Title: Stability conditions and the autoequivalence group on \(\mathbb{K}^3\) surfaces

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Citation: 代数幾何学シンポジウム記録 (2013), 2010: 117-117

Issue Date: 2013-02

URL: http://hdl.handle.net/2433/214922

Type: Departmental Bulletin Paper
§ 1. Stability conditions

Let $X$ be a projective manifold over $\mathbb{C}$.

\[ \mu\text{-stability: } E \in \text{Coh}(X), \omega \in \text{Amp}(X) \]

\[ E \text{ is } \mu\text{-stable } \iff \forall F \subset E, \quad \deg \omega F \frac{\text{rank } F}{\text{rank } E} \]

Generalize

Bridgeland's stability: $E^* \in D^b(\text{Coh}(X)), \sigma \in \text{Stab}(D^b(X))$

$\sigma = (A, Z)$, t-structure of $A : D^b(X) = \text{fullsub abelian category}$, $Z : K(D^b(X)) \to \mathbb{C}$, gorup hom.

$E^*$ is $\sigma$-stable " $\iff$ " $\forall F^* \subset E^*$ $\arg Z(F^*) < \arg Z(E^*)$

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§ 2. Known Results

<table>
<thead>
<tr>
<th>Trivial</th>
<th>non-emptiness of $\text{Amp}(X)$</th>
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<tbody>
<tr>
<td>Difficult</td>
<td>non-emptiness of $\text{Stab}(X) = \text{Stab}(D^b(X))$</td>
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</tbody>
</table>

cf.

- When $X$ is a K3 or an abelian surface $\text{Stab}(X) \neq \emptyset$
- When $X$ is a 3-dim Calabi-Yau ???

In the following, we assume that $X$ is a K3 or an abelian surface.

\[ \text{Stab}(X) \supset \text{Stab}^l(X) \supset \text{U}(X) \]

\[ \{ \sigma \in \text{Stab}(X) | \forall O_x \text{ is } \sigma\text{-stable} \} \]

$X$: K3 surface

$X$ : Abelian surface

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§ 3. Motivation

- $\text{Stab}(X)$ is defined on the category $D^b(X)$.
- For $U(X)$ what's happen when we change $X$ preserving $D(X)$?

\[ \text{cf. For some } X, \exists Y \text{ s.t. } Y \not\cong X \text{ but } \Phi : D^b(Y) \sim D^b(X). \]

Question.

Does there exist an equivalence $\Phi : D^b(Y) \to D^b(X)$, so that

\[ \Phi_* U(Y) = U(X) \text{??} \]

Partial Answer (X: Abelian surface. )

\[ \forall \Phi : D^b(Y) \to D^b(X), \Phi_* (U(Y)) = U(X). \]

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§ 4. Main Results

Theorem 1. (= Answer)

Let $X$ and $Y$ be projective K3 surfaces, and $\Phi : D^b(Y) \sim D^b(X)$ an equivalence. If $\Phi_* (U(Y)) = U(X)$ then

\[ \Phi(-) = M \otimes f_* (-)[n]. \]

where $M \in \text{Pic}(X)$, $f : Y \sim X$, $[n]$: $n$-times shift.

We define the subgroup $\text{Aut} D^b(X)^U(X)$ of $\text{Aut}(D^b(X))$ by

\[ \text{Aut}(D^b(X)^U(X)) := \{ \Phi \in \text{Aut}(D^b(X)) | \Phi_* (U(Y)) = U(X) \}. \]

Corollary 2.

\[ \text{Aut}(D^b(X)^U(X)) = (\text{Aut}(X) \times \text{Pic}(X)) \times \mathbb{Z}[1] \]