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Report on my Stay at the Kyoto Institute of Economic Research

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Abstract. This document is a report on my stay as a visiting fellow at the Kyoto Institute of Economic Research (KIER), financed by the International Research Unit of Advanced Future Studies. I describe my scientific activity at KIER, and provide a non-technical summary of a paper on multiproduct oligopoly pricing.

Keywords: Multiproduct Firms, Aggregative Games, Mergers, Discrete / Continuous Choice

1. Scientific activity

I worked as a visiting fellow at the Kyoto Institute of Economic Research (KIER) from December 16 2015 to February 1 2016. This research stay was financed by the International Research Unit of Advanced Future Studies. I would like to express my gratitude to the Kyoto Institute of Economic research for hosting me, to the International Research Unit of Advanced Future Studies for providing financial support, and to Professor Sekiguchi for inviting me.

During my stay at KIER, I had the opportunity to interact with numerous researchers, such as Professor Hara, Professor Kajii, Professor Sekiguchi, Professor Sano and Dr. Matsuda. I learnt a great deal about many fascinating topics, such as repeated games, auction theory, mechanism design and demand theory. I also received much feedback about my own research on aggregative games and potential games.

On December 24 2015, I presented a paper entitled "Multiproduct-Firm Oligopoly: An Aggregative Games approach" (joint with my colleague Volker Nocke) in the game theory seminar series at KIER. It was a valuable experience for me to present an oligopoly theory paper in front of a game theory audience. The feedback I received there has been instrumental in improving my work.

On January 8 2016, I traveled to the city of Nagoya to present my paper on multiproduct firms in the industrial organization seminar series at Nagoya University. There, I had the opportunity to interact with Professor Adachi and Professor Hanazono, who told me about exciting projects they are currently working on, on the welfare effects of third-degree price discrimination under oligopolistic competition, and on the impact of entry on the profitability of big firms and small firms.

2. Research output

I spent most of my time at KIER working on my paper on multiproduct firms (Nocke, 2015). In the following, I provide a non-technical summary of the main results we obtain in this paper.

2.1. Motivation

Multiproduct firms are very important players in the world economy. In the United States, they account for 91% of total output and 41% of the total number of firms (Bernard, 2010). In addition, many industries appear to be characterized by oligopolistic competition: For instance, in U.S. manufacturing, the average NAICS (North American Industry Classification System) 5-digit industry has a four-firm concentration ratio of about 35%, meaning that the four biggest firms account for 35% of total output. In light of these numbers, it seems to be of first-order importance to be able to analyze oligopolistic competition between multiproduct firms.

However, there is surprisingly little work on this topic in the literature, and much of the literature has focused on the special case where demand is linear (see, for instance, (Eckel, 2010)). The lack of results in the literature may be attributed to a number of technical difficulties, which are inherent to the study of pricing games with multiproduct firms. The first difficulty is the high dimensionality of the firms' strategy spaces: A multiproduct firm sets multiple prices. The second difficulty is that the firms' payoff functions often fail to be quasi-concave in own action. In fact, it is well known that quasi-concavity does not hold when the demand system has been derived from a standard multinomial logit model of choice ((Spady, 1984); (Hanson, 1996)). The third difficulty is that firms' action spaces are unbounded: In principle, a firm should be able to set any price it wants to set, including very high prices. While this issue can sometimes be addressed by finding upper bounds on the firms' best responses, this is not always feasible: For instance, with a constant elasticity of substitution (CES) demand system, the monopoly price is infinite, and the lowest upper bound is therefore infinite as well. The fourth difficulty is that payoff functions often fail to be supermodular or log-supermodular.

These difficulties imply that standard equilibrium existence theorems based on Kakutani or Tarski's fixed-point theorems ((Nash, 1951); (Milgrom, 1990); (Topkis, 1998)) cannot be applied. We develop a multiproduct oligopoly model that takes care of these issues.

2.2. Discrete / continuous choice

The first building block of our analysis is a new class of demand systems. We assume that each consumer makes consumption decisions as follows. The consumer first observes the prices of all products, and an additive taste shock for each product. The consumer then chooses the product that provides her with the highest indirect utility (taking taste shocks into account). After having chosen a product, say product i, the consumer decides how much of product i she wants to consume. Formally, this amount of consumption is computed by applying Roy's identity to the consumer's indirect utility function for product i. This two-tier process is called a discrete / continuous choice model of consumer demand (discrete because the consumer chooses only one product; continuous because the consumer can adjust her consumption of this product).

Under the assumption that the taste shocks are iid type-1 extreme value, the expected demand for each product can be computed in closed-form. In addition, consumer surplus can also be computed in closed form, and is a strictly increasing function of a certain sufficient statistic, which we denote by H. This sufficient statistic also plays a very important in our study of the associated multiproduct-firm pricing game.

2.3. Multiproduct pricing game

A multiproduct pricing game consists of the following elements: a discrete / continuous choice model of consumer demand; a set of firms (which is a partition of the set of products); and a marginal cost for each product. Firms compete by setting their prices simultaneously. Our goal is to characterize the set of pure-strategy Nash equilibria of this game.

We show that such a multiproduct pricing game is aggregative, in the sense that the profit of a firm only depends on the prices set by this firm, and on a uni-dimensional sufficient statistic, which fully summarizes the competitiveness of the industry. This sufficient statistic happens to be equal to H, which, as we saw in the previous subsection, is also a sufficient statistic for consumer surplus.

In addition to the game being aggregative, we also show that firms' best responses can be described in a simple way. Keeping the other firms' prices fixed, a firm's optimal price vector has the property that the Lerner index associated to each price should be equal to a firm-level constant multiplied by a product-level scalar, which describes the local curvature of this product's conditional demand. Therefore, the firm's optimal behavior can be fully summarized by this firm-level constant, which we call the firm's iota-markup in the following.

These properties allow us to re-express the equilibrium existence problem as a nested fixed-point problem: For a given value of H, we can compute each firm's optimal iota-markup by solving for a sequence of independent fixed-point problems; these optimal iota-markups give us in turn a new value for H. We show that this nested fixed-point problem has a solution, and therefore, that the multiproduct pricing game has a Nash equilibrium.

Note that this nested fixed-point structure also gives rise to an efficient algorithm to compute the set of equilibria. In another paper with an additional coauthor (Breinlich, 2015), we use this algorithm to calibrate an international trade model with heterogeneous multiproduct firms, oligopolistic competition and CES demands.

In addition to establishing equilibrium existence, we also derive conditions for equilibrium uniqueness. These conditions are related to the curvature of the products' conditional demands. If the equilibrium is not unique, we are still able to show that the set of equilibria can be ordered, in the following sense: If H and H' are both equilibrium aggregator levels, and if $H>H'$, then consumers all prefer equilibrium H to equilibrium H', and firms all prefer equilibrium H' to equilibrium H. Put differently, equilibrium H is unambiguously more competitive than equilibrium H'.

We also derive interesting comparative statics results. We find that trade liberalization or an increase in the value of the consumers' outside option lowers all prices, makes all consumers better off, makes all firms worse off, and leads to an expansion in the set of products that are supplied in equilibrium. Productivity improvements, on the other hand, have more ambiguous effects. The intuition is the following. Suppose that the marginal cost of product i increases for exogenous reasons. There are two effects. On the one hand, the firm that owns product i, call it firm f, has incentives to increase the price of this product. On the other hand, firm f has less incentive to divert sales towards product i, since this product is now less profitable. This induces firm f to decrease the prices of the other products in its product portfolio. Whether consumers and firm f's rivals end up being better off or worse off depends on which of these two effects dominates.

2.4. Type aggregation with logit and CES demands

Next, we specialize the model to the case of logit and CES demands. We obtain another important property: The product portfolio of a given firm can be fully summarized, in any competitive environment, by a uni-dimensional sufficient statistic, which we call the firm's type. The type of a firm is a strictly increasing function of the quality of this firms' products and of the number of products this firm owns, and a strictly decreasing function of this firms' marginal costs. If two firms have the same type, then these two firms will end up behaving in the exact same way in terms of markups and market shares, no matter what the competitive environment is. In addition, we also find that, to every multiproduct firm, we can associate a single-product firm with the same type. This type aggregation property will end up being extremely useful for our analysis of merger policy.

2.5. Dynamic merger policy under logit and CES demands

In the final part of the paper, we apply the tools we have developed to the analysis of dynamic merger policy. The idea is the following. Consider a dynamic oligopoly framework with strategic and forward-looking firms, and with a strategic and forward-looking antitrust authority. In every period,

some firms receive a merger opportunity. If they decide to use this opportunity to merge, then they need to receive approval from the antitrust authority. Firms maximize the present discounted value of their future profit flows, and the antitrust authority maximizes the present discounted value of consumer surplus. We are interested in finding the equilibria of this game.

This is a difficult problem for the following reasons. The first question is whether potential merger partners want to propose a merger that is profitable given the current market structure. In a static environment, the answer would be yes. However, in a dynamic world, firms realize that a merger today may well affect the set of mergers that will (or will not) take place tomorrow, potentially in an adverse way. This dynamic effect may well induce the potential merger partners not to propose their merger to the antitrust authority, despite this merger being beneficial in the short run. By the same token, some firms may want to propose a merger that is myopically unprofitable, because they expect it to have a positive effect on merger activity in the future.

The second question is whether the antitrust authority wants to approve a merger that raises consumer surplus given the current market structure. Again, the answer to this question would be straightforward in a static world. However, in a dynamic environment, the antitrust authority understands that approving a merger today will have an effect on the set of mergers that will be proposed in the future. Therefore, the antitrust authority may well find it beneficial to block a merger that is consumer surplus-increasing given the current market structure, or to approve a merger that is consumer surplus-decreasing given the current market structure.

In a related paper, (Nocke, 2010) provide a simple answer to this complicated question. They make three assumptions: 1) merger opportunities are disjoint sets, i.e., a firm can take part in at most one merger; 2) if a merger is not approved today, then this merger will remain feasible in all subsequent periods; 3) firms compete in quantities and products are homogeneous. Under these assumptions, they find that, in equilibrium, the antitrust authority optimally adopts a fully myopic merger policy, which involves clearing a merger today if and only if this merger does not lower consumer surplus given the current market structure. In addition, firms also optimally behave in a myopic way, by delegating their merger decisions to the antitrust authority.

We relax the third assumption in (Nocke, 2010), by assuming instead that firms own multiple products, and that they compete in prices with CES or logit demands. Using our aggregative games tools, we find that the results in (Nocke, 2010) survive, i.e., a myopic merger policy continues to be dynamically optimal.

3. Conclusion

My stay at KIER has been extremely fruitful, and I would like to thank once again my sponsor, the International Research Unit of Advanced Future Studies, for providing financial support, Professor Sekiguchi for inviting me, and KIER for their hospitality.

4. References

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