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<th>Stability conditions and the autoequivalence group on K3 surfaces</th>
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Kyoto University
\section{1. Stability conditions}

Let $X$ be a projective manifold over $\mathbb{C}$.

\begin{itemize}
\item \textbf{$\mu$-stability}: $E \in \text{Coh}(X)$, $\omega \in \text{Amp}(X)$
\end{itemize}

Generalize $E$ is $\mu$-stable $\iff \forall F \subset E$, $\deg \omega F < \deg \omega E$

\begin{itemize}
\item Bridgeland’s stability: $E^* \in D^b(\text{Coh}(X))$, $\sigma \in \text{Stab}(D^b(X))$
\end{itemize}

$\sigma = (A, Z)$, t-structure of $A : D^b(X) (\subseteq \text{fullsub abelian category})$, $Z : K(D^b(X)) \to \mathbb{C}$, group hom.

$E^*$ is $\sigma$-stable $\iff \forall F^* \subset E^* \arg Z(F^*) < \arg Z(E^*)$

\section{2. Known Results}

\begin{itemize}
\item \textbf{Trivial} \hspace{1cm} non-emptiness of $\text{Amp}(X)$
\item \textbf{Difficult} \hspace{1cm} non-emptiness of $\text{Stab}(X) (= \text{Stab}(D^b(X)))$.
\end{itemize}

cf. • When $X$ is a K3 or an abelian surface $\text{Stab}(X) \neq \emptyset$

• When $X$ is a 3-dim Calabi-Yau $???

In the following, we assume that $X$ is a K3 or an abelian surface.

$\text{Stab}(X) \supset \text{Stab}^\dagger(X) \supset U(X)$

$\{ \sigma \in \text{Stab}(X) | \forall O \text{ is } \sigma\text{-stable} \}$

\section{3. Motivation}

\begin{itemize}
\item $\text{Stab}(X)$ is defined on the category $D^b(X)$.
\item For $U(X)$ what’s happen when we change $X$ preserving $D(X)$?
\end{itemize}

cf. For some $X$, $\exists Y$ s.t. $Y \not\cong X$ but $\Phi : D^b(Y) \cong D^b(X)$.

\begin{itemize}
\item Question
\end{itemize}

Does there exist an equivalence $\Phi : D^b(Y) \to D^b(X)$, so that $\Phi_* U(Y) = U(X)$?

\begin{itemize}
\item Partial Answer $(X: \text{Abelian surface.})$
\end{itemize}

$\forall \Phi : D^b(Y) \to D^b(X), \Phi_* (U(Y)) = U(X)$.

\section{4. Main Results}

\begin{itemize}
\item \textbf{Theorem 1.} (= Answer)
\end{itemize}

Let $X$ and $Y$ be projective K3 surfaces, and $\Phi : D^b(Y) \cong D^b(X)$ an equivalence. If $\Phi_* (U(Y)) \equiv U(X)$ then

$$\Phi(-) = M \otimes f_*(-)[n].$$

where $M \in \text{Pic}(X)$, $f : Y \cong X$, $[n] : n$-times shift.

We define the subgroup $\text{Aut} D^b(X)^{U(X)}$ of $\text{Aut}(D^b(X))$ by

$$\text{Aut}(D^b(X))^{U(X)} := \{ \Phi \in \text{Aut}(D^b(X)) | \Phi_* (U(X)) = U(X) \}.$$ 

\begin{itemize}
\item \textbf{Corollary 2.}
\end{itemize}

$$\text{Aut}(D^b(X))^{U(X)} = (\text{Aut}(X) \times \text{Pic}(X)) \times \mathbb{Z}[1]$$