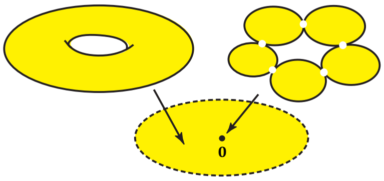
代数幾何学シンポジウム記録
2009年度　pp.115-115

Analytic Neron models as logarithmic manifolds

Tatsuki HAYAMA Osaka University, Japan

1 Classical case.



$J \to \Delta^*$: **Family of elliptic curves,**
(ber over 0) $\cong \mathbb{G}_m(\mathbb{C}) \times \mathbb{Z}/n\mathbb{Z}$.

→ Higher weight case

2 Two ways of construction.

Let $(\mathcal{H}_{\mathbb{Z}}, \mathcal{F})$ be a VHS (**V**ariation of polarized **H**odge **S**tructure) of weight $2n$ 1 over Δ^*.We assume that the monodromy is **unipotent**. Let $J \to \Delta^*$ be the family of **intermediate Jacobians**. Our main concern is

What's the boundary of J?

2.1 Green-Griffiths-Kerr's work.

The Neron model $J^{\mathrm{GGK}} \to \Delta$ by Green-Griffiths-Kerr is roughly

boundary points
= values of ANF at 0.

Here 'ANF' means **A**dmissible **N**ormal **F**unctions.

NF Since the monodromy is unipotent, we have the canonical extension $(\mathcal{H}_e, \mathcal{F}_e)$ over Δ. Let $\mathcal{H}_{\mathbb{Z},e}$ be the direct image of $\mathcal{H}_{\mathbb{Z}}$ by $\Delta^* \hookrightarrow \Delta$. A **N**ormal **F**unction is a section of $\mathcal{F}_e^n \backslash \mathcal{H}_e / \mathcal{H}_{\mathbb{Z},e}$ satisfying transversality condition.

ANF Let $\mathcal{J}_{e,\nabla}$ be the sheaf of NF. The sheaf $\tilde{\mathcal{J}}_{e,\nabla}$ of ANF is characterized by the following exact sequence:

$$0 \to \mathcal{J}_{e,\nabla} \to \tilde{\mathcal{J}}_{e,\nabla} \to G_0 \to 0.$$

Here G_0 is a skyscraper sheaf supported at 0 whose stalk is a **nite abelian group** G. For example, in the case of §1, $G = \mathbb{Z}/n\mathbb{Z}$.

Properties of $J^{\mathrm{GGK}} \to \Delta$ J^{GGK} includes values of ANF at 0. The fiber over 0 is

$$\left(F_{e,0}^{\mathrm{Inv},n} \backslash H_{e,0}^{\mathrm{Inv}} / \mathcal{H}_{e,\mathbb{Z}}\right) \times G,$$

where 'Inv' means the **monodromy invariant part**. Then the dimension of the central fibre may be lower than the dimension of a general fibre. Especially J^{GGK} may **not** be an analytic space.

2.2 Kato-Nakayama-Usui's work.

The Neron model $J^{\mathrm{KNU}} \to \Delta$ by Kato-Nakayama-Usui is roughly

boundary points
= nilpotent orbits.

Let $\Delta^* \to \Gamma\backslash D$ be the period map arising from the VHS. Assume also the monodromy is **unipotent**. By using log Hodge theory, this period map can be extended to the log period map $\Delta \to \Gamma\backslash D_\Sigma$. J^{KNU} is the fiber product

$$\begin{array}{ccc} J^{\mathrm{KNU}} & \longrightarrow & \Gamma'\backslash D'_{\Sigma'} \\ \downarrow & & \downarrow \\ \Delta & \longrightarrow & \Gamma\backslash D_\Sigma. \end{array}$$

in the category of **log manifolds**. Here $\Gamma'\backslash D'_{\Sigma'}$ is the extended period domain of the MHS corresponding to the intermediate Jacobian.

3 Our main result.

Theorem. ([H] arXiv. 0912.4334)

J^{KNU} is homeomorphic to J^{GGK}.

It is well known that ANF are corresponding to AVMHS. L→R is given by taking the **logarithm of the monodromy** and the **limiting HS** of the AVMHS.