代数幾何学シンポジウム記録
1998年度 pp.98-105

Loop groups and elliptic singularities

STEFAN HELMKE

(joint work with PETER SLODOWY)

Abstract. There is a well known relation between simple algebraic groups and simple singularities. The simple singularities appear as the generic singularity in codimension two of the unipotent variety of simple algebraic groups. Furthermore, the semi-universal deformation and the simultaneous resolution of the singularity can be constructed in terms of the algebraic group. The aim of these notes is to extend this kind of relation to loop groups and simple elliptic singularities.

§1 **Simple groups and simple singularities.** Let G be a simple and simply connected algebraic group over $\mathbb{C}$. Then, G acts by conjugation on itself and the ring of invariant functions on G is a polynomial ring in the fundamental characters of G

$$\mathbb{C}[G]^G \;=\; \mathbb{C}[\chi_1,\ldots,\chi_\ell].$$

The induced map

$$G \longrightarrow \operatorname{Spec}\mathbb{C}[\chi_1,\ldots,\chi_\ell] \;\simeq\; \mathbb{C}^\ell$$

is called the *adjoint quotient map* for G. The group G also acts by the adjoint representation on its Lie algebra $\mathfrak{g}$ and the ring of invariant functions on $\mathfrak{g}$ is also a polynomial ring in $\ell = \operatorname{rank} G$ variables, but in this case the generators can not so easily be described. Nevertheless, the map

$$\chi : \mathfrak{g} \longrightarrow \mathfrak{g}/\!/G \;\simeq\; \mathbb{C}^\ell$$

is called the *adjoint quotient map* for $\mathfrak{g}$. The zero fiber $N = \chi^{-1}(0)$ is called the nilpotent variety, since it consists of all the nilpotent elements of $\mathfrak{g}$. Under the action of G the nilpotent variety decomposes into a finite number of orbits. In particular there is one open and dense orbit, the so called regular orbit. Moreover, there is a unique orbit of codimension two in N, the so called subregular orbit. Now let T be a transversal slice to the subregular orbit ($T \simeq \mathbb{C}^{\ell+2}$) and for simplicity assume that G is of type ADE. Then, due to E. Brieskorn, A. Grothendieck and P. Slodowy, the restriction of the adjoint quotient map χ to the transversal slice T

$$\chi|_T : \mathbb{C}^{\ell+2} \longrightarrow \mathbb{C}^\ell$$

is the semi-universal deformation of the simple hypersurface singularity $T \cap N$, where the equation for $T \cap N$ is listed in Figure 1.

Those singularities are characterized in many ways. For example they are the only hypersurface singularities which can be deformed only in a finite number of other

G	$T \cap N$
$A_\ell = SL_{\ell+1}$	$x^{\ell+1} + y^2 + z^2 = 0$
$D_\ell = Spin_{2\ell}$	$x^{\ell-1} + xy^2 + z^2 = 0$
E_6	$x^4 + y^3 + z^2 = 0$
E_7	$x^3y + y^3 + z^2 = 0$
E_8	$x^5 + y^3 + z^2 = 0$

Fig. 1. Simple singularities

singularities. For that reason they are called *simple singularities*. If G is of type $BCFG$, the singularity $T \cap N$ carries an extra finite symmetry and $\chi|_T$ is only the invariant part of the semi-universal deformation of $T \cap N$.

The restriction of the adjoint quotient map for G to a transversal slice at a subregular unipotent orbit is also the semi-universal deformation. But there is an important difference. The map $\chi|_T$ is quasi homogeneous with respect to some $\mathbb{C}^*$-actions on T and $\mathbb{C}^\ell$ with only positive weights. Such $\mathbb{C}^*$-actions don't exist for G.

§2 **Simple elliptic singularities.** Let E be an elliptic curve and $L \longrightarrow E$ be a holomorphic line bundle over E of degree $d < 0$. Then one can contract the zero section of L to a surface singularity X. It turns out that X is a complete intersection, if and only if $d \geqslant -4$. For $d = -4$, the singularity X is the the intersection of two quadrics in $\mathbb{C}^4$. More precisely, X is the cone over the elliptic curve E embedded by a line bundle (L^*) of degree 4 in $\mathbb{P}^3$. This singularity is called of type $\tilde{D}_5$.

For $d \geqslant -3$, the singularity X is a hypersurface singularity of type E_{9+d}. Its equation is listed in Figure 2.

$\tilde{E}_8$	$d = -1$	$x^6 + y^3 + z^2 + \lambda xyz = 0$
$\tilde{E}_7$	$d = -2$	$x^4 + y^4 + z^2 + \lambda xyz = 0$
$\tilde{E}_6$	$d = -3$	$x^3 + y^3 + z^3 + \lambda xyz = 0$

Fig. 2. Simple elliptic singularities

Here, λ is a complex parameter which depends on the elliptic curve E. The deformation theory of those singularities were studied by E. J. Looijenga and K. Saito. It turns out that they deform only in elliptic singularities of the same type with different λ and in simple singularities. For that reason they are called *simple elliptic singularities*.

§3 **Loop groups.** The (holomorphic) *loop group* of the simple algebraic group G is defined by

$$\mathcal{L}\,G \;=\; \{\varphi : \mathbb{C}^* \longrightarrow G \mid \varphi \text{ is holomorphic}\}.$$

The infinite dimensional group $\mathcal{L}\,G$ has a universal central extension

$$1 \longrightarrow \mathbb{C}^* \longrightarrow \tilde{\mathcal{L}}\,G \longrightarrow \mathcal{L}\,G \longrightarrow 1.$$

Now, $\mathbb{C}^*$ acts on $\mathcal{L}\,G$ by the formula $(q\,\varphi)(z) = \varphi(qz)$ and this action can be lifted to $\tilde{\mathcal{L}}\,G$. The semidirect product of $\tilde{\mathcal{L}}\,G$ with $\mathbb{C}^*$ is denoted by

$$\hat{\mathcal{L}}\,G \;=\; \tilde{\mathcal{L}}\,G \rtimes \mathbb{C}^*.$$

It is a certain completion of some affine Kac-Moody group. Such a group has $\ell+1$ fundamental highest weight representations. Their characters are convergent for $(\tilde{\varphi}, q)$ with $q \in D^*$, where $D^* = \{ q \in \mathbb{C} \mid 0 < |q| < 1 \}$. Hence, there is a map

$$\begin{array}{ccc} \hat{\mathcal{L}}G & & \\ \cup & & \\ \tilde{\mathcal{L}}G \times D^* & \xrightarrow{\hat{\chi}} & \mathbb{C}^{\ell+1} \times D^*, \end{array}$$

where $\hat{\chi}$ is given by the $\ell+1$ fundamental characters and the second projection. This map is called the *adjoint quotient map* for $\hat{\mathcal{L}}G$.

§4 **Subregular singularities.** In the sequel we will fix a number $q \in D^*$. The zero fiber $U_q = \hat{\chi}^{-1}(0, q)$ is called the *unstable variety* for reasons we will see later. In this section we are going to describe the singularities of unstable variety in codimension two for all $\hat{\mathcal{L}}G$, where G is of type ADE. For this purpose we will call an $\hat{\mathcal{L}}G$ orbit in U_q *regular*, if it has codimension zero in U_q and it is called *subregular* if it has codimension two in U_q. Actually all orbits in U_q have finite codimension and there is never one of codimension one.

CASE A_1. There is one regular and one subregular orbit in U_q. Let S be the subregular orbit and T_S a transversal slice to S. Then the singularity of $U_q \cap T_S$ is of type $\tilde{D}_5$.

CASE A_ℓ, $\ell > 1$. There are ℓ regular orbits $R_1, \dots, R_\ell$ and each intersection $\bar{R}_i \cap \bar{R}_{i+1}$ contains a 1-parameter family of subregular orbits. For every subrerular orbit, the singularity of $U_q \cap T_S$ is of type A_∞ i.e. two smooth transversally crossing surfaces:

$$(A_\infty) \qquad \{ (x, y, z) \in \mathbb{C}^3 \mid y^2 + z^2 = 0 \}.$$

CASE D_ℓ, $\ell > 5$. There is one regular orbit and a 1-parameter family of subregular orbits. Generically the singularity of $U_q \cap T_S$ is of type A_∞, but for four different subregular orbits it is of type D_∞ (cf. Fig. 3):

$$(D_\infty) \qquad \{ (x, y, z) \in \mathbb{C}^3 \mid xy^2 + z^2 = 0 \}.$$

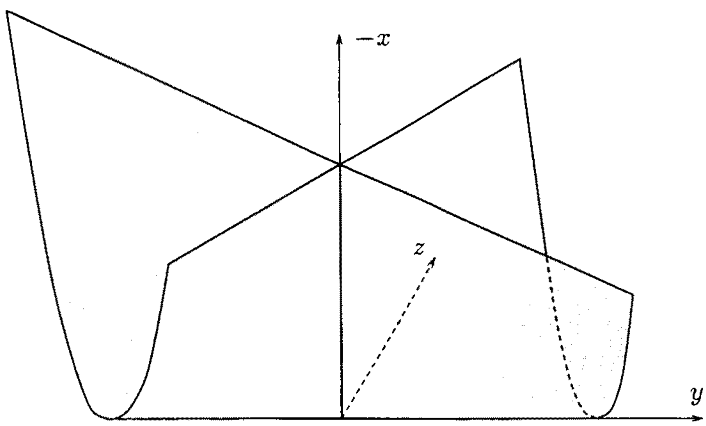


Fig. 3. The Whitney umbrella D_∞

CASE D_4. The same as for D_ℓ with $\ell > 5$, but there are three such 1-parameter families of subregular orbits.

CASE D_5. Beside the 1-parameter family of subregular orbits there are two more subregular orbits. The singularity of $U_q \cap T_S$ for those two orbits is of type $\tilde{D}_5$.

CASE E_6. There is one regular and two subregular orbits. The singularity of $U_q \cap T_S$ is of type $\tilde{E}_6$.

CASE E_7. There is one regular and one subregular orbit. The singularity of $U_q \cap T_S$ is of type $\tilde{E}_7$.

CASE E_8. There is one regular and one subregular orbit. The singularity of $U_q \cap T_S$ is of type $\tilde{E}_8$.

§5 **Deformations.** The restriction of the adjoint quotient map $\hat{\chi}$ to the transversal slice T_S to a subregular orbit S is a deformation of the singularity $U_q \cap T_S$. In the cases E_ℓ and also for the two special subregular orbits of D_5, this turns out to be the semi-universal deformation. But in the other cases, the singularity $U_q \cap T_S$ is non-isolated and the base of its semi-universal deformation is infinite dimensional. But as in the case of an algebraic group of type $BCFG$ there is an extra symmetry and the restriction of the adjoint quotient map seems to be basically the invariant part of the semi-universal deformation.

For example if G is of type A_ℓ with $\ell > 1$, the transversal slice to a subregular orbit is isomorphic to $T_S \simeq \mathbb{C}^{\ell+2} \times \mathbb{C}^* \times D^*$. Now, the elements $(0, q^n\lambda, q)$ are all in the same $\hat{\mathcal{L}}G$-orbit for fixed $\lambda \in \mathbb{C}^*$ and all $n \in \mathbb{Z}$. If one could extend this $\mathbb{Z}$-action to T_S, then

$$T_S/\mathbb{Z} \xrightarrow{\hat{\chi}} \mathbb{C}^{\ell+1} \times D^*$$

would be a deformation of the variety $(U_q \cap T_S)/\mathbb{Z}$ and this has a finite dimensional deformation space. Actually it is isomorphic to the union of two line bundles L' and L'' over the elliptic curve $E = \mathbb{C}^*/q^{\mathbb{Z}}$:

$$(U_q \cap T_S)/\mathbb{Z} \;\simeq\; \{\, (x,y,z) \mid x \in E,\; y \in L'_x,\; z \in L''_x \text{ and } y \cdot z = 0 \,\}.$$

The semi-universal deformation of this variety is basically given by

$$\{\, (x,y,z,s) \mid y \cdot z = s(x) \,\} \longrightarrow \Gamma(E, L),$$

where $L = L' \otimes L''$ and the map is given by the projection onto the last coordinate $s \in \Gamma(E, L)$. Now assume that $\deg L = \ell + 1$. Then the zero divisor of s can be written as

$$(s) \;=\; n_1 P_1 + \cdots + n_r P_r, \qquad n_1 + \cdots + n_r = \ell + 1$$

and $P_i \in E$ with $P_i \neq P_j$ for $i \neq j$ and

$$(*) \qquad n_1 P_1 + \cdots + n_r P_r \;\equiv\; L.$$

The singularities of $\{\, y \cdot z = s(x) \,\}$ are of type $(A_{n_1-1}, \ldots, A_{n_r-1})$. Hence, the projective space of $\Gamma(E, L)$ can be naturally identified with the hyperplane

$$\mathbb{P}(\Gamma(E, L)) \;\subset\; \underbrace{E \times \cdots \times E}_{\ell+1} / S_{\ell+1}$$

given by $(*)$, such that the two stratifications of $\mathbb{P}(\Gamma(E,L))$ according to singularities of $\{\, y \cdot z = s(x) \,\}$ and according to stabilizers of the symmetric group $S_{\ell+1}$ coincide. The last one coincides actually with the stratification of the base of the adjoint quotient map according to stabilizers of $\hat{\mathcal{L}}\, G$. Therefore, the restriction of the adjoint quotient map $\hat{\chi}$ to T_S should be basically the $\mathbb{Z}$-invariant part of the semi-universal deformation of $U_q \cap T_S$.

The case D_ℓ is a little more complicated and we will skip this here.

§6 **Principal bundles over elliptic curves.** Let $P \longrightarrow E$ be a principal G-bundle over the elliptic curve $E = \mathbb{C}^*/q^{\mathbb{Z}}$ and let $\pi : \mathbb{C}^* \longrightarrow E$ be the natural projection. Then, the pull back $\pi^* P$ of P to $\mathbb{C}^*$ is holomorphically trivial since G is connected. Therefore one has $P \simeq (\mathbb{C}^* \times G)/\mathbb{Z}$, where the generator $1 \in \mathbb{Z}$ acts by

$$(z, g) \longmapsto \big(qz, \varphi(z) \cdot g\big)$$

and $\varphi \in \mathcal{L}\, G$. In this way we get a surjective map

$$\begin{array}{ll} \mathcal{L}\, G \rtimes \mathbb{C}^* & \\ \quad \cup & \\ \mathcal{L}\, G \times q & \relbar\joinrel\twoheadrightarrow \{\text{holomorphic } G\text{-bundles over } E\}. \end{array}$$

Actually this map induces a bijection

$$(\mathcal{L}\, G \times q)/\mathcal{L}\, G \xrightarrow{\;1-1\;} \{\text{iso. classes of hol. } G\text{-bundles over } E\}.$$

Now, due to V. Baranovsky and V. Ginzburg an element of $\tilde{\mathcal{L}}\, G \times q$ belongs to the unstable variety U_q if and only if the corresponding G-bundle over E is unstable. For that reason, U_q is called the unstable variety.

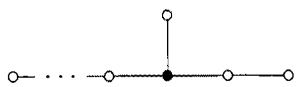
From this we see that there can be no continuous $\mathcal{L}\, G$-invariant function on $\mathcal{L}\, G \times q$. This is the geometric reason, why we need the central extension $\hat{\mathcal{L}}\, G$. In $\tilde{\mathcal{L}}\, G \times q$ all the semistable orbits contain only finitely many orbits in its closure and there is exactly one closed orbit in each such closure.

§7 **Principal bundles and Levi subgroups.** The structure group of a G-bundle over a curve can of course be always reduced to a Borel subgroup of G. But if the bundle is unstable and the curve is elliptic, it can be further reduced. In fact, the instability means that there is some subbundle of positive degree and on an elliptic curve this has to be a direct summand. Therefore the structure group reduces to a Borel subgroup of some Levi subgroup of G (i.e. the centralizer $L = C_G(H)^o$ of some torus $H \subset G$).

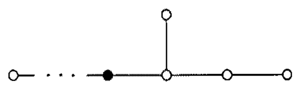
In the other direction, there is some construction of minimally unstable G-bundles whose structure group reduces to a Borel subgroup of a given Levi subgroup. If L is maximal, i.e. $Z(L)^o \simeq \mathbb{C}^*$, this construction usually leads to a unique unstable G-bundle. But if $Z(L)^o \simeq (\mathbb{C}^*)^2$, one gets a 1-parameter family of unstable G-bundles.

For example, if G is of type E_ℓ, the Dynkin diagram of the Levi subgroup corresponding to the regular and subregular orbits are the following

(E_ℓ)



regular orbit

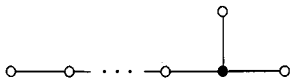


subregular orbit

Here $L = C_G(H)^o$, where H is the subtorus of the maximal torus spanned by the 1-parameter subgroups corresponding to the filled vertices. In the case E_6 there is a symmetry of the diagram which leads to two subregular orbits.

In the case D_ℓ the Dynkin diagram of regular and subregular orbits are

(D_ℓ)

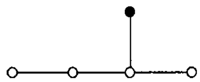


regular orbit

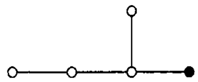
subregular orbit

There is a whole 1-parameter family of subregular orbits. In the case D_4, one gets three such families by the symmetry of the diagram. In the case D_5 there are the following two more subregular elements

(D_5)



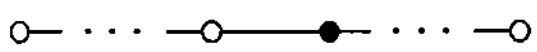
subregular orbit



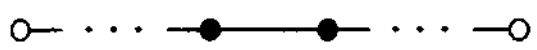
subregular orbit

Finally, in the case A_ℓ with $\ell > 1$ we have

(A_ℓ)



regular orbit



subregular orbit

The case A_1 is a little special. Every unstable rank two vector bundle with trivial determinant is of the form $L \oplus L^*$ for some line bundle L of degree $d > 0$. The regular element corresponds to $d = 1$ and the subregular to $d = 2$.

§8 **The case Spin(10).** There is one subregular $Spin_{10}$-bundle which corresponds to the Levi subgroup $\mathbb{C}^* \cdot SL_5 \subset Spin_{10}$. This is very easy to describe. There is a unique indecomposable rank 5 vector bundle V with $\det V = L^2$, where L is a line bundle of degree 1. The bundle

$$V \oplus V^*$$

is a SO_{10}-bundle with trivial second Stiefel Whitney class. Hence, the structure group can be reduced to $Spin_{10}$ and this is the subregular $Spin_{10}$-bundle we are looking for. The deformations of this bundle are given by extensions

$$0 \longrightarrow V^* \longrightarrow V_\xi \longrightarrow V \longrightarrow 0$$

where the extension class $\xi \in \operatorname{Ext}^1(V, V^*) = H^1(V^* \otimes V^*)$ belongs to the subgroup $H^1(\bigwedge^2 V^*)$. As a $Spin_{10}$-bundle, $V \oplus V^*$ has one more deformation, namely the 1-parameter deformation of L. But the group $\hat{\mathcal{L}}\, G$ contains the translations of the elliptic curve and hence we may ignore those deformations. The transversal slice to the orbit corresponding to $V \oplus V^*$ is therefore

$$T_S \simeq H^1(\textstyle\bigwedge^2 V^*) \times D^*.$$

We want to see that $U_q \cap T_S$ has a $\tilde{D}_5$ singularity. For this purpose we have to find all ξ for which V_ξ is unstable. By definition V_ξ is unstable if and only if there exists an indecomposable proper subbundle $W \subset V_\xi$ of positive degree. Since then one has

$$\operatorname{Hom}(W, V) \neq 0,$$

it follows that the degree of W is 1 and its rank has to be 3, 4 or 5. Actually 4 and 5 can be excluded from the fact that V_ξ is a $Spin_{10}$-bundle. Now, for every indecomposable rank 3 bundle W of degree 1 one has $\mathrm{Hom}(W,V) \simeq \mathbb{C}$ and each nontrivial morphism $W \longrightarrow V$ is injective. From this we get the following commutative diagram

$$\begin{tikzcd}
 & & & 0 & \\
0 \arrow[r] & \mathrm{Hom}(W,V_\xi) \arrow[r] & \mathrm{Hom}(W,V) \arrow[r] & \mathrm{Ext}^1(W,V^*) \arrow[u] & \\
 & & id \in \mathrm{Hom}(V,V) \arrow[u] \arrow[r] & \mathrm{Ext}^1(V,V^*) \arrow[u] \arrow[r, phantom, "\supset"] & H^1(\bigwedge^2 V^*) \ni \xi \\
 & & \mathrm{Hom}(V/W,V) \arrow[u] \arrow[r] & \mathrm{Ext}^1(V/W,V^*) \arrow[u, "F_W"'] \arrow[r, phantom, "\supset"] & H^1(\bigwedge^2 (V/W)^*) \arrow[u, phantom, "\cup"] \\
 & & & 0. \arrow[u] &
\end{tikzcd}$$

It shows that V_ξ is unstable if and only if

$$\xi \in \operatorname{Im} F_W \cap H^1(\textstyle\bigwedge^2 V^*) = H^1(\textstyle\bigwedge^2 (V/W)^*) \simeq \mathbb{C}$$

for some indecomposable vector bundle W of rank 3 and degree 1.

The vector bundle $\bigwedge^2 V^*$ has rank 10 and degree -8. It is actually the direct sum of a vector bundle $\tilde{V}$ of rank 5 and degree -4 with itself. Moreover, one can show that there is a canonially defined subbundle $\tilde{V} \subset \bigwedge^2 V^*$ with the following property. For every choice of W, the line $\operatorname{Im} F_W \cap H^1(\bigwedge^2 V^*)$ is contained in the four dimensional subspace $H^1(\tilde{V}) \subset H^1(\bigwedge^2 V^*)$.

Recall, that due to M. F. Atiyah's work on vector bundles over elliptic curves the map $W \longmapsto \det W$ is a bijection between indecomposable rank 3 vector bundles of degree 1 and $Pic^1(E) \simeq E$. Therefore, the unstable locus in $H^1(\bigwedge^2 V^*)$ is the cone over the image of the natural morphism

$$\begin{aligned} \Phi : E &\longrightarrow \mathbb{P}(H^1(\tilde{V})), \\ P &\longmapsto \operatorname{Im} F_{W_P} \cap H^1(\textstyle\bigwedge^2 V^*), \end{aligned}$$

where W_P is the rank 3 vector bundle with $\det W_P = \mathcal{O}(P)$. Finally one can identify the map Φ with the natural embedding of E in $\mathbb{P}^3$ given by the line bundle L^4. Hence, the singularity $U_q \cap T_S$ is of type $\tilde{D}_5$.

Now, a simple argument using the $\mathbb{C}^*$-action on T_S shows that the restriction of $\hat{\chi}$ to T_S is in fact the semi-universal deformation of the zero fiber $U_q \cap T_S$.

In the cases E_6, E_7 and E_8 it is much harder to see the singularity directly. Instead of this, we classified unstable G-bundles and together with the knowledge of the weights of the $\mathbb{C}^*$-action on T_S, one can identify the singularity $U_q \cap T_S$. But we are not going to discuss this furhter now.

§9 **Double loop algebras.** In contrast to the finite dimensional case, the Lie algebra of $\hat{\mathcal{L}}\,G$ is not usefull to construct simple elliptic singularities. This follows from some work due to I. Frenkel, which relates the orbit structure of the Lie algebra of $\hat{\mathcal{L}}\,G$ with that of the finite dimensional group G. Instead of this, one has to consider double loop algebras. Those are defined by

$$\mathcal{E}\mathfrak{g} \;=\; \{\,\varphi : S^1 \times S^1 \longrightarrow \mathfrak{g} \mid \varphi \text{ is } C^\infty\,\}.$$

There are two derivations $\partial/\partial\alpha$ and $\partial/\partial\beta$ acting on $\mathcal{E}\mathfrak{g}$, where a point in $S^1 \times S^1$ is parameterized by $(e^{i\alpha}, e^{i\beta})$. We are intersted in the conjugacy classes of the semidirect product

$$\mathcal{E}\mathfrak{g} \rtimes \left(\mathbb{C}\,\frac{\partial}{\partial\,\alpha} \oplus \mathbb{C}\,\frac{\partial}{\partial\,\beta}\right).$$

Let us fix a derivation

$$\bar\partial \;=\; \omega\,\frac{\partial}{\partial\,\alpha} + \eta\,\frac{\partial}{\partial\,\beta} \qquad \text{with} \quad Im\,\frac{\omega}{\eta} > 0.$$

Then $\bar\partial$ defines a holomorhic structure on $S^1 \times S^1$ by

$$\begin{array}{l} S^1 \times S^1 \\ \quad \cup \text{ open} \\ f : U \longrightarrow \mathbb{C} \quad \text{is holomorphic} \;:\Longleftrightarrow\; \bar\partial f \;=\; 0. \end{array}$$

With this holomorphic structure $S^1 \times S^1$ becomes an elliptic curve E. Moreover, an element $\varphi \in \mathcal{E}\mathfrak{g}$ defines a holomorphic structure on the topologically trivial $\mathrm{Ad}\,G$-bundle $E \times \mathfrak{g} \longrightarrow E$ by

$$\begin{array}{l} \quad E \\ \quad \cup \text{ open} \\ s : U \longrightarrow \mathfrak{g} \quad \text{is holomorphic} \;:\Longleftrightarrow\; (\bar\partial + \varphi)\, s \;=\; 0, \end{array}$$

where $\varphi\, s$ is defined by matrix multiplication. The adjoint group $\mathcal{E}G$ corresponding to $\mathcal{E}\mathfrak{g}$ acts on $\mathcal{E}\mathfrak{g} \times \bar\partial$ as the gauge group of $E \times \mathfrak{g} \longrightarrow E$ and hence we have a bijection

$$(\mathcal{E}\mathfrak{g} \times \bar\partial)/\mathcal{E}G \xrightarrow{\;1-1\;} \{\text{iso. classes of hol. } G\text{-bundles over } E\}.$$

This suggests that the simple elliptic singularities should also appear in the double loop algebras $\mathcal{E}\mathfrak{g}$. But so far, this is not well understood.

RESEARCH INSTITUTE FOR MATHEMATICAL SCIENCES
Kyoto University, Kyoto 606–8502, Japan
EMAIL ADDRESS: helmke@kurims.kyoto-u.ac.jp