STABLE QUASIMAPS

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Abstract. The moduli spaces of stable quasimaps unify various moduli appearing in the study of Gromov-Witten Theory. This note is a survey article on the moduli of stable quasimaps, based on papers [CK1, CKM, K] as well as author's talk at Kinosaki Algebraic Geometry Symposium 2010.

1. Introduction

A morphism from a variety X to a projective space $\mathbb{P}^n$ is described by a linear system on X, which can also be regarded as a $\mathbb{C}^\times$-bundle P with a section u of $P \times_{\mathbb{C}^\times} \mathbb{A}^{n+1}$ without base points. When X is a curve, one may compactify the morphism space $\mathrm{Mor}(X, \mathbb{P}^n)$ by creating new rational components whenever base points try to appear. This method eventually provides Kontsevich's stable map compactification. There is another compactification, Quot scheme of rank 1 subsheaves of $\mathcal{O}_X^{\oplus n+1}$. The latter's boundary elements allow base points instead of attaching new rational components to X. It turns out that the same idea can be applied to any GIT quotient $W/\!\!/G$ when there are no strictly semistable points ([CK1, CKM]).

The above point of view leads us to:

(1) the notion of a quasimap (P, u), i.e., a pair of a principal G-bundle P on a prestable curve and a section u of $P \times_G W$ with at worst finitely many base points.
(2) New compactifications of moduli of maps from curves to a GIT quotient $W/\!\!/G$ of an affine scheme W. These include intermediate moduli spaces with moduli of stable maps and moduli of stable quasimaps as the asymptotic ones on the parameter space of stabilities (see [MM1, MM2, To] for $\mathbb{P}^n$ and Grassmannians; the investigation of the general case will appear elsewhere). The new spaces are easier to deal with than the stable map spaces in certain cases.

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(3) The virtual smoothness of the Artin stack of the quasimap pairs when W is LCI and $W/\!\!/G$ is smooth.
(4) A new class of examples with symmetric obstruction theory if $W/\!\!/G$ is a Nakajima quiver variety (see [D, K]).
(5) The wall-crossing interpretation of Givental's approach to classical Mirror Conjecture (see [CK2]).

2. Three Quotients

Let W be an affine variety over $\mathbb{C}$ with a linear action by a complex reductive Lie group G. Typical examples of G are products of general linear groups $GL_n(\mathbb{C})$. In this situation one sometimes wants to define a quotient space. There are three approaches.

2.1. **Affine quotients.** Since W is affine, it can be considered as Spec of the ring $\mathbb{C}[W]$ of regular functions on W. Hence, it is natural to define the quotient by Spec of the ring $\mathbb{C}[W]^G$ of G-invariant regular functions on W.

In many cases, this is not interesting. For instance, if the homothety action is included in the G-action, the closure of every G-orbit contains the origin. Therefore every G-invariant function must be constant on W. Thus, $\operatorname{Spec}\mathbb{C}[W]^G = \operatorname{Spec}\mathbb{C}$.

2.2. **GIT quotients.** In the previous example, to obtain an interesting space as a quotient, we need to remove the origin. To do so geometrically, we should also prevent other orbits from approaching the origin or some point. For it, we will regard W as Proj of the graded ring $\mathbb{C}[W \times \mathbb{A}^1]$ where the grading comes from degrees with respect to the extra $\mathbb{A}^1$. Given a character χ of G (i.e., a one-dimensional representation χ of G), we define a G-action on $W \times \mathbb{A}^1_{\chi^{-1}}$, where $\mathbb{A}^1_\alpha$ for character α denotes the one-dimensional representation space of G associated to α. Now it is natural to take Proj of the graded ring $\mathbb{C}[W \times \mathbb{A}]^G$ of G-invariant functions. This quotient is denoted by $W/\!\!/_\chi G$.

To describe the quotient space geometrically, let's recall the following definition. We call a point p:

- χ-*stable* if there is no one-parameter subgroup $\mathbb{C}^\times$ of G such that $\mathbb{C}^\times \cdot (p, 1)$ is closed in $W \times \mathbb{A}^1$.
- χ-*unstable* if there is a one-parameter subgroup $\mathbb{C}^\times$ of G such that the closure of $\mathbb{C}^\times \cdot (p, 1)$ meets $W \times \{0\}$.
- χ-*semistable* if it is not χ-unstable.

It is a theorem that $W/\!\!/_\chi G$ is the categorical quotient of the semistable locus W^{ss}. For a general choice of χ, there is no strictly semistable point. In such a case, the GIT quotient is known to be also a geometric quotient so that $W/\!\!/_\chi G = W^s/G$. (Note that if there is a nontrivial character of G, then G is not semisimple.)

2.3. **Stack quotients.** There is a stack quotient $[W/G]$ which is as a set (over $\operatorname{Spec}\mathbb{C}$) the set W/G of G-orbits, but which keeps the data of isotropy subgroups. The quotient is defined as a category of groupoids over the category Sch of schemes over $\mathbb{C}$, whose objects over a scheme S are pairs (P, u) of principal bundles and G-equivariant morphism u from P to W. The morphism u can be considered also as a section of $P \times_G W$.

2.4. **Relationships.** *Assume from now on that there are no strictly χ-semistable points and G acts on the stable locus freely.* The latter condition is only technical (see [C]).

With this assumption the three quotients are related by the diagram

$$[W/G] \xhookleftarrow[\text{open}]{} W/\!\!/_\chi G \xrightarrow[\text{projective}]{} \operatorname{Spec}\mathbb{C}[W]^G \,.$$

(Note that $W/\!\!/_{\chi=0}G$ coincides with the affine quotient $\operatorname{Spec}\mathbb{C}[W]^G$, but we will not use this notation. For $\chi = 0$, every semistable point is not stable.)

2.5. **Examples.** There are many examples.

Example 2.5.1. Let Y be a projective variety in $\mathbb{P}^n$ and let $C(Y)$ denote the affine cone of Y in $\mathbb{A}^{n+1}$. Then Y is the GIT quotient $C(Y)/\!\!/_{\det}\mathbb{C}^\times$, where the character is defined by the determinant map. Typical examples for this case will be smooth complete intersections Y.

Example 2.5.2. Complete intersections in toric varieties or Grassmannians.

Example 2.5.3. Quiver varieties. In particular, quiver varieties of Nakajima type will be recalled in section 4.

3. Quasimaps

A morphism from X to the GIT quotient W^s/G amounts to a pair (P, u) where P is a principal G-bundle on X and u is a G-equivariant map from P to W whose image is contained in W^s. By Luna's slice theorem, W^s is a principal G-bundle on W^s/G in étale topology. The direction $\Rightarrow$ therefore follows. The other direction holds since $P \to X$ is a categorical quotient. Here we will exchange left and right actions via the inverse map $G \to G, g \mapsto g^{-1}$.

3.1. **Stable quasimaps.** If we allow that u hits the unstable locus, the pair (P, u) is not any more a well-defined map to the GIT quotient, but it is a map $[u]$ to the stack quotient by the very definition of the stack quotient.

Definition 3.1.1. *The pair (P, u) is called a* quasimap *of genus g if:*

- *X is a projective smooth or nodal curve of genus g, with n ordered smooth markings.*
- *The base locus $u^{-1}(P \times_G W^{un})$ consists of finite points.*

The quasimap is called a stable quasimap *if:*

- *$\omega_X \otimes (P \times_G \mathbb{A}^1_\chi)^\epsilon$ is ample for all positive rational number ϵ.*
- *The base points (if any) are smooth and non-marked points.*

3.2. **Degrees.** Note that there is a notion of degree for a map f from curve X to $W/\!\!/G$ by the homomorphism

$$\begin{array}{rcc} \deg(f) : \mathrm{Pic}(W/\!\!/_\chi G) & \to & \mathbb{Z} \\ M & \mapsto & \deg(f^*M) \end{array}.$$

Similarly we define the degree of (P, u) as a homomorphism from the character group of G to $\mathbb{Z}$ by sending $\alpha \mapsto \deg(P \times_G \mathbb{A}^1_\alpha)$. Let β be a group homomorphism from the character group of G to $\mathbb{Z}$.

Theorem 3.2.1. ([CKM]) *The moduli stack $Q_{g,n}(W/\!\!/_\chi G, \beta)$ of stable quasimaps of n-pointed, genus g, degree β to $W/\!\!/_\chi G$ is a finite-type DM stack proper over the affine quotient $\mathrm{Spec}\,\mathbb{C}[W]^G$. Furthermore if the affine scheme W^s is smooth and W is a locally complete intersection scheme, then the moduli stack comes with a natural perfect obstruction theory.*

The map from the moduli stack to the affine quotient is naturally given since $P \to X$ is categorical at the diagram

$$\begin{array}{ccc} P & \longrightarrow & W \\ \downarrow & \searrow & \downarrow \\ X & \dashrightarrow & \mathrm{Spec}\mathbb{C}[W]^G \end{array}$$

and X is a projective scheme over $\mathbb{C}$.

3.3. **Perfect obstruction theory.** It is not difficult to see that the deformations of curves X and principal bundles P on X is unobstructed. Fix X and P and deform only section u of $P \times_G W$. Note that the deformation space $\mathrm{def}(u)$ is $\mathbb{H}^0(X, u^*\mathbf{T}_\rho)$ where $\mathbf{T}_\rho$ is the relative tangent complex of $\rho : P \times_G W \to X$. We may therefore define an obstruction space $\mathrm{ob}(u)$ to be $\mathbb{H}^1(X, u^*\mathbf{T}_\pi)$. The relative virtual dimension is

$\dim \operatorname{def}(u) - \dim \operatorname{ob}(u)$, which is a locally constant. Roughly speaking, this implies that the moduli space is virtually smooth.

3.4. **Historical remarks.** These are limited remarks.

3.4.1. For a projective smooth toric variety, the spaces of stable quasimaps with the fixed domain curve $\mathbb{P}^1$ also become projective smooth toric varieties. The spaces are used to prove the Mirror Theorem for Fano/CY complete intersections by Givental ([G]).

3.4.2. Let $W/\!\!/_{\chi}G = \operatorname{Hom}(\mathbb{C}^r, \mathbb{C}^n)/\!\!/_{\det}GL_r(\mathbb{C}) = \operatorname{Grass}(r, n)$. In this case, a stable quasimap amounts to a rank $n-r$ quotient of $\mathbb{A}^n \otimes \mathcal{O}_X$ on a prestable curve with certain conditions: for example when there is no marking, the conditions are no torsion at nodes and no rational tail. The latter is called a stable quotient and introduced by Marian, Oprea, and Pandharipande [MOP] in 2009.

For a fixed smooth curve X, the moduli spaces of stable quotients are nothing but Quot schemes of rank $n-r$ quotients of $\mathbb{A}^n \otimes \mathcal{O}_X$. Quot schemes have been used and studied in Gromov-Witten theory (for instance, see [Ber, OT]).

3.4.3. For a smooth projective toric variety $\mathbb{A}^N/\!\!/(\mathbb{C}^*)^r$, the theorem is proven in [CK1]. The paper [CK1] shows the idea that all the above constructions can be unified and generalized to any GIT quotient $W/\!\!/G$.

3.5. **Quasimap invariants.** Using the perfect obstruction theory on the moduli space of stable quasimaps, we define the virtual fundamental class of the moduli space. Hence, we can define intersection numbers by integrals of tautological cohomology classes against the virtual fundamental class.

We conjecture that *these invariants and Gromov-Witten invariants for $W/\!\!/G$ carry the same amount of information* ([CK1, CKM, CK2]) A precise formulation of the conjecture is unknown except for the following two cases.

(1) When $W/\!\!/G$ has the property that for every curve C in $W/\!\!/G$, $C \cdot K_{W/\!\!/G} \leq -2$, we expect that both invariants exactly coincide.
(2) The genus zero quasimap invariants should lie on the Lagrangian cone generated by the genus zero gravitational Gromov-Witten invariants (and vice-versa).

3.6. **Some evidence.**

3.6.1. For *fixed* $X = \mathbb{P}^1$ and any toric complete intersection $W/\!\!/G$, (2) is the Mirror Theorem in [G]. For Grassmannian case, (1) is a theorem in [MOP].

3.6.2. In [CK2] we prove (2) for any Fano/CY toric complete intersection in a toric variety and (1) for any Fano toric variety.

4. Stable Quasimaps to Holomorphic Symplectic Quotients

Let V be a smooth affine variety with a holomorphic symplectic form ω (i.e. $\omega \in \Gamma(V, \mathcal{T}_V)$ is a nondegenerate (2,0)-form). In this setup one can define a quotient which is also holomorphic symplectic.

4.1. **Holomorphic symplectic quotients.** Suppose that the G-action V is hamiltonian which means that: the G-action preserves ω and there is a G-equivariant morphism $\mu : V \to \mathfrak{g}^*$ such that $\langle d\mu(\xi), g\rangle = \omega(\xi, d\alpha(g))$ for $\xi \in T_V$ where $\alpha : G \to \mathrm{Aut}V$ is induced from the action. Here $\mathfrak{g}$ denotes the Lie algebra of G. The morphism μ is called a complex moment map.

Define the holomorphic symplectic quotient by $\mu^{-1}(\lambda)/\!\!/_{\chi}G$ where λ is a G-invariant regular value of μ. The quotient is denoted by $V/\!\!/\!\!/_{\lambda,\chi}G$.

4.2. **Symmetry.** Let $\mathbf{F}$ denote the complex

$$[\mathfrak{g} \otimes \mathcal{O}_V \stackrel{d\alpha}{\to} \mathcal{T}_V \stackrel{d\mu}{\to} \mathfrak{g}^* \otimes \mathcal{O}_V]|_{\mu^{-1}(\lambda)}.$$

Note that $\mathbf{F} \cong \mathbf{F}^\vee$ and $\mathbf{F}$ is a monad such that $\mathrm{Ker}d\mu/\mathrm{Im}d\alpha$ in $(\mu^{-1}(\lambda))^s$ is isomorphic to the pullback of the tangent sheaf of holomorphic symplectic quotient. One may also consider $\mathbf{F}$ (more precisely, its mixed construction) as a generalized Euler sequence for the tangent sheaf of $V/\!\!/G$.

4.3. **Symmetric obstruction theory.** Fix a smooth projective curve X. Define $\mathfrak{M}_\beta$ to be the stack of degree β stable quasimaps to the holomorphic symplectic quotient $\mu^{-1}(\lambda)/\!\!/G$ from X. This moduli space has a symmetric obstruction theory if X is an elliptic curve, i.e., the deformation space $\mathbb{H}^0(X, P\times_G \mathbf{F})$ is functorially isomorphic to the dual of the obstruction space $\mathbb{H}^1(X, P\times_G\mathbf{F})$. This follows from Serre duality and $\mathbf{F} \cong \mathbf{F}^\vee$.

Using twisting, this symmetry can be made hold for arbitrary smooth curve X when the quotient is a Nakajima quiver variety.

4.4. **Nakajima's quiver varieties.** A quiver Q is an oriented graph, i.e., data (Q_0, Q_1, h, t) where Q_0 is the set of vertices, Q_1 is the set of arrows, h is the head map, and t is the tail map ($h, t : Q_1 \to Q_0$). Let $\overline{Q}$ be the quiver obtained from Q by adding the opposite arrow $\bar{a}$ for each arrow a in Q_1 (so $|\overline{Q}_1| = 2|Q_1|$). Set $\bar{\bar{a}} = a$.

Fix a distinguished vertex $0 \in Q_0$ and a dimension vector $\mathbf{v} \in \mathbb{N}^{Q_0}$. Let V be the direct sum of $\mathrm{Hom}(\mathbb{A}^{v_{ta}}, \mathbb{A}^{v_{ha}})$ for all $a \in \overline{Q}_1$. V has a decomposition $V_+ \oplus V_-$ based on the arrows in Q_1 and $\overline{Q}_1 \setminus Q_1$ so that it is a symplectic vector space with the canonical symplectic form. Also V comes with a natural action of $G = \Pi_{i \in Q_0, i \neq 0} GL_{v_i}(\mathbb{C})$. It is easy to see that this action is hamiltonian with a moment map

$$\left(\sum_{a \in \overline{Q}_1 : ha = i} (-1)^{|a|} \phi_a \circ \phi_{\bar{a}} \right)_{i \in Q_0, i \neq 0}.$$

Choose $\theta \in \mathbb{Z}^{Q_0 \setminus \{0\}}$ and $\theta : G \to \mathbb{C}^\times$ by $g \mapsto \prod \det g_i^{\theta_i}$. Now we can define $V /\!\!/\!\!/_{\lambda, \theta} G$. Let $\lambda = 0$.

Note that a quasimap data is equivalent to (E_i, ϕ_a) where E_i is a vector bundle $P \times_G \mathbb{A}^{v_i}$ and $\phi_a : E_{ta} \to E_{ha}$ is a homomorphism obtained from u, satisfying the moment map relation and the condition that 'base points' are finite. This therefore motivates the following.

4.5. **Twisted quiver sheaves.** Fix a smooth projective curve X, a line bundle M_a on X for every $a \in \overline{Q}_1$, and an isomorphism $M_a \otimes M_{\bar{a}} \to K_X^{-1}$ for every $a \in Q_1$.

Definition 4.5.1. (E_i, ϕ_a) *is called a* framed-twisted quiver sheaf on X with the moment map relation *if*

1. E_i *is a coherent sheaf on* X.
2. $\phi_a : M_a \otimes E_{ta} \to E_{ha}$ *is an* $\mathcal{O}_X$*-homomorphism.*
3. $\sum_{i \neq 0} \sum_{t\bar{a} = i} (-1)^{|a|} \phi_a \circ (\mathrm{Id}_{M_a} \otimes \phi_{\bar{a}}) = 0$ *in* $\bigoplus_{i \neq 0} \mathrm{End}(K_X^{-1} \otimes E_i, E_i)$.
4. $E_0 = \mathcal{O}_X^{\oplus r}$ *for some integer* r.

Often, we will call it simply twisted quiver sheaf, even just quiver sheaf.

4.6. **Stability Conditions.** Let $\mathcal{A}$ be the abelian category of twisted quiver sheaves with respect to the fixed data above. Let $Z : K(\mathcal{A}) \to \mathbb{C}$ be a homomorphism defined by

$$(E_i, \phi_a) \mapsto \sum_{i \neq 0} \mathrm{rk} E_i + \sqrt{-1} (\sum_{i \neq 0} \deg E_i + \tau \mathrm{rk} E_0).$$

Let $\tau > 0$. Note that $Z(K(\mathcal{A}) \setminus \{0\})$ is contained in the union of the half plane where the real part is positive with the positive y-axis. This is a Bridgeland stability function with Harder - Narasimhan property.

A nonzero twisted quiver sheaf (E_i, ϕ_a) is called τ-(semi)stable if $\mathrm{Arg} Z(E') < (\leq) \mathrm{Arg} Z(E)$ for every nonzero proper subobject E' of E in $\mathcal{A}$.

Proposition 4.6.1. *Fix a dimension vector v with $v_0 = 1$ and a degree vector d. Then there is a positive number τ_0 such that for every $\tau \geq \tau_0$ and for every (E_i, ϕ_a) twisted quiver sheaf with (v, d), TFAE*

1. *τ-semistability.*
2. *τ-stability.*
3. *Stability as a twisted quasimap to $V/\!\!/\!\!/G$ with $\theta = (1, ..., 1)$.*

Hence, given (v, d) there are finite many walls τ_i, $i = 1, ..., N$ such that there are no strictly τ-semistable objects unless $\tau = \tau_i$.

Theorem 4.6.2. *The moduli stack $\mathfrak{M}^{\tau}_{(v,d)}$ of τ-stable quiver sheaves of rank and degree (v, d) is a finite type algebraic space equipped with a symmetric obstruction theory. The stack is proper over* $\mathrm{Spec}\, \mu^{-1}(0)^G$ *if τ is large enough.*

4.7. **Remarks.** When $\overline{Q}$ is the ADHM quiver, the above proposition and theorem were proven in [D] which is one of the main sources of inspiration for the general case.

So far, there are two classes of examples equipped with symmetric obstruction theory: moduli of stable objects in the abelian category of coherent sheaves of a CY 3-fold and representations of a quiver with relations from a superpotential on Q (see [Th, PT, S, JS, KS]).

One can study wall-crossings of topological Euler characteristics of $\mathfrak{M}^{\tau}_{(v,d)}$ weighted by Behrend's constructible functions ([Beh]) using Joyce-Song formula ([JS]) or Kontsevich - Soibelman formula ([KS]). Once again for the ADHM case, it is done in [D, CDP1, CDP2]; and its generalization is a work in progress with H. Lee.

For the use of Joyce - Song theory, one needs to prove:

- $\chi(E, F) := \mathrm{ext}^0(E, F) - \mathrm{ext}^1(E, F) + \mathrm{ext}^1(F, E) - \mathrm{ext}^0(F, E)$ is numerical in certain cases.
- $\mathfrak{M}^{\tau}_{(v,d)}$ is analytic-locally a critical locus of a holomorphic function on a smooth analytic domain.

It would be interesting to relate $\mathcal{A}$ with a CY 3-category.

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References

[Beh] K. Behrend, *Donaldson-Thomas type invariants via microlocal geometry,* Ann. of Math. (2) 170 (2009), no. 3, 1307-1338.

[Ber] A. Bertram, *Quantum Schubert calculus,* Adv. Math. 128 (1997), no. 2, 289-305.

[C] D. Cheong, in preparation.

[CDP1] W-E. Chuang, D.E. Diaconescu, G. Pan, *Chamber Structure and Wallcrossing in the ADHM Theory of Curves II*, `arXiv:0908.1119.`

[CDP2] W-E. Chuang, D.E. Diaconescu, G. Pan, *Rank Two ADHM Invariants and Wallcrossing,* `arXiv:1002.0579.`

[CK1] I. Ciocan-Fontanine, B. Kim, *Stable toric quasimaps*, Advances in Mathematics, Volume 225, Issue 6, 20 December 2010, 3022-3051.

[CK2] I. Ciocan-Fontanine, B. Kim, in preparation.

[CKM] I. Ciocan-Fontanine, B. Kim, D. Maulik, *Stable quasimaps to GIT quotients,* in preparation.

[D] D.-E. Diaconescu, *Chamber structure and wallcrossing in the ADHM theory of curves I,* `arXive:0904.4451.`

[G] A. Givental, *A mirror theorem for toric complete intersections*, in "Topological field theory, primitive forms and related topics (Kyoto, 1996)", Progr. Math., 160, Birkhuser Boston, Boston, MA, 1998, 141–175.

[JS] D. Joyce and Y. Song, *A theory of generalized Donaldson-Thomas invariants,* to appear in Memoirs of the A.M.S, `arXiv:0810.5645.`

[K] B. Kim, *Stable quasimaps to holomorphic symplectic quotients*, `arXiv:1005.4125.`

[KS] M. Kontsevich, Y. Soibelman, *Stability structures, motivic Donaldson-Thomas invariants and cluster transformations,* `arXiv:0811.2435.`

[MOP] A. Marian, D. Oprea, R. Pandharipande, *The moduli space of stable quotients,* `arXiv:0904.2992.`

[MM1] A. Mustaţă, A. Mustaţă, *Intermediate moduli spaces of stable maps,* Invent. Math. 167 (2007), no. 1, 47-90.

[MM2] A. Mustaţă, A. Mustaţă, *The Chow ring of* $M_{0,m}(\mathbb{P}^n, d)$*,* J. Reine Angew. Math. 615 (2008), 93-119.

[OT] C. Okonek, A. Teleman, *Comparing virtual fundamental classes: gauge theoretical Gromov-Witten invariants for toric varieties,* Asian J. Math. 7 (2) (2003), 167-198.

[PT] R. Pandharipande, R.P. Thomas, *Curve counting via stable pairs in the derived category,* Invent. Math. 178 (2009), no. 2, 407-447.

[S] B. Szendröi, *Non-commutative Donaldson-Thomas invariants and the conifold,* Geom. Topol. 12 (2008), no. 2, 1171-1202.

[Th] R.P. Thomas, *A holomorphic Casson invariant for Calabi-Yau 3-folds, and bundles on K3 fibrations,* J. Differential Geom. 54 (2000), no. 2, 367-438.

[To] Y. Toda, *Moduli spaces of stable quotients and the wall-crossing phenomena,* `arXiv:1005.3743.`

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