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McKay quiver representations and tiltings

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Aim

(1) Describe : $\mathcal{M}_{C,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathcal{M}_{C',\mathbf{d}}(Q,R)$
tilting theory.
The isomorphism of moduli spaces.

(2) Generalize : Crawley-Boevey's theorem.

Moduli spaces and Representations

Notations

G : a finite subgroup of $SL(2,\mathbb{C})$.
$\{\rho_0, \cdots, \rho_n\}$: the set of irreducible reps of G.
(Q,R) : the McKay quiver of G.

$\mathbf{d} := (\dim \rho_i)_{i \in Q_0}$: a dimension vector
C : a chamber in the stability space
$\Big\} \overset{GIT}{\rightsquigarrow} \mathcal{M}_{C,\mathbf{d}}(Q,R)$.

$\mathrm{rep}_{C,\mathbf{d}}(Q,R)$: the category of -semistable (Q,R) reps w.r.t. $\in C$ of dimension vector d.

$(\mathrm{rep}_{C,\mathbf{d}}(Q,R)/\simeq) = \mathcal{M}_{C,\mathbf{d}}(Q,R)$ as a set.

Fact

(Kronheimer)
$\mathcal{M}_{C,\mathbf{d}}(Q,R)$ is the minimal resolution.

(Nakajima, Lusztig etc.)
Describe : $S : \mathcal{M}_{C,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathcal{M}_{C',\mathbf{d}}(Q,R)$ as a reflection functor.

Tilting theory and Weyl groups

Tilting modules

Λ : a noetherian algebra (e.g. $\mathbb{C}Q/R$).
T : a right Λ-module.

T : tilting $\overset{def}{\Leftrightarrow}$
- (1) proj.dim$_\Lambda T \leq 1$
- (2) $\mathrm{Ext}^1_\Lambda(T,T) = 0$
- (3) ∃exact sequence $0 \to \Lambda \to T_1 \to T_2 \to 0 \quad (T_i \in \mathrm{add}\, T)$

Theorem (Happel, Rickard)

Tilting modules induce the equivalences

$$\mathcal{D}^{\mathrm{b}}(\mathrm{mod}\,\Lambda) \simeq \mathcal{D}^{\mathrm{b}}(\mathrm{mod}\,\mathrm{End}_\Lambda(T)).$$

Theorem (Iyama-Reiten)

$\mathbb{C}Q/R$: the preprojective algebra.
W : the corresponding Weyl group.
$I_i := \langle 1 - e_i \rangle$: an ideal of $\mathbb{C}Q/R$.
$\mathcal{I} := \{I_{i_1} \cdots I_{i_k} \mid 1 \leq i_1, \ldots, i_k \leq n,\ k \in \mathbb{N}\}$: a set of ideals of $\mathbb{C}Q/R$.

(1) $\mathcal{I} \ni \forall T$ is a tilting module over $\mathbb{C}Q/R$ s.t. $\mathrm{End}_\Lambda(\mathbb{C}Q/R) \cong \mathbb{C}Q/R$.

(2) ∃ a one-to-one correspondence $W \to \mathcal{I}$;
$$w = s_{i_1} \cdots s_{i_k} \mapsto I_w := I_{i_1} \cdots I_{i_k}$$
(a reduced expression)

Main Theorem

C_G : fix the chamber defined by $_0 < 0$ and $_i > 0$ $(i \neq 0)$.
(i.e. $\mathcal{M}_{C_G,\mathbf{d}} = G$-Hilb).

Main Theorem

There is a category equivalence

$$\mathrm{Hom}_{\mathbb{C}Q/R}(I_w, -) : \mathrm{rep}_{C_G,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathrm{rep}_{C_G^w,\mathbf{d}}(Q,R)$$

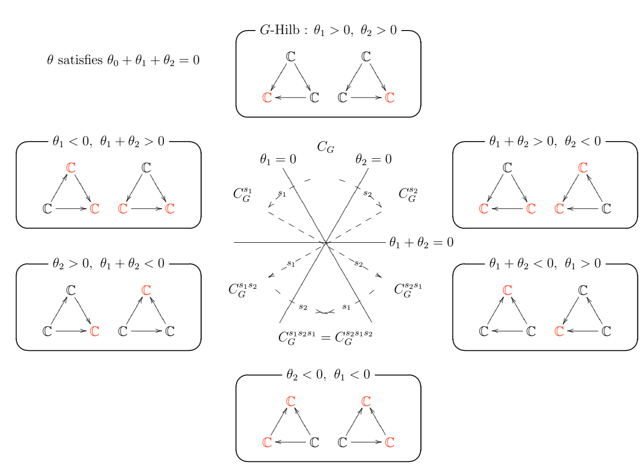
which induces the isomorphism

$$\mathcal{M}_{C_G,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathcal{M}_{C_G^w,\mathbf{d}}(Q,R).$$

Point of proof of Main Theorem

$$\mathbb{R}\mathrm{Hom}_{\mathbb{C}Q/R}(I_i, -) : \mathcal{D}^{\mathrm{b}}(\mathrm{mod}\,\mathbb{C}Q/R) \xrightarrow{\sim} \mathcal{D}^{\mathrm{b}}(\mathrm{mod}\,\mathbb{C}Q/R)$$
$$\cup \qquad\qquad \cup$$
$$\mathrm{Hom}_{\mathbb{C}Q/R}(I_i, -) : \mathrm{rep}_{C,\mathbf{d}}(Q,R) \xrightarrow{\sim} \mathrm{rep}_{C^{s_i},\mathbf{d}}(Q,R)$$

Example of A_2-type



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