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Minimal theory of massive gravity

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ABSTRACT

We propose a new theory of massive gravity with only two propagating degrees of freedom. While the homogeneous and isotropic background cosmology and the tensor linear perturbations around it are described by exactly the same equations as those in the de Rham–Gabadadze–Tolley (dRGT) massive gravity, the scalar and vector gravitational degrees of freedom are absent in the new theory at the fully nonlinear level. Hence the new theory provides a stable nonlinear completion of the self-accelerating cosmological solution that was originally found in the dRGT theory. The cosmological solution in the other branch, often called the normal branch, is also rendered stable in the new theory and, for the first time, makes it possible to realize an effective equation-of-state parameter different from (either larger or smaller than) -1 without introducing any extra degrees of freedom.

1. Introduction

Since the seminal work by Fierz and Pauli [1], especially in the recent years, much theoretical effort in cosmology has been put in order to develop theories of massive gravity [2,3]. These theories were indeed able to introduce, at non-linear level, the desired five modes necessary to describe a massive graviton in a Lorentz invariant way. In other words, these theories are free from the so called Boulware–Deser ghost [4], which had been thought to plague any theories of massive gravity. Together with this first success, much work came in order to see whether these same theories could be viable. Unfortunately, these theories suffer from instability on some key backgrounds, such as the Friedmann–Lemaître–Robertson–Walker (FLRW) universe [5]. In this regard several attempts have been analyzed to find stable cosmological solutions in massive gravity: 1) abandoning the homogeneity and/or the isotropy of cosmological models; 2) changing the theory by introducing new fields interacting with gravity in a way as to save the theory. It proved difficult, even with these attempts, to find a theory of massive gravity with a theoretically consistent and experimentally viable cosmology.

In this letter, we present a new theory of massive gravity which modifies general relativity in a minimal way. We will perform this by looking for a theory with only two tensor modes, which are massive. This will make FLRW backgrounds (including de Sitter) stable and viable as in the standard cosmology. Indeed the tensor modes of the gravity sector will be massive, whereas there are no scalar and vector propagating modes in the gravity sector. In order to achieve this goal we will not impose the Lorentz symmetry, so that a massive graviton does not need to have five degrees of freedom any longer.

2. Precursor theory

In order to define the theory we will make use of the lapse N, shift N^i, and the three-dimensional vielbein $e^I{}_j$ as basic variables. We can then introduce the three-dimensional metric as

$$\gamma_{ij} \doteq \delta_{IJ}\, e^I{}_i\, e^J{}_j\,. \tag{1}$$

Hereafter, $I, J \in \{1, 2, 3\}$ so as i and j. Out of the variables introduced so far, we can build a four-dimensional vielbein as

$$\left\| e^{\mathcal{A}}{}_\mu \right\| = \begin{pmatrix} N & \vec{0}^T \\ e^I{}_i N^i & e^I{}_j \end{pmatrix}, \tag{2}$$

and a four-dimensional metric as

$$g_{\mu\nu} \doteq \eta_{\mathcal{AB}}\, e^{\mathcal{A}}{}_\mu\, e^{\mathcal{B}}{}_\nu\,, \tag{3}$$

where $\eta_{\mathcal{AB}}$ is the Minkowski metric tensor, so that

$$g_{00} = -N^2 + \gamma_{ij} N^i N^j\,,$$
$$g_{0i} = \gamma_{ij} N^j = g_{i0}\,, \quad g_{ij} = \gamma_{ij}\,, \tag{4}$$

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corresponding to the metric tensor written in the ADM variables. We also introduce a non-dynamical four-dimensional vielbein built out of a non-dynamical lapse M, a non-dynamical shift M^i, and a non-dynamical three-dimensional vielbein $E^I{}_j$, as follows:

$$\left\| E^{\mathcal{A}}{}_\mu \right\| \doteq \begin{pmatrix} M & \vec{0}^T \\ E^I{}_i M^i & E^I{}_j \end{pmatrix}. \tag{5}$$

The four-dimensional vielbein of the form (2), often called the ADM vielbein, has 13 independent components, as opposed to 16 independent components of a completely general vielbein in four dimensions. The missing 3 components are the boost parameters that would transform the vielbein of the form (2) to a general vielbein. Therefore, by choosing the form (2) for the vielbein, we introduce a preferred frame and thus explicitly break the local Lorentz symmetry.

We now introduce a precursor action, which will then be used as the starting point to define the theory:

$$\begin{aligned} S_{\rm pre} &= \frac{M_{\rm P}^2}{2}\int d^4x\sqrt{-g}\,\mathcal{R}[g_{\mu\nu}] \\ &+ \frac{M_{\rm P}^2}{2}m^2\int d^4x\left[\frac{c_0}{24}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_\alpha E^{\mathcal{B}}{}_\beta E^{\mathcal{C}}{}_\gamma E^{\mathcal{D}}{}_\delta\right. \\ &+ \frac{c_1}{6}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_\alpha E^{\mathcal{B}}{}_\beta E^{\mathcal{C}}{}_\gamma e^{\mathcal{D}}{}_\delta \\ &+ \frac{c_2}{4}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_\alpha E^{\mathcal{B}}{}_\beta e^{\mathcal{C}}{}_\gamma e^{\mathcal{D}}{}_\delta \\ &+ \frac{c_3}{6}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}E^{\mathcal{A}}{}_\alpha e^{\mathcal{B}}{}_\beta e^{\mathcal{C}}{}_\gamma e^{\mathcal{D}}{}_\delta \\ &\left.+ \frac{c_4}{24}\epsilon_{\mathcal{ABCD}}\epsilon^{\alpha\beta\gamma\delta}e^{\mathcal{A}}{}_\alpha e^{\mathcal{B}}{}_\beta e^{\mathcal{C}}{}_\gamma e^{\mathcal{D}}{}_\delta\right], \end{aligned} \tag{6}$$

where $\mathcal{R}[g_{\mu\nu}]$ is the four-dimensional Ricci scalar for the metric $g_{\mu\nu}$ and the Levi-Civita symbol is normalized as $\epsilon_{0123} = 1 = -\epsilon^{0123}$. The precursor action would be exactly the same as that for the dRGT massive gravity if $e^{\mathcal{A}}{}_\alpha$ were a general, i.e. totally unconstrained, vielbein in four dimensions. At the level of the definition of the precursor theory, the only difference from the dRGT theory is thus that the four-dimensional vielbein is restricted to the form (2).

Having given the action for the precursor theory, it is straightforward to write down its Hamiltonian. The precursor Hamiltonian turns out to be linear in N and N^i and independent of their time derivatives. One can thus safely consider N and N^i as Lagrange multipliers, and the phase space to be considered here then consists of $9 \times 2 = 18$ variables, $e^L{}_k$ and their conjugate momenta $\Pi^k{}_L$. The coefficients of N and N^i define primary constraints, that we denote as $-\mathcal{R}_0$ and $-\mathcal{R}_i$, respectively. The rank of the 4×4 matrix made of Poisson brackets among them is two, leading to two secondary constraints, which we denote as $\tilde{\mathcal{C}}_\tau$ $(\tau = 1, 2)$. Combined with other six (three primary and three secondary) constraints, that we name as $\mathcal{P}^{[MN]}$ and $Y^{[MN]}$, associated with a symmetry condition on the vielbein $e^L{}_k$, it is deduced that the physical phase space is $18 - 6 - 6 = 6$ dimensional and that the number of physical degrees of freedom in the precursor theory is three at the fully nonlinear level.

3. Minimal theory

While the precursor theory itself is interesting, we further proceed to remove one more degree of freedom to define a theory with only two degrees of freedom, that we call the minimal theory of massive gravity. From now on, we will fix the units so that $M_{\rm P}^2 = 2$. Also, we neglect the entirely non-dynamical part proportional to c_0.

The minimal theory is defined in the Hamiltonian language by imposing four constraints, which we denote as $\mathcal{C}_0$ and $\mathcal{C}_i$ and are defined in (9) below, on the precursor theory. Only two combinations among these four constraints are new since the other two independent combinations are $\tilde{\mathcal{C}}_\tau \approx 0$ $(\tau = 1, 2)$, that already exist in the precursor theory. Hence the Hamiltonian of the minimal theory is

$$\begin{aligned} H = \int d^3x[&-N\mathcal{R}_0 - N^i\mathcal{R}_i + m^2 M\mathcal{H}_1 \\ &+ \lambda\mathcal{C}_0 + \lambda^i\mathcal{C}_i + \alpha_{MN}\mathcal{P}^{[MN]} + \beta_{MN}Y^{[MN]}], \end{aligned} \tag{7}$$

where N, N^i, λ, λ^i, α_{MN} (antisymmetric) and β_{MN} (antisymmetric) are 14 Lagrange multipliers. This is a constrained version of the precursor Hamiltonian, because we have added two additional constraints. As a consequence, on the constrained surface the Hamiltonian density reduces only to $H \approx H_1 \doteq \int d^3x m^2 M\mathcal{H}_1$. Each constraint has a specific meaning. The following terms are all derived from the precursor theory,

$$\begin{aligned} \mathcal{R}_0 &= \mathcal{R}_0^{\rm GR} - m^2\mathcal{H}_0, \\ \mathcal{R}_0^{\rm GR} &= \sqrt{\gamma}\,R[\gamma] - \frac{1}{\sqrt{\gamma}}\left(\gamma_{nl}\gamma_{mk} - \frac{1}{2}\gamma_{nm}\gamma_{kl}\right)\pi^{nm}\pi^{kl}, \\ \mathcal{R}_i &= \mathcal{R}_i^{\rm GR} = 2\gamma_{ik}\mathcal{D}_j\pi^{kj}, \\ \mathcal{H}_0 &= \sqrt{\tilde{\gamma}}(c_1 + c_2\,Y_I{}^I) + \sqrt{\gamma}(c_3\,X_I{}^I + c_4), \\ \mathcal{H}_1 &= \sqrt{\tilde{\gamma}}\left[c_1 Y_I{}^I + \frac{c_2}{2}(Y_I{}^I Y_J{}^J - Y_I{}^J Y_J{}^I)\right] + c_3\sqrt{\gamma}, \end{aligned}$$

and

$$\begin{aligned} \mathcal{P}^{[MN]} &= e^M{}_j\,\Pi^j{}_I\delta^{IN} - e^N{}_j\,\Pi^j{}_I\,\delta^{IM}, \\ Y^{[MN]} &= \delta^{ML}\,Y_L{}^N - \delta^{NL}\,Y_L{}^M, \end{aligned}$$

out of which the precursor Hamiltonian is $H_{\rm pre} = \int d^3x[-N\mathcal{R}_0 - N^i\mathcal{R}_i + m^2M\mathcal{H}_1 + \tilde{\lambda}^\tau\tilde{\mathcal{C}}_\tau + \alpha_{MN}\mathcal{P}^{[MN]} + \beta_{MN}Y^{[MN]}]$. Here, $\tau = 1, 2$, $\mathcal{D}_j$ is the spatial covariant derivative compatible with γ_{ij}, $\sqrt{\gamma} = \sqrt{\det\gamma_{ij}}$, $\sqrt{\tilde{\gamma}} = \sqrt{\det\tilde{\gamma}_{ij}}$, $\tilde{\gamma}_{ij} = \delta_{IJ}E^I{}_iE^J{}_j$, $\pi^{jk} = \delta^{IJ}\Pi^j{}_Ie_J{}^k$, $\Pi^j{}_I$ is the canonical momentum conjugate to $e^I{}_j$, and

$$Y_I{}^J = E_I{}^k e^J{}_k, \quad \text{and} \quad X_I{}^J = e_I{}^k E^J{}_k, \tag{8}$$

satisfying $Y_I{}^L X_L{}^J = \delta_I^J$.

Throughout the present letter, for simplicity we adopt the unitary gauge so that M, $M^iE^I{}_i$ and $E^I{}_j$ are only functions of the coordinates. This makes $\mathcal{H}_0$ and $\mathcal{H}_1$ explicitly time-dependent. The remaining constraints, $\mathcal{C}_0$ and $\mathcal{C}_i$, are then defined as

$$\mathcal{C}_0 \doteq \{\mathcal{R}_0, H_1\} + \frac{\partial\mathcal{R}_0}{\partial t}, \quad \mathcal{C}_i \doteq \{\mathcal{R}_i, H_1\}. \tag{9}$$

The two constraints $\tilde{\mathcal{C}}_\tau \approx 0$ $(\tau = 1, 2)$ in the precursor theory can be written as linear combinations of these four constraints. Therefore, only the remaining two combinations are new. In other words, the minimal theory is defined by adding two additional constraints to the precursor theory. The set of two new constraints removes one degree of freedom from the precursor theory. Since the precursor theory has three degrees of freedom, this means that the minimal theory has only two degrees of freedom.

Rigorously speaking, what we have proved here is that there are enough number of constraints, meaning the inequality, (number of d.o.f.) ≤ 2, holds. One might in fact worry that the consistency of the additional two constraints with time evolution might lead to further secondary constraints, overconstraining the theory. This is not the case since, as we shall see explicitly in the next section, there are two (and only two) propagating degrees of freedom around cosmological backgrounds, meaning another inequality, (number of d.o.f.) ≥ 2, holds. By combining the two inequalities, we thus have the equality, (number of d.o.f.) $= 2$, holds. It is also possible to prove the absence of further secondary constraints, and thus the presence of two physical degrees of freedom, in a more formal way by calculating the determinant of the 14×14 matrix-operator made of Poisson brackets among all the fourteen constraints [6].

Having defined the minimal theory with only two degrees of freedom by its Hamiltonian, it is straightforward to calculate the corresponding action via a Legendre transformation. It should be noticed that since in the constraints, e.g. $\mathcal{C}_0$, the canonical momenta are present, on integrating out, e.g. the auxiliary field λ, the resulting Lagrangian (or Hamiltonian) would acquire a structure, which would have a kinetic structure for $\dot{\gamma}_{ij}$ essentially different from the Einstein–Hilbert one. While this modification is essential for the absence of helicity-0 and -1 gravitational modes, the modification to the kinetic term for gravitational waves is suppressed by $m^2/M_{\rm P}^2$ and thus is negligible. A nontrivial point to be noticed in this regard is that the relation between the canonical momenta $\Pi^j{}_I$ and the time derivative of the spatial vielbein $\dot{e}^I{}_j$ is modified due to the additional terms in the Hamiltonian [6]. One can analyze the behavior of the theory, e.g. the cosmological evolution, by using either the Hamiltonian or equivalently the Lagrangian.

4. Cosmology and phenomenology

Let us consider a simple case to show the behavior of the theory, namely a general homogeneous and isotropic (FLRW) cosmological background with flat spatial metric, driven by the graviton mass term and matter fields minimally coupled to the metric $g_{\mu\nu}$. It is rather easy to see that on this background, $\mathcal{C}_i \approx 0$ are trivial (because of homogeneity of the background) and $\mathcal{C}_0 \approx 0$ leads to

$$(c_3 + 2c_2 X + c_1 X^2)(\dot{X} + NHX - MH) = 0\,, \tag{10}$$

where $X \doteq \tilde{a}/a$ is the ratio of the scale factors of the three-dimensional metrics $\tilde{\gamma}_{ij}$ and γ_{ij} respectively, and H is the Hubble expansion rate (not the Hamiltonian), i.e. $H = \dot{a}/(aN)$. This is exactly the same as the well-known constraint equation obtained from the Bianchi identity in the dRGT theory. Two branches of solutions thus exist, corresponding to the two factors of the left hand side of (10). The self-accelerating branch is defined by those values of the constant X which satisfy

$$X = X_\pm \doteq \frac{-c_2 \pm \sqrt{c_2^2 - c_1 c_3}}{c_1}\,, \tag{11}$$

while the normal branch is defined by setting $\dot{X} + NHX - MH = 0$. In order to determine the Lagrange multiplier λ, we demand that

$$\frac{d\mathcal{R}_0}{dt} = \{\mathcal{R}_0, H\} + \frac{\partial \mathcal{R}_0}{\partial t} = \mathcal{C}_0 + \int d^3y \lambda \{\mathcal{R}_0, \mathcal{C}_0(y)\} + \ldots\,,$$

should vanish, where $\{\mathcal{R}_0, \mathcal{C}_0\} \neq 0$ and the neglected part vanishes because of the symmetry of the background. On requiring $\frac{d\mathcal{R}_0}{dt} \approx 0$ and imposing $\mathcal{C}_0 \approx 0$, we then find that $\lambda \approx 0$ on the background. The remaining independent equation is, after re-inserting units,

$$3M_{\rm P}^2 H^2 = \frac{m^2 M_{\rm P}^2}{2}\left(c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3\right) + \rho\,, \tag{12}$$

where ρ is the energy density of matter minimally coupled to the metric $g_{\mu\nu}$. This is exactly the same as the Friedmann equation in the dRGT theory. There is no other independent equation for the background, essentially because both the Bianchi identity and the constraint $\mathcal{C}_0 \approx 0$ lead to the same equation (10). Needless to say, when we constructed the minimal theory in the previous section, we carefully chose $\mathcal{C}_0$ so that this is the case.

While the homogeneous and isotropic cosmological background solutions are exactly the same as those in the dRGT theory, perturbations around them behave completely differently. It is known that, in the standard dRGT theory, all homogeneous and isotropic backgrounds in both branches are plagued by ghosts, either in the helicity-0 or helicity-1 sector, and thus unstable [5]. On the contrary, in the new theory there is no physical degree of freedom in the helicity-0 or helicity-1 sector (namely no scalars or vector modes, according to the standard $1+3$ decomposition of the perturbation for the metric tensor) and thus those (would-be) ghosts are absent, rendering the cosmological background absolutely stable. In the minimal theory, the only existing two propagating modes reduce to the tensor modes, whose quadratic action can be written as

$$S = \frac{M_{\rm P}^2}{8} \sum_{\lambda=+,\times} \int d^4x N a^3 \left[\frac{\dot{h}_\lambda^2}{N^2} - \frac{(\partial h_\lambda)^2}{a^2} - \mu^2 h_\lambda^2\right], \tag{13}$$

where

$$\mu^2 \doteq \frac{1}{2} m^2 X \left[(c_2 X + c_3) + (c_1 X + c_2)\frac{M}{N}\right], \tag{14}$$

gives the mass μ of the tensor modes on this background in both branches, which is in general different from the mass parameter m in the action. This provides a proof of our previous claim that the theory is not overconstrained. Thus, not only the background equation of motion but also the quadratic action for tensor perturbations are exactly the same as those in the dRGT theory [7,8]. On the other hand, for scalar and vector perturbations, there are no additional modes stemming from the gravity sector. This is consistent with what we have found in the previous section, namely the fact that the number of physical degrees of freedom in the gravity sector is only two. (We shall discuss each branch later in this section.) Since the only two physical modes from the gravity sector coincide with the tensor modes, the cosmological background is stable, provided that $\mu^2 > 0$ and that the matter sector is stable (except for the standard Jeans instability that drives the structure formation). In particular, the theory automatically avoids the nonlinear ghost instability found in [5] and the classical Higuchi ghost [9]. This feature of the minimal theory is in a sharp contrast to the dRGT theory.

In the minimal theory, the constraints $\mathcal{C}_0$ and $\mathcal{C}_i$ play key roles in eliminating the unwanted helicity-0 and helicity-1 gravitational modes. Actually, we have chosen $\mathcal{C}_0$ and $\mathcal{C}_i$ so that they always contain the two constraints $\tilde{\mathcal{C}}_\tau$ ($\tau = 1, 2$) in the precursor theory as well as two additional constraints. Moreover, as stressed just after (12), $\mathcal{C}_0 \approx 0$ and the Bianchi identity result in the same equation for the homogeneous and isotropic background. These rather non-trivial properties uniquely characterize $\mathcal{C}_0$ and $\mathcal{C}_i$.

Phenomenology in the self-accelerating branch of the minimal theory is almost the same as the standard ΛCDM cosmology. For the background evolution, since X is set to be a constant by (11), the graviton mass term in (12) behaves as an effective cosmological constant, that can drive the acceleration of the cosmic expansion even without the genuine cosmological constant. Scalar and vector perturbations behave in exactly the same way as in the standard ΛCDM cosmology. Only the tensor perturbations are modified by the graviton mass term, as described above. Therefore, the self-accelerating branch of the minimal theory leads to absolutely stable and phenomenologically viable cosmology.

The normal branch of the minimal theory is also interesting. This branch is defined by $\dot{X} + NHX - MH = 0$, and thus in this branch X is not a constant in general, making the contribution of the graviton mass term to the Friedmann equation (12) dynamical. It is possible to find regimes of parameters in which μ^2 is positive and the effective equation-of-state parameter of the graviton mass term is either larger or smaller than -1. This is quite remarkable both theoretically and observationally. It is commonly believed that the effective equation-of-state different from -1 would imply the existence of extra helicity-0 (and possibly helicity-1) degree(s) of freedom in either the dark sector or the gravity sector. Indeed, this is one of the main motivations for numerous dark energy surveys in the world. On the contrary, in the minimal theory of massive gravity there is no extra degree of freedom, and yet the cosmology with the effective equation-of-state different from -1 is possible and stable.

In the absence of extra gravitational degrees of freedom, in either branch of solutions, the minimal theory is not constrained by fifth force experiments and thus the bound on the graviton mass is relatively weak. We do not even need screen mechanisms such as Vainshtein's one [10]. The strongest bound,

$$\mu_s \lesssim 10^{-5}\ \text{Hz}, \tag{15}$$

comes from modification of the emission rate of the gravitational waves from binary pulsars, which is of order $O(\mu_s^2/\omega^2)$ [11]. Here, μ_s is the mass of gravitational waves at the source position and ω is the characteristic frequency of the system.

The theory opens up new possibilities for gravitational phenomenology. One such example is possible appearance of a sharp peak in the stochastic gravitational wave spectrum [12].

5. Conclusion

We propose here a minimal theory of massive gravity, where only two physical modes in the gravity sector propagate, the gravitational waves, which become massive. This theory, contrary to the dRGT theory, allows stable homogeneous and isotropic (FLRW) cosmologies to exist. In particular, the self-accelerating cosmological solution, that was originally found in the dRGT theory, is now rendered stable in this theory at the fully nonlinear level. Furthermore, we expect that its phenomenology will be closer to General Relativity, as only the tensor modes are dynamical, as in General Relativity. The phenomenology of gravitational waves differs from the one in General Relativity, as they possess a non-zero mass, giving them a different propagation dynamics (e.g. the propagation speed for modes with low frequencies). The normal branch cosmological solution is also rendered stable in the minimal theory and allows the effective equation-of-state parameter to differ from (either larger or smaller than) -1. As far as the authors know, this is the very first example in which the effective equation-of-state parameter is made different from -1 without introducing any extra degrees of freedom. This potentially has rather strong impact on various dark energy surveys in the world.

It is possible to extend this theory to a bigravity theory with only 4 physical degrees of freedom, by promoting M, $M^k E^L{}_k$, and $E^L{}_k$ to be dynamical. Two of the four are massless graviton degrees and the remaining two are massive graviton degrees. It is also straightforward to generalize it to a multi-gravity setup. We hope to extend this method to other cases for which unwanted degrees of freedom can be consistently removed via well-imposed constraints.

Even within the context of single graviton, there are many possibilities for extension of the minimal theory proposed in the present paper. For example, one might be tempted to combine the minimal theory with the idea of the generalized massive gravity [13]. Namely, by introducing Stückelberg fields and promoting the constant coefficients $c_{0,1,2,3,4}$ to functions of them, the dynamics of FLRW background can be altered. It is also possible to promote $c_{0,1,2,3,4}$ to functions of other dynamical scalar fields as in [14,15]. We leave the important discussion of the nature of the Lorentz violations, and their consequences related to the cutoff scale of the theory as a project to be discussed elsewhere. Here, we simply state that classically there is no Lorentz violation in the matter sector and that Lorentz violation via loops should be suppressed by the tiny factor m^2/M_P^2, provided that all matter fields couple minimally to the physical metric $g_{\mu\nu}$ at the classical level.

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