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LOCAL FAMILIES OF K3 SURFACES AND 5 APPLICATIONS

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1. Introduction

A hyperkähler manifold is by the definition a simply connected, compact Kähler manifold F with $H^{2,0}(F) = \mathbf{C}\omega_F$, where ω_F is an everywhere non-degenerate holomorphic 2-form. In this terminology, a K3 surface is nothing but a hyperkähler manifold of dimension 2. Due to the fundamental work by Bogomolov and Beauville [Be], the following results hold for a hyperkähler manifold F of any dimension:

(1) The Kuranishi space of F is smooth and universal;
(2) There is a natural, integral non-degenerate symmetric bilinear form $(*, *)$ of signature $(3, b_2(F) - 3)$ on $H^2(F, \mathbf{Z})$. This bilinear form induces on $H^2(F, \mathbf{C}) = H^{1,1}(F) \oplus \mathbf{C}\omega_F \oplus \mathbf{C}\overline{\omega}_F$ the Hodge structure of weight two;
(3) The local Torelli Theorem holds for the period map given by the Hodge structure on $H^2(F, \mathbf{Z})$ defined in 2. above.

In this report, we consider a smooth family of hyperkähler manifolds $f : \mathcal{X} \to \Delta$ over a disk Δ. In this setting, the following two statements are equivalent:

(1) f is trivial as a family, i.e. isomorphic to the product $F \times \Delta$ over Δ;
(2) all the fibers of f are isomorphic.

We denote by $\rho(F)$ the Picard number of F, i.e. the rank of the Néron-Severi group $NS(F) := \mathrm{Im}(c_1 : H^1(F, \mathcal{O}_F^\times) \to H^2(F, \mathbf{Z})) = (\mathbf{C}\omega_F)^\perp \cap H^2(F, \mathbf{Z})$. Here the last equality is due to the Lefschetz $(1, 1)$-Theorem. Note also that $\mathrm{NS}(F)$ has a natural lattice structure induced by the bilinear form $(*, **)$.

Our starting point is the following:

Theorem 1.1. [Og] *Let $f : \mathcal{X} \to \Delta$ be a non-trivial family of hyperkähler manifolds. Set $m := \min\{\rho(\mathcal{X}_t) | t \in \Delta\}$, $\mathcal{G} := \{t \in \Delta | \rho(\mathcal{X}_t) = m\}$ and $\mathcal{S} := \{t \in \Delta | \rho(\mathcal{X}_t) > m\}$. Then,*

(1) The lattices $NS(\mathcal{X}_t)$ are all isomorphic for $t \in \mathcal{G}$;
(2) $\mathcal{S}$ is a dense countable subset of Δ in the classical topology.

This is a generalization of a result of R. Borcherds, L. Katzarkov, T. Pantev and N. I. Shepherd-Barron [BKPS].

The following example illustrates the phenomenon in the theorem fairly well:

Example 1.2. Let us denote by E_t the elliptic curve of period t. Let Δ be a small disk in the upper half plane $\mathbf{H}$. Then, one has a family of elliptic curves $f_1 : \mathcal{E} \to \Delta$ with the level two structure such that $\mathcal{E}_t = E_t$. Taking a crepant resolution of the quotient of the product $f_2 : \mathcal{E} \times E_{\sqrt{-1}} \to \Delta$ by the inversion, one obtains a family of K3 surfaces $f_3 : \mathcal{X} \to \Delta$ such that $\mathcal{X}_t = \mathrm{Km}(E_t \times E_{\sqrt{-1}})$. This family f_3 satisfies $\rho(\mathcal{X}_t) = 20$ for $t \in \mathbf{Q}(\sqrt{-1})$ and $\rho(\mathcal{X}_t) = 18$ for $t \notin \mathbf{Q}(\sqrt{-1})$. In this example, we have $\mathcal{S} = \Delta \cap \mathbf{Q}(\sqrt{-1})$. □

It is an easy fact that $\mathcal{S}$ is at most countable and that $\mathcal{G}$ is dense and uncountable. Therefore $\mathcal{G}$ is "much bigger" than $\mathcal{S}$. We regard the fibers over $\mathcal{G}$ *general* and the fibers over $\mathcal{S}$ *special*. The essential part of the theorem 1.1 is *the existence of enough special points.*

As it is shown in section 2, the proof of theorem 1.1 is extremely easy. However, by combining theorem 1.1 with other known results, one can obtain several interesting new results. I would like to state 5 of such results without proof. For proof, I refer to the readers the original papers.

1st Application. The first application is the following filling up of Picard numbers:

Application 1.3. [Og] *Let F be a hyperkähler manifold with $b_2(F) = N + 2$. Let $u : \mathcal{U} \to \mathcal{K}$ be the Kuranishi family*[1] *of F. Then, for each integer j such that $0 \le j \le N$, the locus $\{t \in \mathcal{K} | \rho(\mathcal{U}_t) = j\}$ is dense in $\mathcal{K}$.* □

2nd Application. One can also apply theorem 1.1 for clarifying certain relationships among all of the Mordell-Weil lattices of Jacobian K3 surfaces $\varphi : X \to \mathbf{P}^1$. Here the Mordell-Weil group $MW(\varphi)$ with Shioda's positive definite, symmetric bilinear form $\langle *, * \rangle$ is called the Mordell-Weil lattice [Sh 1, 2]. We remark that the Mordell-Weil rank r of a Jacobian K3 surface satisfies $0 \le r \le 18$. By the narrow Mordell-Weil lattice $MW^0(\varphi)$, we mean the sublattice of $MW(\varphi)$ consisting of the sections which pass through the identity component of each fiber. As it is well known, $MW^0(\varphi)$ is of finite index in $MW(\varphi)$. Contrary to the case of rational Jacobian surfaces, the isomorphism classes of both $MW(\varphi)$ and $MW^0(\varphi)$ of Jacobian K3 surfaces are no more finite.

The second application is the following:

Application 1.4. [Og] *For any given Jacobian K3 surface $\varphi : J \to \mathbf{P}^1$ of Mordell-Weil rank $r := r(\varphi)$, there is a sequence $\{\varphi_m : J_m \to \mathbf{P}^1\}_{m=r}^{18}$ of Jacobian K3 surfaces such that*

1. *$\varphi_r : J_r \to \mathbf{P}^1$ is the original $\varphi : J \to \mathbf{P}^1$;*
2. *$r(\varphi_m) = m$ for each m; and*
3. *there is a sequence of isometric embeddings:*

$$MW^0(\varphi) = MW^0(\varphi_r) \subset MW^0(\varphi_{r+1}) \subset \cdots \subset MW^0(\varphi_{17}) \subset MW^0(\varphi_{18}).$$

In particular, the narrow Mordell-Weil lattice of a Jacobian K3 surface is embedded into the Mordell-Weil lattice of some Jacobian K3 surface of rank 18. Conversely, for every sublattice M of the (narrow) Mordell-Weil lattice of a Jacobian K3 surface of rank 18, there is a Jacobian K3 surface whose narrow Mordell-Weil lattice contains M as a sublattice of finite index. Moreover, for each given M, there are at most finitely many isomorphism classes of the Mordell-Weil lattices of Jacobian K3 surfaces which contains M as a sublattice of finite index. □

This coarsely reduces the study of the Mordell-Weil lattices $MW(\varphi)$ to those of the maximal rank 18.

3rd Application. One can also apply theorem 1.1 for studying the behavior of the automorphism groups of K3 surfaces under small one-dimensional projective deformation. More explicitly, one can discuss about the phrase, "*the automorphism*

[1] Note then that $0 \le \rho(\mathcal{U}_t) \le N$ for each $t \in \mathcal{K}$.

groups of projective K3 surfaces become larger at special points in their moduli." from the view of theorem 1.1.

The third application is the following:

Application 1.5. **[Og]** *Let $f : \mathcal{X} \longrightarrow \Delta$ be a smooth projective family of K3 surfaces. The sets $\mathcal{G}$ and $\mathcal{S}$ are the same as in theorem 1.1. Then,*

(1) *There are a (possibly empty) finite subset $\mathcal{F} \subset \mathcal{S}$, a group G^0, and a positive integer N depending only on f such that*

$$G^0 < Aut(\mathcal{X}_t) \text{ for all } t \in \Delta - \mathcal{F}$$

and

$$[Aut(\mathcal{X}_t) : G^0] \leq N \text{ for all } t \in \mathcal{G}.$$

In particular, the map $Aut : \Delta \longrightarrow \{groups\}/ \equiv; t \mapsto [Aut(\mathcal{X}_t)]$ is "upper-semicontinuous" on $\Delta - \mathcal{F}$, in the sense that the map Aut is constant on $\mathcal{G}$ with value $[G^0]$ and the indices $[Aut(\mathcal{X}_t) : G^0]$ can be of infinite order only at the special points $t \in \mathcal{S} - \mathcal{F}$.

(2) *There is a smooth projective family of K3 surfaces $f : \mathcal{X} \longrightarrow \Delta$ such that $\mathcal{F} \neq \emptyset$. More concretely, there is a smooth projective family of K3 surfaces $f : \mathcal{X} \longrightarrow \Delta$ such that $\rho(\mathcal{X}_t) = 2$ and $|Aut(\mathcal{X}_t)| = \infty$ for $t \in \mathcal{G}$ but $\rho(\mathcal{X}_0) = 19$ and $|Aut(\mathcal{X}_0)| < \infty$ at the special point $0 \in \Delta$.*

(3) *There is a subset $\mathcal{D} \subset \mathcal{S}$ such that $\mathcal{D}$ is dense in Δ and such that*

$$|Aut(\mathcal{X}_t)| = \infty \text{ for all } t \in \mathcal{D}. \quad \square$$

The first assertion mostly justifies the phrase quoted at the beginning, while the second denies the phrase in the most strict sense. It is also interesting to compare the second statement with the fact: *If X is a K3 surface, then $|Aut(X)| < \infty$ if $\rho(X) = 1$ and $|Aut(X)| = \infty$ if $\rho(X) = 20$, the maximum value by [SI].*

As a direct consequence of the third assertion, one obtains the following:

Corollary 1.6. *Let $f : \mathcal{X} \longrightarrow \Delta$ be a (not necessarily projective) non-trivial family of projective K3 surfaces. Then, there is a dense subset $\mathcal{D} \subset \Delta$ such that $|Aut(\mathcal{X}_t)| = \infty$ for all $t \in \mathcal{D}$. In particular, the nef cone $\overline{A}(\mathcal{X}_t)$ is not finite rational polyhedral if $t \in \mathcal{D}$.* $\square$

For the statement, the projectivity of K3 surfaces in a family is essential.

The third assertion of our application 1.5 and corollary 1.6 are somewhat surprizing. For instance, let us take the component $\mathcal{H}$ of the Hilbert scheme consisting of quartic K3 surfaces and consider the universal family $u : \mathcal{U} \longrightarrow \mathcal{H}$. Then for any sufficiently general $\Delta \longrightarrow \mathcal{H}$, the induced family $\varphi : \mathcal{X} \longrightarrow \Delta$ satisfies $\mathrm{Pic}(\mathcal{X}_t) = \mathbf{Z}\mathcal{L}_t$ for $t \in \mathcal{G}$, where $\mathcal{L}_t$ is the plane section class of $\mathcal{X}_t \subset \mathbf{P}^3$. So $|\mathrm{Aut}(\mathcal{X}_t)| < \infty$ if $t \in \mathcal{G}$. Application 1.5 or corollary 1.6 claims, however, that there is a dense subset $\mathcal{D}$ such that $|\mathrm{Aut}(\mathcal{X}_t)| = \infty$ for all $t \in \mathcal{D}$ in this family. This also provides an explicit example of a family in which the automorphism groups actually jump above at special points.

On the other hand, one has by Kollár([Bo]): *If $f : \mathcal{Y} \longrightarrow \Delta$ is a smooth family of Calabi-Yau manifolds in $|-K_V|$ of a Fano manifold V, then the nef cones $\overline{A}(\mathcal{Y}_t)$ are finite rational polyhedral, whence $|Aut(\mathcal{Y}_t)| < \infty$, for all $t \in \Delta$ provided that* $\dim V \geq 4$. Thus, the statements similar to application 1.5 and corollary 1.6 do not hold for Calabi-Yau manifolds of higher dimension.

4th Application. As the forth application, one can add some evidence to the following interesting conjecture posed by De-Qi Zhang:

Conjecture 1.7. (De-Qi Zhang) *The universal cover of the smooth locus of a normal K3 surface is a big open set of either a normal K3 surface or of* $\mathbf{C}^2$. *Furthermore, in the latter case, the universal cover factors through a finite étale cover by a big open set of a torus.*

Here, a *normal K3 surface* means a normal surface whose minimal resolution is a K3 surface and a *torus* is a 2-dimensional complex torus. By a *big open set* we mean the complement of a discrete subset.

Let $\overline{S}$ be a normal K3 surface, let $S^0 := \overline{S} - \mathrm{Sing}\,\overline{S}$ and let $\nu : S \to \overline{S}$ the minimal resolution. We set E to be the reduced exceptional divisor $E := \nu^{-1}(\mathrm{Sing}\,\overline{S})$ and decompose E into irreducible components $E := \sum_{i=1}^{r} E_i$. An important invariant is the number $r := r(\overline{S})$ of irreducible components of the exceptional divisor. Clearly, $S^0 = S - E$.

The forth application is the following sharp sufficient condition for the validity of the first alternative in Zhang's conjecture:

Application 1.8. [CKO] *If* $r = r(\overline{S}) \le 15$, *then* $\pi_1(S^0)$ *is finite and the universal cover of* S^0 *is a big open set of a normal K3 surface.* □

This is the best possible uniform bound on r in order that $\pi_1(S^0)$ be finite, in view of the following fact: *A normal Kummer surface* $A/-1$ *satisfies* $r = 16$ *and* $|\pi_1((A/-1)^0)| = \infty$.

Theorem 1.1 is used in reducing the proof to the following:

Theorem 1.9. [CKO] *If the normal K3 surface* $\overline{S}$ *admits an elliptic fibration then either* $\pi_1(S^0)$ *is finite or there is a finite covering of* $\overline{S}$, *ramified only on a finite set, which yields a complex torus*[2]. □

5th Application. As the fifth application of theorem 1.1, one can also show the following result which claims that the moduli space of (unpolarized) K3 surfaces is highly non-Hausdorff:

Application 1.10. [HLOY2] *There are a pair of smooth projective families of K3 surfaces*

$$\mathcal{X} \to \Delta\,,\ \mathcal{Y} \to \Delta$$

and a sequence $\{t_k\} \subset \Delta$ *such that*

$$\lim_{k\to\infty} t_k = 0\,,\ \mathcal{X}_{t_k} \simeq \mathcal{Y}_{t_k}\ \textit{but}\ \mathcal{X}_0 \not\simeq \mathcal{Y}_0\,. \quad \square$$

One can construct such examples by combining theorem 1.1 with theory of Fourier-Mukai partners of K3 surfaces found by Mukai [Mu] and Orlov [Or] and developped further by Hosono-Lian-Oguiso-Yau [HLOY1, 2].

[2]The argument in [CKO] becomes correct if one just erases the following sentence which accidentally inserted there: *Therefore, in the new fibration, for each fibre the G.C.D. of the multiplicities of the components equals* 1.

2. Proof of Theorem 1.1

In the rest, I would like to prove theorem 1.1.

Let us choose a marking $\tau : R^2 f_* \mathbf{Z}_{\mathcal{X}} \simeq \Lambda \times \Delta$, where $\Lambda = (\Lambda, (*.*))$ is a lattice of signature $(3, N-1)$ and consider the period map

$$p : \Delta \to \mathcal{P} := \{[\omega] \in \mathbf{P}(\Lambda \otimes \mathbf{C}) | (\omega.\omega) = 0, (\omega.\overline{\omega}) > 0\} \subset \mathbf{P}(\Lambda \otimes \mathbf{C}) = \mathbf{P}^{N+1}.$$

This map p is defined by $p(t) = \tau_{\mathbf{C}}([\omega_{\mathcal{X}_t}])$. It is known that p is holomorphic. We notice that p is not constant by the local Torelli Theorem.

Let us consider all the primitive sub-lattices Λ^n $(n \in \mathcal{N})$ of Λ. Put $\Delta_n := \{t \in \Delta | \tau(\mathrm{NS}(\mathcal{X}_t)) = \Lambda^n\}$. Then one has a decomposition $\Delta = \sqcup_{n \in \mathcal{N}} \Delta_n$. Since $\mathcal{N}$ is countable but Δ is uncountable, there is an element of $\mathcal{N}$, say 0, such that Δ_0 is uncountable. Since $p(t) \in \Lambda^{0\perp} \otimes \mathbf{C}$ for all $t \in \Delta_0$ and since p is holomorphic, one has then:

$$p(\Delta) \subset \mathcal{P}' := \{[\omega] \in \mathbf{P}(\Lambda^{0\perp} \otimes \mathbf{C}) | (\omega.\omega) = 0, (\omega.\overline{\omega}) > 0\} \subset \mathbf{P}(\Lambda^{0\perp} \otimes \mathbf{C}) = \mathbf{P}^n.$$

Here we regard $\mathbf{P}(\Lambda^{0\perp} \otimes \mathbf{C})$ as a linear subspace of $\mathbf{P}(\Lambda \otimes \mathbf{C})$ defined by $(\Lambda^0.*) = 0$. Set $\mathcal{S} := \Delta - \Delta_0$ and $\mathcal{G} := \Delta_0$. Then, by the Lefschetz $(1,1)$-Theorem, we also have that:

(1) $\Lambda^0 \subset \tau(\mathrm{NS}(\mathcal{X}_t))$ for all $t \in \Delta$ and $\Lambda^0 = \tau(\mathrm{NS}(\mathcal{X}_t))$ for all $t \in \mathcal{G}$;
(2) $t \in \mathcal{S}$ if and only if there is a vector $v \in \Lambda - \Lambda^0$ such that $(v, p(t)) = 0$, i.e. if and only if $\tau(\mathrm{NS}(\mathcal{X}_t))$ is strictly bigger than (the primitive) Λ^0.

Since both $\Lambda - \Lambda^0$ and $\{t \in \Delta | (v, p(t)) = 0\}$ for each $v \in \Lambda - \Lambda^0$ are countable, $\mathcal{S}$ is countable as well.

In order to complete the proof, it remains to show the density of $\mathcal{S}$, i.e. the fact that $\mathcal{S} \cap U \neq \emptyset$ for any sufficiently small disk U.

Claim 2.1. $\mathrm{rank}\, \Lambda^{0\perp} \geq 3$.

Proof. If $\mathrm{rank} \Lambda^{0\perp} \leq 2$, then $\Lambda^{0\perp} \otimes \mathbf{R}$ is spanned by the images of the real and imaginary parts of a holomorphic 2-form. This implies that $\Lambda^{0\perp}$ is a positive definite lattice of rank 2 and that $\mathcal{P}'$ consists of two points. However, the period map p is then constant, a contradiction. □

Let us choose a holomorphic coordinate z of U centered at P. We also choose integral basis of $\Lambda^{0\perp}$ and write $p|U$ as $p(z) = [1 : f_1(z) : f_2(z) : \cdots : f_n(z)]$ with respect to this basis. Here we have $n \geq 2$ by Claim 2.1. We may also assume that $f_1(z)$ is not constant. In what follows, for each $\overrightarrow{a} = (a_0, a_1, a_2, \cdots, a_n) \in \mathbf{R}^{n+1} - \{\overrightarrow{0}\}$, we put:
$f_{\overrightarrow{a}}(z) := a_0 + a_1 f_1(z) + a_2 f_2(z) + \cdots + a_n f_n(z)$;
$l_{\overrightarrow{a}} := a_0 x_0 + a_1 x_1 + a_2 x_2 + \cdots + a_n x_n$, where $[x_0 : x_1 : \cdots : x_n]$ is the homogeneous coordinates of $\mathbf{P}^n$; and
$H_{\overrightarrow{a}} := (l_{\overrightarrow{a}} = 0) \subset \mathbf{P}^n$, the hyperplane defined by the linear form $l_{\overrightarrow{a}}$.

Let k be an element of $\{2, \cdots, n\}$. Since $\dim_{\mathbf{R}} \mathbf{C} = 2$, there is an element $(r_{0,k}, r_{1,k}, r_{2,k}) \in \mathbf{R}^3 - \{\overrightarrow{0}\}$ such that $r_{0,k} \cdot 1 + r_{1,k} f_1(0) + r_{2,k} f_k(0) = 0 - (*)$. Put $\overrightarrow{r_k} := (r_{0,k}, r_{1,k}, 0, \cdots, 0, r_{2,k}, 0, \cdots, 0)$. Here $r_{2,k}$ is located at the same position as x_k in $[x_0 : x_1 : \cdots : x_n]$. In this notation, the equality $(*)$ is rewritten both as $p(0) \in H_{\overrightarrow{r_k}}$ and as $f_{\overrightarrow{r_k}}(0) = 0$.

Claim 2.2. *$p(U)$ is not contained in $\cap_{k=2}^n H_{\overrightarrow{r_k}}$.*

Proof. Assuming to the contrary that $p(U) \subset \cap_{k=2}^{n} H_{\overrightarrow{r_k}}$, we shall derive a contradiction. Since $f_1(z)$ is not constant, we have $r_{2,k} \neq 0$ for each k. Therefore $\cap_{k=2}^{n} H_{\overrightarrow{r_k}}$ is a line $L \simeq \mathbf{P}^1$ defined over $\mathbf{R}$ in $\mathbf{P}^n$. We note that the bilinear form $(*, **)$ is not identically zero on L, because $(\omega, \overline{\omega}) > 0$ for $[\omega] \in p(U)$. This leads the same contradiction as in claim 2.1. □

By claim 2.2, there is k such that $p(U) \not\subset H_{\overrightarrow{r_k}}$, i.e. $f_{\overrightarrow{r_k}}(z) \not\equiv 0$. Since $f_{\overrightarrow{r_k}}(0) = 0$, we may choose a small circle $\gamma \subset U$ around $z = 0$ such that $f_{\overrightarrow{r_k}}(z)$ has no zeros on γ. Set $K := \min\{|f_{\overrightarrow{r_k}}(z)| | z \in \gamma\}$ and $M := \max\{|f_i(z)| | z \in \gamma, i = 0, 1, \cdots, n\}$, where we define $f_0(z) \equiv 1$. Note that $K > 0$ and $M > 0$. Then, by using the triangle inequality, we see that $|f_{\overrightarrow{r_k}}(z) - f_{\overrightarrow{a}}(z)| < |f_{\overrightarrow{r_k}}(z)|$ on γ provided that $|\overrightarrow{a} - \overrightarrow{r_k}| < KM^{-1}(n+1)^{-1}$. Denote by V the open disk such that $\partial V = \gamma$. By the Rouché Theorem, the cardinalities of zeros (counted with multiplicities) on V are the same for $f_{\overrightarrow{r_k}}$ and $f_{\overrightarrow{a}}$. In particular, $f_{\overrightarrow{a}}$ admits a zero on V. Since $\mathbf{Q}^{n+1} - \{\overrightarrow{0}\}$ is dense in $\mathbf{R}^{n+1} - \{\overrightarrow{0}\}$, one can then find an element $\overrightarrow{q} \in \mathbf{Q}^{n+1} - \{\overrightarrow{0}\}$ such that $f_{\overrightarrow{q}}(z)$ has a zero on V. Let us denote this zero by $Q \in V(\subset U)$. We have $f_{\overrightarrow{q}}(Q) = 0$ and $p(Q) \in H_{\overrightarrow{q}}$. Recall that $\Lambda^{0\perp}$ is primitive in Λ, that Λ is non-degenerate, and that our homogeneous coordinates $[x_0 : x_1 : \cdots : x_n]$ are chosen by means of integral basis of Λ and the rational linear equations $(\Lambda^0.*) = 0$. Therefore one can find an element $0 \neq v \in \Lambda$ such that $H_{\overrightarrow{q}} = \{x \in \mathbf{P}^n | (v.x) = 0\}$. Since this v satisfies $(v.p(Q)) = 0$, one has $v \in \tau(NS(\mathcal{X}_Q))$. On the other hand, since $\overrightarrow{q} \neq \overrightarrow{0}$, we have $v \notin \Lambda^0$. Hence this Q satisfies $Q \in S \cap U$. Now we are done. □

References

[Be] A. Beauville, *Variétés Kählerian dont la premiere class de Chern est nulle*, J. Diff. Geom. **18** (1983) 755-782.

[Br] C. Borcea, *Homogeneous vector bundles and families of Calabi-Yau threefolds, II*, Proc. Sym. Pure Math. **52** (1991) 83-91.

[BKPS] R.E. Borcherds, L. Katzarkov, T. Pantev, N. I. Shepherd-Barron, *Families of K3 surfaces*, J. Alg. Geom. **7** (1998) 183-193.

[CKO] F. Catanese, J. Keum, K. Oguiso, *Some remarks on the universal cover of an open K3 surface* , math.AG/0104015, Math. Ann. to appear.

[HLOY1] S. Hosono, B. Lian, K. Oguiso, S.T. Yau, *Autoequivalences of derived category of a K3 surface and monodromy transformations* , math.AG/0201047, J. Alg. Geom. to appear.

[HLOY2] S. Hosono, B. Lian, K. Oguiso, S.T. Yau, *Counting Fourier-Mukai partners and applications*, math.AG/0202014, submitted.

[Mu] S. Mukai, *On the moduli space of bundles on K3 surfaces I*, in Vector bundles on algebraic varieties, Oxford Univ. Press (1987) 341–413.

[Og] K. Oguiso, *Local families of K3 surfaces and applications* , (combined version of preprints math.AG/0011258 and math.AG/0104049) , J. Alg. Geom. to appear.

[Or] D. Orlov, *Equivalences of derived categories and K3 surfaces* , math.AG/9606006, J. Math.Sci. **84** (1997) 1361–1381.

[Sh1] T. Shioda, *On the Mordell-Weil lattices*, Comment. Math. Univ. St. Paul. **39** (1990) 211–240.

[Sh2] T. Shioda, *Theory of Mordell-Weil lattices*: in Proceedings of the International Congress of Mathematicians (Kyoto 1990) Math. Soc. Japan (1991) 473–489.

[SI] T. Shioda, H. Inose, *On singular K3 surfaces*: in Complex analysis and algebraic geometry, Iwanami Shoten (1977) 119–136.

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