

ISSN 1881-6193

**RIMS Kôkyûroku Bessatsu B43**

# Potential Theory and its Related Fields

edited by Kentaro Hirata

September, 2013

Research Institute for Mathematical Sciences

Kyoto University

*RIMS Kôkyûroku Bessatsu B43*

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*September 3 ~7, 2012*

*edited by Kentaro Hirata*

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*Research Institute for Mathematical Sciences*

*Kyoto University, Kyoto, Japan*

The papers presented in this volume of RIMS Kôkyûroku Bessatsu are in final form and refereed.  
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# Preface

This volume collects thirteen selected research papers contributed by the speakers of the RIMS workshop “Potential Theory and its Related Fields”. All papers have been refereed and are in final form.

The workshop was held at Kyoto University from 3 to 7 September, 2012, and was organized by Hiroaki Aikawa (Hokkaido University), Kentaro Hirata (Hiroshima University), Jun Kigami (Kyoto University) and Masaharu Nishio (Osaka City University). The aim was to overview recent developments in potential theory and its related fields. There were 32 invited and contributed lectures and 70 participants during the meeting.

I would like to express my sincere gratitude to all the participants, particularly the speakers, and anonymous referees for their cooperation.

Hiroshima, July 2013

Kentaro Hirata

# RIMS workshop

## Potential Theory and its Related Fields

Dates: September 3 – 7, 2012

Venue: Research Building No. 8 Lecture Room 2,  
Faculty of Engineering, Kyoto University

Organizers: Kentaro Hirata (Akita, Chair), Hiroaki Aikawa (Sapporo),  
Jun Kigami (Kyoto), Masaharu Nishio (Osaka)

### Program

#### Monday, September 3

**10:00 – 10:15**    Opening

**10:15 – 11:15**    **John Lewis**

$p$  harmonic measure in simply connected domains revisited

**11:30 – 12:30**    **Atsushi Kasue**

Quasi-monomorphisms and  $p$ -harmonic functions with finite Dirichlet sum

**14:00 – 15:00**    **Nageswari Shanmugalingam**

Constructing a prime end boundary for non-simply connected domains in Euclidean spaces and metric measure spaces

**15:15 – 15:45**    **Vadim Kaimanovich**

Electrical network reduction and the finite Dirichlet problem

**15:55 – 16:25**    **Hiroaki Masaoka**

On harmonic Hardy-Orlicz spaces

**16:40 – 17:10**    **Ryozi Sakai**

A characterization of entire functions and approximation

**17:20 – 17:50**    **Yûsuke Okuyama**

Equilibrium measures for uniformly quasiregular dynamics

## Tuesday, September 4

**9:15 – 10:15 Masanori Hino**

Geodesic distances and intrinsic distances on some fractal sets

**10:30 – 11:30 Laurent Saloff-Coste**

Heat kernel estimates on inner uniform domains

**11:45 – 12:45 Kazumasa Kuwada**

Applications of Hopf-Lax formulae to analysis of heat distributions

**14:00 – 15:00 Anders Björn**

The Perron method for  $p$ -harmonic functions: Resolutivity and invariance results

**15:15 – 15:45 Tsubasa Itoh**

Modulus of continuity of  $p$ -Dirichlet solutions in a metric measure space

**15:55 – 16:25 Yoshihiro Mizuta**

Sobolev's inequality for Riesz potentials in Lorentz spaces of variable exponent

**16:40 – 17:10 Tanran Zhang**

A potential theoretic approach to the curvature equation

**17:20 – 17:50 Sachiko Hamano**

Variation for the metrics induced by Schiffer and harmonic spans

## Wednesday, September 5

**9:15 – 10:15 Eleutherius Symeonidis**

A concept of harmonicity for families of planar curves

**10:30 – 11:30 Tomas Sjödin**

Two-phase quadrature domains and harmonic balls

## Thursday, September 6

**9:15 – 10:15 John Mackay**

The quasisymmetric geometry of boundaries of relatively hyperbolic groups

**10:30 – 11:30 Bruce Kleiner**

Asymptotic geometry, harmonic functions, and finite generation of isometry groups

**11:45 – 12:45 Eero Saksman**

Rotation of planar quasiconformal maps

**14:00 – 15:00 Mario Bonk**

Non-linear potential theory and the Rickman-Picard theorem

**15:15 – 15:45 Naotaka Kajino**

Weyl's Laplacian eigenvalue asymptotics for the measurable Riemannian structure on the Sierpiński gasket

**15:55 – 16:25 Tetsu Shimomura**

Hardy averaging operator on generalized Banach function spaces

**16:40 – 17:10 Kiyoki Tanaka**

A representation for harmonic Bergman function and its application

**17:20 – 17:50 Fumi-Yuki Maeda**

Mean continuity for potentials of functions in Musielak-Orlicz spaces

## Friday, September 7

**9:15 – 10:15 Jeremy Tyson**

Distortion of dimension by projections and Sobolev mappings

**10:30 – 11:30 Yoshihiro Sawano**

Morrey spaces and fractional integral operators

**11:45 – 12:45 Thomas Ransford**

Computation of capacities

**14:00 – 15:00 Tom Carroll**

Isoperimetric inequalities for a Sobolev Constant

**15:15 – 15:45 Minoru Yanagishita**

The first boundary value problem of the biharmonic equation for the half-space

**15:55 – 16:25 Hiroaki Aikawa**

Extended Harnack inequalities with exceptional sets and a boundary Harnack principle

**16:35 – 17:05 Kentaro Hirata**

Heat kernel estimates and growth estimates of solutions of semilinear heat equations

**17:10 – 17:20** Closing

This workshop is supported by Research Institute for Mathematical Sciences, Kyoto University and the following JSPS Grant-in-Aid for Scientific Research:

(A) #20244007 (Principal researcher: Hiroaki Aikawa, Hokkaido University),

(B) #23340025 (Principal researcher: Jun Kigami, Kyoto University),

(C) #23540220 (Principal researcher: Masaharu Nishio, Osaka City University).

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