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General Heart Construction

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Aim

Give an abstract construction

Triangulated category $\mathcal{C}$ $\rightsquigarrow$ Abelian category 'heart'

Setting

$(\mathcal{U},\mathcal{V})$: a pair of subcategories of $\mathcal{C}$

- $\mathrm{Ext}^1(\mathcal{U},\mathcal{V})=0$
- ${}^\forall C\in\mathrm{Ob}(\mathcal{C})$, ${}^\exists$dist.$\triangle$
  $U\to C\to V[1]\to U[1]$
  $(U\in\mathrm{Ob}(\mathcal{U}), V\in\mathrm{Ob}(\mathcal{V}))$

generalizing both cases:

1. t-structure $(\Leftrightarrow \mathcal{V}\subseteq\mathcal{V}[1])$
   $(\mathcal{T}^{\le 0},\mathcal{T}^{\ge 0})$ $\rightsquigarrow$ $\mathcal{T}^{\le 0}\cap\mathcal{T}^{\ge 0}$
2. cluster tilting subcategory $(\Leftrightarrow \mathcal{V}=\mathcal{U})$
   $\mathcal{T}\subseteq\mathcal{C}$ $\rightsquigarrow$ $\mathcal{C}/\mathcal{T}$

(※Any subcategory is assumed to be full, additive, thick and replete.)

Theorem 1 (N-)

$\mathcal{C}\supseteq\mathcal{H}:=$

Full subcategory

$C\in\mathrm{Ob}(\mathcal{C})$ admitting dist.$\triangle$'s

$V\to W_1\to C\to V[1]$

$U[-1]\to C\to W_2\to U$

$({}^\exists U\in\mathcal{U}, {}^\exists V\in\mathcal{V}, {}^\exists W_1,W_2\in\mathcal{U}\cap\mathcal{V})$

Then

$\underline{\mathcal{H}}:=\mathcal{H}/(\mathcal{U}\cap\mathcal{V})$ is abelian.

generalize 1 & 2

Theorem 2 (Abe, N-)

${}^\exists H:\mathcal{C}\longrightarrow\underline{\mathcal{H}}$ a cohomological functor

Composition of some adjoints and the quotient