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CANONICAL LOCAL HEIGHTS AND MULTIPLICATION FORMULAS FOR THE JACOBIANS OF CURVES OF GENUS 2

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1. Introduction

Let k be a number field. Let C be a curve of genus 2 over k defined by

$$Y^2 = F(X),$$

where $\deg F = 6$, $\operatorname{disc}(F) \neq 0$ and $F \in k[X]$. Note that we cannot transform it into $Y^2 =$ (quintic) over k in general because k is not algebraically closed. Let J be the Jacobian of C. The set of all the k-rational points of J becomes a finitely generated Abelian group. It is called the Mordell-Weil group and denoted by $J(k)$. We use *height functions* to study this group. Height functions are functions from $J(k)$ to $\mathbb{R}$. In this report, we consider two height functions:

- naive height h,
- canonical height $\hat{h}$.

The definitions of these height are given in Section 4. It is known that there exist constants c_1 and c_2 such that

$$c_1 \leq h(P) - \hat{h}(P) \leq c_2$$

for all $P \in J(k)$. We need to compute c_2 when we compute the generators of the Mordell-Weil group $J(k)$. Therefore it is important to compute the bounds c_1 and c_2. We use *multiplication formulas* and *local height functions* to compute the bounds.

Note that the multiplication formulas given here is not on the Jacobians but on the Kummer surfaces associated with the Jacobians. For our purpose, it is sufficient to work on the Kummer surfaces.

Local height functions are functions defined for each place of k and height functions are decomposed to a weighted sum of local height functions.

This report is organized as follows: In Section 2, we review the basic facts of curves of genus 2 and their Jacobians. The multiplication formulas are given in Section 3. In Section 4, we describe height functions on the Jacobians of curves of genus 2. In Section 5, we define local height functions and describe several properties. In Section 6, we give the bounds for the difference $h - \hat{h}$ by using multiplication formulas and local height functions.

The results in this report also hold for elliptic curves. See [10] and [11].

We cannot include detailed proofs in this report. For proofs, see [12]. In [12], we also discuss computing the canonical heights.

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2. Curves of genus 2 and their Jacobians

In this section, we review the basic facts described in [2].

We consider a curve of genus 2 over k

$$C\colon Y^2 = F(X),$$

where

$$F(X) = f_6X^6 + f_5X^5 + f_4X^4 + f_3x^3 + f_2X^2 + f_1X + f_0 \in k[X]$$

is of degree 6 and has no multiple roots. We denote two points at infinity of C by ∞^+ and ∞^-. The points ∞^+ and ∞^- are defined over k or a quadratic extension of k.

Let J be the Jacobian of C. J is an Abelian surface over k. We can identify J with $\mathrm{Pic}^0(C)$ as a group. Let Θ^+, Θ^- be the image of C in J via the embeddings

$$P \mapsto (P) - (\infty^+), \quad P \mapsto (P) - (\infty^-)$$

respectively. Note that Θ^+ and Θ^- may not be defined over k. However, $\Theta^+ + \Theta^-$ is defined over k. $\Theta^+ + \Theta^-$ is linearly equivalent to 2Θ, where Θ is a suitable theta divisor on J. By a theorem of Lefschetz ([7, Section 17, p. 163]), $\Theta^+ + \Theta^-$ is base-point free, and $2(\Theta^+ + \Theta^-)$ is very ample. Since $l(\Theta^+ + \Theta^-) = l(2\Theta) = 4$, we have a morphism $\kappa\colon J \to \mathbb{P}^3$. We call the image of κ the *Kummer surface*, and denote it by K. We write $\kappa(P) = (\xi_1(P), \ldots, \xi_4(P))$. We define an involution $\iota_J\colon J \to J$ by $\iota_J(P) = -P$. Then K is isomorphic to the quotient variety J/ι_J. Note that K has 16 singular points corresponding to elements of order 2 on J. We need not to resolve these singular points here.

The defining equation of K is given by

$$G(\xi_1, \ldots, \xi_4) = R\xi_4^2 + S\xi_4 + T = 0,$$

where R, S, T are homogeneous polynomials of degree 2, 3, 4 respectively. Hence G is a homogeneous polynomial of degree 4. Furthermore, G has coefficients in $\mathbb{Z}[f_0, \ldots, f_6]$. Explicit descriptions of κ and K are given in [2, Chapter 3].

We denote the multiplication-by-m map on J by $[m]$. Since $[m](-P) = -([m]P)$, $[m]$ induces a morphism on K. We denote the morphism on K induced by the duplication map by $\delta = (\delta_1, \ldots, \delta_4)$, that is, $\kappa([2]P) = \delta(\kappa(P))$. Explicit formulas for δ_i are available at [13].

We cannot recover $\kappa(P+Q)$ from only $\kappa(P)$ and $\kappa(Q)$ because P and $-P$ are identified through κ. However, we can obtain an unordered pair $(\kappa(P{+}Q), \kappa(P{-}Q))$ from $\kappa(P)$ and $\kappa(Q)$. Precisely, we have the following result:

Proposition 2.1 ([2, Theorem 3.4.1]). *There are polynomials B_{ij} biquadratic in the $\xi_l(P)$, $\xi_l(Q)$ such that*

$$(\xi_i(P+Q)\xi_j(P-Q) + \xi_j(P+Q)\xi_i(P-Q)) = (2B_{ij}(\kappa(P), \kappa(Q))) \tag{1}$$

as projective coordinates. Furthermore, B_{ij} have coefficients in $\mathbb{Z}[f_0, \ldots, f_6]$.

Explicit formulas for B_{ij} are also available at [13].

3. Multiplication formulas

The following theorem gives the multiplication formulas on the Kummer surface K.

Theorem 3.1. *There exist homogeneous polynomials* $\mu_{m,i} \in k[\xi_1, \ldots, \xi_4]$ $(m \geq 0, i = 1,2,3,4)$ *such that the following conditions are satisfied:*

(i) *We have*

$$\mu_{0,1} = \mu_{0,2} = \mu_{0,3} = 0, \mu_{0,4} = 1,$$
$$\mu_{1,i} = \xi_i,$$
$$\mu_{2m,i} = \delta_i(\mu_m) \quad (m \geq 1), \tag{2}$$
$$\mu_{2m+1,i}\xi_i = B_{ii}(\mu_{m+1}, \mu_m) \quad (m \geq 1) \tag{3}$$

in $k[\xi_1, \ldots, \xi_4]/\langle G \rangle$, *where* $\mu_m = (\mu_{m,1}, \ldots, \mu_{m,4})$ *and* $\langle G \rangle$ *is the ideal generated by the polynomial* G.

(ii) *Let* $\bar{k}$ *be the algebraic closure of* k. *For all* $P \in J(\bar{k})$, *we have*

$$\kappa([m]P) = (\mu_{m,1}(\kappa(P)), \ldots, \mu_{m,4}(\kappa(P))).$$

Furthermore, the polynomials $\mu_{m,i}$ *are uniquely determined modulo* G.

Remark 3.2. In the case $\deg(F) = 5$, Kanayama [5] wrote down the multiplication formulas on the Jacobian J. He used hyperelliptic $\wp$-functions.

Remark 3.3. We can compute $\mu_{m,i}$ inductively. $\mu_{0,i}$ and $\mu_{1,i}$ are defined obviously. $\mu_{2m,i}$ is computed by (2). $\mu_{2m+1,i}$ satisfies (3), that is, there exists a homogeneous polynomial p such that

$$\mu_{2m+1,i}\xi_i + pG = B_{ii}(\mu_{m+1}, \mu_m). \tag{4}$$

Such $\mu_{2m+1,i}$ can be computed by using Gröbner basis (see [1, § 5.6]). We can also compute them by the following elementary operations. In (4), we can assume p does not contain the variable ξ_i by replacing $\mu_{2m+1,i}$ if necessary. We substitute 0 for ξ_i in (4). Then we have $pG_i = b_i$, where G_i (resp. b_i) is the polynomial obtained by substituting 0 for ξ_i in G (resp. $B_{ii}(\mu_{m+1}, \mu_m)$). Hence we can calculate $p = b_i/G_i$ and $\mu_{2m+1,i} = (b_i - pG)/\xi_i$.

We describe some properties of $\mu_{m,i}$.

Proposition 3.4. *For all* $m \geq 0$ *and* $i = 1,2,3,4$, *we can choose* $\mu_{m,i}$ *such that* $\mu_{m,i}$ *has coefficients in* $\mathbb{Z}[f_0, \ldots, f_6]$.

Lemma 3.5. *For all* $m \geq 0$ *and* $i = 1,2,3,4$, $\mu_{m,i}$ *has degree* m^2.

Lemma 3.6. *Let* $O = (0,0,0,1)$. *Then we have*

$$\mu_{m,1}(O) = \mu_{m,2}(O) = \mu_{m,3}(O) = 0, \mu_{m,4}(O) = 1$$

for all $m \geq 0$.

Proposition 3.7. *We have*

$$\mu_{m+n,i}\mu_{m-n,j} + \mu_{m+n,j}\mu_{m-n,i} = 2B_{ij}(\mu_m, \mu_n)$$

in $k[\xi_1, \ldots, \xi_4]/\langle G \rangle$ *for all* $m, n \geq 0$ *such that* $m \geq n$ *and for* $i, j = 1,2,3,4$.

4. Height functions

Let M_k be the set of absolute values which extend the p-adic absolute values or the ordinary absolute value on $\mathbb{Q}$. M_k is naturally identified with the set of places on k. For $v \in M_k$, we denote the completion of k at v by k_v. Let $n_v = [k_v : \mathbb{Q}_v]$.

We describe the elements in M_k closely.

First, let $v \in M_k$ be an Archimedean absolute value. There is an embedding $\sigma\colon k \to \mathbb{C}$ such that

$$|x|_v = |\sigma(x)|$$

for any $x \in k$, where the absolute value in the right-hand side is the ordinary absolute value on $\mathbb{C}$. If k_v is isomorphic to $\mathbb{R}$, then the embedding σ is uniquely determined. If k_v is isomorphic to $\mathbb{C}$, then the embedding σ is determined up to complex conjugates.

Next, let $v \in M_k$ be a non-Archimedean absolute value. Let $\mathcal{O}_v$ be the ring of integers of k_v and π_v be a uniformizer of $\mathcal{O}_v$. We define $\mathrm{ord}_v\colon k_v^\times \to \mathbb{Z}$ by

$$\mathrm{ord}_v(a\pi_v^n) = n, \quad a \in \mathcal{O}_v^\times .$$

Let q_v be the cardinality of the residue field $\mathcal{O}_v/\pi_v\mathcal{O}_v$. Then we have

$$|x|_v = q_v^{-\mathrm{ord}_v(x)/n_v}$$

for all $x \in k^\times$.

We have the product formula:

$$\sum_{v\in M_k} n_v \log |x|_v = 0$$

for all $x \in k^\times$.

We define the *naive height function* $h\colon J(k) \to \mathbb{R}$ by

$$h(P) = \frac{1}{[k:\mathbb{Q}]} \sum_{v\in M_k} n_v \log \max_{1\le i\le 4} |\xi_i(P)|_v.$$

$h(P)$ is independent of the choice of the homogeneous coordinates $(\xi_1(P), \dots, \xi_4(P))$.

We define the *canonical height function* $\hat{h}\colon J(k) \to \mathbb{R}$ by

$$\hat{h}(P) = \lim_{n\to\infty} \frac{1}{4^n} h([2^n]P).$$

The right-hand side converges by general theory (see for instance [4, Theorem B.4.1]).

The canonical height function has a good property with respect to the addition.

Theorem 4.1 (Parallelogram law). *Let $P, Q \in J(k)$. We have*

$$\hat{h}(P+Q) + \hat{h}(P-Q) = 2\hat{h}(P) + 2\hat{h}(Q).$$

In particular, for any integer m,

$$\hat{h}([m]P) = m^2\hat{h}(P).$$

5. Local height functions

In this section, we describe local height functions on the Jacobians of curves of genus 2. Our definitions of local height functions are the almost same as those of Flynn-Smart [3]. We consider the v-adic topology in the rest of this report.

We define divisors Θ_i on J by

$$\Theta_1 = \Theta^+ + \Theta^-, \quad \Theta_i = \Theta_1 + \operatorname{div}\left(\frac{\xi_i}{\xi_1}\right) \quad (i = 2, 3, 4).$$

Note that $P \in \operatorname{supp}(\Theta_i)$ if and only if $\xi_i(P) = 0$.

For $v \in M_k$, we define *naive local height functions* $\lambda_{i,v} \colon J(k_v) \setminus \operatorname{supp}(\Theta_i) \to \mathbb{R}$ by

$$\lambda_{i,v}(P) = \log \max_{1 \le j \le 4} \left| \frac{\xi_j(P)}{\xi_i(P)} \right|_v .$$

$\lambda_{i,v}$ is independent of the choice of homogeneous coordinates for $\kappa(P)$. We have

$$\lambda_{i,v}(P) = \lambda_{j,v}(P) - \log \left| \frac{\xi_i(P)}{\xi_j(P)} \right|_v$$

for any $P \notin \operatorname{supp}(\Theta_i) \cup \operatorname{supp}(\Theta_j)$.

To construct canonical local height functions, we define a function $\Phi_v \colon J(k_v) \to \mathbb{R}$ by

$$\Phi_v(P) = \frac{\max_i |\delta_i(\kappa(P))|_v}{\max_i |\xi_i(P)|_v^4} .$$

The right-hand side is independent of the choice of homogeneous coordinates for $\kappa(P)$ since δ_i are homogeneous polynomials of degree 4. We can show that $\log \Phi_v$ is bounded and continuous on $J(k_v)$. Hence we can define functions $\hat{\lambda}_{i,v} \colon J(k_v) \setminus \operatorname{supp}(\Theta_i) \to \mathbb{R}$ by

$$\hat{\lambda}_{i,v}(P) = \lambda_{i,v}(P) + \sum_{n=0}^{\infty} \frac{1}{4^{n+1}} \log \Phi_v([2^n]P).$$

We call them *canonical local height functions* on J. By definition, we have

$$\hat{\lambda}_{i,v}(P) = \hat{\lambda}_{j,v}(P) - \log \left| \frac{\xi_i(P)}{\xi_j(P)} \right|_v$$

for any $P \notin \operatorname{supp}(\Theta_i) \cup \operatorname{supp}(\Theta_j)$.

As in the case of elliptic curves, we have the quasi-parallelogram law.

Theorem 5.1 (Quasi-parallelogram law). *Let $P, Q \in J(k_v)$. If $P, Q, P+Q, P-Q \notin \operatorname{supp}(\Theta_i)$, then*

$$\hat{\lambda}_{i,v}(P+Q) + \hat{\lambda}_{i,v}(P-Q) = 2\hat{\lambda}_{i,v}(P) + 2\hat{\lambda}_{i,v}(Q) - \log \left| \frac{B_{ii}(\kappa(P), \kappa(Q))}{\xi_i(P)^2 \xi_i(Q)^2} \right|_v .$$

Corollary 5.2. *Let $P \in J(k_v)$ and $m > 0$ be an integer. If $P \notin \operatorname{supp}(\Theta_i)$ and $[m]P \notin \operatorname{supp}(\Theta_j)$, then*

$$\hat{\lambda}_{j,v}([m]P) = m^2 \hat{\lambda}_{i,v}(P) - \log \left| \frac{\mu_{m,j}(\kappa(P))}{\xi_i(P)^{m^2}} \right|_v .$$

Corollary 5.3. *Let $m \geq 2$ be an integer. Define a function $\Phi_{m,v}\colon J(k_v) \to \mathbb{R}$ by*

$$\Phi_{m,v}(P) = \frac{\max_{1\leq i\leq 4} |\mu_{m,i}(\kappa(P))|_v}{\max_{1\leq i\leq 4} |\xi_i(P)|_v^{m^2}}.$$

Then,

$$\hat{\lambda}_{i,v}(P) = \lambda_{i,v}(P) + \sum_{n=0}^{\infty} \frac{1}{m^{2(n+1)}} \log \Phi_{m,v}([m^n]P)$$

for $P \in J(k_v) \setminus \operatorname{supp}(\Theta_i)$.

We consider a relation between global height functions and local height functions. We can decompose these height functions into local height functions.

Theorem 5.4. *Let $P \in J(k) \setminus \operatorname{supp}(\Theta_i)$. Then,*

$$h(P) = \frac{1}{[k:\mathbb{Q}]} \sum_{v\in M_k} n_v \lambda_{i,v}(P),$$

$$\hat{h}(P) = \frac{1}{[k:\mathbb{Q}]} \sum_{v\in M_k} n_v \hat{\lambda}_{i,v}(P).$$

6. Application: height difference bounds

By general theory, it is known that there exists constants c_1 and c_2 such that

$$c_1 \leq h(P) - \hat{h}(P) \leq c_2$$

for all $P \in J(k)$. It is important to estimate the bounds c_1 and c_2 effectively. We consider this problem in this section.

Let $\mathcal{O}_k$ be the ring of integers of k. We assume that the coefficients $f_0, \ldots, f_6$ belong to $\mathcal{O}_k$.

We define a function $\Psi_v\colon J(k_v) \to \mathbb{R}$ by

$$\Psi_v(P) = -\sum_{n=0}^{\infty} \frac{1}{4^{(n+1)}} \log \Phi_v([2^n]P).$$

By definition, we have

$$\lambda_{i,v}(P) - \hat{\lambda}_{i,v}(P) = \Psi_v(P)$$

for all $P \in J(k_v)\backslash\operatorname{supp}(\Theta_i)$. Since $\log \Phi_v$ is bounded and continuous, by Weierstrass M-test, Ψ_v is bounded and continuous on $J(k_v)$ (we consider the v-adic topology). Furthermore, Ψ_v has the maximum and the minimum since $J(k_v)$ is compact. By Theorem 5.4, we have

$$\text{(5)}\quad \frac{1}{[k:\mathbb{Q}]} \sum_{v\in M_k} [k_v : \mathbb{Q}_v] \inf_{Q\in J(k_v)} \Psi_v(Q) \leq h(P) - \hat{h}(P) \leq \frac{1}{[k:\mathbb{Q}]} \sum_{v\in M_k} [k_v : \mathbb{Q}_v] \sup_{Q\in J(k_v)} \Psi_v(Q)$$

for all $P \in J(k)$. By the assumption that $f_0, \ldots, f_6 \in \mathcal{O}_k$, we have

$$\sup_{Q\in J(k_v)} \Phi_v(Q) = 1$$

if v is non-Archimedean (see [3, Lemma 3] or [8, p. 189, the second remark]). Hence we have

$$\inf_{Q\in J(k_v)} \Psi_v(Q) = 0.$$

Therefore the left-hand side of (5) is a finite sum.

For the infimum of Φ_v, the following result is known.

Theorem 6.1 ([8, Theorem 6.1]). *Under the above assumption, if $v \in M_k$ is non-Archimedean, then*

$$\inf_{Q\in J(k_v)} \Phi_v(Q) \geq \left|2^4 \operatorname{disc}(F)\right|_v,$$

where $\operatorname{disc}(F)$ *is the discriminant of* F.

By Theorem 6.1, if $\left|2^4 \operatorname{disc}(F)\right|_v = 1$, then

$$\inf_{Q\in J(k_v)} \Phi_v(Q) = 1.$$

Therefore the right-hand side of (5) is a finite sum. In [8] and [9], further improvements for non-Archimedean absolute values are described.

From now on, we mainly consider Archimedean absolute values although the following results also hold for non-Archimedean absolute values.

By Corollary 5.3, we have

$$\Psi_v(P) = -\sum_{n=0}^{\infty} \frac{1}{m^{2(n+1)}} \log \Phi_{m,v}([m^n]P)$$

for all $P \in J(k_v)$. We define

$$\varepsilon_{m,v}^{-1} = \inf_{Q\in J(k_v)} \Phi_{m,v}(Q), \quad \delta_{m,v}^{-1} = \sup_{Q\in J(k_v)} \Phi_{m,v}(Q).$$

Let

$$S_v(m) = \frac{\log \delta_{m,v}}{m^2-1}, \quad T_v(m) = \frac{\log \varepsilon_{m,v}}{m^2-1}.$$

Then we have the following proposition:

Proposition 6.2. *Let $v \in M_k$ and $m \geq 2$ be an integer. Then, for all $P \in J(k_v)$,*

$$S_v(m) \leq \Psi_v(P) \leq T_v(m).$$

Remark 6.3. This proposition also holds for elliptic curves. See [10, Proposition 3.4].

Corollary 6.4. *Let $m \geq 2$ be an integer. Then*

$$\frac{1}{[k:\mathbb{Q}]} \sum_{v\in M_k} n_v S_v(m) \leq h(P) - \hat{h}(P) \leq \frac{1}{[k:\mathbb{Q}]} \sum_{v\in M_k} n_v T_v(m)$$

for all $P \in J(k)$.

We have the following results for $S_v(m)$ and $T_v(m)$. The same holds for elliptic curves (see [10]). The proofs of them are the same as those in [11].

Proposition 6.5. *Let $m \geq 2$, $l \geq 1$ be integers. Then,*

$$S_v(m) \leq S_v(m^l), \quad T_v(m^l) \leq T_v(m),$$

that is, the bounds in Proposition 6.2 become sharper when we change m to m^l.

We can estimate the differences between the extrema of Φ_v and $S_v(m)$, $T_v(m)$ by the following proposition and its corollaries.

Proposition 6.6. *Let $m \geq 2$ be an integer. Then,*

$$0 \leq \inf_{P \in E(k_v)} \Psi_v(P) - S_v(m) \leq \frac{1}{m^2-1}\left(\sup_{P \in E(k_v)} \Psi_v(P) - \inf_{P \in E(k_v)} \Psi_v(P)\right),$$

$$0 \leq T_v(m) - \sup_{P \in E(k_v)} \Psi_v(P) \leq \frac{1}{m^2-1}\left(\sup_{P \in E(k_v)} \Psi_v(P) - \inf_{P \in E(k_v)} \Psi_v(P)\right).$$

Corollary 6.7.

$$\lim_{m \to \infty} S_v(m) = \inf_{P \in E(k_v)} \Psi_v(P), \quad \lim_{m \to \infty} T_v(m) = \sup_{P \in E(k_v)} \Psi_v(P).$$

We can estimate the difference between the theoretical bounds and the bounds in Proposition 6.2 by the following corollary.

Corollary 6.8.

$$0 \leq \inf_{P \in E(k_v)} \Psi_v(P) - S_v(m) \leq \frac{1}{m^2}\left(T_v(m) - S_v(m)\right),$$

$$0 \leq T_v(m) - \sup_{P \in E(k_v)} \Psi_v(P) \leq \frac{1}{m^2}\left(T_v(m) - S_v(m)\right).$$

As described above, we can estimate Ψ_v with arbitrary accuracy at least theoretically. However, actual computations are quite difficult because the size of $\mu_{m,i}$ increases rapidly.

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