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Non-symplectic automorphisms of 3-power order on $K3$ surfaces

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We classify non-symplectic automorphisms of 3-power order on $K3$ surfaces X which act trivially on the Néron-Severi lattice S_X, i.e., $\varphi\ \omega_X = \zeta\omega_X$.

ω_X: nowhere vanishing holomorphic 2-form.

ζ: primitive 3^k-th root of unity.

$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$: hyperbolic lattice.

A_2, E_6 or E_8: even negative de nite lattice associated with the Dynkin diagram.

Main Theorem

(1) $\exists\ \varphi$ of order 9 $\Leftrightarrow S_X = U \oplus A_2, U \oplus E_8, U \oplus E_6 \oplus A_2$ or $U \oplus E_8 \oplus E_6$.
And

$$X^\varphi = \begin{cases} \{P_1, P_2, \dots, P_6\} & \text{if } S_X = U \oplus A_2, \\ \{P_1, P_2, \dots, P_{10}\} \amalg E_1 & \text{if } S_X = U \oplus E_8 \text{ or } U \oplus E_6 \oplus A_2, \\ \{P_1, P_2, \dots, P_{14}\} \amalg E_1 \amalg E_2 & \text{if } S_X = U \oplus E_8 \oplus E_6. \end{cases}$$

(2) $\exists\ \varphi$ of order 27 $\Leftrightarrow S_X = U \oplus A_2$. And

$$X^\varphi = \{P_1, P_2, \dots, P_6\}.$$

P_i: an isolated point, E_j: a non-singular rational curve.

order 3

Let r be the Picard number of X. We assume that $S_X^*/S_X \simeq (\mathbb{Z}/3\mathbb{Z})^a$.
$22 - r - 2a \geq 0 \Leftrightarrow \exists\ \varphi$ of order 3.

$$X^\varphi = \begin{cases} \{P_1, P_2, P_3\} & \text{if } S_X = U(3) \oplus E_6(3) \\ \{P_1, \dots, P_M\} \amalg C^{(g)} \amalg E_1 \amalg \cdots \amalg E_K & \text{otherwise} \end{cases}.$$

and $M = r/2 - 1$, $g = (22 - r - 2a)/4$, $K = (2 + r - 2a)/4$.
$C^{(g)}$: non-singular curve of genus g.

Examples

S_X	Weierstrass equation and φ
$U \oplus A_2$	$X_1 : y^2 = x^3 + t\prod_{k=1}^{9}(t - \zeta_{27}^{3k})$, $\varphi_1(x, y, t) = (\zeta_{27}^2 x, \zeta_{27}^3 y, \zeta_{27}^6 t)$
$U \oplus E_8$	$X_2 : y^2 = x^3 - t^5\prod_{k=1}^{6}(t - \zeta_6^k)$, $\varphi_2(x, y, t) = (\zeta_9^2 x, \zeta_9^3 y, \zeta_9^6 t)$
$U \oplus E_6 \oplus A_2$	$X_3 : y^2 = x^3 - t^4\prod_{k=1}^{6}(t - \zeta_6^k)$, $\varphi_3(x, y, t) = (\zeta_9 x, \zeta_9^6 y, \zeta_9^3 t)$
$U \oplus E_8 \oplus E_6$	$X_4 : y^2 = x^3 - t^5\prod_{k=1}^{3}(t - \zeta_9^{3k})$, $\varphi_4(x, y, t) = (\zeta_9^2 x, \zeta_9^3 y, \zeta_9^3 t)$

Non-symplectic automorphisms of prime order have been classi ed.

order 2: Nikulin

order 3: Artebani and Sarti, Taki (independently)

order 5, 7: Artebani, Sarti and Taki

order 11: Oguiso and Zhang

order 13, 17, 19: Oguiso and Zhang

References
[1] S. Taki, Classification of non-symplectic automorphisms of order 3 on $K3$ surfaces, to appear in Math. Nachr.
[2] S. Taki, Non-symplectic automorphisms of 3-power order on $K3$ surfaces, preprint.

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