Time as a logical space

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There are two ways of thinking about instants of time: "spatial" accounts emphasize the similarities between instants and *places*; "modal" accounts focus on the parallels between times and *possible worlds*. My aim in this paper is to draw attention to one respect in which times are more similar to possible worlds than they are to places. This is an argument in favor of modal theories of time, but I am not here claiming that it is a decisive one. Other considerations might sway one in favor of spatial theories of time in spite of the differences between times and places. Nor do I claim that times and possible worlds are exactly alike. There are philosophically interesting differences between the two that go beyond the humdrum observation that theories of modality and theories of time differ in terms of their subject matter.

Theories of modality

Theories of modality have three main components. The first component is the set of *possible worlds* W, whose elements are "ways the world might have been," and which serve as positions at which sentences take truth values. Given a possible world w and sentence φ, let us write $\ulcorner w \,|\, \varphi \urcorner$ to say that φ is *true at* w. The true-at operator '|', which is borrowed from George Myro (1986b;a), is defined to interact with truth-functional sentence connectives in the obvious way:

$$\begin{array}{lll} w \,|\, (\varphi \vee \psi) & \text{iff} & \text{either } w \,|\, \varphi \text{ or } w \,|\, \psi \\ w \,|\, \neg\varphi & \text{iff} & w \,|\, \varphi \text{ is not the case} \end{array} \tag{1}$$

The second component are *modal operators* that allow us to make claims about possible worlds other than the one that we are currently considering. These operators are usually introduced in tandem with certain structural relations on the set of possible worlds W. For example, standard modal logic introduces the possibility operator '$\Diamond$' and a binary *accessibility relation* R on W that specifies which worlds are possible relative to a given one. A sentence of the form $\ulcorner\Diamond\varphi\urcorner$ is then said to be true at a possible world w just in case the embedded sentence φ is true at some possible world w' that is R-accessible from w:

$$w \,|\, \Diamond\varphi \quad \text{iff} \quad w' \,|\, \varphi \text{ for some world } w' \text{ such that } Rww' \tag{2}$$

The third component of a theory of modality is the *actual world* $\alpha \in W$, which is the way the world actually is. Usually, we talk about what sentences are true *at* a world. This restriction can be dropped when talking about the actual world α. To be true at the actual world is to be true *simpliciter*:

$$\varphi \quad \text{iff} \quad \alpha \,|\, \varphi \tag{3}$$

Sentences without any modal operators are thus reserved for making claims about the actual world α. To talk about what is the case at the other positions in W, we use modal operators like '$\Diamond$' or expressions of the form "$w \,|\, \ldots$"

Every theory of modality has these three component but there is disagreement about their metaphysical status. *Modal primitivists* take modal operators as conceptually primitive and try to spell out all other modal motions in terms of them, including the notion of a possible world. The *possible worlds analysis* takes the opposite approach and tries to use (2) to eliminate modal operators in favor of quantification over possible worlds. Proponents of the possible worlds analysis disagree amongst themselves about the precise nature of possible worlds, and they also take different sides on the question of whether there is anything metaphysically special about the actual world, or whether it is a world like any other.

Different theories of modality can also offer different views about modal operators and the associated structural relations on the set of all possible worlds. In terms of '$\Diamond$', we can easily define a necessity operator '$\Box$' as shorthand for '$\neg\Diamond\neg$', but it is not clear that all modal operators can be defined in this way. For example, it is widely thought that the counterfactual conditional $\ulcorner\varphi \mathrel{\Box\!\!\rightarrow} \psi\urcorner$ ("If φ were the case then ψ would be the case") resists regimentation in terms of '$\Diamond$' and R alone. To give an acceptable account of '$\Box\!\!\rightarrow$', we are told, we need to postulate a relative similarity relation on W. And even if we restrict ourselves to the standard operator '$\Diamond$', there is still room for disagreement about the structural features of the accessibility relation R. The standard modal system S5 requires every possible world to be accessible form every other one, but there are modal systems that admit non-trivial accessibility relations (Hughes and Cresswell 1996: chs. 2, 3).

Modal theories of time

This is not the place to settle any of these disagreements about the correct theory of modality. The point I want to make is that we can develop a similar theory of temporal distinctions. The first step is to rewrite explicit time references of the form ⌜At time t, φ⌝ as ⌜$t \,|\, \varphi$⌝. In Japanese, this involves interpreting certain occurrences of the particle 'に' as the true-operator '|':

On Wednesday, I go to Kyoto.	水曜日　に　私は京都へいきます。	(4)
Wednesday | I go to Kyoto.	水曜日　|　私は京都へいきます。	

On this view, dates pick out instants of time that serve as positions at which sentences take truth values. There are also time references that describe the time series relative to the present moment. In English, such references often get implemented by the tense of verbs, but similar effects can be achieved by temporal adverbs such as 'yesterday' (昨日) and 'tomorrow' (明日). Since Japanese does not have a future tense, it must use such devices when talking about the future. In a modal theory of time, such temporal references relative to the present get spelled out in terms of sentential tense operators that function in a similar manner as modal operators.

Putting all of this together, a modal theory of time again has three components. Instead of the set of possible worlds W, we have a *time series* T at whose positions sentences take truth values. The true-at operator '|' satisfies a temporal version of (1). *Tense operators* such as 'P' ("it was the case that") and 'F' ("it will be the case that") allow us to express what was or will be the case relative to a given time.

$$\begin{array}{lll} t \,|\, \mathsf{P}\varphi & \text{iff} & t' \,|\, \varphi \text{ for some time } t' \text{ such that } t' < t \\ t \,|\, \mathsf{F}\varphi & \text{iff} & t' \,|\, \varphi \text{ for some time } t' \text{ such that } t < t' \end{array} \tag{5}$$

The *earlier-than relation* $<$ on T serves as an accessibility relation for these tense operators. The *present time* π is that element of the time series that correctly describes how things presently are:

$$\varphi \quad \text{iff} \quad \pi \,|\, \varphi \tag{6}$$

Sentences without tense operators thus make claims about what is true at the present time π. To describe how things are at other times, we use the tense operators ‘P’ and ‘F’ and explicit time references of the form “$t \,|\, \ldots$”

Modal theories of time raise the same kind of questions as theories of modality: should we take tense operators as conceptually primitive or should we use (5) to eliminate them in favor of quantification over times? Is the present time π metaphysically special or is it a time like any other? Different theories of time may also impose different structural constraints on the earlier-than relation $<$, or take different tense operators than ‘P’ and ‘F’ as primitive (Burgess 1984).

Modal theories of individuals

That time can be treated in this way counts in favor of modal theories of time, but it is not quite as conclusive as one might think. Many other structures can be treated in a similar way. For example, one can give a “modal” account of a quantificational logic that only has monadic predicates. Rather than write $\ulcorner Ka \urcorner$ to say that object a is K, we write $\ulcorner a \,|\, K \urcorner$ and treat monadic predicates as sentences that take truth values at individuals. This account can be extended to truth-functional compounds by adopting the clauses (1). In Japanese, this reading might indeed seem quite natural. Take a standard property attribution of the form ‘a は K です’. If we read ‘は’ as the true-at operator ‘|’ and drop the copula ‘です’ then we get $\ulcorner a \,|\, K \urcorner$.

We can split our theory of quantification into three parts that mirror the three components of a theory of modality.[1] In place of the set of possible worlds W, we have the *domain* D of all individuals, which now serve as positions at which sentences take truth values. The existential quantifier ‘$\exists$’ gets treated as a sentential operator that allows us to describe what other individuals are like:

$$d \,|\, \exists\varphi \quad \text{iff} \quad d' \,|\, \varphi \text{ for some object } d' \tag{7}$$

Every individual in D is “accessible” from every other individual and there is no need to

[1] See Prior (1968a;b) and Cresswell (1990; 1996). To extend this to sentences with more than one quantifier, we need a different operator $\exists_x$ for each variable x. The resulting polymodal system is non-trivial; see Kuhn (1980). Montague (1974) develops a different way of treating quantifiers as modal operators. His “possible worlds” are models of predicate logic and he interprets the notion of an *α-variant* of a model as an “accessibility relation” between such “possible worlds.”

specify relations like R in this case. Instead of the actual world, we pick an element a of D to serve as the *focus individual* of our theory:

$$\varphi \quad \text{iff} \quad a \,|\, \varphi \tag{8}$$

To be true at the focus individual is to be true *simpliciter*. Sentences without quantifiers describe the focus individual; to talk about other individuals, we use the sentential operator '$\exists$'.

There are again the by now familiar questions about the metaphysical status of the three components of such a modal theory of quantification. The only difference is that, in this case, these questions are not particularly controversial. There is broad agreement that quantifiers are to be defined in terms of the elements of the domain D, rather than the other way around, and that the focus individual is an individual like all others. But this does not change the fact that there are interesting similarities between W, T, and D. All three form what I call *logical spaces*. The hallmark of a logical space is that every property attribution must be located at some position in such a space.[2] When attributing a property, we must indicate, by our choice of modal auxiliary, whether we are talking about the actual or merely possible possession of the property. To possess a property is to possess it at some world or other. Likewise, we cannot attribute a property without committing ourselves, either explicitly or implicitly, to a time at which the property is being had, and we cannot attribute a property without attributing it to some individual or other. The set of all possible worlds W, the time series T, and the set of individuals D all offer exhaustive ranges of possibilities for property attributions.[3]

Modal theories of space

David Lewis (1983) and others have pointed out that the distinction between extensional and intensional theories is not quite as robust as was once thought. We can rewrite many extensional theories as intensional theories, and vice versa. One might therefore suggest

[2] My use of 'logical space' is thus similar to that of van Fraassen (1970: 100), but unlike that of Gomi (2009), who uses the term to refer to generalized logical systems.

[3] To be precise, only the attribution of ordinary properties is constrained in this way. Once we have modal and tense operators at our disposal, we can also define properties that reflect how an object's ordinary features vary across possible worlds and times. For example, 'is possibly K' and 'is K for seven seconds' do not seem to be features that objects have *at* worlds or times. Similar remarks apply to relational properties such as 'is K thirteen years before World War I'. Thanks to Ikuro Suzuki for pressing me on this point.

that modal theories of times and individuals are merely two specific instances of a general strategy for generating modal theories of an arbitrary subject matter. I do not think this is right, for there is an important limitation to the development of "modal" theories: while we can treat times or individuals like possible worlds, we cannot treat *places* like this.

Philosophers usually distinguish two views about the nature of space. *Spatial substantivalists*, like Isaac Newton, claim that spatial points are metaphysically basic entities that form a three-dimensional spatial manifold $\mathcal{M}$. *Relationists about time*, such as G. W. Leibniz, deny the existence of $\mathcal{M}$. All there is to space, they claim, are material objects that stand in spatial relations to one another. Places are abstractions, mere labels for the positions in the order that spatial relations impose on material objects. There are important differences between these two proposals, but both present a picture of a *geometric space* that is populated by material objects, rather than a logical space at whose positions sentences take truth values.

Some authors have recently promoted a third kind of view, which treats the locations in physical space as positions at which sentences take truth values.[4] Such "modal" accounts of space might initially seem quite promising. A sentence like "There are snakes" can plausibly said to be true in England, '$e \mid S$', and false in Ireland, '$i \mid \neg S$'.[5] In Japanese, this again means reading certain occurrences of the particle 'に' as a true-at operator:

$$\underbrace{\text{イングランド}}_{\text{England}}\ \text{に}\ \underbrace{\text{へびがいます}}_{\text{There are snakes.}} \tag{9}$$

However, these supporting examples all concern claims that simultaneously attribute properties to individuals *and* say something about where they are located. Instead of writing sentences of the special form "a is K *and* a is located at p" as $\ulcorner Ka \wedge Lap \urcorner$, we can perhaps formalize them as $\ulcorner p \mid Ka \urcorner$. But this does not help with simple property attributions that are lacking a location conjunct:

The hotel is clean	ホテルはきれいです。	(10)
Japanese is difficult	日本語はむずかしいです。	

By asserting these sentences, we do not say anything about the location of the hotel (ホテル) or of the Japanese language (日本語). These sentences are of the form 'Ch' and 'Dj', respectively, and there is no spatial parameter in either sentence that could be reinterpreted

[4] See Rescher and Garson (1968), von Wright (1979), and Forbes (1989: sec. 2.4).

[5] According to Catholic legend, St. Patrick banished all snakes from Ireland when they disturbed him during a fast. According to science, there were no snakes in Ireland to begin with.

as a place at which the hotel is said to be clean, or Japanese difficult. This is not to deny that there is some place at which the hotel is located, and perhaps we can even say that Japanese is located wherever there are Japanese speakers. The problem is that our sentences say nothing about either location.

Substantivialists and relationists agree that objects have what one might call *non-geometric* properties. According to spatial substantivalism, there is ultimately only one relation that objects bear to the points on $\mathcal{M}$, namely the *spatial location* relation L. All other geometric properties depend on the location of an object's parts, including an object's shape and its spatial relations to other objects. Instead of L, relationists postulate a range of spatial relations between material objects, but they agree with substantivalists that objects also have non-geometric properties, such as their mass or charge, that are independent of their spatial relations.

For physical space to function as a logical space, every property attribution would have to be located at some place, and that rules out the existence of such non-geometric properties. The modal theory of quantification discussed earlier treats *objects* like possible worlds. What is at issue here, though, are the *places* that material objects occupy, and they cannot be treated in this way.

A spatial substantivalist might reply that it is metaphysically necessary for the hotel to be located somewhere on $\mathcal{M}$, and that 'Ch' entails '$\exists p\, Lhp$'. If that is right then 'Ch' and '$\exists p(Ch \wedge Lhp)$' are necessarily equivalent, and a modal theory of time could then render the latter claim as '$\exists p\; p\,|\,Ch$':

$$Ch \qquad \Longrightarrow \qquad \exists p(Ch \wedge Lhp) \qquad \Longrightarrow \qquad \exists p\; p\,|\,Ch \tag{11}$$

This might yield the desired result, but it does so in the wrong way. What permits the transformation (11) is a metaphysical thesis about the spatial location of objects—namely that every object needs to have one—rather than thesis about the spatial location of property attributions. In the first step of the transformation, we merely add a location claim that has nothing to do with the attribution of C to h, and then try to conceal this fact in the second step of (11).

Spatial theories of time

Modal theories of space run into difficulties because many property attributions are not relativized to a place. Spatial theories of time run into the opposite problem that all property attributions *are* located at some time. Michael Dummett explains:

> A spatial reference ... is most naturally construed as a predicate which is true of a given object at a given time: thus 'There are snakes in England' is of the form 'There are snakes which are in England' ... A temporal reference ... qualifies the entire sentence adverbially: 'John is ill today' plainly cannot be interpreted as of the form 'John is ill and John is today'. (Dummett 1981: 389)

To bring out the impact of these remark, suppose that we adopt spatial substantivalism. 'There are snakes in England' can then be regarded as an existentially quantified conjunction of the form:

$$\exists x\,(Sx \wedge Lxe) \tag{12}$$

The first conjunct attributes the intrinsic property of being a snake, '*Sx*', and the second conjunct ascribes spatial location in England, '*Lxe*'.

In the temporal case, the parallel strategy quickly leads to disaster. Suppose we adopt a *temporal substantivalism* that regards time points as metaphysically basic entities that form a one-dimensional temporal manifold $\mathcal{T}$, and to which objects bear the *exists-at* relation E. Following Dummett's suggestions, we could then try to isolate the temporal location in the sentence

$$\text{John is ill on Monday} \tag{13}$$

by taking it to claim that John is ill and exists on Monday:

$$Ij \wedge Ejm \tag{14}$$

But if this were the logical form of our sentence then John's being healthy on Wednesday would require the truth of '$\neg Ij \wedge Ejw$', and that contradicts (14). It would be impossible for John's health to improve. More generally, if '$Ka \wedge Eat$' were the logical form of "a is K at time t" then no object a could acquire or lose any property K; all change would be logically impossible.

To avoid this problem, Gustav Bergmann (1960: 230) and D. H. Mellor (1981: ch.7) argue that sentences like (13) are actually of the form:[6]

$$Ijm \tag{15}$$

Here 'I' stands for an ill-at *relation* that living creatures bear to times at which they are unwell. There is no conflict between John's being ill on Monday and his being well on

[6] Mellor changed his mind about this issue in his (1998).

Wednesday, which would require the truth of '$\neg Ijw$'. More generally, to say that an object a is K at time t would be to say that a bears the K-relation to that point t on the temporal manifold, Kat.

This view permits the occurrence of change, but it is quite unlike spatial substantivalism that served as its inspiration. All geometric features of an object depend on what points on $\mathcal{M}$ it bears the relation L to, but not all temporal features of an object are determined by what points on $\mathcal{T}$ it bears the relation E to. The most interesting temporal features of an object would rather be a matter of the K-relations that it bears (or doesn't bear) to points on $\mathcal{T}$. Nor is there a temporal counterpart of the non-geometric properties that we have in the spatial case.

David Lewis (1986: 202) objects that the strategy (15) incorrectly treats all changeable features of objects as relations to time points, and that it is therefore unable to permit the possibility of intrinsic change. His counterproposal is to attribute properties to the *temporal parts* of material objects. To say that John is ill on Monday is to say that John has a temporal part *John-on-Monday* that is ill:

$$\exists x(Ix \wedge x = \text{John-on-Monday}) \tag{16}$$

If one accepts the existence of such temporal parts then one could try to develop a *relationism about time* that treats instants of times as classes (or mereological sums) of simultaneous temporal parts.[7] The metaphysical details of this proposal might be different, but we still end up with an account on which property attributions are relativized to a time, and there is still no temporal analogue of non-geometric properties. Lewis's view attributes properties to temporal parts, and to have a property at a time is for some temporal part to have it.

Neither the Bergmann–Mellor nor the Lewis view succeeds in eliminating the temporal location of property attributions; they merely move the temporal parameter to a different place. Nor does either view offer a temporal counterpart of non-geometric properties. This is not to deny that there are important metaphysical differences between these two proposals, nor is it meant as an argument against either of them. My point is merely that neither proposal fully succeeds in treating time on the model of space. We are being offered a choice between different views about the metaphysics of time, all of which treat time like a logical space.

[7] It is more common for relationists to construct times from temporally ordered *events*. Such an event-relationism raises different issues than the temporal-parts relationism discussed here; see Meyer (2011) and Meyer (2013: ch.2) for discussion.

Conclusion

I have argued that times are more similar to possible worlds than they are to places: the time series is a logical space at whose positions sentences take truth values, whereas physical space forms a geometric space that is populated by material objects. We cannot treat spaces like possible worlds, and while we can endow our theory of time with some of the metaphysical trappings of spatial substantivalism or relationism about space, we cannot sever the characteristic link between property-attributions and times. Nor can we obtain a temporal counterpart of the non-geometric properties that feature in theories of physical space.

This does not mean that times and possible worlds are exactly alike. There is no need to adopt the same view about the three components of a modal theory of time as we do for a theory of modality. For example, I argued in Meyer (2006) that one popular view about the metaphysics of modality, *actualism*, has no temporal analogue. There are a number of further issues on which time and modality part company,[8] but that does not change the fact that trying to treat time on the model of *space* is like trying to fit a square peg into a round hole.

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