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Classification of non-symplectic automorphisms of order 3 on $K3$ surfaces

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In this poster, we study non-symplectic automorphisms of order 3 on algebraic $K3$ surface X over $\mathbb{C}$ which act trivially on the Néron-Severi lattice S_X. In particular we shall characterize their fixed locus in terms of the invariants of 3-elementary lattices.

This poster is devoted to study of non-symplectic automorphism φ on X. i.e. $\varphi^*\omega_X = (1+\sqrt{-3}/2)\omega_X$. ($\omega_X$: nowhere vanishing holomorphic 2-form)

Main Theorem

Let ρ be the Picard number of X and let s be the minimal number of generators of S_X^*/S_X.

(1) $22-\rho-2s<0 \Rightarrow \nexists\ \varphi$.

(2) $22-\rho-2s \geq 0 \Rightarrow \exists\ \varphi$.
Moreover $X^\varphi := \{x \in X | \varphi(x) = x\}$ has the form

$$\begin{cases} \coprod_i^3 \{P_i\} \coprod C^{(1)} & \text{if } S_X = U(3) \oplus E_6^*(3) \\ \coprod_i^M \{P_i\} \coprod C^{(g)} \coprod_j^{N-1} E_j & \end{cases}$$

where $M = \rho/2 - 1$, $g = (22-\rho-2s)/4$, $N = (6+\rho-2s)/4$.

Lattices

$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$: hyperbolic lattice.

A_2, E_6 or E_8 an even negative definite lattice associated with the Dynkin diagram of type A_2, E_6 or E_8.

L:lattice. $L^* = \mathrm{Hom}(L, \mathbb{Z})$.

L:3-elementary lattice $\Leftrightarrow L^*/L \simeq (\mathbb{Z}/3\mathbb{Z})^s$.

An even indefinite 3-elementary lattices were classified by A.N. Rudakov and I.R. Shafarevich.

An even indefinite 3-elementary lattice of rank $n(\geq 2)$ is uniquely determined by the number s.

$\Longrightarrow$ We can classify even indefinite 3-elementary lattices admitting primitive imbedding in $U^3 \oplus E_8^2$. $\downarrow$

Let L be an even indefinite 3-elementary lattice admitting primitive imbedding in $U^3 \oplus E_8^2$.
$22 - \mathrm{rank}\, L - 2s < 0 \Rightarrow L$ has no non-trivial isometry of order 3 which acts trivially on L^*/L.

Fixed locus

♠ Let P be an isolated fixed point of φ on X. Then φ^* can be written as $\begin{pmatrix} \zeta^2 & 0 \\ 0 & \zeta^2 \end{pmatrix}$ under some appropriate local coordinates around P.

♠ Let C be a fixed irreducible curve and Q a point on C. Then φ^* can be written as $\begin{pmatrix} 1 & 0 \\ 0 & \zeta \end{pmatrix}$ under some appropriate local coordinates around Q. In particular, fixed curves are smooth.

Hence $X^\varphi = \{P_1\} \amalg \cdots \amalg \{P_M\} \amalg C_1 \amalg \cdots \amalg C_N$, where P_j is an isolated point and C_k is a smooth curve.

♡ Toward a proof of Main Theorem ♡

Lefschetz formula + Hurwitz formula + classification of singular fibers of elliptic pencils by Kodaira + classification tables of 3-elementary lattices.

$\Downarrow \quad \Downarrow \quad \Downarrow$

We can calculate M, N and the genus of C_k.

Examples

We give affine equations of elliptic $K3$ surfaces $X : z^3 = y\left(y^2\prod_{i=1}^{12}(u-a_i) - x^2\right)$ where a_i $(i = 1, 2, \ldots, 12)$ are distinct complex numbers.

$\varphi(x,y,z,u) := (x,y,\zeta z,u)$ (ζ: primitive 3-th root of 1).

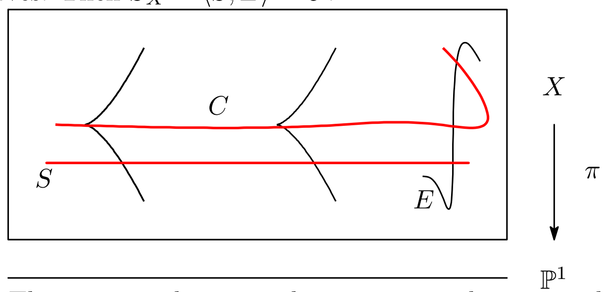
$\pi : X \xrightarrow{u} \mathbb{P}^1$: projection.

$\pi^{-1}(a_i)$: singular fiber of type II.

$X^\varphi = \{y=0\} \coprod \{y^2\prod_{i=1}^{12}(u-a_i) - x^2 = 0\}$.

$C := y^2\prod_{i=1}^{12}(u-a_i) - x^2$: smooth curve.

Now we remark that X has a section. Let S be the section defined by $y = 0$ and E an class of general elliptic curves. Then $S_X = \langle S, E\rangle \simeq U$.



The automorphism φ induces non trivial automorphism of order 3 on E and $\pi^{-1}(a_i)$. Thus we calculate the genus of C by the Hurwitz formula

$$2g(C) - 2 = 2(2g(\mathbb{P}^1) - 2) + 12(2-1).$$

Therefore we can express the fixed locus X^φ as $C^{(5)} \amalg \mathbb{P}^1$.

Remark

$22-\rho-2s \geq 0 \Longrightarrow$ There are some examples of $K3$ surface X with S_X s.t. $\mathrm{rank}\, S_X = \rho$, $S_X^*/S_X \simeq (\mathbb{Z}/3\mathbb{Z})^s$.