. T is the end point of the observation period. It is assumed that the pipe deterioration process consists of compound deterioration process of corrosion of inner surface and degradation of pipe body. For convenience, it is assumed that repair of pipeline has not been conducted not even once from the initial point. When the repair of pipeline is carried out, the calendar time can be considered as a initial point(renewal time).

As shown in Figure 5.3, the inspection of inner corrosion is carried out at the time point $0, t_1^m, \cdots, t_l^m, \cdots$ on the discrete time axis. A local discrete time axis, $u_l^m = 0,1 \cdots T_l^m$, which regards l th inspection time point t_l^m of inner corrosion as $u_l^m = 0$ is introduced. Here, T_l^m denotes a period from the inspection of inner corrosion at t_l^m to next inspection of inner corrosion, $T_l^m = t_{l+1}^m - t_l^m$. The time point u_l^m on local discrete time axis can be called ' m local time point'. As the same way, the inspection of pipe body is carried out at the time point $0, t_1^f, \cdots, t_k^f, \cdots$ on the discrete time axis. A local discrete time axis, $u_k^f = 0,1, \cdots T_k^f$, which regards k th inspection time point t_k^f of inner corrosion as $u_k^f = 0$ is introduced. Here, T_k^f denotes a period from the inspection of inner corrosion at t_k^f to next inspection of inner corrosion, $T_k^f = t_{k+1}^f - t_k^f$. The time point u_k^f on local discrete time axis can be called ' f local time point'

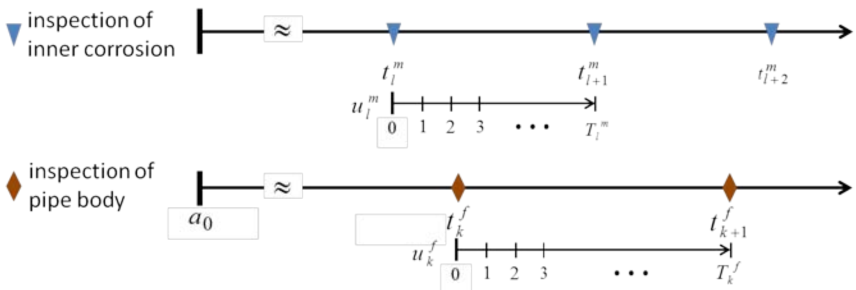

Figure 5.3 Coordinate relations with inspection of inner corrosion and pipe body

The inspection time point of inner surface $t_l^m (l = 0,1,\cdots, N_m)$ and pipe body are not always matched except installation time(or replaced time). Here, relational expressions of the correspondence between ' m local time point' and ' f local time point' are introduced as follows;

$$w^f(u_l^m) = u_k^f \tag{5.1a}$$

$$w^m(u_k^f) = u_l^m \tag{5.1b}$$

The condition state of pipe body at f local time point u_k^f can be expressed by discrete state

variables $g(u_k^f) = s\,(s = 1,\cdots,S; u_k^f = 0,\cdots T_k^f)$. The condition level $s\,(s = 1,\cdots,S)$ means that being high deteriorates the condition state of pipe body. In case of $g(u_k^f) = S$, it means that the condition state of pipe body reaches a service limit status. The condition state of pipe body at initial time $t_0^f = 0$ is $g(0) = 1$. Meanwhile, the condition state of inner surface at m local time point u_l^m can be expressed by discrete state variables $h(u_l^m) = i\,(i = 1,\cdots,I; u_l^m = 0,\cdots T_l^m)$. The condition level $i\,(i = 1,\cdots,I)$ means that being high deteriorates the condition state of inner surface. In case of $h(u_l^m) = I$, it means that the condition state of inner surface reaches a service limit status. The condition state of inner surface at initial time $t_0^m = 0$ is $h(0) = 1$.
Restore of the inner surface may be carried out several times during life time of pipeline. If inner surface is restored it is assumed that the condition state of inner surface is recovered by 1. In this study, the deterioration processes of inner surface and pipe body are expressed by markov chain model and these two models have an interaction each other.

5.4.2. Deterioration process of pipe body

The condition state of pipe body at installation time(or replacement time) $t_0^f\,(u_0^f = 0)$ is observed by $g(0) = 1$. The deterioration progress of pipe body between f local time point u_k^f and $u_k^f + 1$ can be represented by markov transition probability. The interval of unit period $[u_k^f, u_k^f + 1)$ is set by 1. It is impossible to observe the condition state of inner surface i, but it is assumed that we know it. In the local time period $[u_k^f, u_k^f + 1)$ (time period on the local discrete time axis $[t_k^f + u_k^f, t_k^f + u_k^f + 1)$), a markov transition probability which denotes the deterioration process of pipe body can be defined by conditional probability as follows;

$$\text{Prob}[g(u_k^f + 1) = v \mid g(u_k^f) = s,\ h(w^m(u_k^f)) = i] = p^{sv}(i) \tag{5.2}$$

The conditional probability denotes that the condition state of pipe body at f local time point $u_k^f + 1$ is observed by $g(u_k^f + 1) = v$ on condition that condition state of pipe body and inner surface at f local time point u_k^f (time $t_k + u_k$) are observed by $g(u_k^f) = s$ and $h(w^m(u_k^f)) = i$, respectively.

The markov transition probability can be expressed by markov deterioration hazard model proposed by Tsuda [21]. The hazard rate $\lambda^s(i)$ of condition state of pipe body $s\,(s = 1,\cdots,S-1)$ on condition of condition state of inner surface i can be represented as follows;

$$\lambda^s(i) = \beta_0^i x \beta^s = \beta_0^i \lambda^s \tag{5.3}$$

where, $\beta_0^i\ (i = 1,\cdots,I-1)$ is heterogeneity parameters which denotes the heterogeneity of the deterioration rate of pipe body and it depend on condition state of inner surface i. $x = (x_1,\cdots,x_Q)$

and $\beta^s=(\beta_1^s,\cdots,\beta_Q^s)'$ denote Explanatory variables vector and unknown parameter vector respectively. In addition, Q is total number of Explanatory variables and the sign ' denotes transposition. β_0^1 is set 1.

A conditional probability that in condition of i inner surface condition state, the condition state of pipe body s remains between u_k^f and u_k^f+1 can be expressed as follows;

$$p^{ss}(i)=\exp(-\lambda^s(i)) \tag{5.4}$$

Meanwhile, a conditional probability $p^{sv}(i)$ $(s=1,\cdots S-1; v=s+1,\cdots S)$ that in condition of i inner surface condition state, the condition state of pipe body s turn into $v(>s)$ between u_k^f and u_k^f+1 can be expressed as follows;

$$p^{sv}(i)=\sum_{m=s}^{v}\prod_{z=s}^{m-1}\frac{\lambda^z(i)}{\lambda^z(i)-\lambda^m(i)}\prod_{z=m}^{v-1}\frac{\lambda^z(i)}{\lambda^{z+1}(i)-\lambda^m(i)}\exp\{-\lambda^m(i)\} \tag{5.5}$$

where, there are the following conditions;

$$\begin{cases}\prod_{z=s}^{m-1}\frac{\lambda^z(i)}{\lambda^z(i)-\lambda^m(i)}=1 & (m=s)\\ \prod_{z=m}^{v-1}\frac{\lambda^z(i)}{\lambda^{z+1}(i)-\lambda^m(i)}=1 & (m=v)\end{cases}$$

For convenience of representation, equation (5.5) can be simplified as follows;

$$\prod_{z=s}^{m-1}\frac{\lambda^z(i)}{\lambda^z(i)-\lambda^m(i)}\prod_{z=m}^{v-1}\frac{\lambda^z(i)}{\lambda^{z+1}(i)-\lambda^m(i)}\exp\{-\lambda^m(i)\}=\prod_{z=s,\neq m}^{v-1}\frac{\lambda^z(i)}{\lambda^z(i)-\lambda^m(i)}\exp\{-\lambda^m(i)\}$$

In addition, the conditional probability $p^{sS}(i)$ in condition of i inner surface condition state, for any condition state s to go to the absorbing condition state S can be represented as;

$$p^{sS}(i)=1-\sum_{v=s}^{S-1}p^{sv}(i)\ (s=1,\cdots,S-1) \tag{5.6}$$

With the equation (5.2), the markov transition probability at $[u_k^f, u_k^f+1)$ can be defined as;

$$p(i)=\begin{pmatrix}p^{11}(i) & \cdots & p^{1S}(i)\\ \vdots & \ddots & \vdots\\ p^{S1}(i) & \cdots & p^{SS}(i)\end{pmatrix} \tag{5.7}$$

5.4.3. Deterioration process of inner surface of pipeline

The deterioration progress of inner surface between m local time point u_l^m and u_l^m+1 can be represented by markov transition probability. The interval of unit period $[u_l^m, u_l^m+1)$ is set by 1. It is impossible to observe the condition state of pipe body s, but it is assumed that we know it. In the local time period $[u_l^m, u_l^m+1)$ (time period on the local discrete time axis $[t_l^m+u_l^m, t_l^m+u_l^m+1)$), a markov transition probability which denotes the deterioration process of inner surface can be defined by conditional probability as follows;

$$\text{Prob}[h(u_l^m+1)=j \mid h(u_l^m)=i,\ g(w^f(u_l^m))=s]=\pi^{ij}(s) \tag{5.8}$$

The conditional probability denotes that the condition state of inner surface at m local time point u_l^m+1 is observed by $h(u_l^m+1)=j$ on condition that condition state of pipe body and inner surface at m local time point u_l^m (time t_l+u_l) are observed by $g(w^f(u_l^m))=s$ and $h(u_l^m)=i$, respectively.

The hazard rate $\mu^i(s)$ of condition state of inner surface $i\,(i=1,\cdots,I-1)$ on condition of condition state of pipe body s can be represented as follows;

$$\mu^i(s)=\gamma_0^s y\gamma^i=\gamma_0^s\mu^i \tag{5.9}$$

where, γ_0^s $(s=1,\cdots,S-1)$ is heterogeneity parameters which denotes the heterogeneity of the deterioration rate of inner surface and it depend on condition state of pipe body s. $y=(y_1,\cdots,y_V)$ and $\gamma^i=(\gamma_1^i,\cdots,\gamma_V^i)'$ denote Explanatory variables vector and unknown parameter vector respectively. In addition, V is total number of Explanatory variables and the sign ' denotes transposition. γ_0^1 is set 1.

A conditional probability that in condition of s pipe body condition state, the condition state of inner surface i remains between u_l^m and u_l^m+1 can be expressed as follows;

$$\pi^{ii}(s)=\exp(-\mu^i(s)) \tag{5.10}$$

Meanwhile, a conditional probability $\pi^{ij}(s)$ $(i=1,\cdots I-1;\ j=i+1,\cdots I)$ that in condition of s pipe body condition state, the condition state of inner surface i turn into $j(>i)$ between u_l^m and u_l^m+1 can be expressed as follows;

$$\pi^{ij}(s)=\sum_{z=i}^{j}\prod_{r=i,\neq z}^{j-1}\frac{\mu^r(s)}{\mu^r(s)-\mu^z(s)}\exp\{-\mu^z(s)\} \tag{5.11}$$

In addition, the conditional probability $\pi^{iI}(s)$ in condition of s pipe body condition state, for any condition state i to go to the absorbing condition state I can be represented as;

$$\pi^{iI}(s) = 1 - \sum_{v=s}^{I-1} \pi^{ij}(s) \ (i = 1, \cdots I-1) \tag{5.12}$$

With the equation (5.8), the markov transition probability at $[u_l^m, u_l^m + 1)$ can be defined as;

$$\pi(s) = \begin{pmatrix} \pi^{11}(s) & \cdots & \pi^{1S}(s) \\ \vdots & \ddots & \vdots \\ \pi^{S1}(s) & \cdots & \pi^{SS}(s) \end{pmatrix} \tag{5.13}$$

5.4.4. Compound markov deterioration model

Let's suppose the pipeline is replaced at initial time $t = 0$ and then the condition state of pipe body and inner surface are determined by $g(0) = 1$ and $h(0) = 1$, respectively. Thereafter, with the passage of time, corrosion of inner surface and a degradation of pipe body are in progress. The compound deterioration state $x \ (x = 1, \cdots, X)$ at time t is represented by $x(t) = \{\tilde{h}(t), \tilde{g}(t)\}$ using the set of condition state of inner surface $\tilde{h}(t)$ and pipe body $\tilde{g}(t)$. Here, the X is $X = I \times S$. The compound deterioration state $x(t) = x \ (x = 1, \cdots, X)$ correspond to $(1,1), \cdots (1,S), (2,1), \cdots (2,S), (3,1), \cdots (I,S)$ at time t. When the compound deterioration state $x(t) = \{\tilde{h}(t), \tilde{g}(t)\}$ has state variable $x = (i, s)$, The symbols indicating the components $\tilde{h}(t)$ and $\tilde{g}(t)$ of the compound deterioration state $x(t)$ are represented by $\tilde{h}_x(t) = i$ and $\tilde{g}_x(t) = s$, respectively. A frequency distribution of a compound deterioration state is expressed by $\upsilon(t) = \{\upsilon_1(t), \cdots \upsilon_X(t)\}$. Here, the frequency distribution at initial time is $\upsilon(0) = (1, 0, \cdots 0)$. The transition probability matrix $\mathbf{\Omega}$ between the compound deterioration state is defined as follows;

$$\begin{aligned} \mathbf{\Omega} &= \begin{pmatrix} \omega_{11} & \cdots & \omega_{1X} \\ \vdots & \ddots & \vdots \\ \omega_{X1} & \cdots & \omega_{XX} \end{pmatrix} \\ &= \begin{pmatrix} \omega_{11}^{11} & \omega_{11}^{12} & \cdots & \omega_{11}^{jv} & \cdots & \omega_{11}^{IS} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \omega_{is}^{11} & \omega_{is}^{12} & \cdots & \omega_{is}^{jv} & \cdots & \omega_{is}^{IS} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \omega_{IS}^{11} & \omega_{IS}^{12} & \cdots & \omega_{IS}^{jv} & \cdots & \omega_{IS}^{IS} \end{pmatrix} \end{aligned} \tag{5.14}$$

Where, the matrix element $\omega_{is}^{jv} (i, j = 1, \cdots, I; s, v = 1, \cdots, S)$ is defined by;

$$\omega_{is}^{jv} = p^{sv}(i) \pi^{ij}(s) \tag{5.15}.$$

Here, when the condition states show $i > j$ or $s > v$, the element value is $\omega_{is}^{jv} = 0$. If the pipeline is not repaired or replaced, the compound deterioration process can be expressed as follows using markov chain.

$$\upsilon(t) = \upsilon(0)\{\Omega\}^{t} \tag{5.16}$$

5.5. Compound hidden markov deterioration model

5.5.1. Survey strategy

Let's suppose a deterioration state of pipeline is observed by inspection of inner surface corrosion or residual thickness. It is assumed that the inner surface of pipeline is not repaired during target period. If the inner surface is repaired, the condition state of inner surface is regarded as 1 at that time. The inspection of inner surface is carried out at the time point $\bar{t}_1^{m}, \cdots, \bar{t}_{N_m}^{m}$ on discrete time axis, and then the condition state of inner surface $\tilde{h}(\bar{t}_l^{m})$ $(l = 0, \cdots, N_m)$ is observed. Likewise, the inspection of residual thickness is carried out at the time point $\bar{t}_1^{f}, \cdots, \bar{t}_{N_f}^{f}$ on discrete time axis, and then the condition state of pipe body $\tilde{g}(\bar{t}_k^{f})$ $(k = 0, \cdots, N_f)$ is observed. The obtained data from inspection of inner surface and residual thickness is represented by $\tilde{\Xi} = [\{\bar{t}_k^{f}, \tilde{g}(\bar{t}_k^{f})(k = 0, \cdots, N_f)\}, \{\bar{t}_l^{m}, \tilde{h}(\bar{t}_l^{m})\ (l = 0, \cdots, N_m)\}]$. As described before, the local time points m, f which regard the inspection time of inner surface and residual thickness as initial point are defined. The corresponding relation between the local time points m, f is defined in equation (1a,1b). Hereafter, for the convenience of expression, the inspection time of inner surface and residual thickness is rearranged by calendar time and new time point τ_n, $(n = 0, 1, \cdots N)$ is introduced($N = N_m + N_f$). Let's suppose at least one of the inspection of inner surface and residual thickness is carried out at time point τ_n, $(n = 0, 1, \cdots N)$.

The time sets of inspection of inner surface corrosion and residual thickness are represented by ρ^{m} and ρ^{f}, respectively. When only one of the survey results among the inspection of inner surface corrosion and residual thickness can be obtained at time τ_n, the type of inspection $q(\tau_n)$ conducted at τ_n is expressed as follow;

$$q(\tau_n) = \begin{cases} m, \text{ inspection of surface} \\ f, \text{ inspection of FWD} \end{cases} \tag{5.17}$$

Also, the state variables $r(\tau_n)$ obtained at τ_n is expressed as follow;

$$r(\tau_n) = \begin{cases} \tilde{h}(\tau_n), & \tau_n \in \rho^m \\ \tilde{g}(\tau_n), & \tau_n \in \rho^f \end{cases} \tag{5.18}$$

The observed data set which is defined by time $\tau_n, (n = 0,1,\cdots N)$ can be represented by $\Xi = \{\tau_n, q(\tau_n), r(\tau_n)\}$.

5.5.2. Frequency distribution of compound deterioration state

When the compound deterioration process of pipeline proceeds according to equation (5.16), let's derive a observation probability(likelihood) of inspection data. When the initial time is denoted by t_0, the condition state of inner surface and pipe body can be expressed by $\tilde{h}(t_0) = 1$ and $\tilde{g}(t_0) = 1$, respectively and the frequency distribution vector of compound deterioration state is $\upsilon(0) = (1, 0, \cdots 0)$. Let's suppose the time τ_1 belongs to set ρ^m. In other words, the inspection of inner surface corrosion is carried out at time τ_1 ($\tau_1 \in \rho^m$) and then the condition state of inner surface is observed by $\tilde{h}(\tau_1) = i$. The compound deterioration process of pipeline proceeds according to equation (5.16) in time period $[\tau_0, \tau_1)$. The time interval is defined by $\Delta_0 = \tau_1 - \tau_0$. The frequency distribution of compound deterioration state at time τ_1 can be expressed from equation(5.16) as follow;

$$\upsilon(\tau_1) = \upsilon(0)\{\Omega\}^{\Delta_0} \tag{5.19}$$

When the condition state of inner surface is determined by $\tilde{h}(\tau_1) = \bar{i}$, the occurrence frequency $\tilde{\upsilon}_x(\tau_1)$ of compound deterioration state $x(\tau_1) = (\bar{i}, s)$ is defined by;

$$\tilde{\upsilon}_x(\tau_1) = \begin{cases} 0 & i \neq \bar{i} \\ \dfrac{\upsilon_x(\tau_1)}{\sum_{y \in G(\bar{i})} \upsilon_y(\tau_1)} & i = \bar{i} \end{cases} \tag{5.20}$$

where, the set $G(\bar{i})$ is defined by $G(\bar{i}) = \{y \mid y = (\bar{i}, s), (s = 1, \cdots S)\}$.

Let's suppose the time τ_1 belongs to set ρ^f. When the condition state of pipe body is determined by $\tilde{g}(t_1) = \bar{s}$, the occurrence frequency $\tilde{\upsilon}_x(\tau_1)$ of compound deterioration state $x(\tau_1) = (i, \bar{s})$ is defined by;

$$\tilde{\upsilon}_x(\tau_1) = \begin{cases} 0 & s \neq \overline{s} \\ \dfrac{\upsilon_x(\tau_1)}{\sum_{y \in G(\overline{s})} \upsilon_y(\tau_1)} & s = \overline{s} \end{cases} \tag{5.21}$$

where, the set $G(\overline{s})$ is defined by $G(\overline{s}) = \{ y \mid y = (i, \overline{s}), (i = 1, \cdots I) \}$.

The frequency distribution of compound deterioration state at time τ_2 can be expressed from equation(5.16) as follow;

$$\upsilon(\tau_2) = \upsilon(\tau_1) \{\Omega\}^{\Delta_1} \tag{5.22}$$

If we generalize the above argument, when the observed data of time τ_n is obtained, the compound deterioration state $x(\tau_n) = (i, s)$ can be represented as follows;

$(\tau_n \in \rho^m)$

$$\tilde{\upsilon}_x(\tau_n) = \begin{cases} 0 & i \neq \overline{i} \\ \dfrac{\upsilon_x(\tau_n)}{\sum_{y \in G_{\overline{i}}(\tau_n)} \upsilon_y(\tau_n)} & i = \overline{i} \end{cases} \tag{5.23a}$$

$(\tau_n \in \rho^f)$

$$\tilde{\upsilon}_x(\tau_n) = \begin{cases} 0 & s \neq \overline{s} \\ \dfrac{\upsilon_x(\tau_n)}{\sum_{y \in G_{\overline{s}}(\tau_n)} \upsilon_y(\tau_n)} & s = \overline{s} \end{cases} \tag{5.23b}$$

In addition, the frequency distribution of compound deterioration state at time τ_{n+1} can be expressed as follow;

$$\upsilon(\tau_{n+1}) = \upsilon(\tau_n) \{\Omega\}^{\Delta n} \tag{5.24}$$.

5.5.3. Likelihood function

Let's suppose that the observed data $\boldsymbol{\Xi}$ can be obtained through the entire inspection period. The data obtained from initial time to time $\tau_n (0 < n \leq N)$ is expressed by $\xi_n = \{\tau_a, q(\tau_a), r(\tau_a), (a = 0, \cdots, n)\}$. Here, the dummy variable representing the observed data at time τ_n is defined by as follow;

$$
\delta(\tau_n) = \begin{cases} 1 & r(\tau_n) = \overline{i} \ (\tau_n \in \rho^m) \\ 1 & r(\tau_n) = \overline{s} \ (\tau_n \in \rho^f) \\ 0 & \text{otherwise} \end{cases} \tag{5.25}
$$

The probability $\ell_1(\xi_1)$ which obtained data ξ_1 is observed until time can be expressed by as follow;

$$
\ell_1(\xi_1) = \sum_{x=1}^{X} \{\upsilon_x(\tau_1)\}^{\delta(\tau_1)} \tag{5.26}
$$

The observation probability after τ_2 can be formulated recursively as follows;

$$
\ell_2(\xi_2) = \ell_1(\xi_1) \sum_{x=1}^{X} \{\upsilon_x(\tau_2)\}^{\delta(\tau_2)} \tag{5.27a}
$$

$$
\vdots
$$

$$
\ell_N(\xi_N) = \ell_{N-1}(\xi_{N-1}) \sum_{x=1}^{X} \{\upsilon_x(\tau_N)\}^{\delta(\tau_N)} \tag{5.27b}
$$

The likelihood representing the observation probability of the obtained data set $\boldsymbol{\Xi}$ is defined as follows;

$$
\mathcal{L}(\boldsymbol{\Xi} : \boldsymbol{\theta}) = \prod_{n=1}^{N} \sum_{x=1}^{X} \{\upsilon_x(\tau_n : \ )\}^{\delta(\tau_n)} \tag{5.28a}
$$

$$
\upsilon(\tau_{n+1}) = \tilde{\upsilon}(\tau_n) \{\Omega\}^{\Delta n} \tag{5.28b}
$$

where, $\boldsymbol{\theta\beta\gamma}\{\beta_0^s, \ {}^s, \gamma_0^i, \ {}^i : s = 1, \cdots S-1, i = 1, \cdots, I-1\}$ is an unknown parameter vector.

The likelihood function(5.28a) of compound hidden markov deterioration model is high dimensional nonlinear multinomial expression of parameter $\boldsymbol{\theta}$ and the optimization problem has a large number of solutions including complex valued solution. The transition probability ω_{is}^{jv} must have real solutions of between 0 and 1. Using Bayesian estimation method instead of maximum likelihood estimation method can solve the high dimensional nonlinear multinomial expression. But, because the likelihood functions (5.28a,b) have so many terms, it causes the problem of a massive amount of calculation. Therefore, to overcome this problem, we need to establish completion of likelihood function.

5.5.4. The completion of likelihood function

For the completion of likelihood function, the hidden variables are defined. Let's suppose that the observed data $\Xi = \{\bar{\tau}_n, \bar{q}(\bar{\tau}_n), \bar{r}(\bar{\tau}_n), (n = 0, 1, \cdots N)\}$ can be obtained through the entire inspection period. The symbol '-' means an actual observed value. The time interval $[\bar{\tau}_n, \bar{\tau}_{n+1})$ can be explained using m and f local time axis. Let's suppose the inspection of inner surface corrosion is carried out at time $\bar{\tau}_n$, and the time $\bar{\tau}_n - 1$ is expressed by time point $u_l^m - 1$ and $u_k^f - 1$ on the m and f local time axis, respectively. Then, the m and f local time points in time period $[\bar{\tau}_n, \bar{\tau}_{n+1})$ can be represented as follows, respectively;

$$0, 1, \cdots, \cdots, \Delta_n \tag{5.29a}$$

$$u_k^f, u_k^f + 1, \cdots, u_k^f + \Delta_n \tag{5.29b}$$

Meanwhile, let's suppose the inspection of residual thickness is carried out at time $\bar{\tau}_n$. Then, the m and f local time points in time period $[\bar{\tau}_n, \bar{\tau}_{n+1})$ can be represented as follows, respectively;

$$u_l^m, u_l^m + 1, \cdots, u_l^m + \Delta_n \tag{5.30a}$$

$$0, 1, \cdots, \cdots, \Delta_n \tag{5.30b}$$

Here, the transition pattern of condition state of inner surface corrosion in time period $[\bar{\tau}_n, \bar{\tau}_{n+1})$ is expressed using the hidden variable vector.

$$w_n = \begin{cases} (w_0, \cdots, w_{\Delta_n}) & \bar{q}(\bar{\tau}_n) = m \\ (w_{u_l^m}, \cdots, w_{u_l^m + \Delta_n}) & \bar{q}(\bar{\tau}_n) = f \end{cases} \tag{5.31}$$

In addition, the transition pattern of condition state of pipe body in time period $[\bar{\tau}_n, \bar{\tau}_{n+1})$ is expressed using the hidden variable vector.

$$d_n = \begin{cases} (d_{u_k^f}, \cdots, d_{u_k^f + \Delta_n}) & \bar{q}(\bar{\tau}_n) = m \\ (d_0, \cdots, d_{\Delta_n}) & \bar{q}(\bar{\tau}_n) = f \end{cases} \tag{5.32}$$

Where, the hidden variable w_0 is $w_0 = \tilde{h}(\bar{\tau}_n)$ when the type of inspection is $\bar{q}(\bar{\tau}_n) = m$, and also the hidden variable d_0 is $d_0 = \tilde{g}(\bar{\tau}_n)$ when the type of inspection is $\bar{q}(\bar{\tau}_n) = f$.

From the characteristics of deterioration process, if the pipeline is not restored the state of inner surface and pipe body will get worse continuously over time. Thus, the hidden variables have to meet the following conditions;

$$\bar{w}_0 \le \cdots \le w_{u_l^m} \le \cdots \le w_{T_l^m - 1} \le \bar{w}_{T_l^m} \tag{5.33a}$$

$$\bar{d}_0 \le \cdots \le d_{u_k^f} \le \cdots \le d_{T_k^f - 1} \le \bar{d}_{T_k^f} \tag{5.33b}$$

where, $\bar{w}_0 = \tilde{h}(t_l^m)$, $\bar{w}_{T_l^m} = \tilde{h}(T_l^m)$, $\bar{d}_0 = \tilde{g}(t_k^f)$, $\bar{d}_{T_k^f} = \tilde{g}(T_k^f)$.

The condition state vectors of inner surface w_n and pipe body d_n are hidden variables which can't be observed except the $\bar{w}_0$, $\bar{w}_{T_l^m}$, $\bar{d}_0$, and $\bar{d}_{T_k^f}$ but it is assumed that the variables can be observed. The column of hidden variable vector are expressed over the entire observation period as figure 3, and the column of hidden variable vector are obtained by rearranging from time $\tau_0 = 0$ to τ_{N-1}.

$$\tilde{\tilde{w}}_0, \cdots, \tilde{\tilde{w}}_n, \cdots, \tilde{\tilde{w}}_{N-1} \tag{5.34a}$$

$$\tilde{\tilde{d}}_0, \cdots, \tilde{\tilde{d}}_n, \cdots, \tilde{\tilde{d}}_{N-1} \tag{5.34b}$$

where, the hidden variable vector are represented as $\tilde{\tilde{\boldsymbol{w}}} = \{\tilde{\tilde{\boldsymbol{w}}}_n, (n = 0, \cdots, N-1)\}$, $\tilde{\tilde{\boldsymbol{d}}} = \{\tilde{\tilde{\boldsymbol{d}}}_n, (n = 0, \cdots, N-1)\}$.

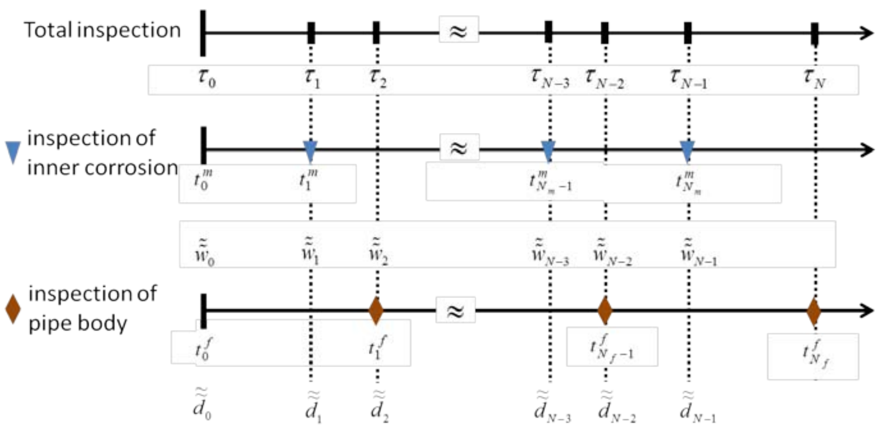

Figure 5.4 Example of hidden variables

The completion of likelihood explained by hidden variable vector $\tilde{\tilde{\boldsymbol{d}}}_1, \tilde{\tilde{\boldsymbol{w}}}_1$ at time τ_1 can be expressed by;

$$\begin{aligned}\tilde{\ell}_1(\tilde{\tilde{\boldsymbol{d}}}_1, \tilde{\tilde{\boldsymbol{w}}}_1, \overline{\xi}_1) &= \tilde{\tilde{\upsilon}}_{\tilde{\tilde{w}}_{\tau_1}, \tilde{\tilde{d}}_{\tau_1}}(\tau_1) \\ &= \prod_{y_0=0}^{T_0-1} \omega_{\tilde{\tilde{w}}_{y_0} \tilde{\tilde{d}}_{y_0}}^{\tilde{\tilde{w}}_{y_0+1} \tilde{\tilde{d}}_{y_0+1}}\end{aligned} \tag{5.35}$$

The completion of likelihood after time τ_2 can be formulated recursively,

$$\tilde{\ell}_2(\tilde{\tilde{\boldsymbol{d}}}_2, \tilde{\tilde{\boldsymbol{w}}}_2, \overline{\xi}_2) = \tilde{\ell}_1(\tilde{\tilde{\boldsymbol{d}}}_1, \tilde{\tilde{\boldsymbol{w}}}_1, \overline{\xi}_1)\, \tilde{\tilde{\upsilon}}_{\tilde{\tilde{w}}_{\tau_2}, \tilde{\tilde{d}}_{\tau_2}}(\tau_2) \tag{5.36a}$$

$$\vdots$$

$$\tilde{\ell}_N(\tilde{\tilde{\boldsymbol{d}}}_N, \tilde{\tilde{\boldsymbol{w}}}_N, \overline{\xi}_N) = \tilde{\ell}_{N-1}(\tilde{\tilde{\boldsymbol{d}}}_{N-1}, \tilde{\tilde{\boldsymbol{w}}}_{N-1}, \overline{\xi}_{N-1})\, \tilde{\tilde{\upsilon}}_{\tilde{\tilde{w}}_{\tau_N}, \tilde{\tilde{d}}_{\tau_N}}(\tau_N) \tag{5.36b}$$

where, the frequency distribution of compound deterioration state can be expressed as follow;

$$\tilde{\tilde{\upsilon}}_{\tilde{\tilde{w}}_{\tau_{n+1}}, \tilde{\tilde{d}}_{\tau_{n+1}}}(\tau_{n+1}) = \prod_{y_n=0}^{T_n-1} \omega_{\tilde{\tilde{w}}_{y_n} \tilde{\tilde{d}}_{y_n}}^{\tilde{\tilde{w}}_{y_n+1} \tilde{\tilde{d}}_{y_n+1}} \tag{5.37}$$

The likelihood representing the observation probability of the obtained data set $\overline{\boldsymbol{\Xi}}$ is defined as follow;

$$\tilde{\mathcal{L}}(\tilde{\tilde{\boldsymbol{d}}}, \tilde{\tilde{\boldsymbol{w}}}, \overline{\boldsymbol{\Xi}} : \boldsymbol{\theta}) = \prod_{n=0}^{N-1} \prod_{y_n=0}^{T_n-1} \omega_{\tilde{\tilde{w}}_{y_n} \tilde{\tilde{d}}_{y_n}}^{\tilde{\tilde{w}}_{y_n+1} \tilde{\tilde{d}}_{y_n+1}} \tag{5.38}$$

The above process is referred to completion of the likelihood function. The complete likelihood function(5.38) is greatly simplified than normal likelihood function (5.28a, b). Here, the hidden variables $\tilde{\tilde{\boldsymbol{d}}}, \tilde{\tilde{\boldsymbol{w}}}$ in the complete likelihood function are unmeasurable variables. If we deploy the complete likelihood function, we can derive the full conditional posterior distribution about hidden variables $\tilde{\tilde{\boldsymbol{d}}}, \tilde{\tilde{\boldsymbol{w}}}$.

5.5.5. The probability distribution of hidden variables

From the characteristics of deterioration process of inner surface, if the pipeline is not restored, the condition (5.33a) is satisfied. If the $\tilde{\tilde{\boldsymbol{w}}}_{-v} = (\tilde{\tilde{w}}_0, \cdots, \tilde{\tilde{w}}_{v-1}, \tilde{\tilde{w}}_{v+1}, \cdots, \tilde{\tilde{w}}_{T_l^m})$ and $\tilde{\tilde{\boldsymbol{w}}}_{-v}^{w} = (\tilde{\tilde{w}}_0, \cdots, \tilde{\tilde{w}}_{v-1}, w, \tilde{\tilde{w}}_{v+1}, \cdots, \tilde{\tilde{w}}_{T_l^m})$ are defined by hidden variables, the conditional probability of $\tilde{\tilde{w}}_v = w(w \in \{\tilde{\tilde{w}}_{v-1}, \cdots, \tilde{\tilde{w}}_{v+1}\})$ can be expressed by as follow;

$$
\begin{aligned}
&\mathrm{Prob}\{\tilde{\tilde{w}}_v = w \mid \tilde{\tilde{\boldsymbol{w}}}_{-v}, \tilde{\tilde{\boldsymbol{d}}}\} \\
&= \frac{\tilde{\mathcal{L}}(\tilde{\tilde{\boldsymbol{w}}}_{-v}^{w}, \tilde{\tilde{\boldsymbol{d}}}, \bar{\Xi}, \boldsymbol{\theta})}{\sum_{w=\tilde{\tilde{w}}_{v-1}}^{\tilde{\tilde{w}}_{v+1}} \tilde{\mathcal{L}}(\tilde{\tilde{\boldsymbol{w}}}_{-v}^{w}, \tilde{\tilde{\boldsymbol{d}}}, \bar{\Xi}, \boldsymbol{\theta})} \\
&= \frac{\chi_w(\tilde{\tilde{w}}_{v-1}, \tilde{\tilde{w}}_{v+1}, \tilde{\tilde{\boldsymbol{d}}})}{\sum_{w=\tilde{\tilde{w}}_{v-1}}^{\tilde{\tilde{w}}_{v+1}} \chi_w(\tilde{\tilde{w}}_{v-1}, \tilde{\tilde{w}}_{v+1}, \tilde{\tilde{\boldsymbol{d}}})}
\end{aligned}
\tag{5.39}
$$

where,

$$
\chi_w(\tilde{\tilde{w}}_{v-1}, \tilde{\tilde{w}}_{v+1}, \tilde{\tilde{\boldsymbol{d}}}) = \omega_{\tilde{\tilde{w}}_{v-1}\tilde{\tilde{d}}_{v-1}}^{w\,\tilde{\tilde{d}}_v} \, \omega_{w\tilde{\tilde{d}}_v}^{\tilde{\tilde{w}}_{v+1}\,\tilde{\tilde{d}}_{v+1}} \tag{5.40}
$$

As the same way, the conditional probability of hidden variables about condition state of pipe body can be obtained. From the characteristics of deterioration process of pipe body, if the pipeline is not restored, the condition (5.33b) is satisfied. If the $\tilde{\tilde{\boldsymbol{d}}}_{-v} = (\tilde{\tilde{d}}_0, \cdots, \tilde{\tilde{d}}_{v-1}, \tilde{\tilde{d}}_{v+1}, \cdots, \tilde{\tilde{d}}_{T_k^f})$ and $\tilde{\tilde{\boldsymbol{d}}}_{-v}^{d} = (\tilde{\tilde{d}}_0, \cdots, \tilde{\tilde{d}}_{v-1}, d, \tilde{\tilde{d}}_{v+1}, \cdots, \tilde{\tilde{d}}_{T_k^f})$ are defined by hidden variables, the conditional probability of $\tilde{\tilde{d}}_v = d(d \in \{\tilde{\tilde{d}}_{v-1}, \cdots, \tilde{\tilde{d}}_{v+1}\})$ can be expressed by as follow;

$$
\begin{aligned}
&\mathrm{Prob}\{\tilde{\tilde{d}}_v = d \mid \tilde{\tilde{\boldsymbol{w}}}, \tilde{\tilde{\boldsymbol{d}}}_{-v}\} \\
&= \frac{\chi_d(\tilde{\tilde{d}}_{v-1}, \tilde{\tilde{d}}_{v+1}, \tilde{\tilde{\boldsymbol{w}}})}{\sum_{d=\tilde{\tilde{d}}_{v-1}}^{\tilde{\tilde{d}}_{v+1}} \chi_d(\tilde{\tilde{d}}_{v-1}, \tilde{\tilde{d}}_{v+1}, \tilde{\tilde{\boldsymbol{w}}})}
\end{aligned}
\tag{5.41}
$$

where,

$$
\chi_d(\tilde{\tilde{d}}_{v-1}, \tilde{\tilde{d}}_{v+1}, \tilde{\tilde{\boldsymbol{w}}}) = \omega_{\tilde{\tilde{w}}_{v-1}\tilde{\tilde{d}}_{v-1}}^{\tilde{\tilde{w}}_v\,d} \, \omega_{\tilde{\tilde{w}}_v\,d}^{\tilde{\tilde{w}}_{v+1}\,\tilde{\tilde{d}}_{v+1}} \tag{5.42}
$$

5.6. Model estimation

5.6.1. MCMC method

To estimate the compound deterioration hazard model, it is difficult to apply normal maximum likelihood method or Bayesian estimation method since the compound hidden markov deterioration model which is having the form of mixture distribution model is high dimensional nonlinear multinomial expression. Kobayashi et al proposed a method to estimate the mixture distribution model in compound hidden markov model[19] and Hierarchical hidden markov deterioration model[18] using Bayesian MCMC method and complete likelihood function.

In this study, the M-H method is used to perform sampling unknown parameter β, γ from an empirical distribution that is similar to posterior distribution and accordingly obtains samples from the original distribution [22]. Furthermore, a random walk is used to improve the efficiency of sampling. The M-H method is described below. The parameter $\hat{\beta}^s = (\beta_0^s, \boldsymbol{\beta}^s) = (\beta_0^s, \beta_1^s, \cdots, \beta_Q^s)$ $(s = 1, \cdots, S-1)$ contained in hazard rate(5.3) of pipe body condition is unknown parameter. And it is assumed that the prior probability density function of this unknown parameter follows the normal distribution, $\hat{\beta}^s \approx \mathcal{N}_{Q+1}(\zeta^{s,\hat{\beta}}, \Sigma^{s,\beta})$ $\hat{\beta}^s \approx \mathcal{N}_{Q+1}(\zeta^{s,\hat{\beta}}, \Sigma^{s,\beta})$. Here, the probability density function of $Q+1$ dimensional normal distribution $\mathcal{N}_{Q+1}(\zeta^{s,\hat{\beta}}, \Sigma^{s,\beta})$ is ;

$$\phi(\hat{\beta}^s \mid \boldsymbol{\zeta}^{s,\hat{\beta}}, \boldsymbol{\Sigma}^{s,\beta}) = \frac{1}{(2\pi)^{\frac{Q+1}{2}} \sqrt{|\boldsymbol{\Sigma}^{s,\hat{\beta}}|}} \text{ex} \left\{ -\frac{1}{2} (\beta^s \text{p} - \boldsymbol{\zeta}^{s,\beta}) (\boldsymbol{\Sigma}^{s,\beta})^{-1} (\beta^s - \boldsymbol{\zeta}^{s,\beta})' \right\} \tag{5.43}$$

where, the $\boldsymbol{\zeta}^{s,\hat{\beta}}$ and $\boldsymbol{\Sigma}^{s,\hat{\beta}}$ denote the prior expectations vector and prior variance-covariance matrix of $\mathcal{N}_{Q+1}(\zeta^{s,\hat{\beta}}, \Sigma^{s,\beta})$, respectively.

As the same way, it is assumed that the prior probability density function of the unknown parameter, $\hat{\boldsymbol{\gamma}}^i = (\gamma_0^i, \gamma_1^i, \cdots, \gamma_V^i)$ $(i = 1, \cdots, I-1)$ contained in hazard rate(5.9) of inner surface condition, follows the normal distribution, $\hat{\boldsymbol{\gamma}}^i \approx \mathcal{N}_{V+1}(\boldsymbol{\zeta}^{i,\hat{\gamma}}, \boldsymbol{\Sigma}^{i,\gamma})$. Here, the $\boldsymbol{\zeta}^{i,\hat{\gamma}}$ and $\boldsymbol{\Sigma}^{i,\hat{\gamma}}$ denote the prior expectations vector and prior variance-covariance matrix of $\mathcal{N}_{V+1}(\boldsymbol{\zeta}^{i,\hat{\gamma}}, \boldsymbol{\Sigma}^{i,\gamma})$, respectively.

The parameter vector $\hat{\boldsymbol{\beta}}$ is sampled using random walk MH method. The stride of the random walk is assumed to follow a normal distribution with a mean of 0 and a variance of σ_i^2.

$$\beta_g^{(m)} - \beta_g^{(m-1)} \approx N(0, \sigma_g) \tag{5.44a}$$

$$\gamma_y^{(m)} - \gamma_y^{(m-1)} \approx N(0, \sigma_y) \tag{5.44b}$$

where, m is the number of sampling.

The sampling process of parameters $\hat{\boldsymbol{\beta}}$, $\hat{\boldsymbol{\gamma}}$ with random walk MH method is described below.

step1. Set the initial value

The variance parameters σ_g, σ_y of empirical distribution(5.44a,b) are set randomly. The initial value of hidden variable $\tilde{\tilde{\boldsymbol{w}}}^{(0)} = (\tilde{\tilde{w}}_0^{(0)}, \cdots, \tilde{\tilde{w}}_{u_l^m}^{(0)}, \cdots, \tilde{\tilde{w}}_{T_l^m}^{(0)})$ and $\tilde{\tilde{\boldsymbol{w}}}^{(0)} = (\tilde{\tilde{w}}_0^{(0)}, \cdots, \tilde{\tilde{w}}_{u_l^m}^{(0)}, \cdots, \tilde{\tilde{w}}_{T_l^m}^{(0)})$ are set with the constraints(5.33a,b). In addition, the estimation value of unknown parameter $\hat{\beta}^{(0)}$, $\hat{\boldsymbol{\gamma}}^{(0)}$ are set randomly. The influence of the initial value is gradually gone away according to accumulation of simulation number of MCMC methods. The sampling number m is 1.

step 2. Sampling parameter $\hat{\beta}^{(m)}$

The parameter $\hat{\boldsymbol{\beta}}^{s,(m)} = (\beta_0^{s,(m)}, \cdots, \beta_Q^{s,(m)})$ $(s = 1, \cdots, S-1)$ defined about degradation of pipe body is sampled using random walk MH method.

step2-1

With the hidden variable vectors $\tilde{\tilde{\boldsymbol{w}}}^{(m-1)}$ and $\tilde{\tilde{\boldsymbol{d}}}^{(m-1)}$, parameter vectors $\hat{\boldsymbol{\beta}}^{(m-1)}$, $\hat{\boldsymbol{\gamma}}^{(m-1)}$

step 2-2

A parameter vector of the sampling number m and sub-step g is defined as;

$$\hat{\boldsymbol{\beta}}_{g-1}^{s,(m)} = (\beta_0^{s,(m)}, \cdots, \beta_{g-1}^{s,(m)}, \beta_g^{s,(m-1)}, \cdots, \beta_Q^{s,(m-1)})' \tag{5.45}$$

The random walk vector $\boldsymbol{\iota}_g^{s,(m)} = (0, \cdots, 0, \iota_g^{s,(m)}, 0, \cdots, 0)'$ of sub-step g is defined. The stride of the random walk is assumed to follow a normal distribution with a mean of 0 and a variance of $(\sigma)^2$.

$$\boldsymbol{\iota}_g^{s,(m)} \approx N(0,(\sigma_g)^2)$$

The parameter vector $\hat{\beta}_g^{s,(m)}$ is defined by;

$$\hat{\hat{\boldsymbol{\beta}}}_g^{s,(m)} = \boldsymbol{\beta}_{g-1}^{s,(m)} + \boldsymbol{\iota}_g^{s,(m)} \tag{5.46}$$

The parameter vector $\hat{\boldsymbol{\beta}}_{(s,g)}^{(m)}$ is expressed by;

$$\hat{\boldsymbol{\beta}}_{(s,g)}^{\widehat{(m)}} = (\boldsymbol{\beta}^{1,(m)},\cdots,\boldsymbol{\beta}_g^{s,(m)},\boldsymbol{\beta}^{s+1,(m-1)},\cdots,\boldsymbol{\beta}^{S-1,(m-1)}) \tag{5.47}$$

The acceptance probability $\Upsilon_{(s,g)}^{(m)}$ is calculated as follows:

$$\Upsilon_{(s,g)}^{(m)} = \min\left[\frac{\tilde{\mathcal{L}}(\hat{\boldsymbol{\beta}}_{(s,g)}^{(m)},\, \bar{\Xi})}{\tilde{\mathcal{L}}(\hat{\boldsymbol{\beta}}_{(s,g-1)}^{(m)},\, \bar{\Xi})}, 1\right] \tag{5.48}$$

where, $\tilde{\mathcal{L}}(\hat{\boldsymbol{\beta}}_{(s,g)}^{(m)},\, \bar{\Xi})$ is complete likelihood function expressed in equation(5.38).

step. 2-3

The uniform distribution $u \sim \boldsymbol{\mathcal{U}}(0,1)$ is generated, and then the sample $\boldsymbol{\beta}^{m,g}$ is determined by applying the following condition:

$$\boldsymbol{\beta}_g^{s,(m)} = \begin{cases} \hat{\boldsymbol{\beta}}_{g-1}^{s,(m)} + \boldsymbol{\iota}_g^{s,(m)} & u \le \Upsilon_{(s,g)}^{(m)} \\ \hat{\boldsymbol{\beta}}_{g-1}^{s,(m)} & \mathit{otherwise}, \end{cases} \tag{5.49}$$

The above procedure is carried out from $g = 0$ to $g = Q$.

step 3. Sampling parameter $\boldsymbol{\gamma}^{(m)}$

The parameter $\boldsymbol{\gamma}^{(m-1)} = (\gamma_1^{(m-1)},\cdots,\gamma_G^{(m-1)})$ defined about corrosion of inner surface is sampled using random walk MH method.

step. 3-1

With the hidden variable vectors $\tilde{\tilde{\boldsymbol{w}}}^{(m-1)}$ and $\tilde{\tilde{\boldsymbol{d}}}^{(m-1)}$, parameter vectors $\hat{\boldsymbol{\beta}}^{(m)}$, $\hat{\boldsymbol{\gamma}}^{(m-1)}$

step 3-2

A parameter vector of the sampling number m and sub-step y is defined as;

$$\boldsymbol{\gamma}_{y-1}^{i,(m)} = (\boldsymbol{\gamma}_{1}^{i,(m)}, \cdots, \boldsymbol{\gamma}_{y-1}^{i,(m)}, \boldsymbol{\gamma}_{y}^{i,(m)}, \cdots, \boldsymbol{\gamma}_{V}^{i,(m)})' \tag{5.50}$$

The random walk vector $\boldsymbol{\iota}_{y}^{i,(m)} = (0, \cdots, 0, \iota_{y}^{i,(m)}, 0, \cdots, 0)'$ of sub-step g is defined. The stride of the random walk is assumed to follow a normal distribution with a mean of 0 and a variance of $(\sigma)^2$.

$$\boldsymbol{\iota}_{y}^{i,(m)} \approx N(0, (\sigma_y)^2)$$

The parameter vector $\boldsymbol{\gamma}_{y}^{i,(m)}$ is defined by;

$$\boldsymbol{\gamma}_{y}^{i,(m)} = \boldsymbol{\gamma}_{y-1}^{i,(m)} + \boldsymbol{\iota}_{y}^{i,(m)} \tag{5.51}$$

The parameter vector $\boldsymbol{\gamma}_{(i,y)}^{(m)}$ is expressed by;

$$\boldsymbol{\gamma}_{(i,y)}^{(m)} = (\boldsymbol{\gamma}^{i,(m)}, \cdots, \boldsymbol{\gamma}^{i,(m)}, \boldsymbol{\gamma}^{i+1,(m-1)}, \cdots, \boldsymbol{\gamma}^{I-1,(m-1)}) \tag{5.52}$$

The acceptance probability $\Upsilon_{(s,g)}^{(m)}$ is calculated as follows:

$$\Upsilon_{i,y}^{(m)} = \min\left[\frac{\tilde{\boldsymbol{\mathcal{L}}}(\boldsymbol{\gamma}_{y}^{i,(m)}, \bar{\Xi})}{\tilde{\boldsymbol{\mathcal{L}}}(\boldsymbol{\gamma}_{y-1}^{i,(m)}, \bar{\Xi})}, 1\right] \tag{5.53}$$

step 3-3

The uniform distribution $u \sim \boldsymbol{\mathcal{U}}(0,1)$ is generated, and then the sample $\boldsymbol{\gamma}_y^{i,(m)}$ is determined by applying the following condition:

$$\boldsymbol{\gamma}_y^{i,(m)} = \begin{cases} \boldsymbol{\gamma}_{y-1}^{i,(m)} + \boldsymbol{\iota}_y^{i,(m)} & u \leq \Upsilon_{(i,y)}^{(m)} \\ \boldsymbol{\gamma}_{y-1}^{i,(m)} & \mathit{otherwise}, \end{cases} \tag{5.54}$$

The above procedure is carried out from $y = 0$ to $y = V$.

step 4 Sampling hidden variable $\tilde{\boldsymbol{w}}^{(m)}$

New hidden variable $\tilde{\tilde{\boldsymbol{w}}}^{(m)}$ is sampled with given condition, hidden variable $\tilde{\tilde{\boldsymbol{w}}}^{(m-1)}, \tilde{\tilde{\boldsymbol{d}}}^{(m-1)}$ and parameter $\hat{\boldsymbol{\beta}}^{(m)}, \hat{\boldsymbol{\gamma}}^{(m)}$. With the updated parameter estimates $\hat{\boldsymbol{\beta}}^{(m)}, \hat{\boldsymbol{\gamma}}^{(m)}$, the markov transition probability $\omega_{is}^{jv}(m)$ of compound deterioration process is defined using equation (5.15).

The new hidden variable $\tilde{\tilde{\boldsymbol{w}}}^{(m)}$ is randomly sampled base on full conditional posterior probability (5.39). A hidden variable vector $\tilde{\tilde{\boldsymbol{w}}}_{-v}^{(m)} = (\tilde{\tilde{w}}_1^{(m)}, \cdots, \tilde{\tilde{w}}_{v-1}^{(m)}, \tilde{\tilde{w}}_{v+1}^{(m-1)}, \cdots, \tilde{\tilde{w}}_{T_l^m}^{(m-1)})$ is defined in a certain period of time $[t_l^m, t_{l+1}^m)$ $(l = 0, \cdots, N^m - 1)$.

The full conditional posterior probability of $\tilde{\tilde{w}}_v^{(m)} = w \in \{\tilde{\tilde{w}}_{v-1}^{(m)}, \cdots, \tilde{\tilde{w}}_{v+1}^{(m-1)}\}$ is expressed by;

$$\begin{aligned} &\mathrm{Prob}\{\tilde{\tilde{w}}_v = w \mid \tilde{\tilde{\boldsymbol{w}}}_{-v}^{(m)}, \tilde{\tilde{\boldsymbol{d}}}^{(m-1)}\} \\ &= \frac{\chi_w(\tilde{\tilde{w}}_{v-1}^{(m)}, \tilde{\tilde{w}}_{v+1}^{(m-1)}, \tilde{\tilde{\boldsymbol{d}}}^{(m-1)})}{\sum_{w=\tilde{\tilde{w}}_{v-1}^{(m)}}^{\tilde{\tilde{w}}_{v+1}^{(m-1)}} \chi_w(\tilde{\tilde{w}}_{v-1}^{(m)}, \tilde{\tilde{w}}_{v+1}^{(m-1)}, \tilde{\tilde{\boldsymbol{d}}}^{(m-1)})} \end{aligned} \tag{5.55}$$

where,

$$\chi_w(\tilde{\tilde{w}}_{v-1}^{(m)}, \tilde{\tilde{w}}_{v+1}^{(m-1)}, \tilde{\tilde{\boldsymbol{d}}}^{(m-1)}) = \omega_{\tilde{\tilde{w}}_{v-1}^{(m)} \tilde{\tilde{d}}_{v-1}^{(m-1)}}^{w \tilde{\tilde{d}}_v^{(m-1)}}(m)\, \omega_{w \tilde{\tilde{d}}_v^{(m-1)}}^{\tilde{\tilde{w}}_{v+1}^{(m-1)} \tilde{\tilde{d}}_{v+1}^{(m-1)}}(m) \tag{5.56}$$

For all the $l(l = 0, \cdots, N^m - 1)$, a successive hidden variable $\tilde{\tilde{w}}_v^{(m)}$ $(v = 0, \cdots, T_l^m)$ is obtained from $v = 0$.

step 5 Sampling hidden variable $\tilde{\tilde{\boldsymbol{d}}}^{(m)}$

New hidden variable $\tilde{\tilde{\boldsymbol{d}}}^{(m)}$ is sampled with given condition, hidden variable $\tilde{\tilde{\boldsymbol{w}}}^{(m-1)}, \tilde{\tilde{\boldsymbol{d}}}^{(m-1)}$ and parameter $\hat{\boldsymbol{\beta}}^{(m)}, \hat{\boldsymbol{\gamma}}^{(m)}$. With the updated parameter estimates $\hat{\boldsymbol{\beta}}^{(m)}, \hat{\boldsymbol{\gamma}}^{(m)}$, the markov transition probability $\omega_{is}^{jv}(m)$ of compound deterioration process is defined using equation (5.15).

The new hidden variable $\tilde{\boldsymbol{d}}^{(m)}$ is randomly sampled base on full conditional posterior probability (5.39). A hidden variable vector $\tilde{\tilde{\boldsymbol{d}}}_{-v}^{(m)} = (\tilde{\tilde{d}}_1^{(m)}, \cdots, \tilde{\tilde{d}}_{v-1}^{(m)}, \tilde{\tilde{d}}_{v+1}^{(m-1)}, \cdots, \tilde{\tilde{d}}_{T_k^f}^{(m-1)})$ is defined in a certain period of time $[t_k^f, t_{k+1}^f)$ $(l = 0, \cdots, N^f - 1)$.

The full conditional posterior probability of $\tilde{\tilde{d}}_v^{(m)} = d \in \{\tilde{\tilde{d}}_{v-1}^{(m)}, \cdots, \tilde{\tilde{d}}_{v+1}^{(m-1)}\}$ is expressed by;

$$
\begin{aligned}
&\mathrm{Prob}\{\tilde{\tilde{d}}_v = d \mid \tilde{\tilde{\boldsymbol{w}}}^{(m)}, \tilde{\tilde{\boldsymbol{d}}}_{-v}^{(m)}\} \\
&= \frac{\chi_d(\tilde{\tilde{\boldsymbol{w}}}^{(m)}, \tilde{\tilde{d}}_{v-1}^{(m)}, \tilde{\tilde{d}}_{v+1}^{(m-1)})}{\sum_{\tilde{\tilde{d}}_{v-1}^{(m)}}^{\tilde{\tilde{d}}_{v+1}^{(m-1)}} \chi_d(\tilde{\tilde{\boldsymbol{w}}}^{(m)}, \tilde{\tilde{d}}_{v-1}^{(m)}, \tilde{\tilde{d}}_{v+1}^{(m-1)})}
\end{aligned}
\tag{5.57}
$$

where,

$$
\chi_d(\tilde{\tilde{\boldsymbol{w}}}^{(m)}, \tilde{\tilde{d}}_{v-1}^{(m)}, \tilde{\tilde{d}}_{v+1}^{(m-1)}) = \omega_{\tilde{\tilde{w}}_{v-1}^{(m)} \tilde{\tilde{d}}_{v-1}^{(m-1)}}^{\tilde{\tilde{w}}_v^{(m)} d}(m) \omega_{\tilde{\tilde{w}}_v^{(m-1)} d}^{\tilde{\tilde{w}}_{v+1}^{(m)} \tilde{\tilde{d}}_{v+1}^{(m-1)}}(m)
\tag{5.58}
$$

For all the $k(k = 0, \cdots, N^f - 1)$, a successive hidden variable $\tilde{\tilde{d}}_v^{(m)}$ $(v = 0, \cdots, T_k^f)$ is obtained from $v = 0$.

step6. Final Judgment of the Algorithm

The updated parameter estimates $\hat{\boldsymbol{\beta}}^{(m)}, \hat{\boldsymbol{\gamma}}^{(m)}$ and hidden variable $\tilde{\tilde{\boldsymbol{w}}}^{(m)}, \tilde{\tilde{\boldsymbol{d}}}^{(m)}$ with the above algorithm is accumulated. In case of $m \le \bar{m}$, return to step 2 with $m = m + 1$. Otherwise, the algorithm is terminated.

In the initial stage of the algorithm, there remains the influence of the initial value of the parameters. Thus, the samples are then accumulated except for those that were generated during the burn-in period $\underline{m}$. If the number of samples m is sufficiently large, the parameters estimated by using the above algorithm will converge on the estimated value of the posterior distribution. It is possible to estimate the statistics about the posterior distribution of parameter vector $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$

using the obtained sample $\boldsymbol{\beta}^{(m)}$, $\boldsymbol{\gamma}^{(m)}$ $(m = \underline{m}+1, \underline{m}+2, \cdots, \overline{m})$ from MH method.

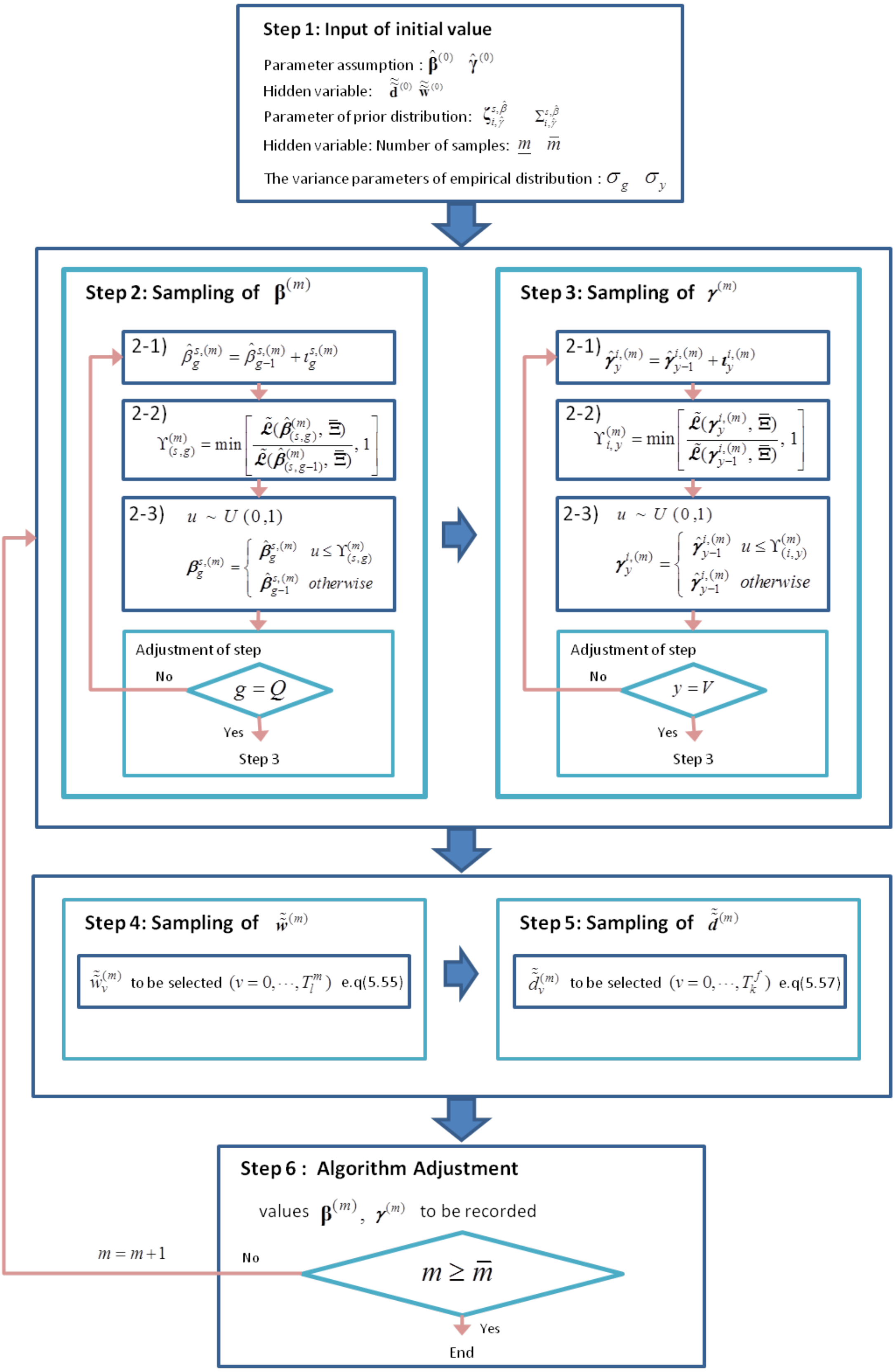

Figure 5.5 Flowchart of Bayesian Estimation for Compound Hidden Markov Deterioration Model

5.6.2. Posterior distribution statistic

Statistical testing for parameters $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ can be carried out based on the samples obtained from MCMC method. It is difficult to represent the a posterior probability density function of the parameters by analytical function with MCMC method. The non-parametric distribution function and the density function are estimated using the obtained sample. The obtained sample from MCMC method is expressed by $\boldsymbol{\theta}^{(m)} = (\boldsymbol{\beta}^{(m)}, \boldsymbol{\gamma}^{(m)})$ $(m = 1, \cdots, \overline{m})$.

Among generated samples, the first $\underline{m}$ samples which considered as set of the convergence process are removed from the sample set. A new set of samples is then defined as a replacement with its subscriptions as $\mathcal{M} = \{\underline{m}+1, \cdots, \overline{m}\}$.

Because the statistics of parameter $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ can be defined in the same way, we focus on parameter $\boldsymbol{\beta}$ below. The joint probability distribution function $G(\boldsymbol{\beta})$ of parameter $\boldsymbol{\beta}$ can be defined;

$$G(\boldsymbol{\beta}) = \frac{\#(\boldsymbol{\beta}^{(m)} \leq \boldsymbol{\beta}, m \in \mathcal{M})}{\overline{m} - \underline{m}} \tag{5.59}$$

where, $\#(\boldsymbol{\beta}^{(m)} \leq \boldsymbol{\beta}, m \in \mathcal{M})$ is total number of samples which meet logical expression $\boldsymbol{\beta}^{(m)} \leq \boldsymbol{\beta}, m \in \mathcal{M}$. In addition, the expectation vector $\tilde{\boldsymbol{\zeta}}^s(\boldsymbol{\beta}^s)$ and variance-covariance matrix $\tilde{\boldsymbol{\Sigma}}^s(\boldsymbol{\beta}^s)$ of the posterior distribution of $\boldsymbol{\beta}^s$ are expressed, respectively;

$$\begin{aligned} \tilde{\boldsymbol{\zeta}}^s(\boldsymbol{\beta}^s) &= (\tilde{\zeta}(\beta_0^s), \cdots, \tilde{\zeta}(\beta_Q^f))' \\ &= \left(\sum_{m=\underline{m}+1}^{\overline{m}} \frac{\beta_0^{s(m)}}{\overline{m} - \underline{m}}, \cdots, \sum_{m=\underline{m}+1}^{\overline{m}} \frac{\beta_Q^{s(m)}}{\overline{m} - \underline{m}} \right)' \end{aligned} \tag{5.60a}$$

$$\tilde{\boldsymbol{\Sigma}}^s(\boldsymbol{\beta}^s) = \begin{pmatrix} \tilde{\sigma}^2(\beta_0^s) & \cdots & \tilde{\sigma}^2(\beta_0^s \beta_Q^s) \\ \vdots & \ddots & \vdots \\ \tilde{\sigma}^2(\beta_Q^s \beta_0^s) & \cdots & \tilde{\sigma}^2(\beta_Q^s) \end{pmatrix} \tag{5.60b}$$

where, with the $r, q = 0, \cdots, Q$,

$$\tilde{\sigma}^2(\beta_r^s) = \sum_{m=\underline{m}+1}^{\overline{m}} \frac{\{\beta_r^{s(m)} - \tilde{\zeta}(\beta_r^s)\}^2}{\overline{m} - \underline{m}} \tag{5.61a}$$

$$\tilde{\sigma}(\beta_Q^s \beta_0^s) = \sum_{m=\underline{m}+1}^{\bar{m}} \frac{\{\beta_r^{s(m)} - \tilde{\zeta}(\beta_r^s)\}\{\beta_q^{s(m)} - \tilde{\zeta}(\beta_q^s)\}}{\bar{m} - \underline{m}} \tag{5.61b}$$

Credible intervals of parameter $\boldsymbol{\beta}$ can be defined by using samples generated from MH method. For example, $100(1-2\varepsilon)\%$ credible interval of parameter $\boldsymbol{\beta}$ is defined by using statistical sampling order $(\underline{\beta}_q^{s,\varepsilon}, \bar{\beta}_q^{s,\varepsilon})$ $(s = 1,\cdots,S-1;\ q = 0,\cdots,Q)$ with $\underline{\beta}_q^{s,\varepsilon} < \beta_q^s < \bar{\beta}_q^{s,\varepsilon}$;

$$\underline{\beta}_q^{s,\varepsilon} = \arg\max_{\beta_q^{s*}} \left\{ \frac{\#(\beta_q^{s,(m)} \le \beta_q^{s,*}, m \in \mathcal{M})}{\bar{m} - \underline{m}} \le \varepsilon \right\} \tag{5.62a}$$

$$\bar{\beta}_q^{s,\varepsilon} = \arg\max_{\beta_q^{s**}} \left\{ \frac{\#(\beta_q^{s,(m)} \le \beta_q^{s,**}, m \in \mathcal{M})}{\bar{m} - \underline{m}} \le \varepsilon \right\} \tag{5.62b}$$

In MCMC method, there is no guarantee that the initial value of parameter $\boldsymbol{\theta}^{(0)}$ is a sample from the posterior distribution. Thus, it is required to consider $\bar{m}$ samples generated from MH method as posterior distribution of the first $\underline{m}$ set $\boldsymbol{\theta}^{(m)} = (\boldsymbol{\beta}^{(m)}, \boldsymbol{\gamma}^{(m)})$ $(m = 1,\cdots,\underline{m})$. And then, the samples after $\underline{m}+1$ are adopted. About the adopted samples after $\underline{m}+1$, a hypothetical test using the Geweke statistical test [23] is carried out to verify whether samples are generated from the posterior distribution. Among the samples $\boldsymbol{\theta}^{(m)}$ $(m = 1,\cdots,\bar{m})$ generated from MH method, the first m_1 data and the last m_2 are adopted. Geweke recommended the ranges as $m_1 = 0.1(\bar{m} - \underline{m})$ and $m_2 = 0.5(\bar{m} - \underline{m})$ for the two subsets respectively. According to Chib [24] and Newey and West [25], the Geweke statistical test(referred as Z-score)used to verify values of parameter β can be represented as follows;

$$Z_{\beta_q^s} = \frac{{}_1\bar{\beta}_q^s - {}_2\bar{\beta}_q^s}{\sqrt{\upsilon_1^2(\beta_q^s) + \upsilon_2^2(\beta_q^s)}} \approx \mathcal{N}(0,1) \tag{5.63}$$

$${}_1\bar{\beta}_q^s = \frac{\sum_{m=\underline{m}+1}^{\underline{m}+m_1} \beta_q^{s(m)}}{m_1}, \quad {}_2\bar{\beta}_q^s = \frac{\sum_{m=\bar{m}-m_2+1}^{\bar{m}} \beta_q^{s(m)}}{m_2}$$

$$\upsilon_1^2(\beta_q^s) = \frac{2\pi \hat{f}_{\beta_q^s}^1(0)}{m_1}, \quad \upsilon_2^2(\beta_q^s) = \frac{2\pi \hat{f}_{\beta_q^s}^2(0)}{m_2}$$

where, $f_{\beta_q^s}^{l}(0)$ $(l=1,2)$ is the probability density function and the value of $2\pi f_{\beta_q^s}^{l}(0)$ is estimated from the following equations;

$$2\pi \hat{f}_{\beta_q^s}^{l}(0) = {}_{l}\hat{w}_0 + 2\sum_{s=1}^{q} w(s,q)\,{}_{l}w_q^s \tag{5.64}$$

$${}_{1}\hat{w}_0 = m_1^{-1} \sum_{m=\underline{m}+1}^{\underline{m}+m_1} (\beta_q^{s(m)} - {}_{1}\overline{\beta}_q^s)^2$$

$${}_{2}\hat{w}_0 = m_2^{-1} \sum_{m=\overline{m}-m_2+1}^{\overline{m}} (\beta_q^{s(m)} - {}_{2}\overline{\beta}_q^s)^2$$

$${}_{1}\hat{w}_q^s = m_1^{-1} \sum_{m=\underline{m}+s+1}^{\underline{m}+m_1} (\beta_q^{s(m)} - {}_{1}\overline{\beta}_q^s)(\beta_q^{s(m-s)} - {}_{1}\overline{\beta}_q^s)$$

$${}_{2}\hat{w}_q^s = m_2^{-1} \sum_{m=\overline{m}-m_2+s+1}^{\overline{m}} (\beta_q^{s(m)} - {}_{2}\overline{\beta}_q^s)(\beta_q^{s(m-s)} - {}_{2}\overline{\beta}_q^s)$$

$$w(s,q) = 1 - \frac{f}{v+1}$$

The parameter v which denotes the approximate value of spectrum density is assigned by 20 as recommended in the Geweke statistical test. The null hypothesis H_0 and alternative hypothesis H_1 concerning the invariance distribution of setting-values for parameter β_q^s can be defined as;

$$\begin{cases} H_0 : |Z_{\beta_q^s}| \le Z_{v/2} \\ H_1 : |Z_{\beta_q^s}| > Z_{v/2} \end{cases} \tag{5.65}$$

where, $Z_{v/2}$ is the critical value to be applied for dismissing the null hypothesis. In case of the statistical hypothesis testing for the null hypothesis by a significant level v%, the $Z_{v/2}$ can be defined by a value which satisfies $v/2\% = 1 - \Phi(Z_{v/2})$. Here, the $\Phi(Z)$ is the distribution function of the standard normal distribution.

5.7. Empirical study

5.7.1. Overview of Empirical study

The application of compound deterioration model is carried out with 3 observed data set from "precision safety diagnosis of water pipe supply system" which has been conducted every 5 years. Basic information about the data shown in Table 1 below.

Table 5.1 Features of inspection data

Features	value	
Material	Ductile cast iron	
Years laid(average age)	From 1963 to 2013	
Diameter/mm	100~1200	
Number of pipes	1,896	
Number of samples	Inspection of inner surface	2,545
	Inspection of pipe body	1,203

To express deterioration process with markov chain model, it is required to present the pipe condition by discrete condition indications. Because the observed data in this empirical study is obtained by continuous measurement values, it is necessary to convert continuous values to discrete condition values. In this study, we categorized the condition of inner surface into five ratings shown in table 2 which is proposed by Korea Ministry of Construction and Transportation[26].

Table 5.2 Description of Condition States.

Condition state	Range of inner corrosion area
1	No corrosion
2	5% below
3	15-5%
4	30-15%
5	30% excess

In addition, the condition of pipe body is categorized into five ratings shown in table 3.

Table 5.3 Description of Condition States

Condition state	Range of residual thickness rate
1	t>=1.0
2	0.95-1.0
3	0.8-0.95
4	0.7-0.8
5	T<0.7

* t(Residual thickness rate)= Residual thickness / Required thickness

5.7.2. Estimation results

The exponential hazard function of condition state of inner surface and pipe body are specified respectively as follows;

$$\lambda^s(i) = \beta_0^i \exp(\beta_1^s + \beta_2^s x_2) \ (i,\ s = 1,\cdots,4) \tag{5.66a}$$

$$\mu^i(s) = \gamma_0^s \exp(\gamma_1^i + \gamma_2^i y_2) \ (i,\ s = 1,\cdots,4) \tag{5.66b}$$

The unknown parameter β_1^s and γ_1^i are constant terms, β_2^s and γ_2^i represent the pipe diameter. In this study, other characteristic variables that reflect the influence of soil unit weight, top traffic volume, and so on were neglected, either because of their small impacts or because data were unavailable. Each hazard model has heterogeneity parameters β_0^i $(i = 1,\cdots, I-1)$ which denote the heterogeneity of the deterioration rate of pipe body and it depend on condition state of inner surface i and γ_0^s $(s = 1,\cdots,S-1)$ which denotes the heterogeneity of the deterioration rate of inner surface and it depend on condition state of pipe body s. Here, the parameter β_0^1 and γ_0^1 are normalized to 1. For all of $i\,(i = 2,\cdots, I-1)$ and $s\,(s = 2,\cdots,S-1)$, if $\beta_0^i = 1$ and $\gamma_0^s = 1$ are satisfied, the hazard model of inner surface corrosion and pipe body degradation hazard are independence of each other. In other words, the hypothesis of compound deterioration which inner surface corrosion and pipe body degradation effect each other is rejected. The estimated results which don't satisfy the condition of credible intervals not containing zero and Geweke test are excluded and finally, the results which maximize the Bayesian factors(5.11 & 5.26-28) are adopted.

To conduct the M-H method, the number of iteration required to reach a steady state (the burn-in period) was set to $\underline{N} = 10,000$ and the number of iterations for parameter sampling was set to $\bar{N} = 20,000$. The 10,000 burn-in samples were omitted and the remaining 10,000 parameter samples were used to carry out the estimation.

The estimation results of compound deterioration hidden markov model for the hazard model of pipe body degradation and inner surface corrosion are shown in Table 5.4 and 5.5, respectively.

Table 5.4 Estimation results for pipe body degradation

Condition state	Constant term β_1^s	Diameter β_2^s	Hazard rate $E[\lambda^s(1)]$	Life expectancy $ET^s(1)$
	-3.810	-		
1	(-3.920, -3.718)	(-)	0.022	45.163
	1.543	-		
	-3.224	-1.187		
2	(-3.404, -3.086)	(-1.590, -0.899)	0.035	28.306
	1.324	*1.924*		
	-2.745	-1.180		
3	(-2.959, -2.596)	(-1.674, -0.822)	0.057	17.522
	1.359	*1.205*		
	-2.936	-		
4	(-3.229, -2.718)	(-)	0.053	18.838
	0.347	-		

Notes: Values in $(\cdot)$ show 95% credible intervals and values shown in italic type in each row are the Geweke statistical test.

Table 5.5 Estimation results inner surface corrosion

Condition state	Constant term γ_1^i	Diameter γ_2^i	Hazard rate $E[\mu^i(1)]$	Life expectancy $ET^i(1)$
	-2.472	-1.143		
1	(-2.644, -2.305)	(-1.514, -0.780)	0.062	16.012
	0.916	*0.943*		
	-2.450	-1.692		
2	(-2.740, -2.163)	(-2.470, -0.966)	0.055	18.106
	1.893	*1.880*		
	-1.896	-1.244		
3	(-2.326, -1.488)	(-2.305, -0.205)	0.108	9.249
	0.593	*0.636*		
	-2.220	-		
4	(-2.864, -1.603)	(-)	0.109	9.207
	0.536	-		

Notes: Values in $(\cdot)$ show 95% credible intervals and values shown in italic type in each row are the Geweke statistical test.

In addition, the estimation results of heterogeneity parameters β_0^i and γ_0^s are shown in Table 5.6. The estimations obtained by M-H method show the probability distribution of the parameters. In the Table 5.4-7, the values estimated by Bayesian estimation method are the sample average of parameters, and the values in parentheses refer 95% credible intervals. All the credible intervals of estimated parameters don't contain zero. The credible intervals not containing zero imply that there is a statistically significant (Wu and Hamada, 2009). The absolute value of the Geweke test statistics shown in italic type are all less than 1.96, so the convergent hypothesis cannot be dismissed at a significance level of 5%.

Table 5.6 Estimation results of heterogeneity parameters

Condition state	β_0^s	Condition state	γ_0^i
	1.008		1.058
$i=2$	(1.001, 1.019)	$s=2$	(1.003, 1.166)
	0.452		*0.107*
	1.068		1.246
$i=3$	(1.005, 1.152)	$s=3$	(1.014, 1.690)
	1.615		*1.758*
	1.194		1.978
$i=4$	(1.027, 1.355)	$s=4$	(1.090, 3.297)
	0.590		*0.902*

Notes: Values in $(\cdot)$ show 95% credible intervals and values shown in italic type in each row are the Geweke statistical test.

With the hazard model of pipe body degradation $\lambda^s(i)$ and inner surface corrosion $\mu^i(s)$, with given condition of inner surface corrosion i, the life expectancy $ET^s(i)$ of pipe body condition s and with given condition of pipe body condition s, the life expectancy $ET^i(s)$ of inner surface condition i can be defined by as follows, respectively;

$$ET^s(i) = \int_0^\infty \exp(-\lambda^s(i)y^s)dy^s = \frac{1}{\lambda^s(i)} \tag{5.67a}$$

$$ET^i(s) = \int_0^\infty \exp(-\mu^i(s)y^i)dy^i = \frac{1}{\mu^i(s)} \tag{5.67b}$$

The Table 5.4 shows the expected value of pipe body degradation hazard rate of each condition state and the life expectancy of pipe body condition s when inner surface corrosion state is $i=1$.

In addition, The Table 5.5 shows the expected value of inner surface corrosion hazard rate of each condition state and the life expectancy of inner surface corrosion state i when pipe body condition state is $s=1$.

The expected value of pipe body degradation hazard rate of each condition state and the life expectancy of pipe body condition state s obtained for each inner surface corrosion state i are presented in Table 5.7. As shown in Table 5.7, it can be seen that the more inner surface condition get worse, more the life expectancy of pipe body condition state s is short. Likewise, the expected value of inner surface corrosion hazard rate of each condition state and the life expectancy of inner surface corrosion condition i obtained for each pipe body condition state s are presented in Table 5.8. It also can be seen that the more pipe body condition get worse, more the life expectancy of inner surface corrosion state i is short.

On the assumption that the inner surface corrosion state is staying in $i\,(i=2,\cdots,I)$, the average time $E[T](s\,|\,i)$ which pipe body condition state reaches $s\,(s=2,\cdots,S)$ from pipe construction time can be defined by as follow;

$$E[T](s\,|\,i)=\sum_{k=1}^{s-1}\frac{1}{\lambda^{k}(i)} \tag{5.68}$$

Likewise, on the assumption that the pipe body degradation state is staying in $s\,(s=2,\cdots,S)$, the average time $E[T](i\,|\,s)$ which the inner surface corrosion state reaches $i\,(i=2,\cdots,I)$ from pipe construction time can be defined by as follow;

$$E[T](i\,|\,s)=\sum_{l=1}^{i-1}\frac{1}{\mu^{l}(s)} \tag{5.69}$$

Table 5.7 Estimation results of life expectancy(year) of pipe body for each inner surface corrosion state i

s	$i=1$		$i=2$		$i=3$		$i=4$	
	$E[\lambda^s(1)]$	$ET^s(1)$	$E[\lambda^s(i)]$	$ET^s(i)$	$E[\lambda^s(i)]$	$ET^s(i)$	$E[\lambda^s(i)]$	$ET^s(i)$
1	0.022	45.163	0.023	42.691	0.028	36.239	0.044	22.831
2	0.035	28.306	0.037	26.757	0.044	22.713	0.070	14.309
3	0.057	17.522	0.060	16.563	0.071	14.060	0.113	8.858
4	0.053	18.838	0.056	17.807	0.066	15.116	0.105	9.523

Table 5.8 Estimation results of life expectancy(year) of inner surface for each pipe body condition degradation state s

i	$s=1$		$s=2$		$s=3$		$s=4$	
	$E[\mu^i(1)]$	$ET^i(1)$	$E[\mu^i(s)]$	$ET^i(s)$	$E[\mu^i(s)]$	$ET^i(s)$	$E[\mu^i(s)]$	$ET^i(s)$
1	0.062	16.012	0.096	10.384	0.119	8.381	0.149	6.696
2	0.055	18.106	0.085	11.742	0.106	9.477	0.132	7.572
3	0.108	9.249	0.167	5.999	0.207	4.842	0.259	3.868
4	0.109	9.207	0.167	5.971	0.207	4.820	0.260	3.851

With the equation (5.68-69), we can obtain performance curve of inner surface corrosion and pipe body degradation, respectively as shown in Figure 5.5 and 5.6. With a given condition as inner surface condition state $i=1$, we could verify that the time to reach service limit status of pipe body degradation ($s=5$) is about 109.83years. In addition, we could confirm that the more inner surface condition get worse, the pipe body degradation rate will accelerate. Like, with a given condition as pipe body condition state $s=1$, we could verify that the time to reach service limit status of inner surface corrosion($i=5$) is about 52.57years and the more pipe body condition get worse, the inner surface corrosion rate will accelerate. In addition, the Figures 5.5 and 5.6 show that the time to reach service limit status of inner surface is shorter than life expectancy of pipe body. In addition we were able to verify that the effect from inner surface corrosion to pipe body degradation is smaller than the effect from pipe body degradation to the inner surface corrosion. Because the condition of inner surface corrosion can effect pipe body degradation, it is required to maintain the inner surface of pipe with rehabilitation method such as lining. Therefore, the compound deterioration hidden markov model can allow to establish optimal maintenance strategy

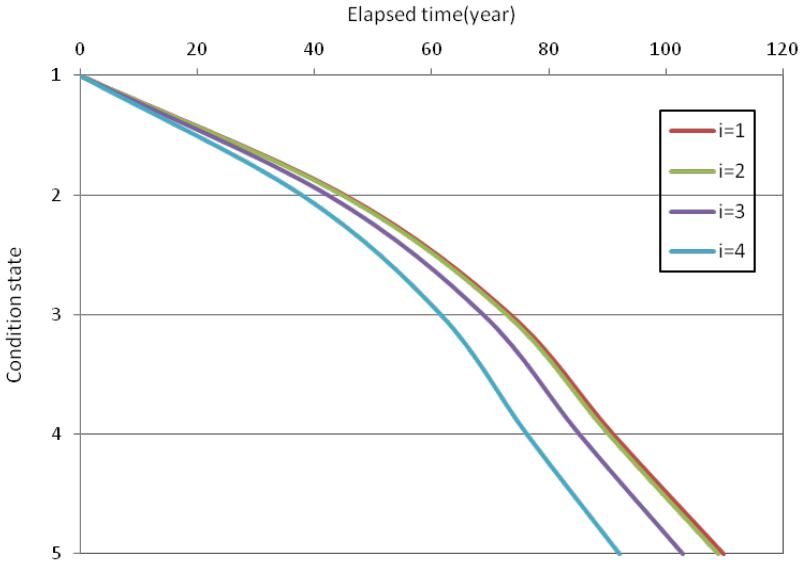

Figure 5.6 A Performance curve of pipe body for each condition state of inner surface

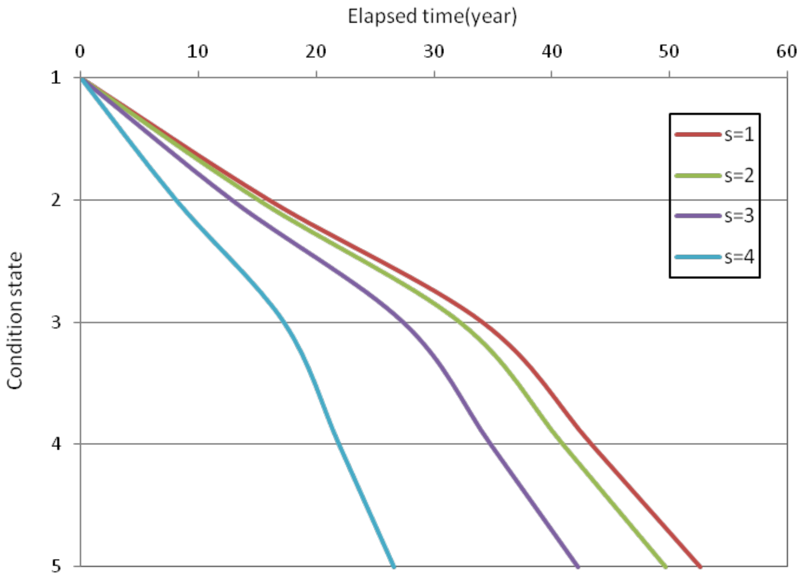

Figure 5.7 A Performance curve of inner surface for each condition state of pipe body

5.8. Conclusion

Maintenance strategy of pipeline system is required to achieve not only economics but also water quality and structural stability of pipeline. Deterioration process of pipeline is compound phenomenon consisting of deterioration of inner surface and pipe body. Thus, for maintenance of pipeline, deterioration forecasting model is required considering deterioration process of inner surface and body of pipe at the same time. But, because deterioration process of buried pipe has many uncertainties and with incomplete information, it is difficult to forecast the deterioration process of pipeline. In this study, in case of incomplete data caused by temporal mismatch in the data of the inner surface corrosion and pipe body condition, the compound deterioration process of pipeline is explained with compound hidden markov deterioration model.

The compound hidden markov deterioration model is estimated using Bayesian estimation method. The empirical study was carried out by using an inspection dataset of real pipeline system. With the estimated results of empirical study, we could verify the compound deterioration relation in inner surface corrosion and pipe body degradation. The more pipe body condition get worse, the inner surface corrosion rate will accelerate. Likewise, the more inner surface condition get worse, the pipe body degradation rate will accelerate. In addition, the results show that the effect from inner surface corrosion to pipe body degradation is smaller than the effect from pipe body degradation to the inner surface corrosion.

Because the inner surface corrosion can cause not only contamination of drinking water and degradation of water flow capacity but also acceleration of pipe body degradation, it needs to be properly maintained. In addition, because the corrosion rate of inner surface of which the pipe body is degraded severely can be accelerated, a replacement of pipeline may be economical rather than rehabilitation of inner surface. We believe that our new model can be extended to other items of infrastructure and will contribute to advancing asset management.

In this regard, the compound hidden markov deterioration model can be applied to establishing optimal maintenance strategy for rehabilitation of inner surface and replacement of pipeline. For future extension of our study, it is required to develop a method of life cycle cost analysis which consider the inner surface corrosion and pipe body degradation at once. In addition, there is also a need for studies to determine the optimal pipe replacement time and rehabilitation time for inner surface.

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6. Conclusion

6.1. A Brief Summary

Pipeline system is an important infrastructure to supply purified water for maintenance and development of city. Pipeline system was built at the high economic growth intensively and now, it has emerged as a major problem to the water authorities worldwide to maintain these enormous aging pipelines. Thus, because an enormous cost is required to rehabilitate the aging pipeline, it is important to establish the optimum maintenance strategy.

Decision making for pipeline system management that is policy variables for determining a significant strategy for the timing of rehabilitation is greatly affected by deterioration prediction model. It is therefore important to know how the deterioration of the system proceeds. This research has proposed a probabilistic deterioration forecasting model of pipeline based on statistical methods with inspection data for optimal rehabilitation strategy of the pipeline system. The deterioration process of pipeline is formulated by a hazards model.

In chapter 3, we have developed a competing deterioration-hazard model that considers competition among several types of burst in pipeline systems and the proposed model allows us to determine the probability of burst for each type of bursts. In real pipeline system, pipe failures caused by deterioration appear in various forms. We therefore classified pipeline failures as ‘B-burst’, which occur in the pipe body, or ‘C-burst’, which occur in pipe-connection parts. The Weibull deterioration-hazard model is used to address the lifetime pipeline, and takes into account the nature of the competition between several types of failure by using a competing deterioration-hazard model. The deterioration of the pipeline is predicted by developing a competing deterioration-hazard model that considers competition between C- burst and B- burst. The competing deterioration hazard model is estimated using Bayesian estimation method. The empirical study was carried out by using an inspection dataset of real pipeline system.

In chapter 5, in case of incomplete data caused by temporal mismatch in the data of the inner surface corrosion and pipe body condition, the compound deterioration process of pipeline is explained with compound hidden markov deterioration model. Deterioration process of pipeline is compound phenomenon consisting of deterioration of inner surface and pipe body. Thus, for maintenance of pipeline, deterioration forecasting model is required considering deterioration process of inner surface and body of pipe at the same time. This study formulates compound deterioration process considering interaction of deterioration of inner surface and pipe body and systematic loss of data to compound hidden markov deterioration model. The compound hidden markov deterioration model is estimated using Bayesian estimation method. The empirical study was carried out by using an inspection dataset of real pipeline system.

In chapter 4, the study targeted mainly on development of methodology to apply deterioration hazard model proposed in chapter 3. The pipe failure is briefly classified into burst and leakage, and the deterioration procedure of burst and leakage is forecasted by using competing deterioration hazard model. The time to burst and leak are explained by using Weibull distribution and exponential distribution, respectively and the deterioration model is estimated using Bayesian estimation method. Estimation for optimal replacement time and expected life cycle cost are carried out in the second phase after estimating the competing deterioration hazard model. The least life cycle cost analysis is conducted on the basis of maintenance strategy that repair of leakage and pipe replacement due to burst are carried out on an as needed basis. The empirical application of the proposed model was carried out to the real pipeline system, S city in Korea.

6.2. Conclusions

The competing deterioration-hazard model allows us to determine the probability density of bursts in pipe body and connection. In comparison with the conventional Weibull deterioration hazard, the competing deterioration hazard model can improve the quality of deterioration forecasting. Because the choice of a maintenance and repair method will depend on the type of burst, therefore the competing deterioration hazard model enables us to establish an optimum maintenance strategy of pipeline system. (Chapter 3)

For pipeline system, the optimal rehabilitation model allows us to determine optimal replacement timing based on stochastic forecasting model and life cycle cost analysis. With a empirical study, we could confirm that the DCIP is more beneficial type of pipe than CIP in asset management of the pipeline system. From the application view points, we believe that our new model can be extended to other items of infrastructure and will contribute to advancing asset management. (Chapter 4)

Compound hidden markov deterioration model is suitable methodology for estimation of compound deterioration relation in inner surface corrosion and pipe body degradation and can be applied with incomplete data caused by temporal mismatch. We could verify that the degradation of pipe body and corrosion of inner surface influence each other with complex interaction. The compound hidden markov deterioration model can be applied to establishing optimal maintenance strategy for rehabilitation of inner surface and replacement of pipeline. For future extension of the study, it is required to develop a method of life cycle cost analysis which consider the inner surface corrosion and pipe body degradation at once. In addition, there is also a need for studies to determine the optimal pipe replacement time and rehabilitation time for inner surface.

Appendix A

The appendix shows a solving process of integral equation (4.11).

Let, $z+t=\tau$ then, $dt=d\tau$ and $t=\tau-z$. (A.1)

The equation (4.11) can be arranged as follows;

$$\begin{aligned} L(z) &= \int_0^{T-z} \{c_l + I_l + L(z+t)\} \cdot \gamma_l \exp(-\gamma_l t) \cdot \exp(-\rho t) d\ t \\ &= \int_z^{T} \{c_l + I_l + L(\tau)\} \cdot \gamma_l \exp(-\gamma_l (\tau - z)) \cdot \exp(-\rho(\tau - z)) d\tau \\ &= -\int_T^{z} \{c_l + I_l + L(\tau)\} \cdot \frac{d}{d\tau}\left[-\exp(\gamma_l (z-\tau))\right] \cdot \exp(\rho(z-\tau)) d\tau \end{aligned} \tag{A.2}$$

Let, $\mu(z) = c_l + I_l + L(z)$ also, $\mu(\tau) = c_l + I_l + L(\tau)$

and let, $K(z,\tau) = \frac{d}{d\tau}\left[-\exp(\gamma_l (z-\tau))\right] \cdot \exp(\rho(z-\tau))$ as a kernel function. Then, the $\mu(z)$ can be expressed as following;

$$\mu(z) = c_l + I_l - \int_T^{z} \mu(\tau) k(z,\tau) d\tau = c_l + I_l + \left[\int_0^T - \int_0^z \right] \mu(\tau) k(z,\tau) d\tau \tag{A.3}$$

let, $g(z) = c_l + I_l + \int_0^T \mu(\tau) k(z,\tau) d\tau$ then, the $\mu(z)$ can be simplified as following;

$$\mu(z) = g(z) - \int_0^z \mu(\tau) k(z,\tau) d\tau \tag{A.4}$$

Since $k(z,\tau)$ has the form $k(z-\tau)$, the integral form can be transformed by convolution of $\mu(\tau)$ and $k(z)$.

$$\int_0^z \mu(\tau) k(z,\tau) d\tau = \int_0^z \mu(\tau) k(z-\tau) d\tau = \mu(z) * k(z) \tag{A.5}$$

Thus, the $\mu(z)$ can be expressed as following;

$$\mu(z) = g(z) - \mu(z) * k(z) \tag{A.6}$$

Take Laplace transform to (A.6), then, we could arrange it as follows;

$$La\left[\mu(z)\right] = La\left[g(z) - \mu(z) * k(z)\right] = La\left[g(z)\right] - La\left[\mu(z)\right] La\left[k(z)\right]$$

$$La\left[\mu(z)\right] = \frac{La\left[g(z)\right]}{1 + La\left[k(z)\right]}$$

$$\mu(z) = La^{-1}\left[\frac{La\left[g(z)\right]}{1 + La\left[k(z)\right]}\right] \tag{A.7}$$

since, $\mu(z) = c_l + I_l + L(z)$

$$\begin{aligned} L(z) &= La^{-1}\left[\frac{La\left[g(z)\right]}{1 + La\left[k(z)\right]}\right] - c_l - I_l \\ &= \frac{1}{2\pi i}\int_{r-i\infty}^{r+i\infty} \frac{g(s)}{1+k(s)} e^{zs} ds - c_l - I_l \quad : \ \text{Bromwich integral} \end{aligned} \tag{A.8}$$

where, $g(s) = La\left[g(z)\right]$, and $k(s) = La\left[k(z)\right]$.

Here, the $g(z)$ and $k(z)$ are represented again as follows;

$$g(z) = c_l + I_l + \int_0^T \mu(\tau) k(z,\tau) d\tau = c_l + I_l + \int_0^T [c_l + I_l + L(\tau)] \gamma_l \exp((z-\tau)(\gamma_l + \rho)) d\tau$$

$$k(z) = \gamma_l \exp(z(\gamma_l + \rho))$$

With Laplace transform, the $k(s)$ can be expressed as follow;

$$\begin{aligned} k(s) = La[k(z)] &= \int_0^\infty \gamma_l \, \mathrm{e}^{z(\gamma_l+\rho)} e^{-sz} dz = \gamma_l \int_0^\infty \mathrm{e}^{z(\gamma_l+\rho-s)} dz \\ &= \left[\frac{\gamma_l}{\gamma_l + \rho - s} e^{z(\gamma_l+\rho-s)}\right]_0^\infty = -\frac{\gamma_l}{\gamma_l + \rho - s} \end{aligned} \tag{A.9}$$

Because from definition of Laplace transform the condition $s > \gamma_l + \rho$ is satisfied, the (A.9) can be arranged as following

$$1 + La(k(z)) = \frac{\rho - s}{\gamma_l + \rho - s} = \frac{s - \rho}{s - (\gamma_l + \rho)} \tag{A.10}$$

Meanwhile, the $g(s)$ can be expressed as follow;

$$g(s) = La(g(z)) = \int_0^\infty e^{-sz}\left[c_l + I_l + \int_0^T (c_l + I_l + L(\tau))\gamma_l e^{(\gamma_l+\rho)(z-\tau)} d\tau\right] d$$
$$= \int_0^\infty (c_l + I_l) e^{-sz} dz + \int_{\tau=0}^T \int_{z=0}^\infty (c_l + I_l + L(\tau))\gamma_l e^{(\gamma_l+\rho-s)z} e^{-\tau(\gamma_l+\rho)} dz d\tau \qquad \text{(A.11)}$$

With interchange of integration order $d\tau dz$ to $dzd\tau$, the (A.11) can be arranged as followings;

$$La(g(z)) = \left[\frac{-(c_l + I_l)}{s} e^{-sz}\right]_{z=0}^\infty + \int_{\tau=0}^T \left[(c_l + I_l + L(\tau))\frac{\gamma_l}{\gamma_l + \rho - s} e^{(\gamma_l+\rho-s)z}\right]_{z=0}^\infty e^{-\tau(\gamma_l+\rho)} d\tau$$
$$= \frac{(c_l + I_l)}{s} + \int_0^T (c_l + I_l + L(\tau))\frac{-\gamma_l}{\gamma_l + \rho - s} e^{-\tau(\gamma_l+\rho)} d\tau \qquad \text{(A.12)}$$

Because $g(0) = c_l + I_l + \int_0^T (c_l + I_l + L(\tau))\gamma_l e^{-\tau(\gamma_l+\rho)} d\tau$, the (A.12) can be arranged as followings;

$$La(g(z)) = \frac{(c_l + I_l)}{s} - \left(\frac{g(0 - (c_l) + I_l)}{\gamma_l + \rho - s}\right) \qquad \text{(A.13)}$$

Therefore, with (A.10) and (A.13),

$$\frac{La(g(z))}{1 + La(k(z))} e^{sz} = \left(\frac{s - \rho - \gamma_l}{s - \rho}\right)\left[\frac{(c_l + I_l)}{s} - \left(\frac{g(0 - (c_l) + I_l)}{\gamma_l + \rho - s}\right)\right] e^{sz} \qquad \text{(A.14)}$$

Since, $\mu(z) = c_l + I_l + L(z) = g(z) - \int_0^z \mu(\tau)k(z-\tau)d\tau$ the following is satisfied.

$$\mu(0) = c_l + I_l + L(0) = g(0) - 0$$

$$\therefore g(0) - (c_l + I_l) = L(0) \qquad \text{(A.15)}$$

Substitute equation (A.15) into (A.14), following result is obtained.

$$\frac{La(g(z))}{1 + La(k(z))} e^{sz} = \left(1 - \frac{\gamma_l}{s - \rho}\right)\left[\frac{(c_l + I_l)}{s} + \frac{L(0)}{s - (\gamma_l + \rho)}\right] e^{sz}$$
$$= \left[\frac{(c_l + I_l)}{s} + \frac{L(0)}{s - (\gamma_l + \rho)} - \frac{\gamma_l (c_l + I_l)}{s(s - \rho)} - \frac{\gamma_l L(0 \qquad)}{(s - \rho)(s - \gamma_l - \rho)}\right] e^{sz} \qquad \text{(A.16)}$$

With partial fraction $\left[\frac{1}{AB} = \frac{1}{B - A}\left(\frac{1}{A} - \frac{1}{B}\right)\right]$, the equation (A.16) can be simplified as following;

$$\frac{La(g(z))}{1+La(k(z))}e^{sz} = e^{sz}\left[\frac{(c_l + I_l) + \frac{\gamma_l(c_l+I_l)}{\rho}}{s} + \frac{L(0) - \frac{\gamma_l(c_l+I_l)}{\rho}}{s-\rho}\right] \quad \text{(A.17)}$$

Let,

$$h(s) = \frac{La(g(z))e^{sz}}{1+La(k(z))}$$

and let,

$$h_1(s) = e^{sz}\left(\frac{c_l + I_l + \frac{\gamma_l(c_l+I_l)}{\gamma_l}}{s}\right),\ h_2(s) = e^{sz}\left(\frac{L(0) - \frac{\gamma_l(c_l+I_l)}{\rho}}{s-\rho}\right)$$

Since, $h(s) = h_1(s) + h_2(s),\ \sum \mathrm{Res}(h(s)) = \sum \mathrm{Res}(h_1(s)) + \sum \mathrm{Res}(h_2(s))$

Also,

$$\sum \mathrm{Res}(h_1(s)) = c_l + I_l + \frac{\gamma_l(c_l + I_l)}{\rho} \quad \text{(A.18a)}$$

$$\sum \mathrm{Res}(h_2(s)) = e^{\rho z}\left(L(0) - \frac{\gamma_l(c_l+I_l)}{\rho}\right) \quad \text{(A.18b)}$$

The equation (A.8) can be arranged by Jordan's Lemma;

$$\begin{aligned}
L(z) &= \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} h(s)ds - c_l - I_l \\
&= \frac{1}{2\pi i}\oint h(s)ds - c_l - I_l, \\
&= \left[\frac{1}{2\pi i}(2\pi i)\sum \mathrm{Res}(h(s))\right] - c_l - I_l \\
&= \frac{\gamma_l(c_l + I_l)}{\rho}(1-e^{\rho z}) + L(0)e^{\rho z}
\end{aligned} \quad \text{(A.19)}$$

Here, since, $L(T) = 0 = \frac{\gamma_l(c_l + I_l)}{\rho}(1-e^{\rho T}) + L(0)e^{\rho T}$ the $L(0)$ can be obtained by;

$$L(0) = \frac{\gamma_l(c_l + I_l)}{\rho}(1-e^{-\rho T}) \quad \text{(A.20)}$$

Therefore, substitute the equation (A.20) into (A.19), we could get $L(z)$ as following;

$$L(z) = \frac{\gamma_l (c_l + I_l)}{\rho}\left[1 - e^{\rho z} + e^{\rho z} - e^{\rho(z-T)}\right] = \frac{\gamma_l (c_l + I_l)}{\rho}\left[1 - e^{\rho(z-T)}\right] \qquad \text{(A.21)}$$

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