



Resonant primordial gravitational waves amplification



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ABSTRACT

We propose a mechanism to evade the Lyth bound in models of inflation. We minimally extend the conventional single-field inflation model in general relativity (GR) to a theory with non-vanishing graviton mass in the very early universe. The modification primarily affects the tensor perturbation, while the scalar and vector perturbations are the same as the ones in GR with a single scalar field at least at the level of linear perturbation theory. During the reheating stage, the graviton mass oscillates coherently and leads to resonant amplification of the primordial tensor perturbation. After reheating the graviton mass vanishes and we recover GR.

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1. Introduction

Inflation [1] is the leading paradigm of very early universe cosmology, but its physical origin is still mysterious. The generation of primordial gravitational waves is a generic prediction of the inflationary universe. It leads to B mode polarization in the CMB, and provides an important window to the physics of very early universe. It was reported that the primordial tensor-to-scalar ratio is $r < 0.11$ (95% CL), based on *Planck* full sky survey [2]. Several next-generation satellite missions (CMBPol, CoRE and LiteBIRD) as well as the ground based experiments (AdvACT, CLASS, Keck/BICEP3, Simons Array, SPT-3G) and balloons (EBEX, Spider), are aimed at measuring primordial gravitational waves down to $r \sim 10^{-3}$. See Ref. [3] for a recent updated forecast on these future experiments.

According to the Lyth bound [4], the tensor-to-scalar ratio is proportional to the variation of the inflaton field during inflation, i.e. $\Delta\phi/M_p \simeq \int dN \sqrt{\dot{\tau}/8}$. The threshold $\Delta\phi = M_p$ then corresponds to $r = 2 \times 10^{-3}$, assumed that tensor power spectrum is nearly scale-invariant. The sizeable amplitude of the primordial gravitational waves requires a super-Planckian excursion of the inflaton, i.e. $\Delta\phi > M_p$.

In quantum field theory, the naturalness principle tells us that the variation of a field ϕ over the distance greater than the cutoff scale is generally regarded as being out of the validity of the theory. In a gravitational system, we take the Planck mass as the UV cutoff scale, because gravity strongly couples to the matter sector and the graviton-graviton scattering violates unitarity above this

scale. Thus the inflationary prediction may not be reliable in the case of a super-Planckian excursion. Therefore, the detection of the primordial tensor perturbation with its amplitude larger than the threshold value $r = 2 \times 10^{-3}$ has a profound impact on our understanding of fundamental physics. It implies that either quantum field theory or gravity may be modified in the very early universe.

In this letter, by means of modifying gravity, we propose a new mechanism to evade the Lyth bound. We consider a minimal extension of GR with a non-vanishing graviton mass term in the very early universe. Specifically we propose a model in which the graviton mass is proportional to the inflaton during reheating. Then the coherent oscillation of the inflaton induces that of graviton mass and gives rise to resonant amplification of the primordial tensor perturbation. This is a broad parametric resonance which includes all long wavelength modes, given the graviton mass is much greater than the Hubble constant during reheating. After reheating, the graviton mass vanishes as the inflaton decays and we recover GR.

2. A massive gravity theory

The theoretical and observational consistency of massive gravity has been a longstanding problem, the pioneering attempt could be traced back to Fierz and Pauli's work in 1939 [5]. However, Fierz–Pauli's theory and its non-linear completion, the so-called dRGT massive gravity [6], suffer from many pathologies [7–11]. The origin of these pathologies is probably the Poincare symmetry of the Stückelberg scalar field configuration.

Away from the Poincare symmetry, a broad class of massive gravity theories have been discussed in the literature [12–18]. In

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this letter, we consider a massive gravity theory with the internal symmetry [12,19]

$$\varphi^i \rightarrow \Lambda_j^i \varphi^j, \quad \varphi^i \rightarrow \varphi^i + \Xi^i(\varphi^0), \quad (1)$$

where Λ_j^i is the $SO(3)$ rotational operator, $\Xi^i(\varphi^0)$ are three arbitrary functions of their argument, φ^i and φ^0 are four Stückelberg scalars with non-trivial VEVs,

$$\varphi^0 = f(t), \quad \varphi^i = x^i, \quad i = 1, 2, 3. \quad (2)$$

These non-trivial VEVs give a non-vanishing graviton mass, due to the presence of preferred space-time frame. At the first derivative level, there are two combinations of the Stückelberg fields that respect this symmetry,

$$X = g^{\mu\nu} \partial_\mu \varphi^0 \partial_\nu \varphi^0, \\ Z^{ij} = g^{\mu\nu} \partial_\mu \varphi^i \partial_\nu \varphi^j - \frac{g^{\mu\nu} \partial_\mu \varphi^0 \partial_\nu \varphi^i \cdot g^{\lambda\rho} \partial_\lambda \varphi^0 \partial_\rho \varphi^j}{X}. \quad (3)$$

Note that in the language of ADM formalism, in the unitary gauge we have $X = N^{-2}$ and $Z^{ij} = h^{ij}$, where N is lapse and h^{ij} is the spatial metric. We will see below that the Z^{ij} term gives rise to a non-vanishing mass to gravitational waves. The graviton mass term could be written as a generic scalar function of the above two ingredients.

Due to the internal symmetry $\varphi^i \rightarrow \varphi^i + \Xi^i(\varphi^0)$, there are only 3 dynamical degrees of freedom (DOF) in our theory, i.e. 2 tensor modes, and 1 scalar mode. In the language of ADM formalism or the $(3+1)$ -decomposition of space-time, we find that these two ingredients in Eq. (3) are free from the shift N^i and thus the associated Hamiltonian of gravity is linear in N^i . This implies that 3 momentum constraints and the associated secondary constraints eliminate 3 DOF in h_{ij} , and the number of residual DOF is thus 3 [15].

Now we apply this massive gravity theory to the early universe. To minimize our model, we identify the time-like Stückelberg scalar with the inflaton scalar field ϕ , i.e. $\varphi^0 = \phi$. By doing this, we achieve a minimal model of massive gravity, in which only the tensor modes receive a modification, while the scalar and vector modes remain the same as the ones in the single scalar model in GR.

To be specific, we consider the following action with enhanced global symmetry $\varphi^i \rightarrow \text{constant} \cdot \varphi^i$,

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} \mathcal{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right. \\ \left. - \frac{9}{8} M_p^2 m_g^2(\phi) \frac{\bar{\delta} Z^{ij} \bar{\delta} Z^{ij}}{Z^2} \right], \quad (4)$$

where $V(\phi)$ is the inflaton potential, the numerical factor $9/8$ is inserted for later convenience, and $\bar{\delta} Z^{ij}$ is a traceless tensor defined by [18]

$$\bar{\delta} Z^{ij} \equiv Z^{ij} - 3 \frac{Z^{ik} Z^{kj}}{Z}, \quad (5)$$

where Z^{ij} is defined by Eq. (3) with φ^0 replaced by ϕ , $Z \equiv Z^{ij} \delta_{ij}$, and the summation over repeated indices is understood. Note that the 2nd line of Eq. (4) is the graviton mass term, which does not contribute to the background energy momentum tensor. Its non-trivial contribution starts from the quadratic action in perturbations.

The scalar functional dependence of graviton mass $m_g^2(\phi)$ is still left undecided due to our ignorance of underlying fundamental theory. However, from the particle physics perspective, particle

masses can be neglected at high energies, so that physics would become scale invariant and weakly coupled. We could reasonably expect that on the Hubble scale, i.e. one of the typical scales during inflation, physics is still weakly coupled. We thus assume the following scalar dependence

$$m_g^2(\phi) = \frac{\lambda \phi^2}{1 + (\phi/\phi_*)^4}, \quad (6)$$

where ϕ_* is the inflaton field value at the end of inflation. Without loss of generality, we assume $\phi = 0$ is the minimum of the potential at which the inflaton settles down after reheating. During inflation, $\phi \gg \phi_*$ and graviton becomes massless on the scale that we are interested in, different polarizations of graviton are thus weakly coupled.

As usual, we consider a flat FLRW background,

$$ds^2 = -dt^2 + a^2 d\mathbf{x}^2. \quad (7)$$

Due to the $SO(3)$ rotational symmetry of the 3-space, we can decompose the metric perturbation into scalar, vector, and tensor modes. These modes are completely decoupled at linear order. We define the metric perturbation variables as

$$g_{00} = -(1 + 2\alpha), \\ g_{0i} = a(t) (S_i + \partial_i \beta), \\ g_{ij} = a^2(t) \left[\delta_{ij} + 2\psi \delta_{ij} + \partial_i \partial_j E + \frac{1}{2} (\partial_i F_j + \partial_j F_i) + \gamma_{ij} \right], \quad (8)$$

where α , β , ψ and E are scalars, S_i and F_i are vectors, and γ_{ij} is tensor. The vector modes satisfy the transverse condition, $\partial_i S^i = \partial_i F^i = 0$, and the tensor modes satisfy the transverse and traceless condition, $\gamma_i^i = \partial_i \gamma^{ij} = 0$.

3. Tensor perturbation

The action for the tensor perturbation reads

$$S_T^{(2)} = \frac{M_p^2}{8} \int dt d^3x a^3 \left[\dot{\gamma}_{ij} \dot{\gamma}^{ij} - \left(\frac{k^2}{a^2} + m_g^2 \right) \gamma_{ij} \gamma^{ij} \right]. \quad (9)$$

We see that the graviton receives a mass correction. We quantize the tensor mode as

$$\gamma_{ij}(x) = \sum_{s=\pm} \int d^3k \left[a_{\mathbf{k}} e_{ij}(\mathbf{k}, s) \gamma_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + h.c. \right], \quad (10)$$

where $a_{\mathbf{k}}$ is the annihilation operator and $e_{ij}(\mathbf{k}, s)$ is the transverse and traceless polarization tensor which we normalize as

$$e_{ij}(\mathbf{k}, s) e^{ij}(\mathbf{k}, s') = \delta_{ss'}. \quad (11)$$

The equation of motion for the tensor modes reads

$$\ddot{\gamma}_{\mathbf{k}} + 3H \dot{\gamma}_{\mathbf{k}} + \left(\frac{k^2}{a^2} + m_g^2 \right) \gamma_{\mathbf{k}} = 0. \quad (12)$$

During inflation, the universe undergoes a superluminal expansion with a nearly constant Hubble parameter. The vacuum fluctuations are stretched and frozen on super-horizon scales. At this stage, the graviton mass is

$$m_g^2 \simeq \lambda \phi_*^2 (\phi_*/\phi_I)^2, \quad \text{because } \phi_*^4 \ll \phi_I^4, \quad (13)$$

where the subscript “ I ” is for “inflation”. The inflaton fluctuation and metric perturbations are mixed on the scale $\epsilon^{1/2} H_I$ during inflation [20]. In our framework, it is natural to expect that inflation and massive gravity as new physics appear on the same scale.

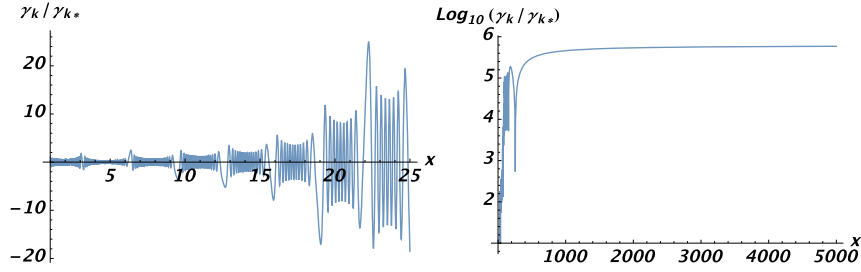


Fig. 1. The resonant amplification of tensor modes during reheating. The horizontal axis is $x \equiv Mt$ and the vertical axis is the relative amplitude of the tensor modes γ_k/γ_{k*} , where γ_{k*} is the amplitude at the end of inflation. Note that on the right figure the x -scale is much larger than the left one, and the vertical axis is logarithm of the growth rate. The parameters are $\xi = 10^6$ and $\Gamma = 0.05$, with the initial condition $d\gamma_k/dx|_{x=1} = 0$.

The mixings between different polarizations of massive graviton are characterized by the graviton mass scale. Therefore it amounts to that graviton mass during inflation satisfies

$$m_g^2 \simeq \lambda \phi_*^2 (\phi_*/\phi_1)^2 \sim H_1^2 \epsilon \ll H_1^2, \quad (14)$$

where $\epsilon \equiv -\dot{H}/H^2$ is the slow-roll parameter. As we shall see below, this condition can be easily satisfied in our model. In passing, we note that as the scalar and vector modes are the same as in GR, our theory is free from the Higuchi ghost [21] even for $m_g < 2H$ in the de Sitter space-time.

Given the small but non-vanishing mass, the inflationary tensor spectrum is calculated by

$$P_\gamma = \frac{2H^2}{\pi^2 M_p^2} \left(\frac{k}{aH} \right)^{2m_g^2/3H^2}, \quad (15)$$

with the tilt

$$n_t \simeq -2\epsilon + \frac{2m_g^2}{3H^2}, \quad (16)$$

Significantly, the tensor spectrum has the blue tilt if $m_g^2 > 3H^2\epsilon$.

At the end of inflation, the slow-roll condition breaks down and the universe undergoes reheating. At the reheating stage, the inflaton oscillates around the potential minimum, and gradually decays to radiation. The potential is expanded around the minimum as

$$V(\phi) \simeq \frac{1}{2} M^2 \phi^2 + \dots, \quad (17)$$

where M is the mass of the inflaton during reheating and the dots stand for higher order corrections which are irrelevant at low energy scale. Asymptotically for large $Mt \gg 1$, we have

$$\phi_r \simeq \frac{\phi_*}{\sqrt{3\pi} Mt} \sin(Mt) \exp\left(-\frac{1}{2}\Gamma Mt\right), \quad (18)$$

where subscript “r” stands for “reheating”. The Einstein equations tell us $H \simeq \frac{2}{3r}$ at this stage. To include the effect of decaying inflaton, we have simply added a decaying factor $e^{-\Gamma Mt}$ into the above solution, without specifying the detailed model of reheating.

During the reheating stage, we have

$$\phi_r \ll \phi_*, \quad \text{and thus} \quad m_g^2 \simeq \lambda \phi_r^2. \quad (19)$$

The equation of motion of gravitational waves (12) becomes a Mathieu-type equation,

$$\frac{d^2 \gamma_k}{dx^2} + \frac{2}{x} \frac{d\gamma_k}{dx} + \frac{\xi \cdot e^{-\Gamma x}}{x^2} \sin^2(x) \gamma_k = 0, \quad (20)$$

where $x \equiv Mt$ and $\xi \equiv \frac{\lambda \phi_*^2}{3\pi M^2}$. Note that we have neglected the spatial gradient term since we are interested in the long wavelength modes.

It is well known that the Mathieu equation has a very efficient and broad parametric resonance if $\frac{\xi \cdot e^{-\Gamma x}}{x^2} \gg 1$, i.e. the graviton mass must be much greater than Hubble parameter during reheating. Let us check whether this condition can be satisfied. Note that the Friedmann equation tells us $M_p^2 H_r^2 \sim M^2 \phi_r^2$, where H_r is the Hubble parameter during reheating. Generally we expect that $M^2 \sim H_1^2$ due to the breaking of the slow-roll condition at the end of inflation. We thus get

$$m_g^2 \simeq \lambda \phi_r^2 \sim \lambda \cdot \frac{M_p^2}{H_1^2} \cdot H_r^2. \quad (21)$$

Demanding that the graviton mass be much greater than the Hubble parameter during reheating yields the condition,

$$\lambda \frac{M_p^2}{H_1^2} \gg 1. \quad (22)$$

Combining conditions (14) and (22), we get

$$\frac{H_1^2}{M_p^2} \ll \lambda \ll \frac{H_1^2}{M_p^2} \cdot \frac{M_p^2 \phi_1^2}{\phi_*^4}. \quad (23)$$

Thus $\frac{M_p^2 \phi_1^2}{\phi_*^4} \gg 1$, i.e. $\phi_* \ll M_p$ is required for the self-consistency of the above inequality. Note that $\phi_* \ll M_p$ is also the condition of the validity of our effective field theory, which is automatically satisfied for many small field inflationary models.

We have numerically solved Eq. (20). The results are plotted in Figs. 1 and 2. The resonant amplification factor depends on the value of ξ and the decay rate Γ . We can also read off the threshold for a significant resonant amplification is roughly $\xi > 10^3$. The tensor modes stop growing in the large Mt limit due to the decay of the inflaton. For those long wavelength modes whose gradient term is always negligible during reheating, the final power spectrum is still almost scale-invariant, as long as the graviton mass during inflation is small enough and thus the tensor tilt in Eq. (16) is small.

Note that the kinetic term of inflaton field is canonical at leading order, which implies that in the massless limit $m_g^2 \rightarrow 0$, scalar and tensor just simply decouple. On the other hand, it has been previously proven that this theory smoothly reduces to GR in the massless limit due to the absence of vDVZ discontinuity [12]. After reheating, $\phi \rightarrow 0$, the graviton becomes massless and we recover GR.

4. Vector perturbation

To calculate the vector perturbation, we adopt the unitary gauge, in which the fluctuations of $SO(3)$ Stückelberg scalar fields are fixed to be zero, i.e. $\delta\varphi^i = 0$. The quadratic action of the vector

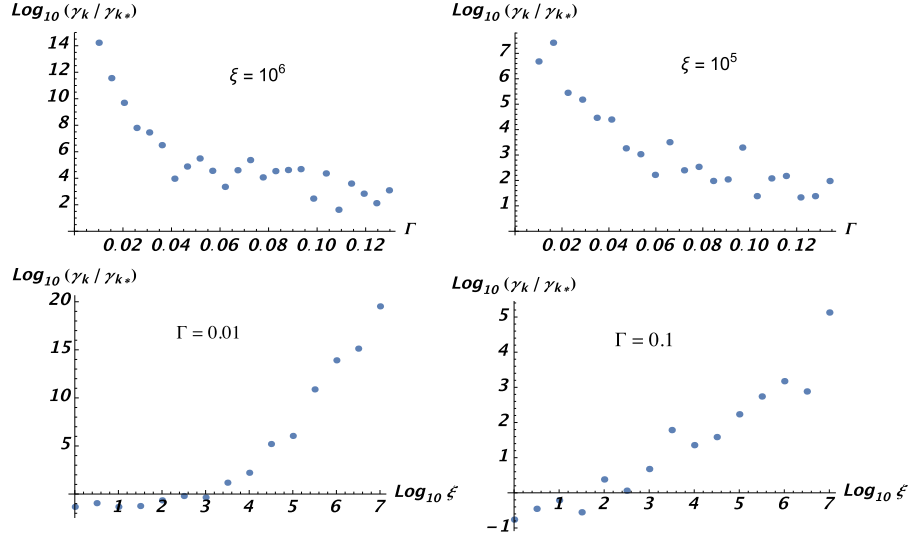


Fig. 2. The parameter dependence of the resonant amplification. The horizontal axis is Γ for the upper panels and ξ for the lower panels. The vertical axis is $\text{Log}_{10}(\gamma_k/\gamma_{k*})$ evaluated at $\chi = 1000$.

perturbation reads (in momentum space)

$$S_V^{(2)} = \frac{M_p^2}{16} \int a^3 k^2 \left[\dot{F}_i \dot{F}_i - m_g^2 F_i F_i - \frac{4S_i \dot{F}_i}{a} + \frac{4S_i S_i}{a^2} \right]. \quad (24)$$

After integrating out S_i , we get

$$S_V^{(2)} = -\frac{1}{16} M_p^2 m_g^2 \int a^3 k^2 F_i F_i. \quad (25)$$

This clearly shows that the kinetic term for vector perturbation was canceled out. It is by no mean of an accident, because the kinetic terms of vector modes are prohibited by internal symmetry $\varphi^i \rightarrow \varphi^i + \Xi^i(\varphi^0)$.

5. Scalar perturbation

In the scalar sector, α , β and E are non-dynamical. After integrating them out, the quadratic action for the scalar perturbation in the uniform ϕ gauge (i.e. $\delta\phi = 0$) reads

$$S_s^{(2)} = M_p^2 \int a^3 \epsilon \left(\dot{\psi}^2 - \frac{k^2}{a^2} \psi^2 \right). \quad (26)$$

This is exactly the same as the one in GR with a single scalar field. In this gauge, ψ is identical to the curvature perturbation on the comoving slicing, \mathcal{R}_c , and the power spectrum is given by the same formula [22,23],

$$P_{\mathcal{R}} = \frac{H^2}{8\pi^2 \epsilon M_p^2}. \quad (27)$$

Thus the tensor-to-scalar ratio we observe today is

$$r = \frac{A \times P_\gamma}{P_{\mathcal{R}}} = 16\epsilon \times A, \quad (28)$$

where P_γ is the power spectrum of tensor perturbation produced in inflation, which is given by Eq. (15) with graviton mass correction on the tilt. A is the resonant amplification factor of the tensor modes during reheating. For instance, with the parameters choice in Fig. 1, the factor A could be of the order of 10^{11} . The variation of the inflaton per e -fold is

$$\frac{d\phi}{dN} = \frac{\dot{\phi}}{H} = \pm \sqrt{\frac{r}{8A}}. \quad (29)$$

Thus during 60 e -folds, ϕ traverses a distance $\Delta\phi \simeq 15M_p \sqrt{2r/A}$. Hence a sizeable tensor-to-scalar ratio is possible even for a sub-Planckian excursion. We conclude that the Lyth bound can be explicitly evaded.

6. Conclusion and discussion

In this letter, we have minimally extended GR to a theory with a non-vanishing graviton mass term and proposed a mechanism to enhance the inflationary tensor perturbation. In our model, only the tensor perturbation is affected, while the scalar and vector perturbations remain the same as the ones in GR. The graviton mass is assumed to be proportional to the inflaton during reheating, and hence its coherent oscillations give rise to a significant resonant amplification for all long wavelength modes on super-horizon scales. Then we have numerically studied the dependence of the amplification factor on the graviton mass and the decay rate of inflaton during reheating. We find that the Lyth bound can be explicitly evaded in our model.

Our model contains three non-dynamical spacelike Stückelberg fields φ^i , which may formally become dynamical if we include higher order derivative terms. During inflation, however, the would-be new degrees of freedom are supermassive and exponentially decay away. Thus we can safely integrate out these modes at low energy scale. On the other hand, to screen these would-be new degrees at late time, if necessary, we can simply add a tiny but non-zero constant to the mass term in Eq. (6). The current upper bound of the graviton mass m_g is about 10^{-20} eV, from observation of Hulse–Taylor binary pulsar, PSR B 1913 + 16 [24].

Note that our results do not crucially depend on the assumption of Eq. (6). The broad parametric resonance of Mathieu equation requires only that graviton mass must be much greater than Hubble constant at the beginning of reheating. In this sense, the possibilities of functional dependence of m_g^2 on inflaton are very rich. It would be very interesting to ask what types of functional dependence of m_g^2 could be naturally induced from some more fundamental physics. However, this question is beyond the scope of this paper and we will leave it as one of possible directions to explore.

At the nonlinear perturbation level, we expect that the graviton mass term will introduce several new interaction terms. It will be interesting to study its possible imprints in, e.g. the non-Gaussianity of CMB anisotropies. As for models beyond the minimal model, a massive graviton generically induces non-trivial

scalar and vector perturbations. We plan to study such possibilities and their possible observational effects in future.

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References

- [1] A.H. Guth, *Phys. Rev. D* 23 (1981) 347.
- [2] Planck Collaboration, arXiv:1502.02114 [astro-ph.CO].
- [3] P. Creminelli, D.L. Nacir, M. Simonović, G. Trevisan, M. Zaldarriaga, arXiv:1502.01983 [astro-ph.CO].
- [4] D.H. Lyth, *Phys. Rev. Lett.* 78 (1997) 1861.
- [5] M. Fierz, W. Pauli, *Proc. R. Soc. Lond. A* 173 (1939) 211–232.
- [6] C. de Rham, G. Gabadadze, A.J. Tolley, *Phys. Rev. Lett.* 106 (2011) 231101, arXiv:1011.1232 [hep-th].
- [7] H. van Dam, M.J.G. Veltman, *Nucl. Phys. B* 22 (1970) 397–411.
- [8] V.I. Zakharov, *JETP Lett.* 12 (1970) 312.
- [9] D.G. Boulware, S. Deser, *Phys. Rev. D* 6 (1972) 3368–3382.
- [10] A.E. Gumrukcuoglu, C. Lin, S. Mukohyama, *J. Cosmol. Astropart. Phys.* 1203 (2012) 006, arXiv:1111.4107 [hep-th].
- [11] A. De Felice, A.E. Gumrukcuoglu, S. Mukohyama, *Phys. Rev. Lett.* 109 (2012) 171101.
- [12] S.L. Dubovsky, *J. High Energy Phys.* 0410 (2004) 076.
- [13] V.A. Rubakov, arXiv:hep-th/0407104.
- [14] D. Comelli, F. Nesti, L. Pilo, *J. High Energy Phys.* 1307 (2013) 161, arXiv:1305.0236 [hep-th].
- [15] D. Comelli, F. Nesti, L. Pilo, *J. Cosmol. Astropart. Phys.* 1411 (11) (2014) 018, arXiv:1407.4991 [hep-th].
- [16] C. Lin, *Phys. Lett. B* 727 (2013) 31, arXiv:1305.2069.
- [17] C. Lin, *Phys. Lett. B* 738 (2014) 386, arXiv:1307.2574.
- [18] C. Lin, L.Z. Labun, arXiv:1501.07160 [hep-th].
- [19] S.L. Dubovsky, P.G. Tinyakov, I.I. Tkachev, *Phys. Rev. Lett.* 94 (2005) 181102, arXiv:hep-th/0411158.
- [20] C. Cheung, P. Creminelli, A.L. Fitzpatrick, J. Kaplan, L. Senatore, *J. High Energy Phys.* 0803 (2008) 014, arXiv:0709.0293 [hep-th].
- [21] A. Higuchi, *Nucl. Phys. B* 282 (1987) 397.
- [22] M. Sasaki, *Prog. Theor. Phys.* 76 (1986) 1036.
- [23] V.F. Mukhanov, *Sov. Phys. JETP* 67 (1988) 1297; V.F. Mukhanov, *Zh. Eksp. Teor. Fiz.* 94 (7) (1988) 1.
- [24] P.J. Sutton, L.S. Finn, *Class. Quantum Gravity* 19 (2002) 1355, arXiv:gr-qc/0112018.