Vertical Separation between Competing Manufacturers and Their Retailers

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ABSTRACT

We examine an industry in which manufacturers set their wholesale prices (i.e., they engage in price competition) and retailers select order sizes (i.e., they engage in quantity competition). We consider the simplest example of such price-quantity competition, in which each of two manufacturers of a homogenous product sells through a different retailer, and the two retailers together face a non-linear demand function. We show that under this price-quantity competition, manufacturers vertically separate from their retailers and equilibrium wholesale prices are set below the marginal cost of production. The manufacturers’ profits are lower than under vertical integration (i.e., a prisoner’s dilemma occurs) and economic welfare is greater. Equilibrium wholesale prices decrease with demand expansion. The wholesale prices set by the manufacturers are strategic substitutes.

Keywords: Price-quantity competition; Vertical separation; Vertical integration; Franchise fee

JEL Classification: L22, L23, L13

1 Introduction

Why might competing manufacturers vertically separate from their retailers? Many papers consider this question and reach conflicting answers. Coase (1937) and Williamson (1975, 1985, 1986, 1989), for example, argue that vertical integration can reduce transaction costs and increase channel profit, but that vertical integration itself has inherent costs that might negate its otherwise propitious...
effects. Meyer et al. (1992) argue that vertical separation occurs if the influence cost, which represents loss of efficiency of organization activities, is large. Strategic effects among horizontal competitors also matter. Bonanno and Vickers (1988), Rey and Stiglitz (1988), and Bettignies (2006) point out, while considering the strategic relationships between channels, that manufacturers may vertically separate from their retailers to soften their competition with one another.

Our paper offers a new perspective on manufacturers’ vertical separation from their retailers. Its starting point is the very natural presumption that manufacturers set their wholesale prices (i.e., they engage in price competition) and retailers select order sizes (i.e., they engage in quantity competition). We consider the simplest example of such price-quantity competition, in which each of two manufacturers of a homogeneous product sells through a different retailer, and the two retailers together face a non-linear demand function. Saggi and Vettas (2002), which is most related to our paper, posit a similar vertical channel facing a linear demand function (see also Flath, 1983; and Motta, 2004). In this paper, we extend their model to a more general framework and examine the manufacturers’ channel strategy: vertical separation with franchise fee, vertical separation without franchise fee, or vertical integration.

Our main result is that, with price-quantity competition, the equilibrium outcome is for the two manufacturers to vertically separate from their retailers. In this equilibrium, the wholesale prices are below the marginal production cost. The manufacturers’ profits are lower than under vertical integration, and economic welfare is greater. In effect, vertical separation in this setting amounts to a prisoner’s dilemma. Moreover, when the inverse demand function is linear, equilibrium wholesale prices decrease with demand expansion. The wholesale prices set by the manufacturers are strategic substitutes.

The rest of the paper is organized as follows. Section 2 shows the model, section 3 examines whether manufacturers separate their retailers, and the last section concludes our study.

2 The model

Two manufacturers and their two retailers operate in a market. Each manufacturer $i$ ($i = 1, 2$) produces a homogeneous good and sells it to consumers through corresponding retailer $i$ ($i = 1, 2$). The inverse demand function of the retail market is given by

$$p = p(Q, a), \quad p_Q < 0, \quad p_a > 0$$

(1)

where $p$ is the retail price, $Q = q_1 + q_2$ the total sales, $q_i$ the sales of retailer $i$, and $a$ a parameter representing demand size. The symbols $p_Q$ and $p_a$ indicate the partial derivative with respect to $Q$ and $a$, respectively. We assume that the marginal cost of production is a positive constant, $c$. 

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We consider a two-stage game. In the first stage, the manufacturers simultaneously determine their wholesale prices and franchise fees. We describe this pricing strategy as a two-part pricing system, denoted by $F$. In the second stage, the retailers choose their sales quantities. We solve the game by backward induction to obtain the sub-game perfect Nash equilibrium.

2.1 The second stage

In the second stage, retailer $i$, after accepting the wholesale price $w_i$ and the franchise fee $F_i$, determines the sales quantity $q_i$, given the rival’s sales quantity $q_j$. The decision problem of retailer $i$, to maximize its profit $\pi_i$, is

$$\max_{q_i} \pi_i = (p(Q, a) - w_i)q_i - F_i.$$ 

The first- and second-order conditions are

$$p + p_q q_i - w_i = 0, \quad i = 1, 2 \quad (2)$$

$$2p_{qq} + p_{qq} q_i < 0. \quad (3)$$

From (2), we obtain the sales quantity of retailer $i$, $\hat{q}(w', w_1)$, as a function of $w_i$ and $w_j$. We denote the industry sales quantity and the retail price by $\hat{Q}(w', w_1)$ and $\hat{p}(w', w_1)$, where the hat (’) represents the equilibrium values in the second stage. Assuming the interior solutions are at equilibrium, we have $\hat{p} + \hat{p}_w q_i - w_i = \hat{p} + \hat{p}_w q_i - w_i = 0$. Therefore, we have $\hat{q} - \hat{q}' = (w' - w_i)/\hat{p}_w$ and $\hat{q} < \hat{q}'$ if and only if $w' < w_i$. That is, the retailer who faces a lower wholesale price has a larger share of the market.

Differentiating (2) with respect to $w_1$, $w_2$, and $a$, we obtain

$$\begin{bmatrix} 2\hat{p}_w + \hat{q}_w \hat{p}_w & \hat{p}_w + \hat{q}_w \hat{p}_w \\ \hat{p}_w + \hat{q}_w \hat{p}_w & 2\hat{p}_w + \hat{q}_w \hat{p}_w \end{bmatrix} \begin{bmatrix} dq_i \\ dq_i' \end{bmatrix} = \begin{bmatrix} dw_1 - (\hat{p}_w + \hat{p}_w \hat{q}_w)da \\ dw_2 - (\hat{p}_w + \hat{p}_w \hat{q}_w)da \end{bmatrix}$$

where $p_{qq}$ is the second-order partial derivative with respect to $Q$. Assuming that Hahn’s stability condition, $2\hat{p}_w + \hat{q}_w \hat{p}_w < \hat{p}_w + \hat{q}_w \hat{p}_w < 0$ holds (see Hahn, 1962), we have

$$2\hat{p}_w + \hat{q}_w \hat{p}_w < \hat{p}_w + \hat{q}_w \hat{p}_w < 0$$

$$D \equiv (2\hat{p}_w + \hat{q}_w \hat{p}_w)(2\hat{p}_w + \hat{q}_w \hat{p}_w) - (\hat{p}_w + \hat{q}_w \hat{p}_w)(\hat{p}_w + \hat{q}_w \hat{p}_w) = \hat{p}_w (3\hat{p}_w + \hat{Q}_w \hat{p}_w) > 0.$$ 

\(^1\)For notational simplicity, we do not write $a$. 

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Then, we have

\[
\frac{d\hat{q}_i}{dw_i} = \frac{(2\hat{p}_o + \hat{q}'\hat{p}_o)}{D} < 0
\] (5)

\[
\frac{d\hat{q}_i}{dw_i} = -\frac{(\hat{p}_o + \hat{q}'\hat{p}_o)}{D} > 0
\] (6)

\[
\frac{d\hat{Q}}{dw} = \hat{p}_o / D < 0
\] (7)

Equations (5)–(7) imply that the quantity sold in the channel and the total industry sales quantity both decrease as own-wholesale price rises, while the rival’s sales quantity increases.

If the symmetric equilibrium \( (\hat{q}_i = \hat{q}_j) \) prevails, we have

\[
\frac{d\hat{q}_i}{da} = -\frac{\hat{p}_o (\hat{p}_a + \hat{q}'\hat{p}_o)}{D} > 0 \quad \text{iff} \quad \frac{dMR}{da} = \frac{d(p + q'p_o)}{da} > 0
\] (8)

where \( MR_i \) is the marginal revenue of retailer \( i \) and \( p_{oa} \) is the cross partial derivative with respect to \( Q \) and \( a \). Equation (8) shows that demand expansion increases both retailers’ sales quantities if and only if the expansion raises the conjectural marginal revenues of the retailers.

The gross profits of retailer \( i \) is given by

\[
\hat{\pi}_i (w_i, w’) = \hat{q}_i (w_i, w’) \left( \hat{p}_o (\hat{q}_i (w_i, w’) + \hat{q}_j (w_i, w’)) - w’ \right).
\]

Therefore, from (2), we have

\[
\hat{\pi}_i' = \hat{p}_o \hat{q}_i \hat{q}_i’ - \hat{q}_i' < 0
\] (9)

\[
\hat{\pi}_i’ = \hat{q}_i’ \left( \hat{p}_o (Q) - w’ \right) + \hat{q}_j’ \left( \hat{p}_o (\hat{q}_i’ + \hat{q}_j’) \right)
\]

\[
= -\hat{q}_i’ \hat{p}_o \hat{q}_i’ + \hat{q}_j’ \left( \hat{p}_o (\hat{q}_i’ + \hat{q}_j’) \right)
\]

\[
= \hat{p}_o \hat{q}_i’ \hat{q}_j’ > 0.
\] (10)

Equations (9) and (10) indicate that a rise in the wholesale price reduces the affiliated retailer’s profit but enhances the rival’s profit.

### 2.2 The first stage

In the first stage, manufacturer \( i \) determines the wholesale price \( w’ \) and the franchise fee \( F’ \) under the condition that the affiliated retailer earns a non-negative profit. The decision problem of manufacturer \( i \) is

\[
\max_{w’} \Pi’ = \hat{q}’ (w’ - c) + F’, \ \text{s.t.} \ \pi’ \geq 0.
\]
Since the above constraint is binding in equilibrium, the decision problem reduces to\(^3\)

\[
\max_{w^i} \Pi^i = \hat{q}^i(w^i, w^i)(\hat{p}(\hat{q}^i(w^i, w^i) + \hat{q}^i(w^i, w^i)w^i, w^i)) - c.
\]

In the first stage, the first- and second-order conditions are

\[
\Pi''_{ii} = \hat{q}^i\hat{p}_q(\hat{q}^i_\omega + \hat{q}^i_\sigma) + (\hat{p} - c)\hat{q}^i_\omega = 0, \quad i = 1, 2 \tag{11}
\]

\[
\Pi''_{w_i} < 0. \tag{12}
\]

From (11), we have the equilibrium wholesale price \(w^i_F\) in the first stage\(^3\).

From (2), we can rearrange (11) as follows:

\[
\Pi''_{w_i} = (\hat{p} + \hat{q}^i\hat{p}_q - c)(dq^i / dw^i) + \hat{q}^i\hat{p}_q(dq^i / dw^i)
\]

\[
= (\hat{p} + \hat{q}^i\hat{p}_q - w^i)(dq^i / dw^i) + (w^i - c)(dq^i / dw^i) + \hat{q}^i\hat{p}_q(dq^i / dw^i)
\]

\[
= (w^i - c)\hat{q}^i_{\omega w} + \hat{q}^i\hat{p}_q\hat{q}^i_{\omega i} = 0. \tag{13}
\]

From (13), we obtain

\[
w^i - c = -\hat{q}^i\hat{p}_q\hat{q}^i_{\omega i} < 0. \tag{14}
\]

Therefore, we have \(w^i < c\). That is, the wholesale price is set below the marginal cost of production \(c\). As long as \(w^i < c\) holds, the two manufacturers charge positive franchise fees to earn non-negative profits. Thus, we have the following proposition:

**Proposition 1.** When price-quantity competition occurs, wholesale prices are set below the marginal cost of production, and the franchise fees are positive.

The impact of the demand expansion on the wholesale price is given by

\[
dw^i / da = \left(\Pi''_{w_i w^i} - \Pi''_{w_i w^i}\right)\Pi''_{w_i a} / E
\]

\[
= -\Pi''_{w_i a} / \left(\Pi''_{w_i w^i} + \Pi''_{w_i w^i}\right)
\]

where \(E \equiv \Pi''_{w_i w^i} + \Pi''_{w_i w^i} - \Pi''_{w_i w^i} - \Pi''_{w_i w^i}\) (see Appendix 1). From Appendix 1, if \(\Pi''_{w_i} < 0 (\Pi''_{w_i} > 0)\) holds, then the wholesale prices are strategic substi-
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Proposition 2. Suppose that the stability condition $E > 0$ holds. Then, demand expansion causes each manufacturer to lower its wholesale price when $\Pi_{w_i}^i < 0$ and raise its wholesale price when $\Pi_{w_i}^i > 0$.

The intuition behind proposition 2 is as follows. Suppose that the demand expands. Then, manufacturers who can charge franchise fees increase their retail sales quantities to maximize their own-channel profit. However, because sales quantities are determined by the retailers, the manufacturers must lower the wholesale prices to increase orders. They can extract the resulting retailer profit through franchise fees. Moreover, an increase in one’s wholesale price implies a demand expansion for the rival manufacturer (channel). Thus, the manufacturer cuts the wholesale price in the face of the rival’s wholesale price increase. This is a completely different scenario compared to usual price competition.

Suppose that the inverse demand function is linear; i.e., $p(Q, a) = a - bQ$, $(a > 0, b > 0)$. Then, the marginal revenue of retailers in the second stage becomes $MR^i = a - 2Q$, and thus, we have $dMR^i/da = 1 > 0$ and $dq/da = -\hat{p}_Q(\hat{p}_Q + \hat{q}_Q)/D = 1/(3b) > 0$. In addition, the profit of manufacturer $i$ in the first stage is written as $\Pi = (a + w^i + w^j - 3c)(a - 2w^i + w^j)/(9b)$, so we have the reaction function $w^i(w^j) = (a - w^j + 6c)/4$ and the equilibrium wholesale price $w^i = (a + 6c)/5$. Therefore, we obtain $dw^i/dw^j = -1/4 < 0$, $dw^i/da = -1/5 < 0$, and $\Pi_{w_i}^i = -1/(9b) < 0$. We summarize these results as follows:

Corollary 1. If the inverse demand function is linear, the equilibrium wholesale prices are lowered by demand expansion and are strategic substitutes. The equilibrium wholesale prices are below the marginal cost (i.e., $w < c$).

3 Endogenous determination of channel structure

In this section, we investigate whether each upstream manufacturer chooses vertical separation or vertical integration in equilibrium. To address this issue, we first examine the equilibrium in the following two cases: (1) linear pricing, denoted by $L$, where each manufacturer vertically separates from its own retailer but cannot charge franchise fees; (2) vertical integration, denoted by $V$, where each manufacturer vertically integrates with its own retailer.
### 3.1 Linear pricing: L

Even without franchise fees, the retailer’s profit-maximizing problem is the same as the problem with franchise fees. The problem of manufacturer $i$ is given by

$$\max_{w'} \Pi^i = \hat{q}^i(w', w)(w' - c).$$

The first- and second-order conditions are

$$\Pi^i_{w'} = \hat{q}^i + (w' - c) \hat{q}^i_{w'} = 0$$

$$\Pi^i_{w'', w'} < 0.$$  \tag{15} \tag{16}

From (15), we have equilibrium wholesale prices $(w^i_L, w^j_L)$, where the superscript $L$ indicates equilibrium values in linear pricing. Equation (15) also yields

$$w^i_L - c = -\hat{q}^i / \hat{q}^i_{w'} > 0.$$  

This means that $w^i_L > c$; i.e., the wholesale price is set above the marginal cost of production.

### 3.2 Vertical integration: V

Suppose that the manufacturers integrate with their retailers and sell their goods directly to consumers, engaging in Cournot competition in the product market. This vertical integration is the same as vertical separation with franchise fees in which the manufacturers set their own wholesale price at the marginal cost of production $c$; that is, $w^i = w' - c$. The equilibrium output is given by $\hat{q}(c,c)$.

By comparing the three cases, linear pricing, two-part pricing, and vertical integration, we have $w^F < c < w^L$. From (7), we also obtain $\hat{Q}(w^F, w^F) > \hat{Q}(c, c) > \hat{Q}(w^i_L, w^j_L)$, or $Q^F > Q^V > Q^L$ and $p^F > p^V > p^L$, where the superscript $V$ indicates equilibrium values in vertical integration.

We denote economic welfare, that is, the sum of the profits of manufactures and retailers plus consumer surplus ($CS$) $(\pi^i + \pi^j + \Pi^i + \Pi^j + CS)$, as $W$. We know that $W(Q) = \int_{0}^{Q} p(s) ds - cQ$. Differentiating $W(Q)$, we obtain

$$W'(Q) = p(Q) - c \quad \text{and} \quad W''(Q) = p'(Q) < 0.$$  

Thus, we find that $W(Q)$ is concave.
Let us consider two-part pricing. From (15) and (2), we know
\[
p(Q) - c = -p_0 q' - q' p_0 q_t' q_w',
\]
\[
- p_0 q' \left(1 + q_w' q_w'\right)
\]
\[
= -p_0 q' \left(d w'/dq'\right) \left(q_w' + q_w'\right)
\]
\[
= -p_0 q' \left(d w'/dq'\right) Q_{w'} > 0.
\]

Therefore, \(W'(Q)|_{Q=Q^F} > 0\) holds where \(Q^F\) is the total output under two-part pricing. Recall that \(Q^F > Q^r > Q^L\). Thus, we find \(W^r > W^r > W^L\).

We summarize these results as follows:

**Proposition 3.** Economic welfare is maximized by two-part pricing.

The underlying logic is as follows. Because the wholesale price is set below marginal cost, two-part pricing lowers the resale price and strengthens competition in the retail market. That is desirable from the economic welfare viewpoint but undesirable from the channel-profit viewpoint. On the other hand, because the wholesale price is set below the marginal cost, linear pricing raises the retail price and softens competition. Note that this result does not hold where the manufacturers and retailers engage in price competition in both markets.

Let us consider which organization of the distribution channel the manufacturers employ in equilibrium: two-part pricing (strategy F), linear pricing (strategy L), or vertical integration (strategy V). We develop the following three-stage game: In the first stage, each manufacturer chooses the mode of distribution. In the second stage, each manufacturer \(i\) sets the wholesale price \(w_i^r\) and franchise fee \(F_i\) (if possible). In the third stage, each retailer determines the sales quantity \(q_i\). The second and third stages of this game are the same as those examined in the previous section. Therefore, the game reduces to the following 3 \(\times\) 3 matrix game with three strategies: F, L, and V (see Table 1). Thus, we establish the following proposition:

**Table 1.** 3 \(\times\) 3 game about organization of the distribution channel

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>V</th>
<th>F</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((\Pi_{1V,1V}), (\Pi_{2V,2V}))</td>
<td>((\Pi_{1F,1F}), (\Pi_{2F,2F}))</td>
<td>((\Pi_{1L,1L}), (\Pi_{2L,2L}))</td>
<td></td>
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</tr>
<tr>
<td>2</td>
<td>((\Pi_{2V,2V}), (\Pi_{1V,1V}))</td>
<td>((\Pi_{2F,2F}), (\Pi_{1F,1F}))</td>
<td>((\Pi_{2L,2L}), (\Pi_{1L,1L}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>((\Pi_{1V,1V}), (\Pi_{2V,2V}))</td>
<td>((\Pi_{1F,1F}), (\Pi_{2F,2F}))</td>
<td>((\Pi_{1L,1L}), (\Pi_{2L,2L}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>((\Pi_{2V,2V}), (\Pi_{1V,1V}))</td>
<td>((\Pi_{2F,2F}), (\Pi_{1F,1F}))</td>
<td>((\Pi_{2L,2L}), (\Pi_{1L,1L}))</td>
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<tr>
<td>L</td>
<td>((\Pi_{1V,1V}), (\Pi_{2V,2V}))</td>
<td>((\Pi_{1F,1F}), (\Pi_{2F,2F}))</td>
<td>((\Pi_{1L,1L}), (\Pi_{2L,2L}))</td>
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</tbody>
</table>

**Note:** \(\Pi_{ij}\) shows the equilibrium profit of manufacturer 1 when manufacturer 1 employs strategy \(i\) while manufacturer 2 adopts strategy \(j\), where \(i, j = V, F, L\).
Proposition 4. Suppose that (i) the wholesale prices are strategic substitutes when each manufacturer employs strategy F and (ii) the equilibrium output (sales volume) is positive under every possible pair of strategies. Each manufacturer employs strategy F in equilibrium.

Proof: First, we prove that strategy L is strongly dominated by strategy V. We denote the equilibrium output of manufacturer $i$ under strategy L for the rival’s output by $\bar{q}_i$. Under strategy V, manufacturer $i$ can realize $\bar{q}_i$ irrespective of the rival’s output because the retail prices and outputs realized under strategy V are the same as those under strategy L. Thus, the realized profit $\pi$ is also the same as that under strategy L. By employing strategy V, manufacturer $i$ can always earn at least the profit realized under strategy L.

By eliminating the strongly dominated strategy L from the original $3 \times 3$ matrix game, we obtain a reduced $2 \times 2$ game consisting of strategies F and V. We show that the unique Nash equilibrium of the game is given by $\{F, F\}$. Suppose that each manufacturer employs strategy F. If manufacturer $j$ switches to strategy V, the marginal cost becomes $c$. Manufacturer $i$, who employs strategy F, decreases the wholesale price considering that wholesale prices are assumed to be strategic substitutes under strategy F. The residual demand of manufacturer $j$ decreases, and so does the profit. Thus, neither manufacturer has an incentive to deviate from $\{F, F\}$.

Next, we show that the strategy pair $\{V, V\}$ does not constitute a Nash equilibrium. Suppose that each manufacturer employs strategy V. By deviating to strategy F, a manufacturer can realize the Stackelberg leader position of this $2 \times 2$ game if another manufacturer continues to employ strategy V. The deviating manufacturer can increase profits because the first-mover advantage works in this Cournot competition game with a homogeneous product. Thus, each manufacturer has an incentive to deviate from $\{V, V\}$.

It is also straightforward that the strategy pairs $\{F, V\}$ and $\{V, F\}$ do not constitute a Nash equilibrium. (Q.E.D.)

In the case of homogeneous goods, Cournot competition generally means that the choice variables (quantities) are strategic substitutes. In our setting, wholesale prices become strategic substitutes (see Nariu and Watanabe, 2005). Therefore, the assumptions of proposition 4 are almost always true. Moreover, propositions 3 and 4 imply that the equilibrium mode of distribution, i.e., two-part pricing, achieves higher economic welfare than do the other modes. On the other hand, the manufacturers’ profit is the highest when both manufacturers employ vertical integration. Each manufacturer is stuck in a prisoner’s dilemma.

We examine the above $3 \times 3$ game in the previous linear example when the demand function is $p = a - bQ$ and the cost function is $c(q) = cq'$. Table 2 shows the $3 \times 3$ game when the demand function is given by $p = 1 - Q$ and $c(q) = q$. We find that strategy L is strongly dominated by strategy V because both $1/16 > 6/289$ and $25/144 > 2/27$ hold.
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Strategy V is strongly dominated by strategy F in the reduced $2 \times 2$ game. Note that strategy V is not dominated by strategy F in the original $3 \times 3$ game because $25/144 > 50/289$ holds. The remaining strategy pair $\{F, F\}$ constitutes a Nash equilibrium.

4 Conclusion

Traditional economic theory holds that manufacturers earn zero profit under Bertrand competition with homogeneous goods, even when only two manufacturers exist. In reality, however, oligopolistic Bertrand competitors can control their products’ prices and earn some profit margin. In this paper, we reach a contrary result by positing that Bertrand-competing manufacturers are selling not to final demanders but to Cournot-competing retailers. We show that under general assumptions, manufacturers who determine their wholesale prices and sell their goods through affiliated retailers earn positive profits, regardless of whether they charge franchise fees. The findings in this paper shed light on a new aspect of vertical dealings.

References


Appendix 1. Stability condition

If the rival manufacturer $j$ changes its wholesale price $w^j$, manufacturer $i$ revises its wholesale price $w^i$ as follows:

$$\Pi_i w^i \partial w^j + \Pi_i w^j \partial w^i = 0.$$ 

Therefore, we have

$$\partial w^i / \partial w^j = -\Pi_i w^j / \Pi_i w^i > 0, \quad \text{iff} \quad \Pi_i w^j > 0.$$ 

The above inequality indicates that wholesale prices are strategic substitutes if and only if $\Pi_i w^j < 0$. Differentiating (A-4), we obtain

$$\begin{bmatrix} \Pi_i w^j & \Pi_i w^j \\ \Pi_i w^j & \Pi_i w^j \end{bmatrix} \begin{bmatrix} \partial w^i \\ \partial w^j \end{bmatrix} = -\Pi_i w^j da.$$ 

We know from the stability condition that $E = \Pi_i w^j - \Pi_i w^j > 0$. Because $\Pi_i w^j = \Pi_i w^j$ and $\Pi_i w^j = \Pi_i w^j$ prevail in a symmetric equilibrium, we obtain

$$E = \left(\Pi_i w^j - \Pi_i w^j\right)\left(\Pi_i w^j - \Pi_i w^j\right) > 0.$$