<table>
<thead>
<tr>
<th>Title</th>
<th>Explicit formulas for Hasse-Witt invariants of cyclotomic function fields with conductor of degree two (Algebraic Number Theory and Related Topics 2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>SHIOMI, Daisuke</td>
</tr>
<tr>
<td>Citation</td>
<td>数理解析研究所講究録別冊 = RIMS Kokyuroku Bessatsu (2013), B44: 213-222</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2013-12</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/209073">http://hdl.handle.net/2433/209073</a></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
</tr>
</tbody>
</table>

Kyoto University
Explicit formulas for Hasse-Witt invariants of cyclotomic function fields with conductor of degree two

By

Daisuke SHIOMI *

§ 1. Introduction

Let $p$ be a prime. Let $\mathbb{F}_q$ be the field with $q = p^r$ elements. Let $k = \mathbb{F}_q(T)$ be the rational function field over $\mathbb{F}_q$, and let $A = \mathbb{F}_q[T]$ be the polynomial subring of $k$. For a monic polynomial $m \in A$, we denote the $m$-th cyclotomic function field by $K_m$. For definitions and basic properties of cyclotomic function field, see [Go], [Ha], and [Ro].

Let us denote by $J_m$ the Jacobian of $K_m\overline{\mathbb{F}}_q$, where $\overline{\mathbb{F}}_q$ is an algebraic closure of $\mathbb{F}_q$. For a prime $l$, it is well-known that the $l$-primary subgroup $J_m(l)$ of $J_m$ satisfies

$$J_m(l) \cong \begin{cases} \bigoplus_{i=1}^{2g_m} \mathbb{Q}_l / \mathbb{Z}_l & \text{if } l \neq p, \\ \bigoplus_{i=1}^{\lambda_m} \mathbb{Q}_p / \mathbb{Z}_p & \text{if } l = p, \end{cases}$$

where $g_m$ is the genus of $K_m$, and $\lambda_m$ is an integer where $0 \leq \lambda_m \leq g_m$. The integer $\lambda_m$ is called the Hasse-Witt invariant of $K_m$.

Kida and Murabayashi gave an explicit formula for $g_m$ for all monic polynomial $m \in A$ as Corollary 1 in the section 2 of [K-M]. Applying their genus formula to the cases of $\deg m = 1$ and $\deg m = 2$, we have gotten

Theorem 1.1. Let $m \in A$ be a monic polynomial.

(1) If $\deg m = 1$, then we have $g_m = 0$. 

2000 Mathematics Subject Classification(s): 11M38, 11R60. 
Key Words: cyclotomic function field, Jacobian variety. 
*Department of Mathematical Sciences, Faculty of Science, Yamagata University, 1-4-12 Kojirakawamachi, Yamagata, 990-8560, Japan. e-mail: shiomi@sci.kj.yamagata-u.ac.jp

© 2013 Research Institute for Mathematical Sciences, Kyoto University. All rights reserved.
(2) If \( \deg m = 2 \), then we have

\[
g_m = \begin{cases} 
\frac{(q-2)(q+1)}{2} & \text{if } m \text{ is irreducible}, \\
\frac{(q-2)(q-1)}{2} & \text{if } m = P^2 \text{ where } P \text{ is a monic polynomial of degree one,} \\
\frac{(q-2)(q-3)}{2} & \text{if } m = PQ \text{ where } P, Q \text{ are distinct monic polynomials of degree one.}
\end{cases}
\]

Next we consider the Hasse-Witt invariant case. The main theorem of this paper is the following results.

**Theorem 1.2.** Let \( m \in A \) be a monic polynomial, and \( q = p^r \).

(1) If \( \deg m = 1 \), then we have \( \lambda_m = 0 \).

(2) If \( \deg m = 2 \), then we have

\[
\lambda_m = \begin{cases} 
\left( \frac{p(p+1)}{2} \right)^r - q - 1 & \text{if } m \text{ is irreducible,} \\
0 & \text{if } m = P^2 \text{ where } P \text{ is a monic polynomial of degree one,} \\
\left( \frac{p(p+1)}{2} \right)^r - 3q + 3 & \text{if } m = PQ \text{ where } P, Q \text{ are distinct monic polynomials of degree one.}
\end{cases}
\]

**Remark.** Noting that \( \lambda_m \leq g_m \), we have \( \lambda_m = 0 \) if \( \deg m = 1 \). Hence we obtain the first assertion of Theorem 1.2.

We call \( K_m \) ordinary if \( \lambda_m = g_m \). By comparing Theorem 1.1 and 1.2, we obtain the following results.

**Corollary 1.3.** Let \( m \in A \) be a monic polynomial of degree two.

(1) Assume that \( q = p \). Then \( K_m \) is ordinary if and only if one of the following conditions holds: (a) \( q = 2 \), (b) \( m \) is irreducible, (c) \( m = PQ \) where \( P, Q \) are distinct polynomials of degree one.

(2) Assume that \( q = p^r (r \geq 2) \). Then \( K_m \) is not ordinary.
The second assertion of Corollary 1.3 is generalized as follows:

**Theorem 1.4.** Assume that \( q = p^r (r \geq 2) \), and \( m \in A \) is a monic polynomial. Then \( K_m \) is ordinary if and only if \( \deg m = 1 \). In this case, \( K_m \) is rational.

### §2. Preparations

In this section, we review some basic facts for zeta functions, \( L \)-functions, and power residue symbols.

Let us define the zeta function of \( K_m \) as follows

\[
\zeta(s, K_m) = \prod_{p: \text{prime}} \left(1 - \frac{1}{Np^s}\right)^{-1},
\]

where \( p \) runs through all primes of \( K_m \), and \( Np \) is the number of elements of the reduce class field of \( p \). By the standard fact about the zeta function, there is a polynomial \( Z_m(u) \in \mathbb{Z}[u] \) such that

\[
\zeta(s, K_m) = \frac{Z_m(q^{-s})}{(1-q^{-s})(1-q^{1-s})}.
\]

Then we have the following relation between \( \lambda_m \) and \( Z_m(u) \).

**Theorem 2.1.** (cf. Proposition 11.20 in [Ro]).

\[
\lambda_m = \deg \overline{Z}_m(u),
\]

where \( \overline{Z}_m(u) \in \mathbb{F}_p[u] \) is the reduction of \( Z_m(u) \) modulo \( p \).

Let \( X_m \) be the group of all primitive Dirichlet characters modulo \( m \). Then \( \zeta(s, K_m) \) can be written as follows

\[
\zeta(s, K_m) = \left\{ \prod_{\chi \in X_m} L(s, \chi) \right\} (1-q^{-s})^{-\frac{[K_m:k]}{q-1}},
\]

where \( L(s, \chi) = \sum_{a \in \mathbb{F}_q^\times} \chi(a)q^{-s\deg a} \). (cf. Lemma 2.1 in [Sh1]).

For a character \( \chi \in X_m \), we call \( \chi \) real if \( \chi(a) = 1 \) for all \( a \in \mathbb{F}_q^\times \). Otherwise, we call \( \chi \) imaginary.

Let \( m \in A \) be a monic polynomial of degree \( d \). For \( \chi \in X_m \), we put

\[
s_i(\chi) = \sum_{a: \text{monic, deg } a = i} \chi(a).
\]

Then it is known that
• \( s_i(\chi) = 0 \) if \( \chi \) is non-trivial and \( i \geq \deg f_\chi \),

• \( \sum_{i=0}^{d-1} s_i(\chi) = 0 \) if \( \chi \) is non-trivial and real,

where \( f_\chi \) is the conductor of \( \chi \) (cf. section 3 in [G-R]). Assume that \( d = 2 \). Then \( L(s, \chi) \) can be calculated as follows

\[
L(s, \chi) = \begin{cases} 
(1 - q^{1-s})^{-1} & \text{if } f_\chi = 1, \\
1 & \text{if } \deg f_\chi = 1, \\
1 - q^{-s} & \text{if } f_\chi = m, \text{ and } \chi \text{ is real,} \\
1 + s_1(\chi)q^{-s} & \text{if } f_\chi = m, \text{ and } \chi \text{ is imaginary.}
\end{cases}
\]

Hence we obtain

\[
(2.1) \quad Z_m(u) = \prod_{\chi \text{ imaginary}} (1 + s_1(\chi)u).
\]

In the end of this section, we review a power residue symbol. For an integer \( n \geq 2 \), let \( W_n \) be the set of all \( n \)-th roots of unity. Let \( K \) be a number field containing \( W_n \), and let \( \mathcal{O}_K \) be the ring of integers of \( K \). Let \( \mathfrak{p} \) be a prime ideal of \( K \) not dividing \( n \).

For \( \alpha \in \mathcal{O}_K \) which is prime to \( \mathfrak{p} \), there exists uniquely \( \left( \frac{\alpha}{\mathfrak{p}} \right)_n \in W_n \) satisfying

\[
\left( \frac{\alpha}{\mathfrak{p}} \right)_n \equiv \alpha^{(N\mathfrak{p} - 1)/n} \text{ mod } \mathfrak{p}.
\]

We call \( \left( \frac{\alpha}{\mathfrak{p}} \right)_n \) the power residue symbol mod \( \mathfrak{p} \) of order \( n \).

\section{§ 3. A proof of Theorem 1.2}

The purpose of this section is to prove the second assertion of Theorem 1.2.

\subsection{§ 3.1. The case (I)}

Assume that \( m \) is a monic irreducible polynomial of degree two. Take \( \gamma \in \mathbb{F}_{q^2} \) so that \( m(\gamma) = 0 \). Then \( f(T) \mapsto f(\gamma) \) gives rise to an isomorphism \( A/mA \sim \mathbb{F}_{q^2} \). Now let \( \mathfrak{p} \) be a prime ideal of \( K = \mathbb{Q}(e^{2\pi i/(q^2 - 1)}) \) dividing \( p \), and let \( \chi_\mathfrak{p} = \left( \frac{\gamma}{\mathfrak{p}} \right)_{q^2 - 1} \) be the power residue symbol mod \( \mathfrak{p} \) of order \( q^2 - 1 \). We see that \( \chi_\mathfrak{p}^n \) is real if and only if \( n \) is divisible by \( q - 1 \). Therefore, by the equality (2.1), we have

\[
(3.1) \quad Z_m(u) = \prod_{0 \leq n \leq q^2 - 2 \atop n \not\equiv 0 \text{ mod } q - 1} (1 + s_1(\chi_\mathfrak{p}^n)u).
\]
Under the identification $A/mA = \mathbb{F}_{q^2}$, we have an equality
\[ s_1(\chi_p^n) = \sum_{\alpha \in \mathbb{F}_q} (\gamma + \alpha)^n \]
in $\mathbb{F}_{q^2} = \mathcal{O}_K/p$.

For $1 \leq n \leq q^2 - 2$ ($n \equiv 0 \mod q - 1$), we consider the $q$-adic expansion $n = a(n) + b(n)q$. By the Newton formula, we have gotten
\[ \sum_{\alpha \in \mathbb{F}_q} (T + \alpha)^n = -\binom{b(n)}{q - 1 - a(n)} (T^q - T)^{a(n)+b(n)-(q-1)}, \]
as was verified by Gekeler (cf. Corollary 3.14 in [Ge]). Here $\binom{\ast}{\ast}$ is a binomial coefficient. This implies an equality
\[ s_1(\chi_p^n) = -\binom{b(n)}{q - 1 - a(n)} (\gamma^q - \gamma)^{a(n)+b(n)-(q-1)} \]
in $\mathbb{F}_{q^2}$. Notice that $\binom{b(n)}{q - 1 - a(n)} \equiv 0 \mod p$ for $a(n) + b(n) < q - 1$. Therefore, by Theorem 2.1 and the equality (3.1), we obtain
\[ (3.2) \quad \lambda_m = \# \left\{ 1 \leq n \leq q^2 - 2 : a(n) + b(n) > q - 1, \binom{b(n)}{q - 1 - a(n)} \not\equiv 0 \mod p \right\}, \]
where $\# S$ is the number of elements of a set $S$. Next we will calculate the right side of the equality (3.2). For $1 \leq n \leq q^2 - 2$, we put
\[ a(n) = a_0(n) + a_1(n)p + \cdots + a_{r-1}(n)p^{r-1}, \]
\[ b(n) = b_0(n) + b_1(n)p + \cdots + b_{r-1}(n)p^{r-1}, \]
where $0 \leq a_i(n), b_i(n) \leq p - 1$ ($i = 0, 1, ..., r - 1$). Since
\[ q - 1 - a(n) = (p - 1 - a_0(n)) + (p - 1 - a_1(n))p + \cdots + (p - 1 - a_{r-1}(n))p^{r-1}, \]
we have
\[ \binom{b(n)}{q - 1 - a(n)} \equiv \prod_{i=0}^{r-1} \binom{b_i(n)}{p - 1 - a_i(n)} \mod p. \]
Hence we obtain the following equivalence
\[ \binom{b(n)}{q - 1 - a(n)} \not\equiv 0 \mod p \iff a_i(n) + b_i(n) \geq p - 1 \quad (0 \leq i \leq r - 1). \]
We see that
\[
\left( \frac{p(p+1)}{2} \right)^r = \#\{ n \in [0, q^2 - 1] : a_i(n) + b_i(n) \geq p - 1 \ (0 \leq i \leq r - 1) \},
\]
\[
q = \#\{ n \in [0, q^2 - 1] : a(n) + b(n) = q - 1 \},
\]
\[
1 = \#\{ n \in [0, q^2 - 1] : a(n) + b(n) = 2(q - 1) \},
\]
where \([0, q^2 - 1] = \{0, 1, 2, \ldots, q^2 - 1\}\). Therefore we have
\[
\lambda_m = \left( \frac{p(p+1)}{2} \right)^r - q - 1.
\]

\section{3.2. The case (II)}

Let \( \alpha \in \mathbb{F}_q \) and \( m(T) = (T - \alpha)^2 \). Let \( \varepsilon \) denote the image of \( T - \alpha \) in \( A/\mathfrak{m}A \). Then \( f(T) \mapsto f(\alpha) + f'(\alpha)\varepsilon \) gives rise to an isomorphism \( A/\mathfrak{m}A \cong \mathbb{F}_q[\varepsilon] \). It follows that any character \( \chi : (A/\mathfrak{m}A)^\times \to \mathbb{C}^\times \) is given by \( f(T) \mapsto \eta(f(\alpha))\psi(f'(\alpha)/f(\alpha)) \), where \( \eta \) is a multiplicative character of \( \mathbb{F}_q \), and \( \psi \) is an additive character of \( \mathbb{F}_q \). Furthermore \( s_1(\chi) \) is nothing but the Gauss sum \( G(\eta^{-1}, \psi) \). It is readily seen that

- \( \chi \) is trivial \( \iff \eta \) is trivial and \( \psi \) is trivial,
- \( \deg f_{\chi} = 1 \iff \eta \) is non-trivial and \( \psi \) is trivial,
- \( f_{\chi} = m \) and \( \chi \) is real \( \iff \eta \) is trivial and \( \psi \) is non-trivial,
- \( f_{\chi} = m \) and \( \chi \) is imaginary \( \iff \eta \) is non-trivial and \( \psi \) is non-trivial.

By the equality (2.1), we have
\[
Z_m(u) = \prod (1 + G(\eta^{-1}, \psi)u),
\]
where \( \eta \) runs through all non-trivial multiplicative characters of \( \mathbb{F}_q \), and \( \psi \) runs through all non-trivial additive characters of \( \mathbb{F}_q \).

Let \( p \) be a prime ideal of \( \mathbb{Q}(e^{2\pi i/p}, e^{2\pi i/(q-1)}) \) dividing \( p \). If \( \eta \) is non-trivial and \( \psi \) is non-trivial, then we have \( G(\eta^{-1}, \psi) \in \mathfrak{p} \) by the Stickelberger theorem for Gauss sums (cf. Theorem 11.2.1 in [B-E-W]). Hence we obtain \( \lambda_m = 0 \) by Theorem 2.1. This completes the proof of the case (II).

\textbf{Remark.} I appreciate that the referee taught me the above proof. We can generalize the case (II) as follows: \( \lambda_{P^n} = 0 \ (n \geq 0) \) if \( P \) is a monic polynomial of degree one (cf. Proposition 3.2 in [Sh1]).
§ 3.3. The case (III)

Let $\alpha, \beta \in \mathbb{F}_q$ $(\alpha \neq \beta)$ and $m(T) = (T - \alpha)(T - \beta)$. Then $f(T) \mapsto (f(\alpha), f(\beta))$ gives rise to an isomorphism $(A/mA)^\times \rightarrow \mathbb{F}_q^\times \times \mathbb{F}_q^\times$. It follows that any character $\chi : (A/mA)^\times \rightarrow \mathbb{C}^\times$ is given by $f(T) \mapsto \chi_1(f(\alpha))\chi_2(f(\beta))$, where $\chi_1$ and $\chi_2$ are the multiplicative characters of $\mathbb{F}_q$. Furthermore we have an equality

$$s_1(\chi) = \chi_2(-1)(\chi_1 \chi_2)(\alpha - \beta)J(\chi_1, \chi_2),$$

where $J(\chi_1, \chi_2)$ denotes the Jacobi sum associated to $\chi_1$ and $\chi_2$. It is readily seen that

- $\chi$ is trivial $\Leftrightarrow \chi_1$ and $\chi_2$ are trivial,
- $\deg f_\chi = 1 \Leftrightarrow$ one of $\chi_1$ and $\chi_2$ is non-trivial and the other is trivial,
- $f_\chi = m$ and $\chi$ is real $\Leftrightarrow \chi_1$ and $\chi_2$ are non-trivial and $\chi_1 \chi_2$ is trivial,
- $f_\chi = m$ and $\chi$ is imaginary $\Leftrightarrow \chi_1$, $\chi_2$, and $\chi_1 \chi_2$ are non-trivial.

Let $\mathfrak{p}$ be a prime ideal of $\mathbb{Q}(e^{2\pi i/(q-1)})$ above $p$. Let $\chi_\mathfrak{p} = \left(\frac{-1}{\mathfrak{p}}\right)_{q-1}$ be the power residue symbol mod $\mathfrak{p}$ of order $q - 1$. Then we have the following one to one corresponding

$$\left\{ \chi \in X_m : \chi \text{ is imaginary of conductor } m \right\} \overset{1:1}{\longleftrightarrow} \left\{ (\chi_\mathfrak{p}^{n_1}, \chi_\mathfrak{p}^{n_2}) : 1 \leq n_1, n_2 \leq q - 2, \text{ } n_1 + n_2 \not\equiv 0 \text{ mod } q - 1 \right\}.$$

By the equalities (2.1) and (3.3), we have

$$\lambda_m = \deg(Z_m(u) \mod \mathfrak{p}) = \sum_{\substack{1 \leq n_1 \leq q - 2 \\ 1 \leq n_2 \leq q - 2 \\ n_1 + n_2 \not\equiv 0 \mod q - 1}} \deg(1 + \chi_\mathfrak{p}^{n_2}(-1)\chi_\mathfrak{p}^{n_1+n_2}(\alpha - \beta)J(\chi_\mathfrak{p}^{n_1}, \chi_\mathfrak{p}^{n_2})u \mod \mathfrak{p}).$$

Next we will calculate $\text{ord}_\mathfrak{p}J(\chi_\mathfrak{p}^{n_1}, \chi_\mathfrak{p}^{n_2})$, where $\text{ord}_\mathfrak{p}$ is the valuation of $\mathfrak{p}$. For an integer $n \in \mathbb{Z}$, we define $L(n) \in \mathbb{Z}$ as follows

$$0 \leq L(n) < q - 1, \quad L(n) \equiv n \mod q - 1.$$

We consider the $p$-adic expansion

$$L(n) = a_0(n) + a_1(n)p + \cdots + a_{r-1}(n)p^{r-1} \quad (0 \leq a_i(n) < p).$$

Define $l(n)$ as follows

$$l(n) = a_0(n) + a_1(n) + \cdots + a_{r-1}(n).$$
For $1 \leq n_1, n_2 \leq q - 2 \ (n_1 + n_2 \neq q - 1)$, it is known as the Stickelberger theorem that
\[
\text{ord}_p J(\chi_p^{n_1}, \chi_p^{n_2}) = r - \frac{l(n_1) + l(n_2) - l(n_1 + n_2)}{p - 1}
\]

\[
= r - \# \left\{ 0 \leq i \leq r - 1 : L(n_1p^i) + L(n_2p^i) > q - 1 \right\}
\]

(cf. Corollary 11.2.4 and Theorem 11.2.9 in [B-E-W]). Noting that
\[
J(\chi_p^{n_1}, \chi_p^{n_2})J(\chi_p^{q-1-n_1}, \chi_p^{q-1-n_2}) = q,
\]
we obtain
\[
\lambda_m = \# \left\{ (n_1, n_2) \in [1, q-2]^2 : n_1 + n_2 \not\equiv 0 \mod q-1, \right. \\
\left. \text{ord}_p J(\chi_p^{n_1}, \chi_p^{n_2}) = 0 \right\}
\]
\[
= \# \left\{ (n_1, n_2) \in [1, q-2]^2 : n_1 + n_2 \not\equiv 0 \mod q-1, \right. \\
\left. \text{ord}_p J(\chi_p^{n_1}, \chi_p^{n_2}) = r \right\}
\]
\[
= \# \left\{ (n_1, n_2) \in [1, q-2]^2 : n_1 + n_2 \not\equiv 0 \mod q-1, \right. \\
\left. l(n_1) + l(n_2) = l(n_1 + n_2) \right\}
\]

by the Stickelberger theorem. We see that
\[
l(n_1) + l(n_2) = l(n_1 + n_2)
\]
\[
\iff L(n_1p^{r-1-i}) + L(n_2p^{r-1-i}) \leq q - 1 \ (i = 0, 1, 2, ..., r - 1)
\]
\[
\iff a_i(n_1) + a_i(n_2) \leq p - 1 \ (i = 0, 1, 2, ..., r - 1).
\]

Therefore we have
\[
\lambda_m = \# \left\{ (n_1, n_2) \in [1, q-2]^2 : n_1 + n_2 \not\equiv 0 \mod q-1, \\
a_i(n_1) + a_i(n_2) \leq p - 1 \ (0 \leq i \leq r - 1) \right\}.
\]

Notice that
\[
\left( \frac{p(p + 1)}{2} \right)^r = \# \left\{ (n_1, n_2) \in [0, q-1]^2 : a_i(n_1) + a_i(n_2) \leq p - 1 \ (0 \leq i \leq r - 1) \right\},
\]
\[
3q - 3 = \# \left\{ (n_1, n_2) \in [0, q-1]^2 : n_1 = 0 \text{ or } n_2 = 0 \text{ or } n_1 + n_2 = q - 1 \right\}.
\]

Hence we have
\[
\lambda_m = \left( \frac{p(p + 1)}{2} \right)^r - 3q + 3.
\]

§ 4. A proof of Theorem 1.4

The purpose of this section is to prove Theorem 1.4.
Lemma 4.1. Let $m_1, m_2$ be monic polynomials such that $m_1 | m_2$. If $K_{m_2}$ is ordinary, then $K_{m_1}$ is also ordinary.

This follows from the following general result.

Lemma 4.2. Let $k$ be a field of characteristic $p$, and let $\pi : Y \to X$ be a finite covering of projective non-singular curves over $k$. If $Y$ is ordinary, then $X$ is also ordinary.

Proof. We give a proof for the reader’s convenience. Let $A$, $B$ be the Jacobians of $X$, $Y$, respectively. Then $\pi$ induces the homomorphism of abelian varieties $\pi^* : A \to B$, and the embedding of $p$-divisible groups $\pi^* : T_p A \to T_p B$. Assume that $Y$ is ordinary. Then each slope of $T_p B$ is only 0 or 1. Hence each slope of $T_p A$ is only 0 or 1. Therefore $X$ is also ordinary.

\[ \square \]

Now we prove Theorem 1.4.

Proof. Assume that $\deg m = 1$. Then we have $g_m = \lambda_m = 0$. Hence $K_m$ is ordinary. Conversely, we assume that $K_m$ is ordinary. Let $m = Q_1^{n_1}Q_2^{n_2} \cdots Q_t^{n_t}$ be the irreducible factorization, where $Q_1, Q_2, ..., Q_t$ are distinct monic polynomials. By Lemma 4.1, $K_{Q_i^{n_i}}$ is ordinary for each $i$. It follows from Corollary 1.3 and Corollary 3.1 in [Sh2] that $\deg Q_i = 1$ and $n_i = 1$. Now suppose that $t \geq 2$. Again by Lemma 4.1, $K_{Q_1Q_2}$ is ordinary, which contradicts to the second assertion of Corollary 1.3.

\[ \square \]

Acknowledgements

The author wish to thank the referee for many helpful suggestions.

References


