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Kyoto University
On codim4 Q-Fano 3 Folds with Fano index 2

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Algebraic Geometry Symposium Kinosaki

1. Main Results

We study Fano 3-folds with Fano index 2: that is, 3-folds X with Picard rank 1, Q-factorial terminal singularities and −K_X = 2A for an ample Weil divisor A. We gave a first classification of all possible Hilbert series of such polarised varieties X, A and list 33 families that can be realised in codimension up to 4. See, Brown-Suzuki, Fano 3-folds with divisible anticanonical class, Manuscripta Math. 123, 2007, pp.37-51.

We work over the complete number field C.

2. The Classification of Q-Fano 3-folds

The Fano index f = f(X) of a Fano 3-fold X is the largest positive integer such that −K_X = fA for some Weil divisor A. Equality of divisors denotes linear equivalence of some multiple. A Weil divisor A is called a primitive ample divisor.

A choice of minimal homogeneous generating set x_1, ..., x_5 ∈ R(X, A) determines an embedding X → P^N = P(n_0, ..., n_5) for some weighted projective space P^N, where x_i ∈ H^0(X, O_X(α_iA)). With this embedding, we say that X has codimension N = N.

Ex. 1 Codimension 1 hypersurface case
B = \{\phi_{1,2,1,4,2,1,8,2}\}
P_X(t, \alpha) = \frac{1}{1 + t} \prod_{i=1}^{6} (1 - t^{n_i})

It is known that the embedding of codimension 2 [resp. 3] is defined by the complete intersection [resp. five maximal Pfaffians of a 5 × 5 skew matrix]. 36 can be realised by (X, A) of codimension up to 3.

3. The graded ring method

A Fano 3-fold X with primitive ample divisor A has a graded ring \( R(X, A) = \bigoplus_{n \geq 0} H^0(X, O_X(nA)) \). This graded ring is finitely generated. The Hilbert series of X, A is \( P_X(t, \alpha) = \sum_{n=0}^{\infty} \chi(X, nA) t^n \).

Each \( \chi(X, nA) \) is given by the singular Riemann-Roch formula ([YPG, Su2]) and then

\[ \chi(X, nA) = \frac{1}{1 - t^{nA}} \prod_{i=1}^{6} (1 - t^{n_i}) \]

References

[03] M. Reid, Young people's guide to canonical singularity, Lecture Notes, 1996.

\[ \chi(X, nA) = \frac{1}{1 - t^{nA}} \prod_{i=1}^{6} (1 - t^{n_i}) \]

Then we have whole possible candidates of B and their Hilbert series. As a result, we have the following list of Q-Fano 3-folds in codimension 4.

\[ \chi(O_X(-A)) = 0 \]

The example problem of two remaining candidates

\[ 1. \chi = 0, A^2 = 1 \]

\[ 2. \chi = 0, A^2 = 1 \]

is still open, but both expected not to exist.