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Stability conditions and the autoequivalence group on K3 surfaces

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§ 1. Stability conditions

Let X be a projective manifold over $\mathbb{C}$.

$\boxed{\mu\text{-stability: } E \in \mathrm{Coh}(X),\ \omega \in \mathrm{Amp}(X)}$

$$E \text{ is } \mu\text{-stable} \overset{\text{def}}{\iff} \forall F \subset E,\ \frac{\deg_\omega F}{\mathrm{rank} F} < \frac{\deg_\omega E}{\mathrm{rank} E}$$

$\boxed{\text{Bridgeland's stability: } E^\bullet \in D^b(\mathrm{Coh}(X)),\ \sigma \in \mathrm{Stab}(D^b(X))}$

$\sigma = (\mathcal{A}, Z)$, t-structure of $\mathcal{A} : D^b(X)(\fallingdotseq$ fullsub abelian category),
$Z : K(D^b(X)) \to \mathbb{C}$, gorup hom.

$$E^\bullet \text{ is } \sigma\text{-stable } “\overset{\text{def}}{\iff}” \ \forall F^\bullet \subset E^\bullet \arg Z(F^\bullet) < \arg Z(E^\bullet)$$

§ 2. Known Results

Trivial	non-emptiness of $\mathrm{Amp}(X)$
Difficult	non-emptiness of $\mathrm{Stab}(X)(= \mathrm{Stab}(D^b(X)))$.

cf.
- When X is a K3 or an abelian surface $\mathrm{Stab}(X) \neq \emptyset$
- When X is a 3-dim Calabi-Yau ???

In the following, we assume that X is a K3 or an abelian surface.

$$\begin{array}{ccccc} \mathrm{Stab}(X) & \underset{??}{\supset} & \mathrm{Stab}^\dagger(X) & \supset & U(X) \\ & & & & \| \\ & & & & \{\sigma \in \mathrm{Stab}(X) \mid \forall \mathcal{O}_x \text{ is } \sigma\text{-stable}\} \end{array}$$

- Conn. & Open
- Explicit locus

X:K3 surface	$\mathrm{Stab}^\dagger(X) \supsetneq U(X)$
X : Abelian surface	$\mathrm{Stab}^\dagger(X) = U(X)$

§ 3. Motivation

- $\mathrm{Stab}(X)$ is defined on the category $D^b(X)$.
- For $U(X)$ what's happen when we change X preserving $D(X)$?

cf. For some X, $\exists Y$ s.t. $Y \ncong X$ but $\Phi : D^b(Y) \xrightarrow{\sim} D^b(X)$.

Question.

Does there exist an equivalence $\Phi : D^b(Y) \to D^b(X)$, so that $\Phi_* U(Y) = U(X)$??

Partial Answer (X:Abelian surface.)

$$\forall \Phi : D^b(Y) \to D^b(X), \Phi_*(U(Y)) = U(X).$$

§ 4. Main Results

Theorem 1. (= Answer)

Let X and Y be projective K3 surfaces, and $\Phi : D^b(Y) \xrightarrow{\sim} D^b(X)$ an equivalence. If $\Phi_*(U(Y)) = U(X)$ then

$$\Phi(-) = M \otimes f_*(-)[n].$$

where $M \in \mathrm{Pic}(X)$, $f : Y \xrightarrow{\sim} X$, $[n]$:n-times shift.

We define the subgroup $\mathrm{Aut}\, D^b(X)^{U(X)}$ of $\mathrm{Aut}(D(X))$ by

$$\mathrm{Aut}(D^b(X))^{U(X)} := \{\Phi \in \mathrm{Aut}(D^b(X)) \mid \Phi_*(U(X)) = U(X)\}.$$

Corollary 2.

$$\mathrm{Aut}(D^b(X))^{U(X)} = (\mathrm{Aut}(X) \ltimes \mathrm{Pic}(X)) \times \mathbb{Z}[1]$$