

# K3 surfaces and log del Pezzo surfaces of index three

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We want to classify log del Pezzo surfaces of index  $k$ .

**History of classification**

- $k = 1$ : classical result
- $k = 2$ : Alexeev and Nikulin, Nakayama

Generalize the idea of [AN] to the  $k = 3$  case!

**Review of [AN] ( $k = 2$  case)**

- Smooth Divisor Theorem  
 $\exists C \in |-2K_Z|$  s.t.  $C$  : smooth curve and  $C \not\ni$  singularities.
- Right resolution  
In general, we get the following dual graph by the minimal resolution.  

$$\textcircled{-3} - \textcircled{-2} - \dots - \textcircled{-2} - \textcircled{-3}$$
 $\uparrow$ : blow up at all intersection points  

$$\textcircled{-4} - \textcircled{-1} - \textcircled{-4} - \dots - \textcircled{-1} - \textcircled{-4}$$
- Classification of non-symplectic involutions on  $K3$  surfaces by Nikulin

We get a correspondence between  $K3$  surfaces with a non-symplectic involution and log del Pezzo surfaces of index 2.

**Main Theorem ( $k = 3$  case)**

There exists a correspondence between  $K3$  surfaces with a non-symplectic automorphisms of order 3 and log del Pezzo surfaces of index 3.

- **Multiple Smooth Divisor Property**  
 $\exists 2C \in |-3K_Z|$  s.t.  $C$  : smooth curve and  $C \not\ni$  singularities.
- Right resolution  
It is a successive union of the unit chain  

$$\textcircled{-3} - \textcircled{-1} - \textcircled{-6}$$
- Classification of non-symplectic automorphisms of order 3 on  $K3$  surfaces by Artebani and Sarti, **Taki** (independently)

There exists a log del Pezzo surface of index 3 which does not satisfy **MSDP**. (ex.  $\mathbb{P}(1, 1, 3)$ ) Thus the observation does not give the complete classification.

**Example**

