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Classification of non-symplectic automorphisms of order 3 on $K3$ surfaces

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In this poster, we study non-symplectic automorphisms of order 3 on algebraic $K3$ surfaces $X$ over $\mathbb{C}$ which act trivially on the Néron-Severi lattice $S_X$. In particular, we shall characterize their fixed locus in terms of the values of 3-elementary lattices.

This poster is devoted to study of non-symplectic automorphisms $\varphi$ on $X$. i.e. $\varphi^*\omega_X = (1 + \sqrt{-3}/2)\omega_X$. ($\omega_X$: nowhere vanishing holomorphic 2-form).

Main Theorem

Let $\rho$ be the Picard number of $X$ and let $s$ be the minimal number of generators of $S_X^* / S_X$.

(1) $22 - \rho - 2s < 0 \implies \exists \varphi \neq 0$.

(2) $22 - \rho - 2s \geq 0 \implies \exists \varphi \neq 0$ Moreover $X^\varphi := \{ x \in X \mid \varphi(x) = x \}$ has the form

$$\bigcup_i \{ P_i \} \bigcup C^{(1)}$$

if $S_X = U(3) \oplus E_8^*(3)$

$$\bigcup_i \{ P_i \} \bigcup C^{(\varphi)} \bigcup \bigcup_i \{ E_i \}$$

where $M = \rho/2 - 1$, $g = (22 - \rho - 2s)/4$, $N = (6 + \rho - 2s)/4$.

Let $L$ be an even indefinite 3-elementary lattice admitting primitive embedding in $U^3 \oplus E_8^*$. \\

Fixed locus

- Let $P$ be an isolated fixed point of $\varphi$ on $X$. Then $\varphi^*$ can be written as $\begin{pmatrix} \zeta^2 & 0 \\ 0 & \zeta \end{pmatrix}$ under some appropriate local coordinates around $P$.

- Let $C$ be a fixed curve and $Q$ a point on $C$. Then $\varphi^*$ can be written as $\begin{pmatrix} 1 & 0 \\ 0 & \zeta \end{pmatrix}$ under some appropriate local coordinates around $Q$. In particular, fixed curves are smooth.

Hence $X^\varphi = \{ P_i \} \sqcup \{ C_j \}$ $\sqcup \{ C_k \}$ where $P_i$ is an isolated point and $C_k$ is a smooth curve.

Lefschetz formula + Hurwitz formula + classification of singular fibrations of elliptic pencils by Kodaira + classification tables of 3-elementary lattices.

\[ \begin{array}{cccc} \text{Lattice} & U & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \text{hyperbolic lattice.} \\ A_2 & E_6 & \text{or} E_8 \text{an even negative definite lattice associated} & \text{with the Dynkin diagram of type } A_2, \text{or } E_6 \text{or } E_8. \\ L^:* & \text{Hom}(L, Z) & \text{L: lattice.} \\ L:3\text{-elementary lattice} & \L^* \cup L \simeq (Z/3Z)^* & \end{array} \]

An even indefinite 3-elementary lattices were classified by A.N. Rudakov and I.R. Shafarevich.

Examples

We give affine equations of elliptic $K3$ surfaces $X$.

- $X = y \left( y^2 \prod_{i=1}^{12} (u - a_i) - x^2 \right)$ where $a_i (i = 1, \ldots, 12)$ are distinct complex numbers.

- $\varphi(x, y, z, u) := (x, y, \zeta z, u)$ ($\zeta$: primitive 3-th root of 1).

- $\pi : X \to \mathbb{P}^1$: projection.

- $\pi^{-1}(a_i)$: singular fiber of type II.

- $X^\varphi = \{ y = 0 \} \bigcup \{ y^2 \prod_{i=1}^{12} (u - a_i) - x^2 = 0 \}$.

- $C := y^2 \prod_{i=1}^{12} (u - a_i) - x^2$: smooth curve.

Now we remark that $X$ has a section. Let $S$ be the section defined by $y = 0$ and $E$ an class of general elliptic curves. Then $S_X = (S, E) \simeq U$.

The automorphism $\varphi$ induces non trivial automorphism of order 3 on $E$ and $\pi^{-1}(a_i)$. Thus we calculate the genus of $C$ by the Hurwitz formula

$$2g(C) - 2 = 2(2g(\mathbb{P}^1) - 2) + 12(2 - 1).$$

Therefore we can express the fixed locus $X^\varphi$ as $C(5) \sqcup \mathbb{P}^1$.

Remark

$\implies$ There are some examples of $K3$ surfaces $X$ with $S_X$ s.t. rank $S_X = \rho$, $S_X^* / S_X \simeq (\mathbb{Z}/3\mathbb{Z})^*$. 

We can calculate $M$, $N$ and the genus of $C_k$. 

$\square$